**INFINITY PART 1 Robert, David, Brandon**

**Definitions to know and explain:**

**• Cardinality** -The number of elements in a finite set |A|

**• Countable, Uncountable** - a countable set is a [set](https://en.wikipedia.org/wiki/Set_(mathematics)) with the same [cardinality](https://en.wikipedia.org/wiki/Cardinality) ([number](https://en.wikipedia.org/wiki/Cardinal_number) of elements) as some [subset](https://en.wikipedia.org/wiki/Subset) of the set of [natural numbers](https://en.wikipedia.org/wiki/Natural_number).

**Examples of:**

**• A countable set - ℕ, ℤ,** Finite sets, such as: {1,2,3}, {a,b,c,d}, {red,green,blue}, {horse,dog}, etc.

**• An uncountable set - ℝ**

**Prove:**

The cardinality of the naturals is equal to the cardinality of the integers - **Robert**

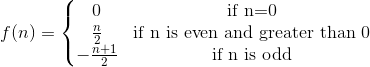
Recall that we say that two sets A and B have the same cardinality if there exists a bijective function from A to B. If you haven't come across bijective functions before, you can think of a bijective function as a function assigning each element in B to an unique element in A.

More formally, a bijective function is a function from A to B that is injective (maps each element from A to an unique element in B) and surjective (for every element b in B, there exists an element a in A such that f(a)=bf(a)=b).

For example, {3,5,7,9} and {a,b,c,d} have the same cardinality as we can define a bijective function mapping 3→a, 5→b, 7→c and 9→d.

Similarly, we can define a bijective function /one to one correspondence between the set of integers and the set of natural numbers by the following

f:N→Zf:N→Z



This map maps 0 to 0, the nth even number to n and the nth odd number to −n.

Note that the definition of natural numbers I used included 0, but the argument is effectively the same for natural numbers without 0.

The cardinality of the naturals is equal to the cardinality of the rationals. - **David**

* Define cardinality if not already defined.
* Define bijection if not already defined. 1:1 correspondence.
* Show an example of what it means for 2 sets to have the same cardinality. E.g. |{a,b,c}|=|{1,2,3}|.
* Define the natural numbers:**ℕ**={1,2,3,4,5,...}
* Define the rational numbers: **ℚ**={±m/n : m**ℕ** and n**ℕ**}
* Elucidate that both of these sets have an infinite number of elements.
* Show that there is a bijection between **ℕ** and **ℚ** such that there is a one to one correspondence between the elements of **ℕ** and **ℚ**.

The cardinality of the naturals is not equal to the cardinality of the reals - **Brandon**

* Define the reals
* Talk about Cantor’s Theorems
  + Cantor’s Diagonal Theorem
* The real numbers ℝ are more numerous than the natural numbers ℕ. Moreover ℝ has the same number of elements as the power set of ℕ. Symbolically, if the cardinality of ℕ is denoted as א‎0, the cardinality of of the real numbers is: |ℝ| = 2^א‎0>א‎0