Adversarial Search - Bayesian Networks

Assignment 3

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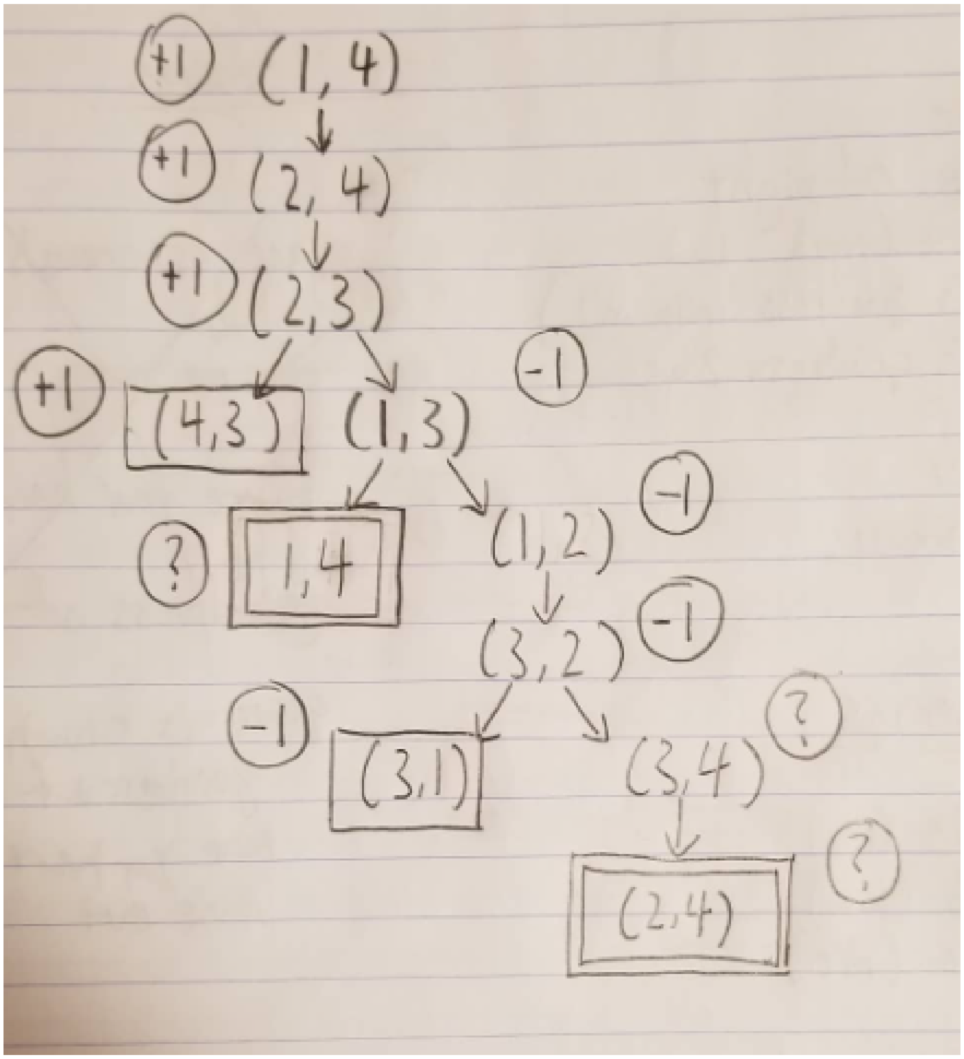
Problem 1



"?" values are used to annotate loop states. When an agent has to choose between a +1 value and a "?" value (+1,?) the max player will always choose +1 while the min player will choose ?, and vice versa for (-1,?), with the min player choosing -1 and the max player choosing "?". Since the game value of a loop state is unknown, it is best for a max/min player to choose the corresponding max/min value if it is available, however if that value is not available the "?" state is a viable option to explore as it cannot be worse than an immediate loss. If all the successors of a state have a "?" value, the backed-up value is also "?".

Problem 2

(a) Computing the joint probability distribution:

$$P(a, b, c, d, e) = P(a) * P(b) * P(c) * P(d|a, b) * P(e|b, c)$$
$$P(a, b, c, d, e) = 0.2 * 0.5 * 0.8 * 0.1 * 0.3$$
$$P(a, b, c, d, e) = 0.0024$$

(b) Computing the joint probability distribution:

$$P(\neg a, \neg b, \neg c, \neg d, \neg e) = P(\neg a) * P(\neg b) * P(\neg c) * P(\neg d|\neg a, \neg b) * P(\neg e|\neg b, \neg c)$$
$$P(\neg a, \neg b, \neg c, \neg d, \neg e) = 0.8 * 0.5 * 0.2 * 0.1 * 0.8$$
$$P(\neg a, \neg b, \neg c, \neg d, \neg e) = 0.0064$$

(c) Using the conditional probability rule $P(A|B) = \frac{P(A \bigcap B)}{P(B)}$

$$P(\neg a|b, c, d, e) = \frac{P(\neg a, b, c, d, e)}{P(b, c, d, e)}$$
$$P(\neg a|b, c, d, e) = \frac{P(\neg a, b, c, d, e)}{P(a, b, c, d, e) + P(\neg a, b, c, d, e)}$$
$$P(\neg a|b, c, d, e) = \frac{P(\neg a) * P(b) * P(c) * P(d|\neg a, b) * P(e|b, c)}{0.0024 + P(\neg a) * P(b) * P(c) * P(d|\neg a, b) * P(e|b, c)}$$
$$P(\neg a|b, c, d, e) = \frac{0.8 * 0.5 * 0.8 * 0.6 * 0.3}{0.0024 + 0.8 * 0.5 * 0.8 * 0.6 * 0.3}$$
$$P(\neg a|b, c, d, e) = \frac{0.0576}{0.0024 + 0.0576}$$
$$P(\neg a|b, c, d, e) = 0.96$$

Problem 3

(a)

$$P(b|j, m) = \alpha * P(b, j, m)$$
$$\alpha = \frac{1}{P(\neg b, j, m) + P(b, j, m)}$$

$$P(b,j,m) = \sum_e \sum_a P(b,j,m,e,a)$$
$$P(b,j,m) = P(b) \sum_e P(e) \sum_a P(a|b,e)P(j|a)P(m|a)$$
$$P(b,j,m) = 0.001 * 0.002 * 0.95 * 0.9 * 0.7 + 0.001 * 0.002 * 0.05 * 0.05 * 0.01 +$$
$$0.001 * 0.998 * 0.94 * 0.9 * 0.7 + 0.001 * 0.998 * 0.06 * 0.05 * 0.01$$
$$P(b,j,m) = 0.00059224259$$

$$P(\neg b,j,m) = \sum_e \sum_a P(\neg b,j,m,e,a)$$
$$P(\neg b,j,m) = P(\neg b) \sum_e P(e) \sum_a P(a|\neg b,e)P(j|a)P(m|a)$$
$$P(\neg b,j,m) = 0.999 * 0.002 * 0.29 * 0.9 * 0.7 + 0.999 * 0.002 * 0.71 * 0.05 * 0.01 +$$
$$0.999 * 0.998 * 0.001 * 0.9 * 0.7 + 0.999 * 0.998 * 0.999 * 0.05 * 0.01$$
$$P(\neg b,j,m) = 0.00149185764$$

$$\alpha = \frac{1}{P(\neg b,j,m) + P(b,j,m)}$$
$$\alpha = \frac{1}{0.00149185764 + 0.00059224259}$$
$$\alpha = \frac{1}{0.00208410023}$$

$$P(b|j,m) = \alpha * P(b,j,m)$$
$$P(b|j,m) = \frac{0.00059224259}{0.00208410023}$$
$$P(b|j,m) = 0.28417183659$$

Therefore, the probability of a burglary given that John and Mary both called, is 0.28417183659.

(b) In a Bayesian network in the form of a chain, using the enumeration method has a worst case runtime of $\mathcal{O}(2^n)$, as moving down each level in the chain generates another level in a binary tree.

Using the variable elimination method, moving down each level in the Bayesian network chain only adds one additional problem to compute (point-wise product of factors), as each node in the network only has one parent. Thus, the variable elimination method has a worst case runtime of $\mathcal{O}(n)$.

Problem 4

(a)

(b)