Adversarial Search - Bayesian Networks

Assignment 3

Michael Li - 192008938
Daniel Liu - 184007283
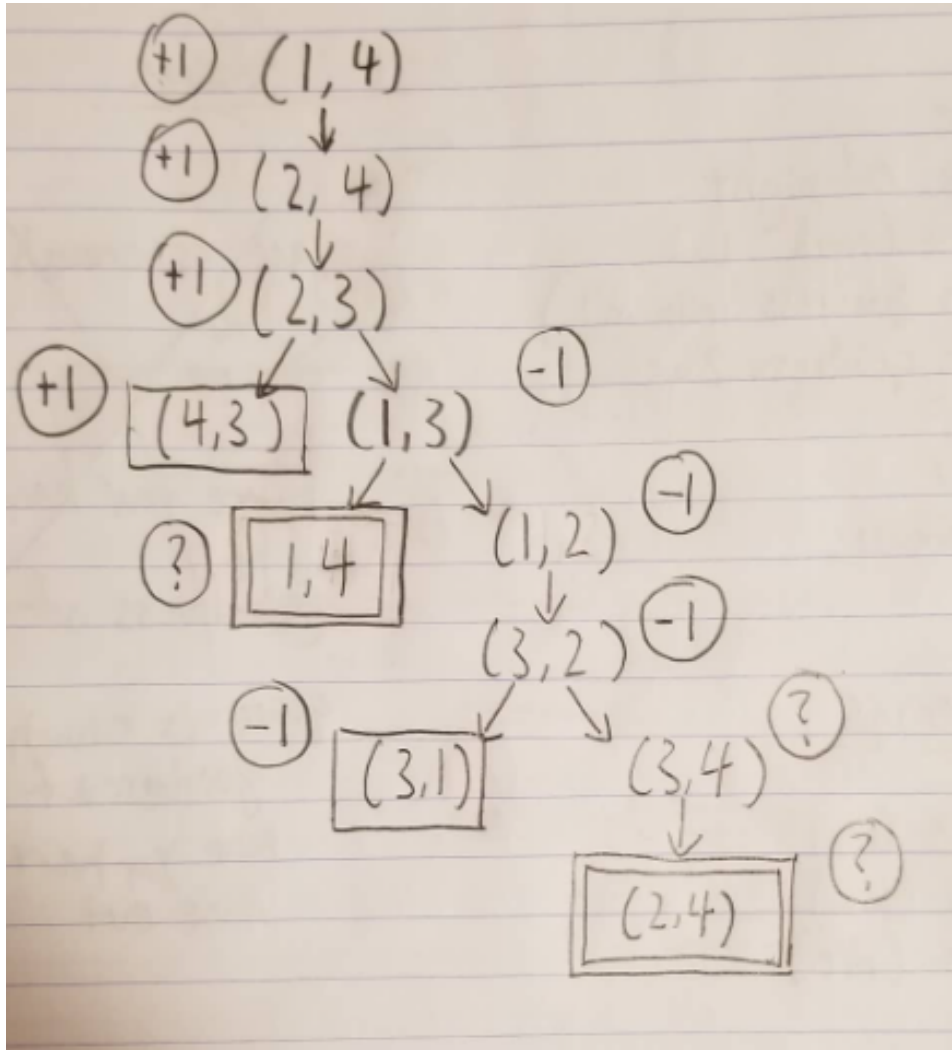**Robert Kulesa - 185009892**

Problem 1

(+1) (1, 4)
↓
(+1) (2, 4)
↓
(+1) (2,3)
↙   ↘
(+1) [(4,3)] (1,3)   (-1)
        ↓   ↘
(?) [1,4]   (1,2)   (-1)
               ↓
            (3,2)   (-1)
        ↙   ↘
(-1) [(3,1)]   (3,4)   (?)
                  ↓
              [(2,4)]   (?)

"?" values are used to annotate loop states. When an agent has to choose
between a +1 value and a "?" value (+1,?) the max player will always
choose +1 while the min player will choose ?, and vice versa for (-1,?), with
the min player choosing -1 and the max player choosing "?". Since the game
value of a loop state is unknown, it is best for a max/min player to choose
the corresponding max/min value if it is available, however if that value is
not available the "?" state is a viable option to explore as it cannot be worse
than an immediate loss. If all the successors of a state have a "?" value, the
backed-up value is also "?".

Problem 2

(a) Computing the joint probability distribution:

$$P(a, b, c, d, e) = P(a) * P(b) * P(c) * P(d|a, b) * P(e|b, c)$$
$$P(a, b, c, d, e) = 0.2 * 0.5 * 0.8 * 0.1 * 0.3$$
$$P(a, b, c, d, e) = 0.0024$$

(b) Computing the joint probability distribution:

$$P(\neg a, \neg b, \neg c, \neg d, \neg e) = P(\neg a) * P(\neg b) * P(\neg c) * P(\neg d|\neg a, \neg b) * P(\neg e|\neg b, \neg c)$$
$$P(\neg a, \neg b, \neg c, \neg d, \neg e) = 0.8 * 0.5 * 0.2 * 0.1 * 0.8$$
$$P(\neg a, \neg b, \neg c, \neg d, \neg e) = 0.0064$$

(c) Using the conditional probability rule $P(A|B) = \frac{P(A \bigcap B)}{P(B)}$

$$P(\neg a|b, c, d, e) = \frac{P(\neg a, b, c, d, e)}{P(b, c, d, e)}$$
$$P(\neg a|b, c, d, e) = \frac{P(\neg a, b, c, d, e)}{P(a, b, c, d, e) + P(\neg a, b, c, d, e)}$$
$$P(\neg a|b, c, d, e) = \frac{P(\neg a) * P(b) * P(c) * P(d|\neg a, b) * P(e|b, c)}{0.0024 + P(\neg a) * P(b) * P(c) * P(d|\neg a, b) * P(e|b, c)}$$
$$P(\neg a|b, c, d, e) = \frac{0.8 * 0.5 * 0.8 * 0.6 * 0.3}{0.0024 + 0.8 * 0.5 * 0.8 * 0.6 * 0.3}$$
$$P(\neg a|b, c, d, e) = \frac{0.0576}{0.0024 + 0.0576}$$
$$P(\neg a|b, c, d, e) = 0.96$$

Problem 3

(a)

$$P(b|j, m) = \alpha * P(b, j, m)$$
$$\alpha = \frac{1}{P(\neg b, j, m) + P(b, j, m)}$$

$$P(b,j,m) = \sum_e \sum_a P(b,j,m,e,a)$$

$$P(b,j,m) = P(b) \sum_e P(e) \sum_a P(a|b,e)P(j|a)P(m|a)$$

$$P(b,j,m) = 0.001 * 0.002 * 0.95 * 0.9 * 0.7 + 0.001 * 0.002 * 0.05 * 0.05 * 0.01+$$

$$0.001 * 0.998 * 0.94 * 0.9 * 0.7 + 0.001 * 0.998 * 0.06 * 0.05 * 0.01$$

$$P(b,j,m) = 0.00059224259$$

$$P(\neg b,j,m) = \sum_e \sum_a P(\neg b,j,m,e,a)$$

$$P(\neg b,j,m) = P(\neg b) \sum_e P(e) \sum_a P(a|\neg b,e)P(j|a)P(m|a)$$

$$P(\neg b,j,m) = 0.999 * 0.002 * 0.29 * 0.9 * 0.7 + 0.999 * 0.002 * 0.71 * 0.05 * 0.01+$$

$$0.999 * 0.998 * 0.001 * 0.9 * 0.7 + 0.999 * 0.998 * 0.999 * 0.05 * 0.01$$

$$P(\neg b,j,m) = 0.00149185764$$

$$\alpha = \frac{1}{P(\neg b,j,m) + P(b,j,m)}$$

$$\alpha = \frac{1}{0.00149185764 + 0.00059224259}$$
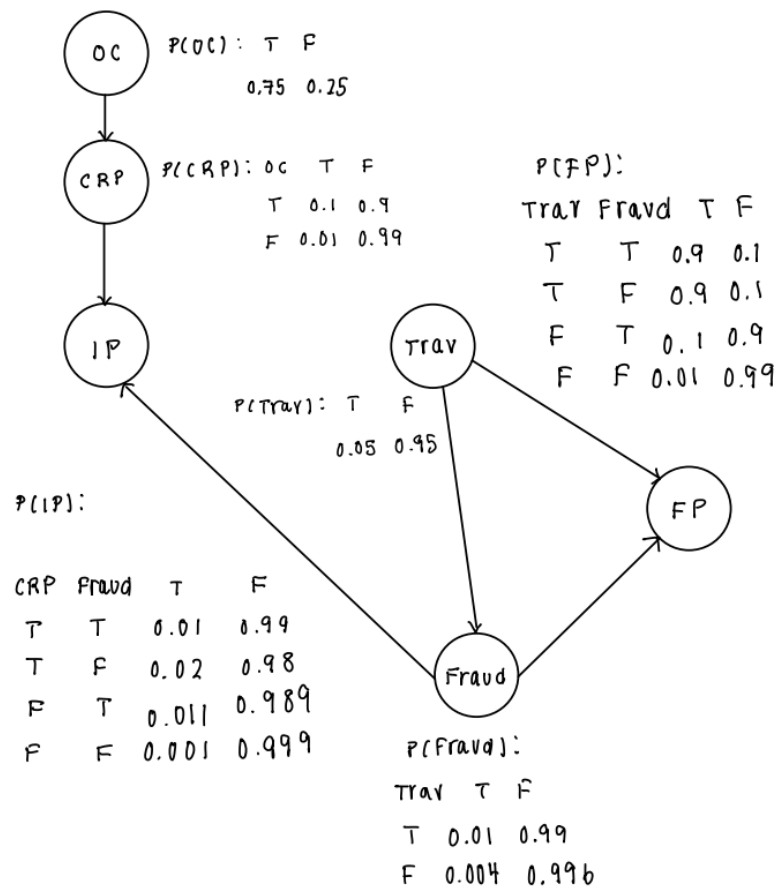
$$\alpha = \frac{1}{0.00208410023}$$

$$P(b|j,m) = \alpha * P(b,j,m)$$

$$P(b|j,m) = \frac{0.00059224259}{0.00208410023}$$

$$P(b|j,m) = 0.28417183659$$

Therefore, the probability of a burglary given that John and Mary both called, is 0.28417183659.

(b) In a Bayesian network in the form of a chain, using the enumeration method has a worst case runtime of $\mathcal{O}(2^n)$, as moving down each level in the chain generates another level in a binary tree.

Using the variable elimination method, moving down each level in the Bayesian network chain only adds one additional problem to compute (point-wise product of factors), as each node in the network only has one parent. Thus, the variable elimination method has a worst case runtime of $\mathcal{O}(n)$.

Problem 4

OC P(OC): T F
0.75 0.25

CRP P(CRP): OC T F
T 0.1 0.9
F 0.01 0.99

IP

P(Trav): T F
0.05 0.95

P(IP):

CRP	Fraud	T	F
T	T	0.01	0.99
T	F	0.02	0.98
F	T	0.011	0.989
F	F	0.001	0.999

Trav

P(FP):

Trav	Fraud	T	F
T	T	0.9	0.1
T	F	0.9	0.1
F	T	0.1	0.9
F	F	0.01	0.99

FP

Fraud

P(Fraud):

Trav	T	F
T	0.01	0.99
F	0.004	0.996

(a)

(b) Probability the current transanction is a fraud prior to searching for previous computer related purchases and before verifying if it is a foreign and/or an internet purchase:

4

$$P(Fraud) =$$
$$P(Fraud = True | Trav = True) * P(Trav = True) +$$
$$P(Fraud = True | Trav = False) * P(Trav = False) =$$
$$0.01 * 0.05 + 0.004 * 0.95 =$$
$$0.0043$$

Probability the current transaction is a fraud given that it is a foreign transaction, not an internet purchase, and card holder purchased computer related purchases in the past week:

$$P(Fraud = True | IP = False, CRP = True, OC = True) =$$
$$P(IP = False | CRP = True, Fraud = True) * P(CRP = True) +$$
$$P(IP = False | CRP = False, Fraud = True) * P(CRP = False) =$$
$$P(IP = False | CRP = True, Fraud = True) * (P(CRP = True | OC = True)*$$
$$P(OC = True) +$$
$$P(CRP = True | OC = False) * P(OC = False)) +$$
$$P(IP = False | CRP = False, Fraud = True) * (P(CRP = False | OC = True)*$$
$$P(OC = True) +$$
$$P(CRP = False | OC = False) * P(OC = False)) =$$
$$0.99 * (0.1 * 0.75 + 0.01 * 0.25) + 0.89 * (0.9 * 0.75 + 0.99 * 0.25) =$$
$$0.89775$$

$$P(Fraud = True | FP = True) =$$
$$P(Fraud = True | Trav = True) * P(FP = True | Trav = True, Fraud = True)*$$
$$P(Trav = True) +$$
$$P(Fraud = True | Trav = False) * P(FP = True | Trav = False, Fraud = True)*$$
$$P(Trav = False) =$$
$$0.01 * 0.90 * 0.05 + 0.004 * 0.10 * 0.95 =$$
$$0.00083$$

$$P(Fraud = True | IP = False, CRP = True, OC = True, FP = True) =$$
$$0.89775 * 0.00083 =$$
$$0.000745$$