

SIMULATION RESULTS

We have here simulation results based on the JINI method. We compare with some other methods where possible.

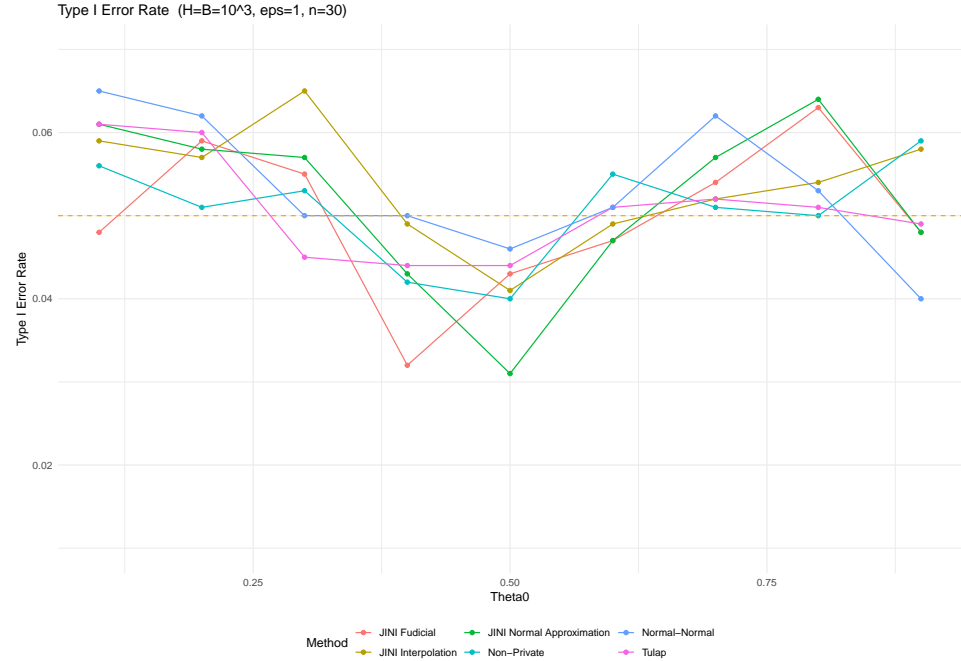
1. SIMPLE HYPOTHESIS TESTING

Given the observed sample $X_1, \dots, X_n \sim \text{Ber}(\theta_0)$. We test the following hypothesis in differential privacy framework:

$$H_0 : \theta_0 \leq 0.9 \quad vs \quad H_1 : \theta_0 > 0.9.$$

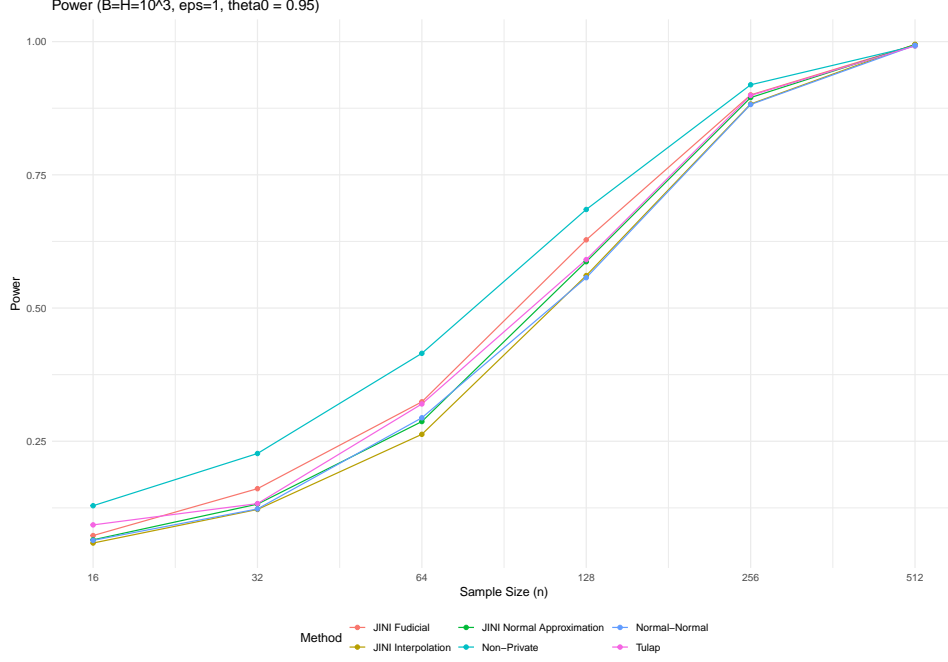
We study the power and level of the test. In what follows, B is the number of JINI bootstrap and H is the number of simulation.

1.1. Level Analysis. Simulation setting: $n = 30, B = 10^3, H = 10^3, \epsilon = 1, \theta_0 = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$.



1.2. Power Analysis. Simulation setting:

$$\theta_0 = 0.95, B = 10^3, H = 10^3, \epsilon = 1, n = 16, 32, 64, 128, 256, 512.$$



2. MULTIPLE COMPARISON

Here we consider comparing three groups. Given $X_{1,j}, \dots, X_{n,j} \sim \text{Ber}(\theta_{0_j})$, $j = 1, 2, 3$. Without loss of generality we set Group 1 as the reference group and define the log odd ratio of Groups 2 and 3 with respect to Group 1 as follows:

$$\beta_2 = \log \left(\frac{\theta_{0_2}}{1 - \theta_{0_2}} \right) - \log \left(\frac{\theta_{0_1}}{1 - \theta_{0_1}} \right)$$

$$\beta_3 = \log \left(\frac{\theta_{0_3}}{1 - \theta_{0_3}} \right) - \log \left(\frac{\theta_{0_1}}{1 - \theta_{0_1}} \right)$$

2.1. **Case 1:** We test the hypotheses:

$$H_0 : \beta_i = 0 \quad vs \quad H_1 : \beta_i \neq 0, \quad i = 2, 3$$

Simulations Setting: $\theta_{0_1} = 0.7, \theta_{0_2} = 0.5, \theta_{0_3} = 0.68; n_1 = 200, n_2 = 100, n_3 = 120; \alpha = 0.05, seed = 1234, B = 10^3, H = 10^3$

3. CONFIDENCE INTERVALS

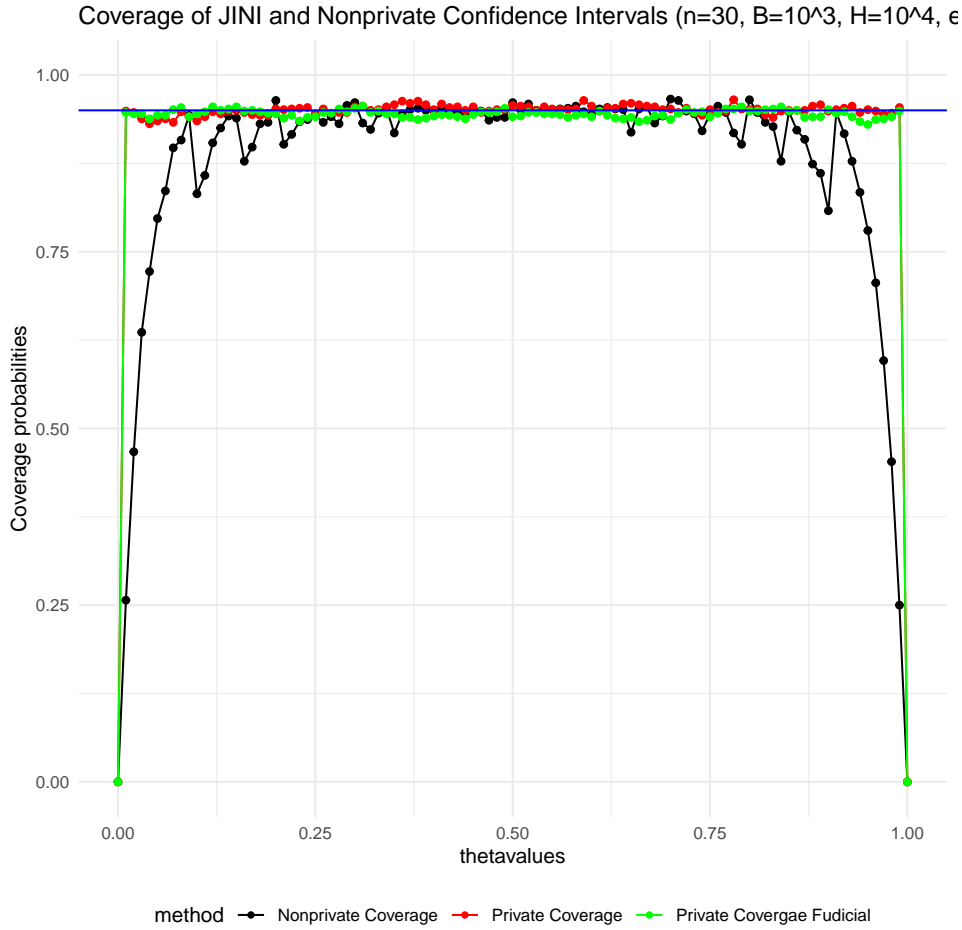
We use JINI method to build confidence interval for a population parameter in the DP framework.

Group	true β	est. $\hat{\beta}$	p-value	Reject H_0
Group 2	-0.8473	-0.5585	0.0009	TRUE
Group 3	-0.0935 2	-0.5273	0.0006	TRUE

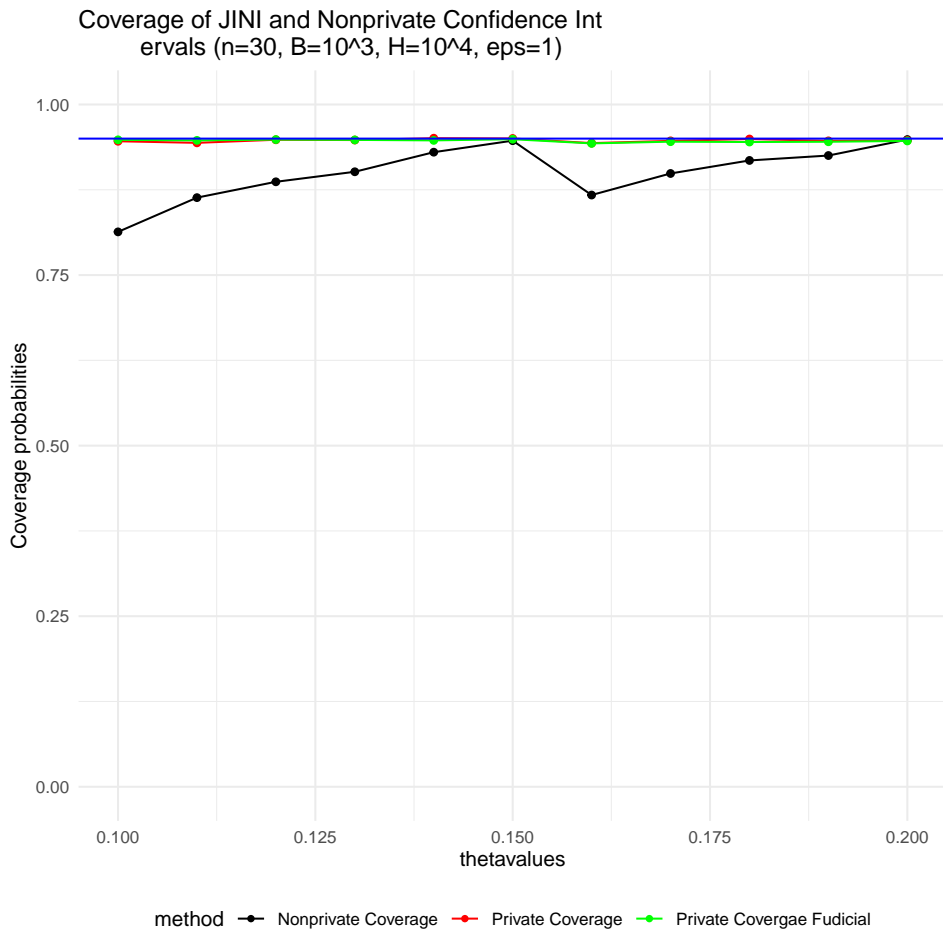
TABLE 1. Hypothesis testing.

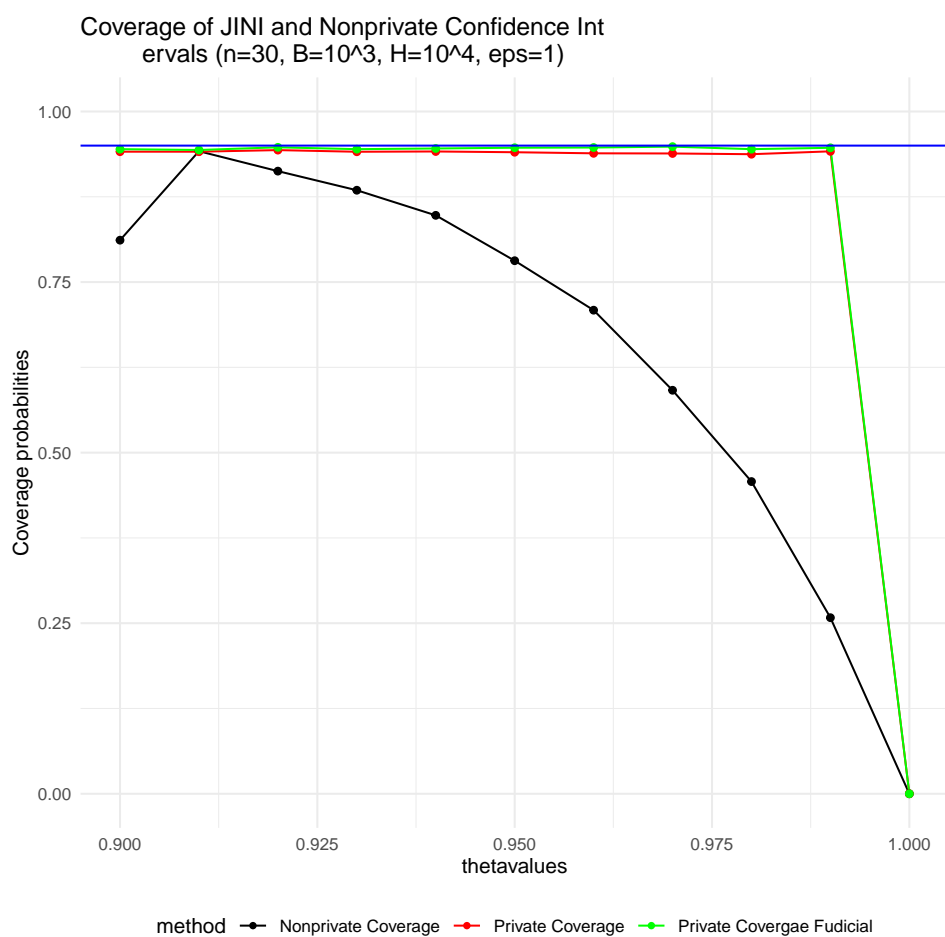
3.1. Coverage Analysis. Consider $X_1, \dots, X_n \sim \text{Ber}(\theta_0)$. We study the coverage of the CI for θ_0 in DP setting.

Simulation Setting: $\theta_0 = 0.000.010.020.030.040.05, \dots, 1, n = 30, B = 10^3, H = 10^4, \epsilon = 1, \alpha = 0.05, \text{seed} = 345$

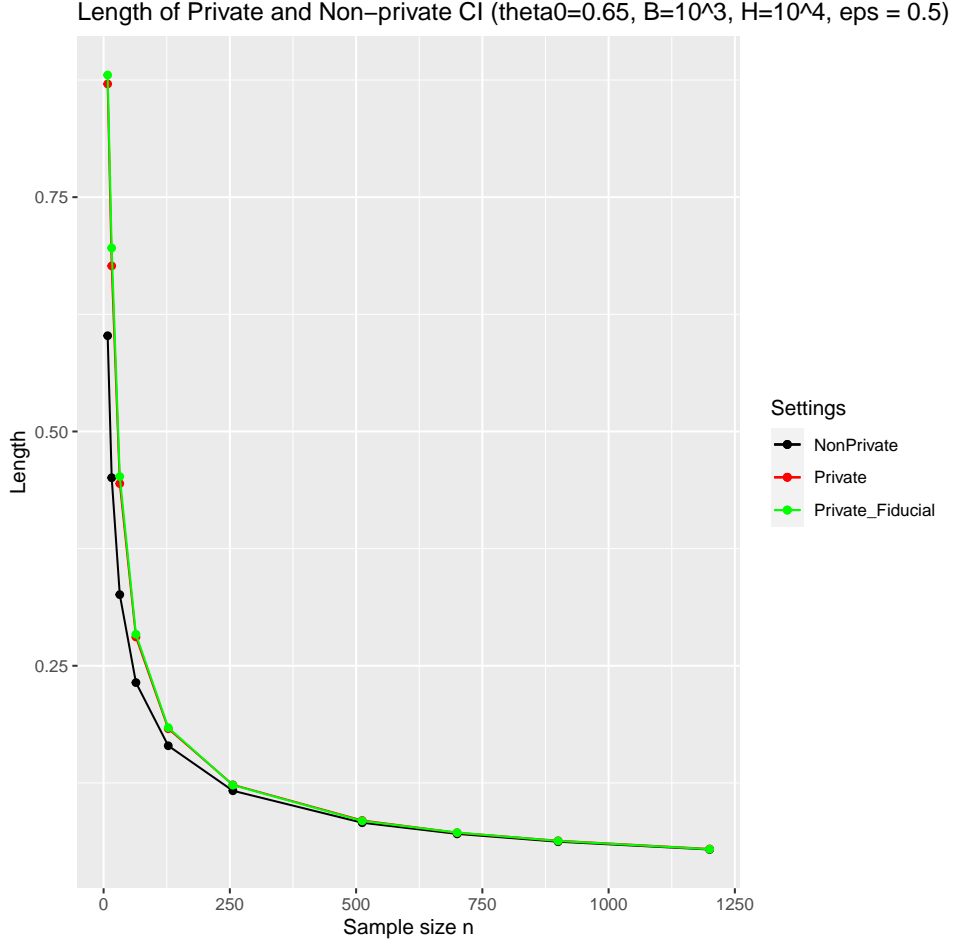


From the above coverage plot, the JINI private methods appear to be doing much better than the non-private method for values of θ_0 near 0 and 1.

3.1.1. Coverage for θ_0 near θ .

3.1.2. Coverage for θ_0 near 1.

3.2. Length of Confidence Intervals.



3.3. Multiple Comparison. Here we are comparing three groups in terms of their log odd ratios. Precisely, we set Group 1 as reference and compare the other two groups with Group 1.

Given $X_{1,j}, \dots, X_{n,j} \sim Ber(\theta_{0_j})$, $j = 1, 2, 3$. Define the log odd ratio of Groups 2 and 3 relative to Group 1 as follows:

$$\beta_2 = \log \left(\frac{\theta_{0_2}}{1 - \theta_{0_2}} \right) - \log \left(\frac{\theta_{0_1}}{1 - \theta_{0_1}} \right)$$

$$\beta_3 = \log \left(\frac{\theta_{0_3}}{1 - \theta_{0_3}} \right) - \log \left(\frac{\theta_{0_1}}{1 - \theta_{0_1}} \right)$$

We build confidence intervals for β_1 and β_2 .

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