SIMULATION RESULTS

We have here simulation results based on the JINI method. We compare with some other methods where possible.

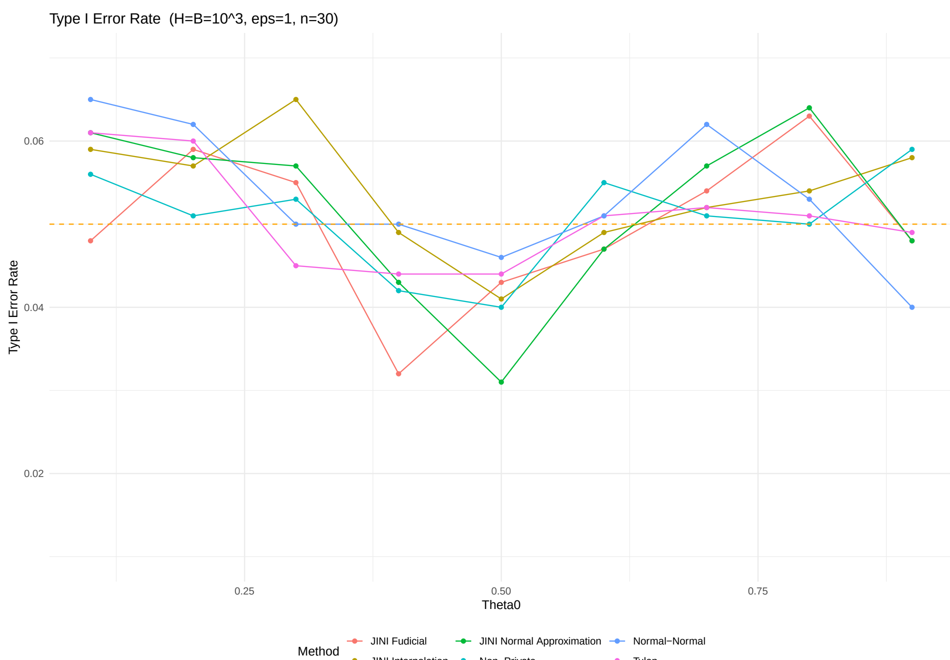
1. Simple Hypothesis Testing

Given the observed sample $X_1, \dots, X_n \sim Ber(\theta_0)$. We test the following hypothesis in differential privacy framework:

$$H_0 : \theta_0 \leq 0.9 \qquad vs \qquad H_1 : \theta_0 > 0.9.$$

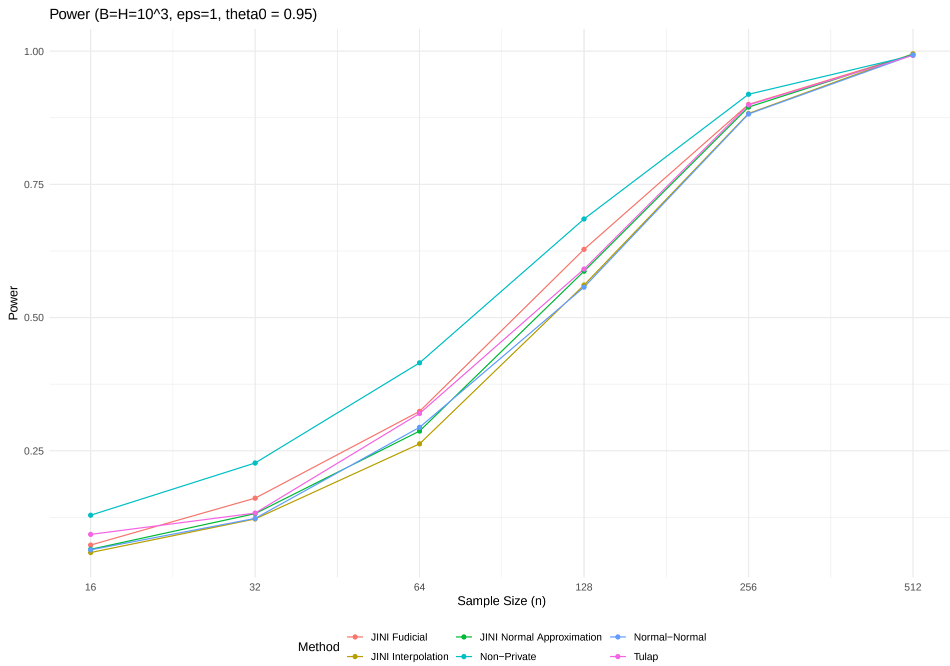
We study the power and level of the test. In what follows, B is the number of JINI bootstrap and H is the number of simulation.

1.1. **Level Analysis. Simulation setting:** $n = 30, B = 10^3, H = 10^3, \epsilon = 1, \theta_0 = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9.$



1.2. **Power Analysis.** Simulation setting:

$$\theta_0 = 0.95, B = 10^3, H = 10^3, \epsilon = 1, n = 16, 32, 64, 128, 256, 512.$$



2. Multiple Comparison

Here we consider comparing three groups. Given $X_{1,j}, \ldots, X_{n,j} \sim Ber(\theta_{0_j}), \quad j = 1, 2, 3$. Without loss of generality we set Group 1 as the reference group and define the log odd ratio of Groups 2 and 3 with respect to Group 1 as follows:

$$\beta_2 = \log\left(\frac{\theta_{0_2}}{1-\theta_{0_2}}\right) - \log\left(\frac{\theta_{0_1}}{1-\theta_{0_1}}\right)$$
$$\beta_3 = \log\left(\frac{\theta_{0_3}}{1-\theta_{0_3}}\right) - \log\left(\frac{\theta_{0_1}}{1-\theta_{0_1}}\right)$$

2.1. **Case 1:** We test the hypotheses:

$$H_0 : \beta_i = 0 \qquad vs \qquad H_1 : \beta_i \neq 0, \quad i = 2, 3$$

Simulations Setting: $\theta_{0_1} = 0.7, \theta_{0_2} = 0.5, \theta_{0_3} = 0.68; n_1 = 200, n_2 = 100, n_3 = 120; \alpha = 0.05, seed = 1234, B = 10^3, H = 10^3$

Group	true β	est. $\hat{\beta}$	p-value	Reject H_0
Group 2	-0.8473	-0.5585	0.0009	TRUE
Group 3	-0.0935 2	-0.5273	0.0006	TRUE

Table 1. Hypothesis testing.

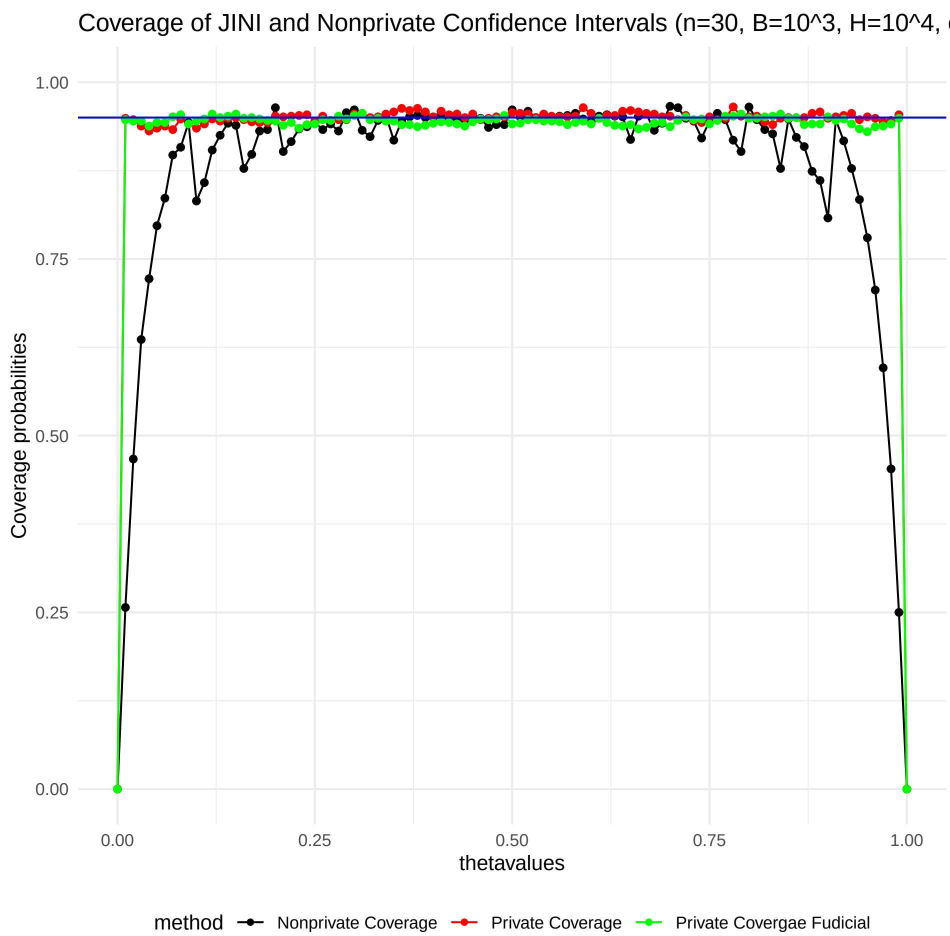
3. Confidence Intervals

We use JINI method to build confidence interval for a population parameter in the DP framework.

3.1. Coverage Analysis.

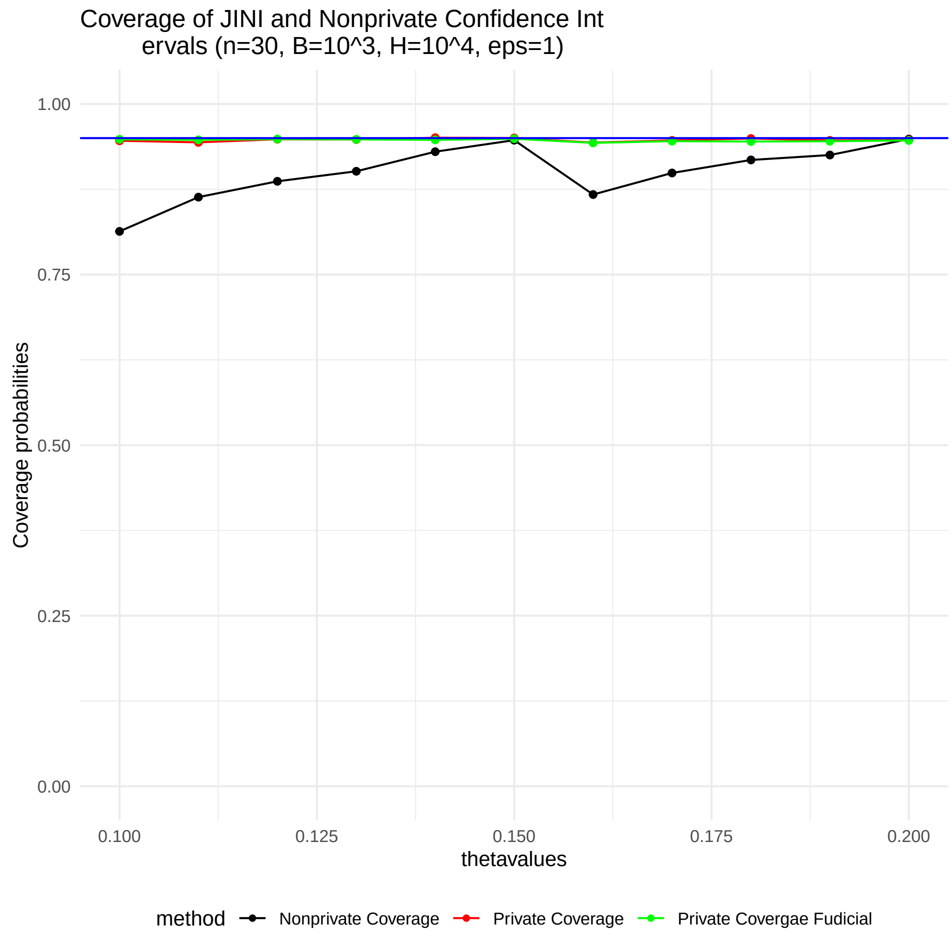
Consider $X_1, \ldots, X_n \sim Ber(\theta_0)$. We study the coverage of the CI for θ_0 in DP setting.

Simulation Setting: $\theta_0 = 0.00, 0.01, 0.02, 0.03, 0.04, 0.05, \ldots, 1, n = 30, B = 10^3, H = 10^4, \epsilon = 1, \alpha = 0.05, seed = 345$

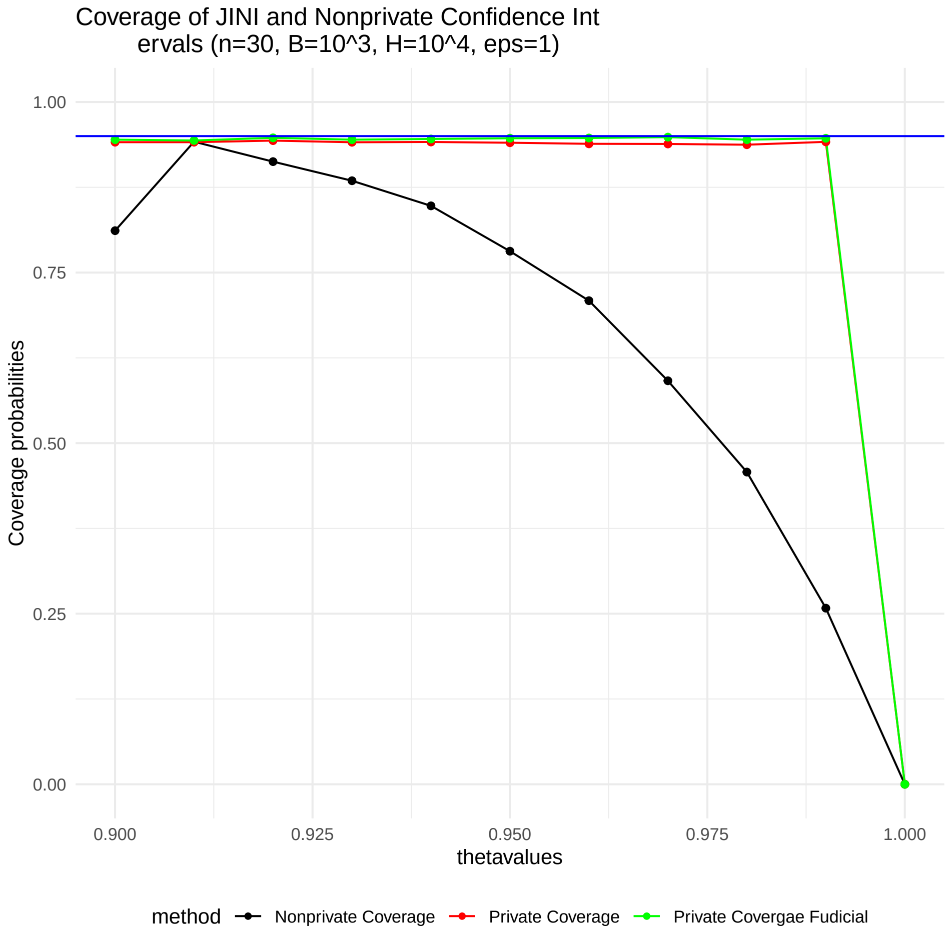


From the above coverage plot, the JINI private methods appear to be doing much better than the non-private method for values of θ_0 near 0 and 1.

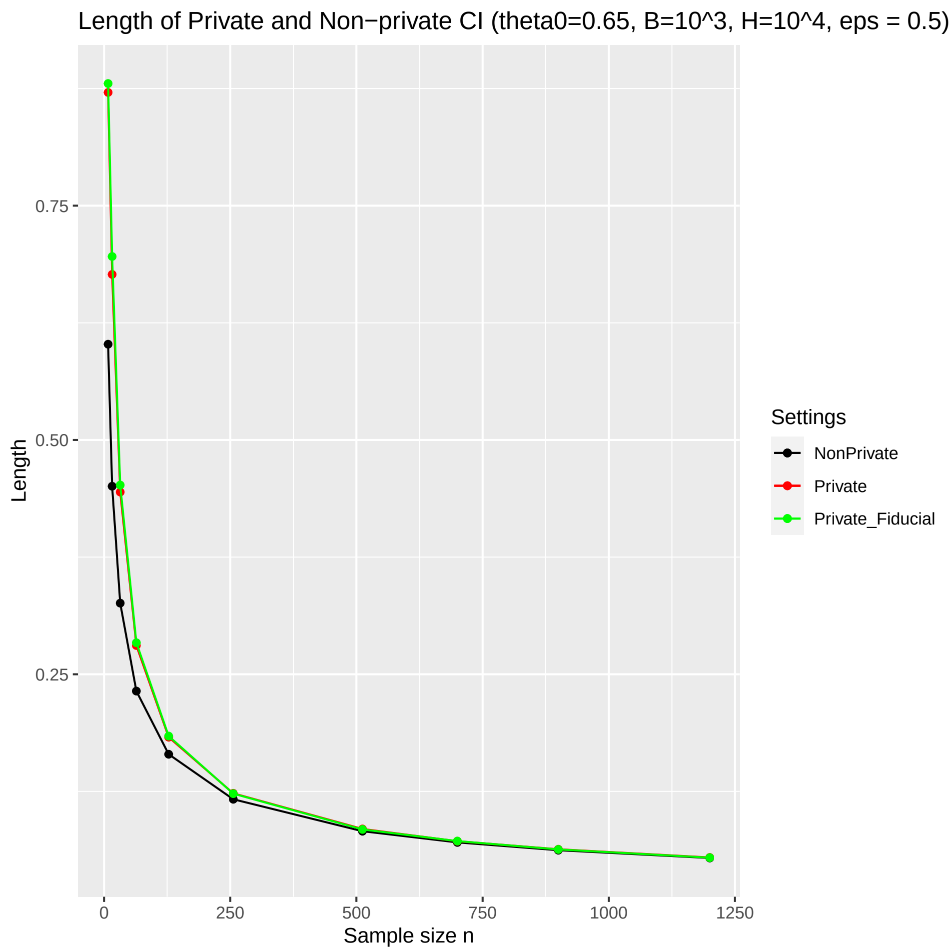
3.1.1. *Coverage for θ_0 near 0.*



3.1.2. *Coverage for θ_0 near 1.*



3.2. Length of Confidence Intervals.



3.3. Multiple Comparison.

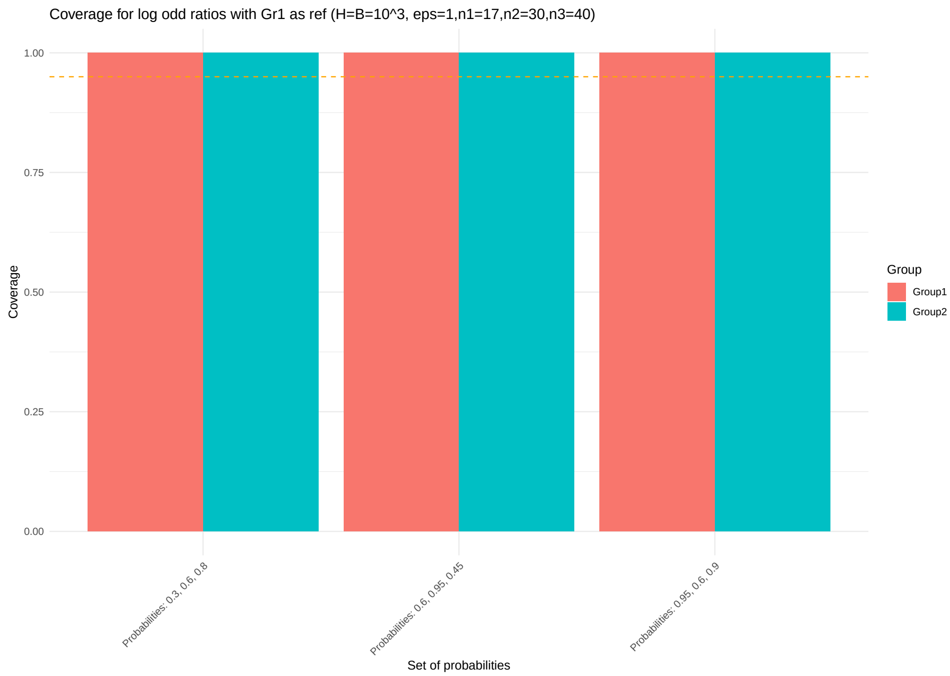
Here we are comparing three groups in terms of their log odd ratios. Precisely, we set Group 1 as reference and compare the other two groups with Group 1.

Given $X_{1,j}, \ldots, X_{n,j} \sim Ber(\theta_{0_j}), \quad j = 1, 2, 3$. Define the log odd ratio of Groups 2 and 3 relative to Group 1 as follows:

$$\beta_2 = \log\left(\frac{\theta_{0_2}}{1-\theta_{0_2}}\right) - \log\left(\frac{\theta_{0_1}}{1-\theta_{0_1}}\right)$$

$$\beta_3 = \log\left(\frac{\theta_{0_3}}{1-\theta_{0_3}}\right) - \log\left(\frac{\theta_{0_1}}{1-\theta_{0_1}}\right)$$

We build confidence intervals for β_1 and β_2.


Coverage for log odd ratios with Gr1 as ref (H=B=10^3, eps=1,n1=17,n2=30,n3=40)
Coverage
1.00
0.75
0.50
0.25
0.00
Probabilities: 0.3, 0.6, 0.8
Probabilities: 0.6, 0.95, 0.45
Probabilities: 0.95, 0.6, 0.9
Set of probabilities
Group
Group1
Group2