



UNIVERSITY OF AMSTERDAM  
Informatics Institute

# Contact Tracing with Differential Privacy Guarantees

Protect your score

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AAAI 2024



Brain, Behavior, and Immunity

Volume 89, October 2020, Pages 531-542





Review Article

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Nina Vindegaard, Michael Eriksen Benros  

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Best Practice & Research Clinical Anaesthesiology

Volume 35, Issue 3, October 2021, Pages 293-306



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## Economic impact of COVID-19 pandemic on healthcare facilities and systems: International perspectives

Alan D. Kaye MD, PhD (Provost & Vice Chancellor of Academic Affairs).<sup>a</sup>  ,  
Chikezie N. Okeagu MD (Assistant Professor).<sup>b</sup>  , Alex D. Pham MD (Resident Physician).<sup>c</sup>  ,  
Rayce A. Silva (Medical Student).<sup>d</sup>  , Joshua J. Hurley MD, PGY-1 (Resident Physician).<sup>e</sup>  ,  
Brett L. Arron MD (Associate Professor).<sup>f</sup>  , Noeen Sarfraz MD MPH (Resident Physician).<sup>g</sup>  ,  
Hong N. Lee MD (Assistant Professor).<sup>h</sup>  , G.E. Ghali DDS, MD, FACS, FRCS(Ed) (Chancellor).<sup>i</sup>  ,  
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Richard D. Urman MD (Associate Professor).<sup>k</sup>  ,  
Elyse M. Cornett PhD (Assistant Professor).<sup>h</sup>  

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# Covid-19: Cities fear 'huge' economic impact of restrictions

 29 September 2020

BBC, Sept 2020

# Covid had negative impact on children's reading - Estyn

 4 May

BBC, May 2023

# This interactive tool tracks covid-19 travel restrictions by country

Skyscanner's detailed travel map is color-coded in stoplight-style green, yellow and red

Washington Post, December 2020

Rob Romijnders - April 19, 2024

# Privacy is important

“The top reasons against app use were as follows: mistrusting the government, concerns about data security and **privacy**, and doubts about efficacy.” Jones et al. 2021

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## Privacy is important

“The top reasons against app use were as follows: mistrusting the government, concerns about data security and **privacy**, and doubts about efficacy.” Jones et al. 2021

“The most cited reasons for not downloading were related to **data (...)** **concerns**” Gao et al. 2022

“The main reasons for not downloading and using the app were (...) **worries about privacy**” Walrave et al. 2022

## The Lancet, 2020

*“most of the applications in use or under consideration have an impact on individual privacy that democratic societies would normally consider to be **unacceptably high**”*

## Research question

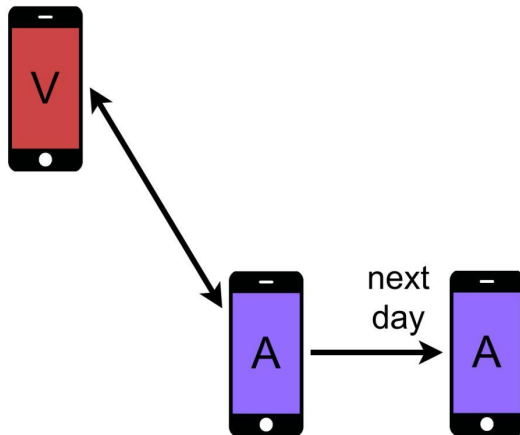
Low peak infection rate

under reasonable differential privacy

# Attack Scenario

Privacy with respect to  
released covidscore

V is victim, A is attacker



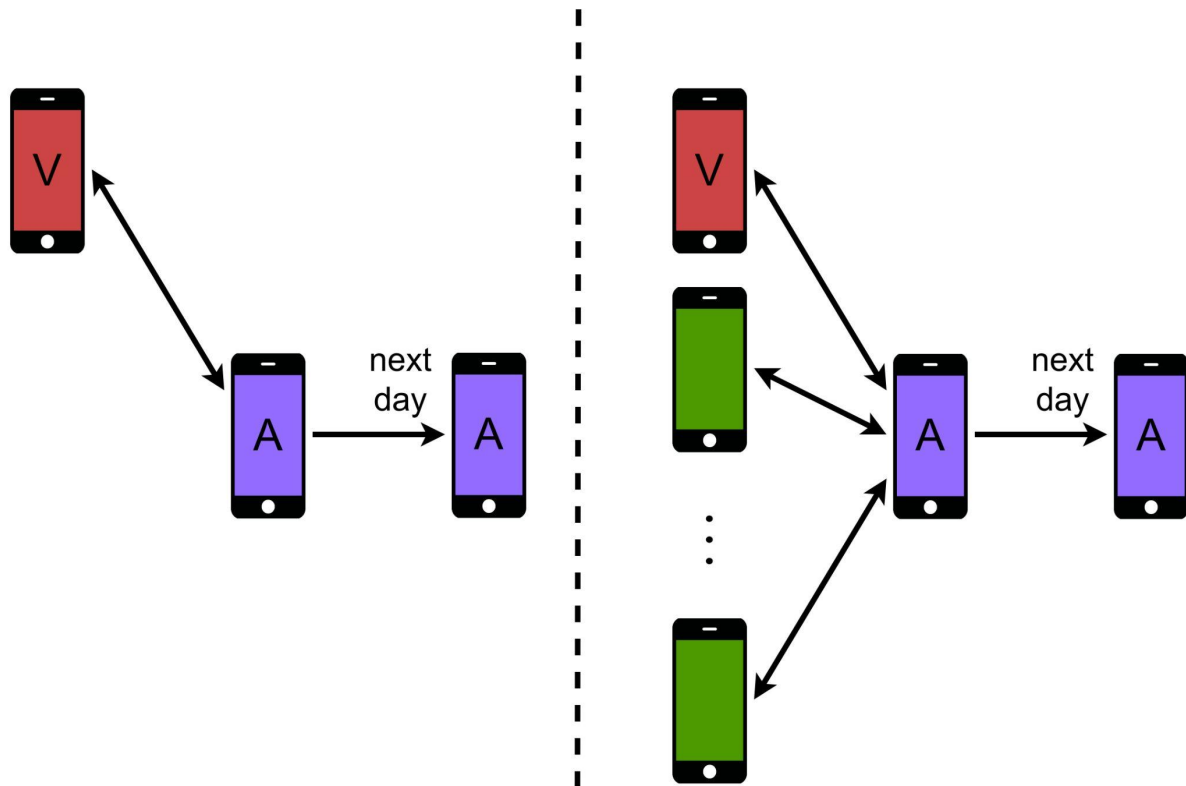


# Attack Scenario

Privacy with respect to  
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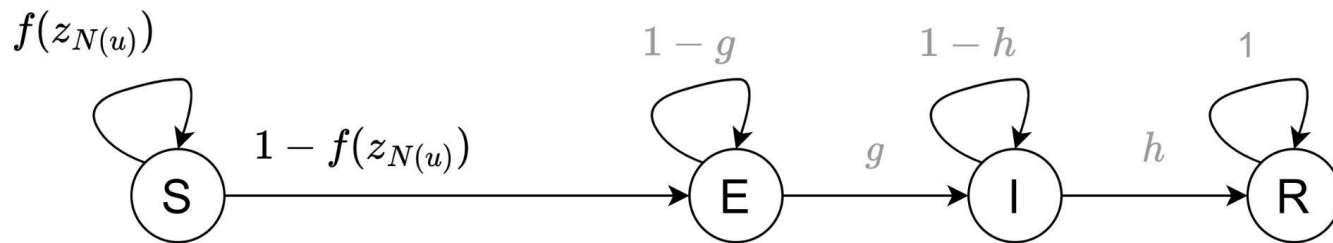
V is victim, A is attacker

Green phones are agents  
with 'known' score



# SEIR transitions are a Markov chain

Susceptible - Exposed - Infected - Recovered



$$f(z_{N(u)}) = (1 - p_0)(1 - p_1)^{|\{z \in z_{N(u)} : z=I\}|}$$

# Dynamics model

Susceptible

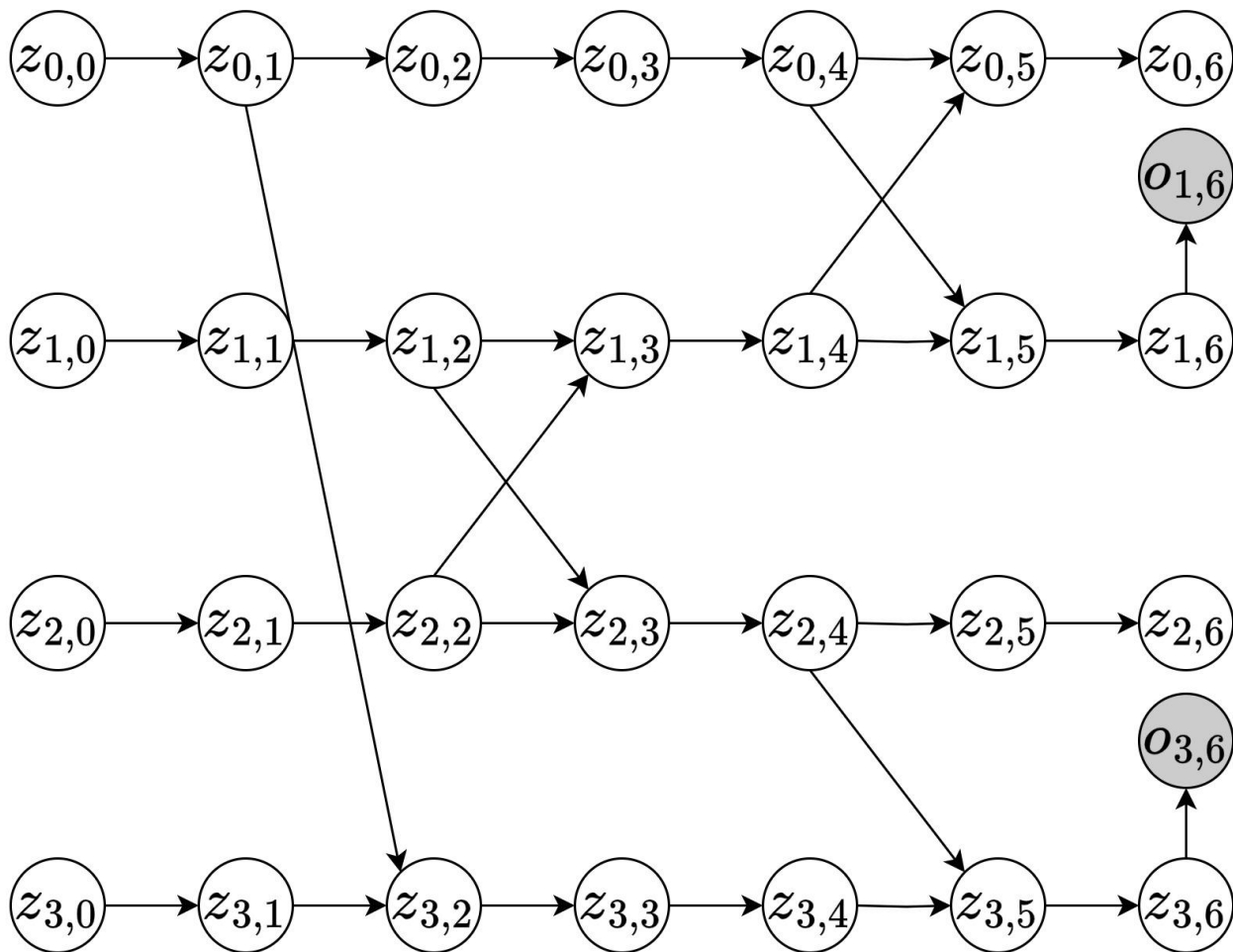
Exposed

Infected

Recovered

$$P(z_{u,t+1} | \mathcal{Z}_t) = \begin{cases} f(u, t, \mathcal{Z}_t) & \text{if } z_t = S \wedge z_{t+1} = S \\ 1 - f(u, t, \mathcal{Z}_t) & \text{if } z_t = S \wedge z_{t+1} = E \\ 1 - g & \text{if } z_t = E \wedge z_{t+1} = E \\ g & \text{if } z_t = E \wedge z_{t+1} = I \\ 1 - h & \text{if } z_t = I \wedge z_{t+1} = I \\ h & \text{if } z_t = I \wedge z_{t+1} = R \\ 1 & \text{if } z_t = R \wedge z_{t+1} = R \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$$f(u, t, \mathcal{Z}_t) = (1 - p_0)(1 - p_1)^{|\{(v, u, t) \in \mathcal{D} : z_{v,t} = I\}|} \quad (2)$$



# Exact inference is intractable!

$p(\text{joint distribution}) =$

$$\begin{aligned}
 & p(z_{0,0}) \cdot p(z_{0,1}|z_{0,0}) \quad \cdot p(z_{0,2}|z_{0,1}) \quad \cdot p(z_{0,3}|z_{0,2}) \cdot p(z_{0,4}|z_{0,3}) \cdot p(z_{0,5}|z_{0,4}, z_{1,4}) \cdot p(z_{0,6}|z_{0,5}) \cdot \\
 & p(z_{1,0}) \cdot p(z_{1,1}|z_{1,0}) \quad \cdot p(z_{1,2}|z_{1,1}) \cdot p(z_{1,3}|z_{1,2}, z_{2,2}) \cdot p(z_{1,4}|z_{1,3}) \cdot p(z_{1,5}|z_{1,4}, z_{0,4}) \cdot p(z_{1,6}|z_{1,5}) \cdot \\
 & p(z_{2,0}) \cdot p(z_{2,1}|z_{2,0}) \quad \cdot p(z_{2,2}|z_{2,1}) \cdot p(z_{2,3}|z_{2,2}, z_{1,2}) \cdot p(z_{2,4}|z_{2,3}) \quad \cdot p(z_{2,5}|z_{2,4}) \cdot p(z_{2,6}|z_{2,5}) \cdot \\
 & p(z_{3,0}) \cdot p(z_{3,1}|z_{3,0}) \cdot p(z_{3,2}|z_{3,1}, z_{0,1}) \quad \cdot p(z_{3,3}|z_{3,2}) \cdot p(z_{3,4}|z_{3,3}) \cdot p(z_{3,5}|z_{3,4}, z_{2,4}) \cdot p(z_{3,6}|z_{3,5}) \cdot \\
 & \quad p(o_{3,6}|z_{3,6}) \quad \cdot p(o_{1,6}|z_{1,6})
 \end{aligned} \tag{146}$$

$$p(z_{2,6}|o_{1,6}, o_{3,6}) = \frac{p(\text{joint distribution})}{\sum_{z_{0,0}} \sum_{z_{0,1}} \sum_{z_{0,2}} \sum_{z_{0,3}} \sum_{z_{0,4}} \sum_{z_{0,5}} \sum_{z_{0,6}} \sum_{z_{1,0}} \sum_{z_{1,1}} \sum_{z_{1,2}} \sum_{z_{1,3}} \sum_{z_{1,4}} \sum_{z_{1,5}} \sum_{z_{1,6}} \sum_{z_{2,0}} \sum_{z_{2,1}} \sum_{z_{2,2}} \sum_{z_{2,3}} \sum_{z_{2,4}} \sum_{z_{2,5}} \sum_{z_{2,6}} \sum_{z_{3,0}} \sum_{z_{3,1}} \sum_{z_{3,2}} \sum_{z_{3,3}} \sum_{z_{3,4}} \sum_{z_{3,5}} \sum_{z_{3,6}} p(\text{joint distribution})} \tag{147}$$

# Inference

Gibbs sampling

$$p(z_u | \hat{z}_{\neg u}, \mathcal{O}).$$

Belief Propagation

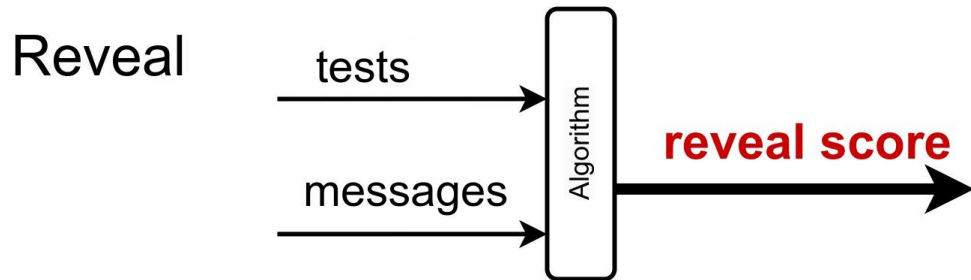
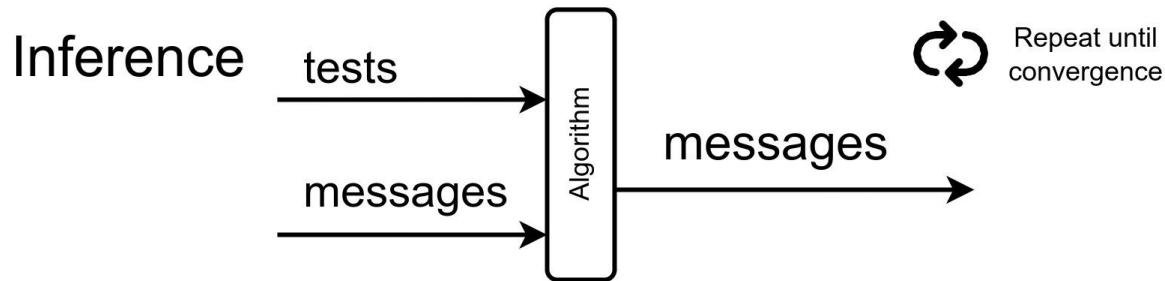
$$\mu_{f_s \rightarrow z_{u,t}}(z_{u,t}) = \sum_{z_s} f_s(z_s, z_{u,t}) \prod_{k \in \text{Nb}(f_s) \setminus z_{u,t}} \mu_{z_k \rightarrow f_s}$$

$$\mu_{z_{u,t} \rightarrow f_s}(z_{u,t}) = \prod_{k \in \text{Nb}(z_{u,t}) \setminus f_s} \mu_{f_k \rightarrow z_{u,t}}$$

# Factorised neighbours

$$\begin{aligned} b_u(z_u) &= \sum_{z_{N(u)}} P(z_u | z_{N(u)}, \mathcal{O}) B_{N(u)}(z_{N(u)}) \\ &= E_{B_{N(u)}(z_{N(u)})} [P(z_u | z_{N(u)}, \mathcal{O})]. \end{aligned}$$

# Modular view of contact tracing

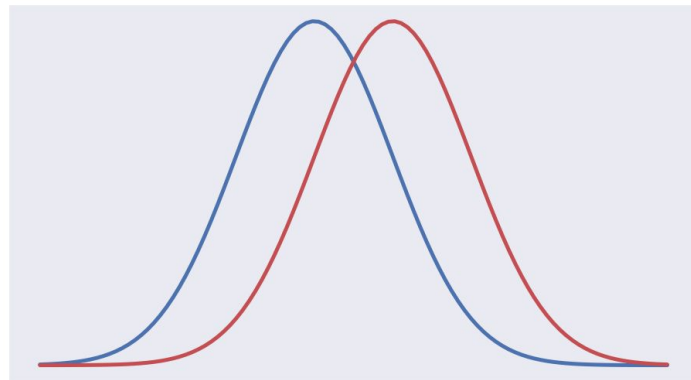
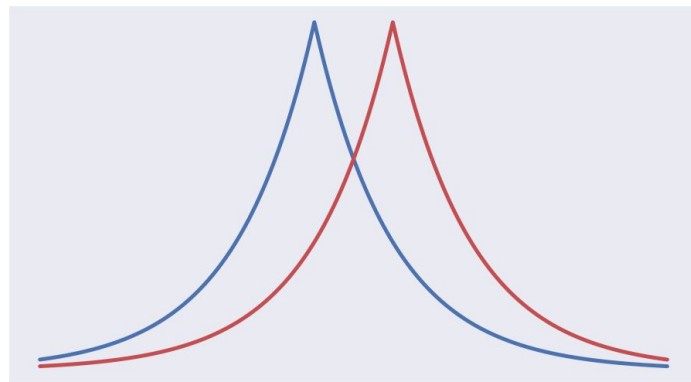




# Differential privacy

Definition of  $(\varepsilon, \delta)$  differential privacy (Dwork and Roth 2014):  
for every  $\varepsilon > 0$ ,  $\delta \in [0, 1)$ , a mechanism  $f(\cdot)$ , for any outcome  $\Phi$  in the range of  $f(\cdot)$ , and any two adjacent data sets  $D, D'$  that differ in at most one element, satisfies the constraint:

$$p(f(D) \in \Phi) \leq e^\varepsilon p(f(D') \in \Phi) + \delta$$

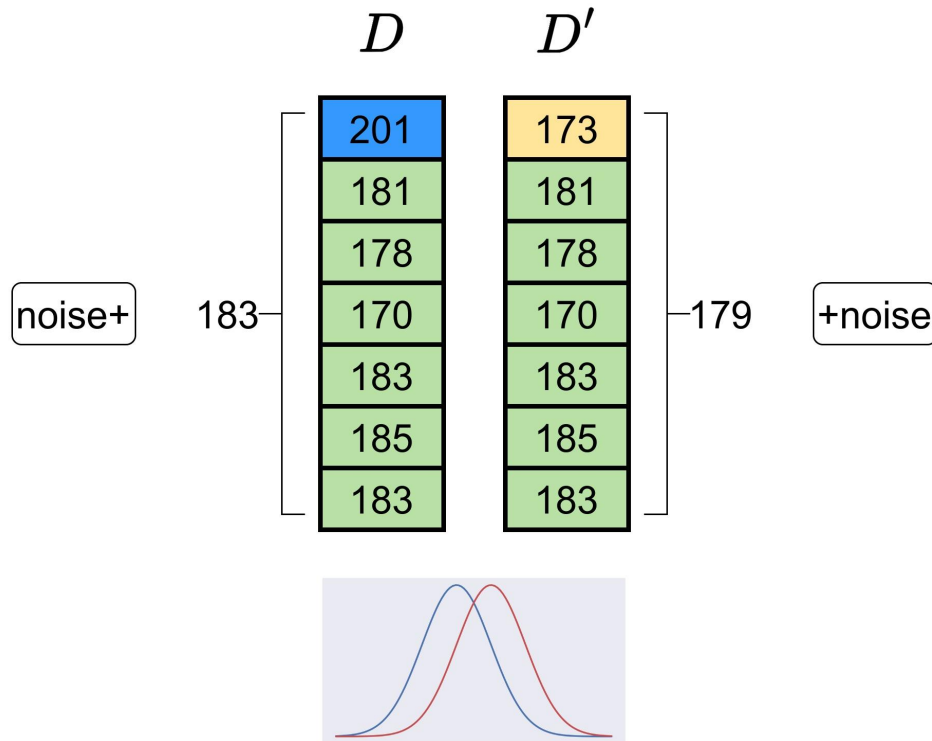


# Differential privacy

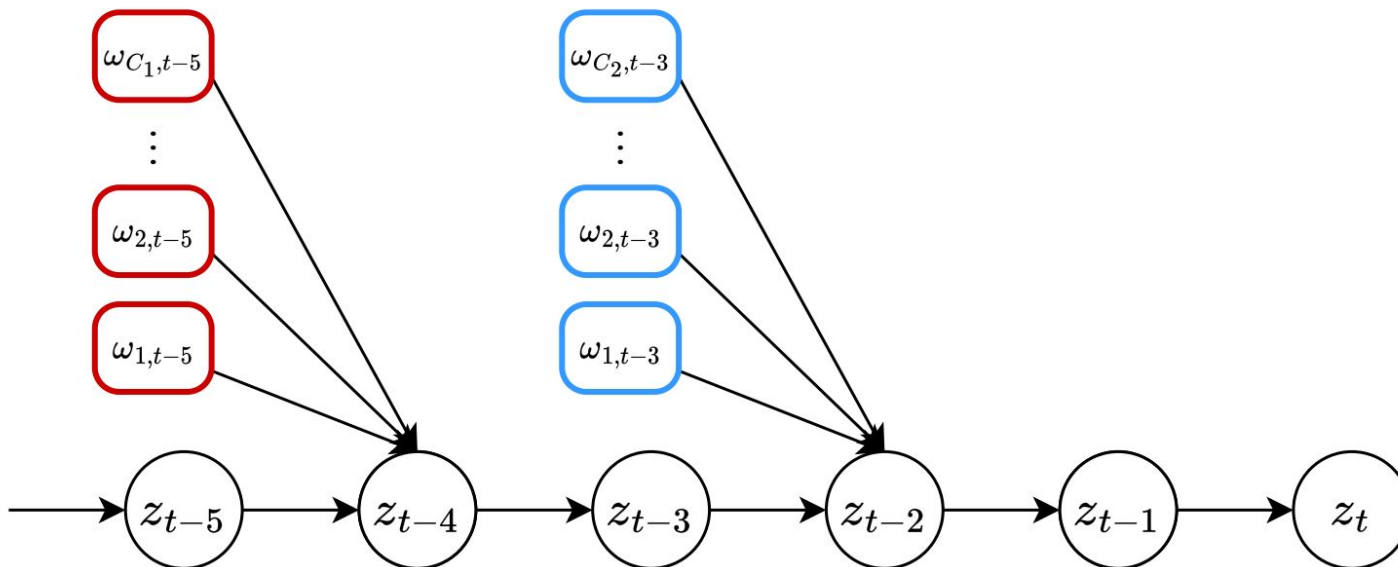
Gaussian Mechanism:

Add noise according to ‘sensitivity’

$$\sigma > \frac{\Delta}{\varepsilon} \left( 2 \log \left( \frac{1.25}{\delta} \right) \right)^{\frac{1}{2}}$$



# Example graph for a user over seven days



# FN can be written as function of product factors

$$\phi_{u,t} = F_1(\omega_{1,t-5}, \omega_{2,t-5}, \dots, \omega_{C_1,t-5}, \\ \omega_{1,t-3}, \omega_{2,t-3}, \dots, \omega_{C_2,t-3})$$

# FN can be written as function of product factors

$$\phi_{u,t} = F_1(\omega_{1,t-5}, \omega_{2,t-5}, \dots, \omega_{C_1,t-5}, \\ \omega_{1,t-3}, \omega_{2,t-3}, \dots, \omega_{C_2,t-3})$$

$$\phi_{u,t} = F_2\left(\prod_{i=1}^{C_1} \omega_{i,t-5}, \quad \prod_{i=1}^{C_2} \omega_{i,t-3}\right)$$

# Log-normals have closed form Renyi divergence

$$\begin{aligned}
 D_a(p_u|p_v) &= \underbrace{\log \left( \frac{\sigma_v}{\sigma_u} \right) + \frac{1}{2(a-1)} \log \left( \frac{\sigma_v^2}{\sigma_*^2} \right)}_{\text{equals 0}} \\
 &\quad + \frac{a}{2\sigma_*^2} \cdot (\mu_u - \mu_v)^2 \\
 &= \frac{a}{2\sigma_*^2} (\mu_u - \mu_v)^2 = \frac{a}{2C\sigma^2} (\mu_u - \mu_v)^2
 \end{aligned}$$

# Optimize $a, \rho$ from RDP to DP

## Optimization-problem 1.

$$\min_{a, \rho} \frac{a}{\rho}$$

*Such that:*

$$\rho + \frac{\log \frac{1}{\delta}}{a - 1} - \varepsilon = 0$$

$$a = 1 + \frac{d + \sqrt{d(d + \varepsilon)}}{\varepsilon}$$

# Renyi Differential Privacy between log-normals

$$\sigma^2 \geq \frac{a}{2C_\rho} \left( \log(1 - \gamma_u p_1) - \log(1 - \gamma_l p_1) \right)^2$$



## Methods to compare with in experimental results

- Traditional contact tracing (Baker et al. 2021)
- Gibbs sampling (Herbrich et al. 2020; Wang et al. 2015)
- Per-message noising of FN (Romijnders et al. 2023)
- DPFN (ours)

## Two simulators

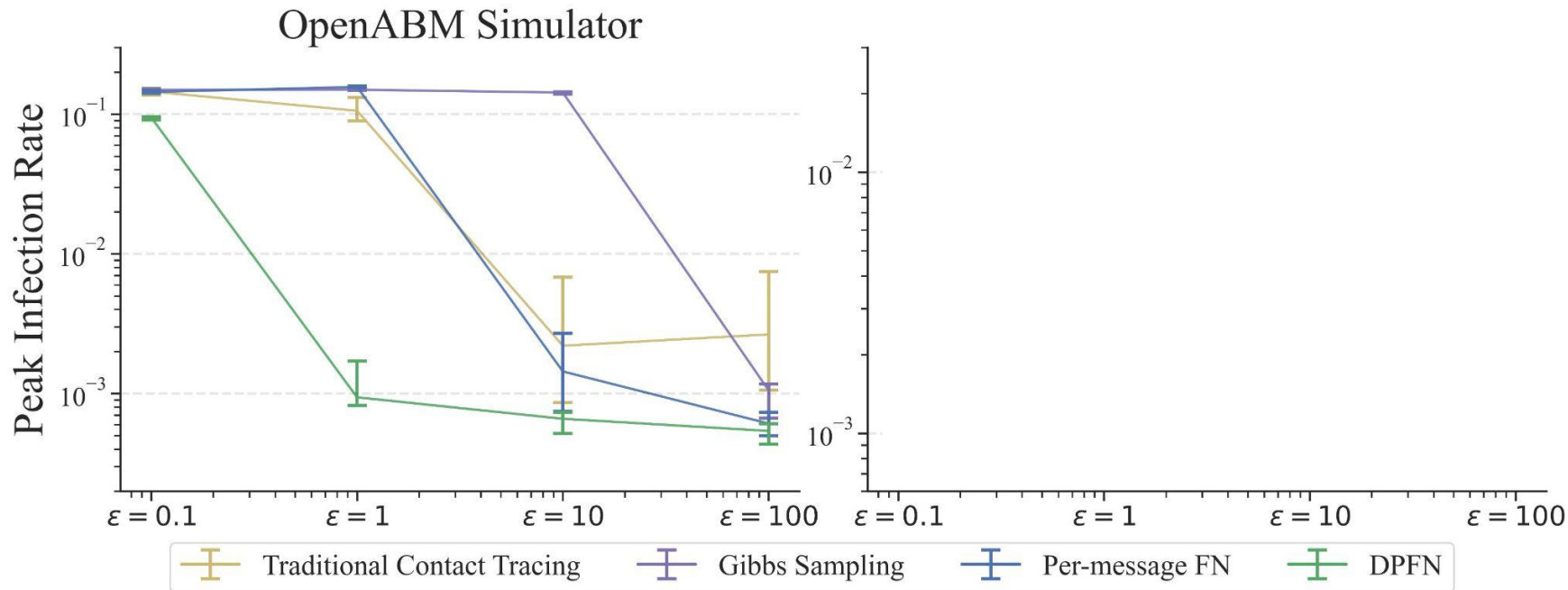
### OpenABM (Hinch et al. 2021)

- Stratifying for 9 age categories, 3 occupations, and 6 household types
- 150 parameters calibrated against a typical city in the UK

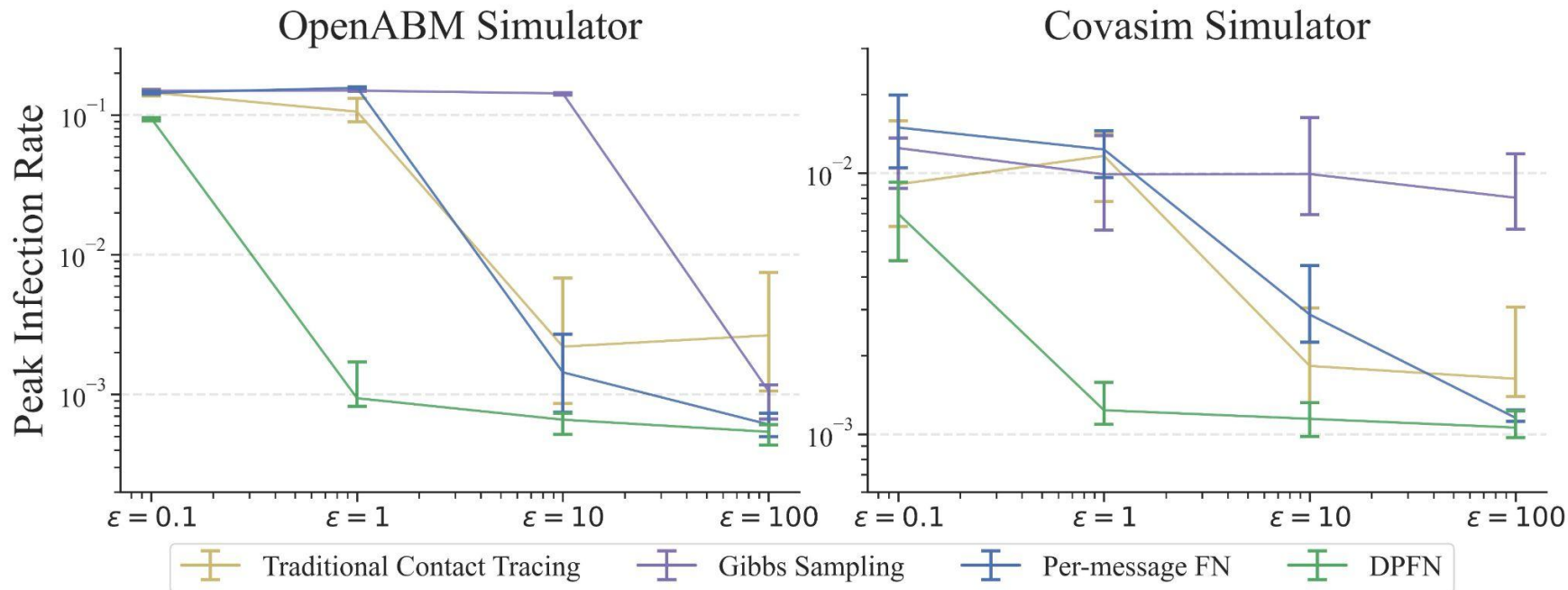
### COVASIM (Kerr et al. 2021)

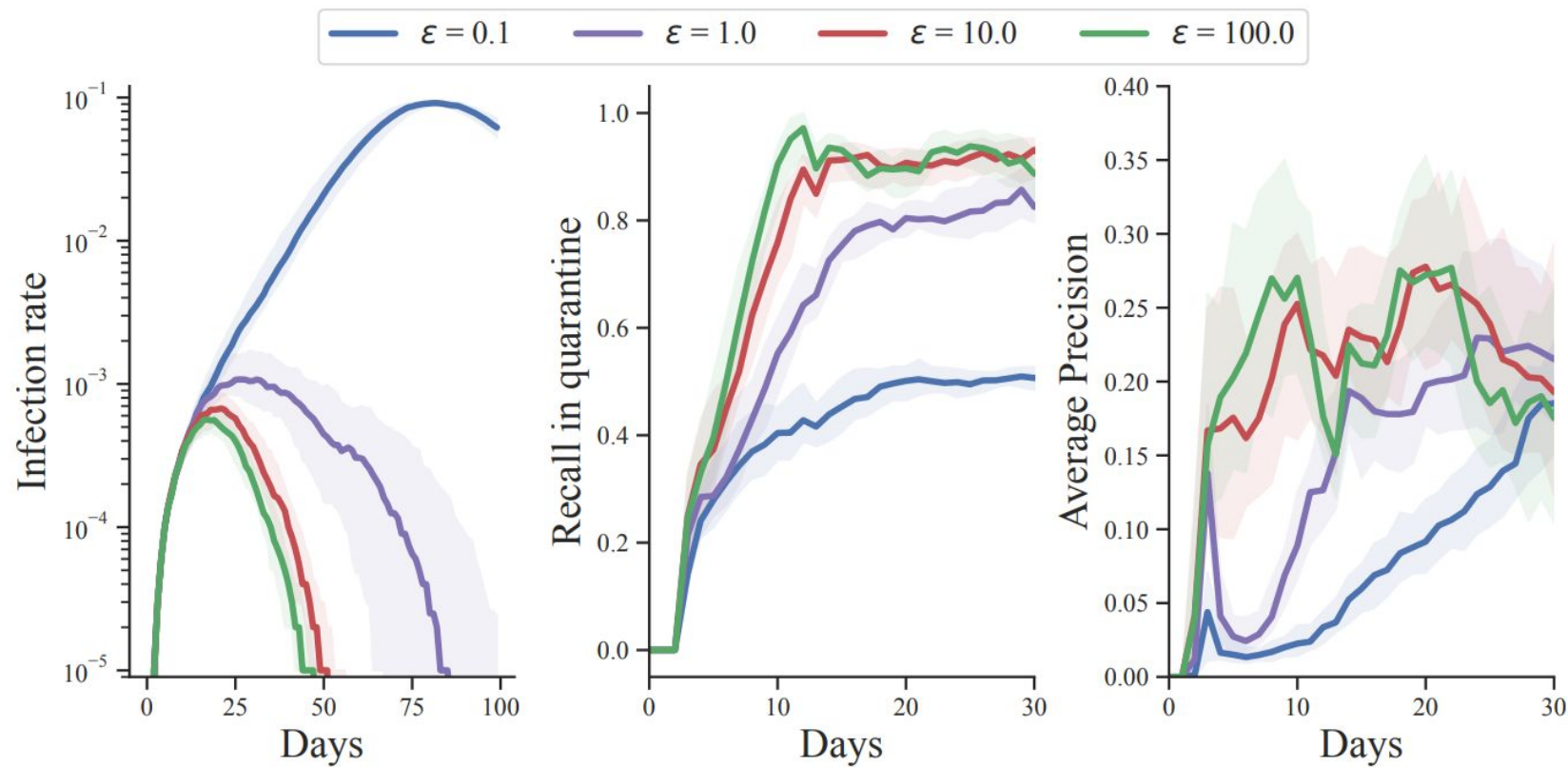
- Contact patterns in layers like households, schools, workplaces, and social communities.
- Calibrated against a typical city in the USA

## Results on two widely used simulators

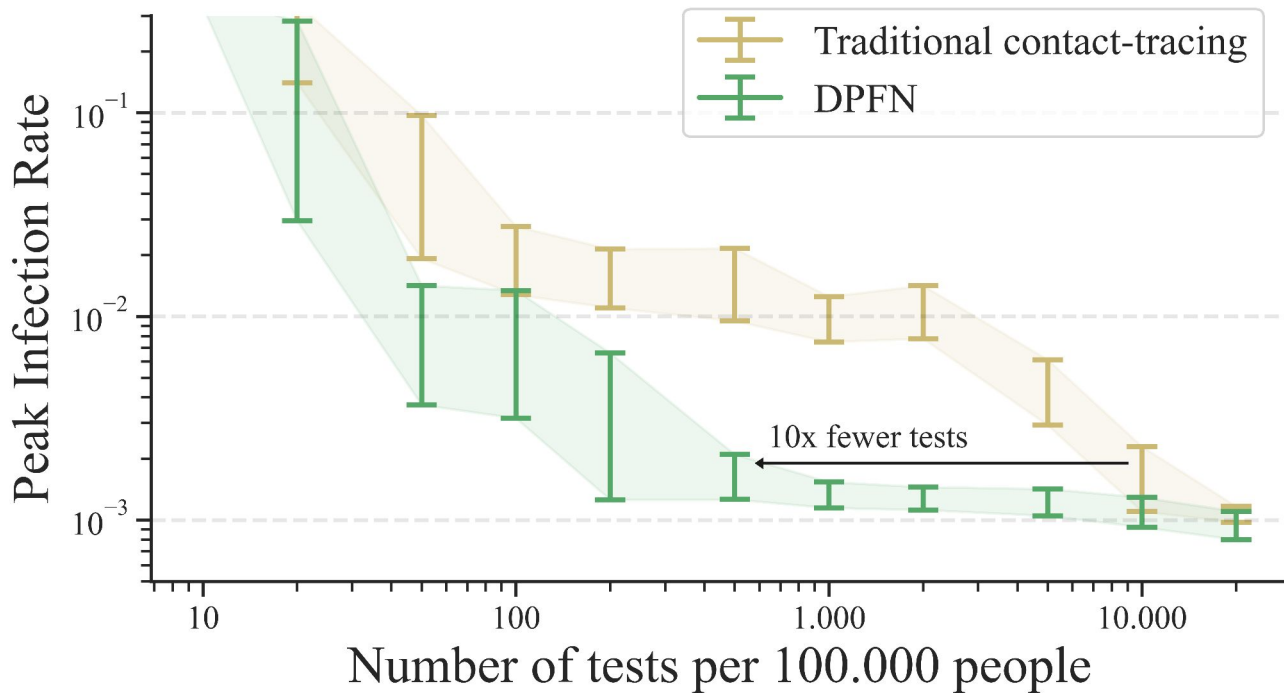


## Results on two widely used simulators





## Our method has low PIR with fewer tests



## More testing

Noise effects of DP can be counteracted with additional testing budget

Test setup	No privacy	DPFN	DPFN+
(fpr 0.0%; fnr 0.0%)	0.5 [0.5,0.6]	1.1 [0.7,1.4]	0.6 [0.5,0.7]
(fpr 1%; fnr 0.1%)	0.5 [0.4,0.6]	1.1 [0.9,1.7]	0.6 [0.5,0.6]
(fpr 10%; fnr 1%)	0.6 [0.5,0.8]	17.6 [11.4,20.4]	0.9 [0.7,1.0]
(fpr 25%; fnr 3%)	0.6 [0.5,0.8]	46.6 [40.4,48.0]	0.7 [0.6,0.8]
No testing	200 [190,212]		

## Conclusion

- Attack model against contact tracing
- Novel decentralised, differentially private algorithm
- Pareto optimal; save 10x and 2.5x testing budget at low PIR
- Future work:
  - Partial adoption, decentralized reinforcement learning, prosocial participation





# Protect Your Score: Contact Tracing with Differential Privacy Guarantees

[github.com/robromijnders/dpfn\\_aaai](https://github.com/robromijnders/dpfn_aaai)

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