APMA2822B Homework 1

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1 APMA2822B Homework 1 - Hammad Izhar + Robert Scheidegger

In this report we plan to analyze the performance of matrix-vector multiplication using a variety of memory allocation patterns and multiplication methods. These experiments were conducted using Brown's compute grid OSCAR using 32 cores on an **Intel Xeon Platinum 8268 CPU** and 16GB of DDR4 RAM.

A total of 2016 configurations were run varying memory allocators, multiplication methods, size of matrix, and number of threads. A summary of configuration variables is given below:

Memory Allocator	Description
`Disjoint`	Allocates `matrix`, `vector`, and `output` in separate contiguous blocks
`DisjointRow`	Allocates each row of `matrix`, `vector`, and `output` in their own contiguous blocks
`Contiguous`	Allocates all of `matrix`, `vector`, and `output` in the same contiguous memory block
`Mmap`	Allocates a new address space for `matrix`, `vector`, and `output` which is a contiguous memory block

Multipliers	Description
RowColumnMultiplier	Iterate over the rows of `matrix` to compute the output
ColumnRowMultiplier	Iterate over the columns of `matrix` to compute the output

Matrices of size n-by-m were multiplied where $n \in \{10^i \mid 0 \leq i \leq 5\}$ and $m \in \{10^i \mid 0 \leq i \leq 4\}$.

The number of threads `n_threads` varied in the set $\{1, 2, 4, 8, 16, 32, 64\}$.

[ ]:
```
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np
```

[ ]:
```
df = pd.read_csv('../data/oscar_data.csv', encoding='latin-1')
df['flops'] = 2 * df['n'] * df['n'] / df['time_us'] * 1e6
df['gflops'] = df['flops'] / 1e9
df['iops'] = 5 * df['n'] * df['m'] / df['time_us'] * 1e6
```

1.1 Roofline Analysis

For the purposes of this analysis, we will look at the results from 100000-by-10000 matrices allocated using the `DisjointMemoryAllocator` and multiplied using the `RowColumnMultiplier`. These are the largest matrices we allocated for these experiments.

Excluding timing and parallelization primitives, the code of `RowColumnMultiplier` is as follows:

```
// include/multipliers.hpp

for (uint32_t i = 0; i < n; i++) {
    for (uint32_t j = 0; j < m; j++) {
        output[i] += matrix[i][j] * vector[j];
    }
}
```

To compute the arithmetic intensity, we first count the number of I/O operations (memory accesses) required:

1. Load `matrix[i]` a pointer to the column, 8 bytes
2. Load `matrix[i][j]` a float, 4 bytes
3. Load `vector[j]` a float, 4 bytes
4. Save `matrix[i][j] * vector[j]` into a temporary variable a float, 4 bytes
5. Load `output[i]` a float, 4 bytes
6. Save `output[i]` a float, 4 bytes

This is a total of 6 I/O operations totalling 28 bytes transferred. We then count the number of floating point operations:

1. Multiply `RESULT = matrix[i][j] * vector[j]`
2. Add `RESULT + output[i]`

Therefore, the total arithmetic intensity is given by:

$$\text{Arithmetic Intensity} = \frac{2\ \text{FLOPS}}{28\ \text{Accesses}} = \frac{1}{14}\ \frac{\text{FLOPS}}{\text{byte}}$$

From the Ark Spec for the Xeon Platinum 8268 the max turbo frequency is 3.90 GHz. According to this community post Intel Skylake-X processors (the 8268 implements the Cascade Lake architecture) can perform 24 floating point operations per clock cycle per core. Therefore using 32 cores, we can achieve a maximum FLOP rate of

$$\text{Maximum FLOP Rate} = 32\ \text{cores} \cdot \frac{2.90 \cdot 10^9\ \text{cycle}}{1\ \text{second}} \cdot \frac{24\ \text{FLOPS}}{1\ \text{cycle-core}} = 2.227\ \text{TFLOPS}$$

The Xenon Platinum 8268 supports DDR4 RAM with maximum speeds of 2933 MHz and 6 memory channels. Therefore, the maximum memory bandwidth of the processor is given by:

$$\text{Maximum Memory Bandwidth} = 8\ \text{bytes} \cdot \frac{2.933\ \text{GHz}}{1\ \text{channel}} \cdot 6\ \text{channel} = 140.78\ \text{GHz} = 140.78\ \text{GB/s}$$

The ridge point of our roofline plot is therefore given by:

$$I^* = \frac{\text{Maximum FLOP Rate}}{\text{Maximum Memory Bandwidth}} = \frac{2.227 \text{ TFLOPS}}{140.78 \text{ GB/s}} = 15.81 \ \frac{\text{FLOPs}}{\text{byte}}$$

The predicted FLOP rate is given by:

$$\text{Predicted FLOP Rate} = \frac{1 \text{ FLOPS}}{14 \text{ byte}} \cdot \frac{140.78 \cdot 10^9 \text{ bytes}}{1 \text{ second}} = 100.05 \text{ GFLOPs}$$

Therefore, we are in bandwidth-limited region of the roof-line plot. This is within the margin of error of the experimental results we determined.

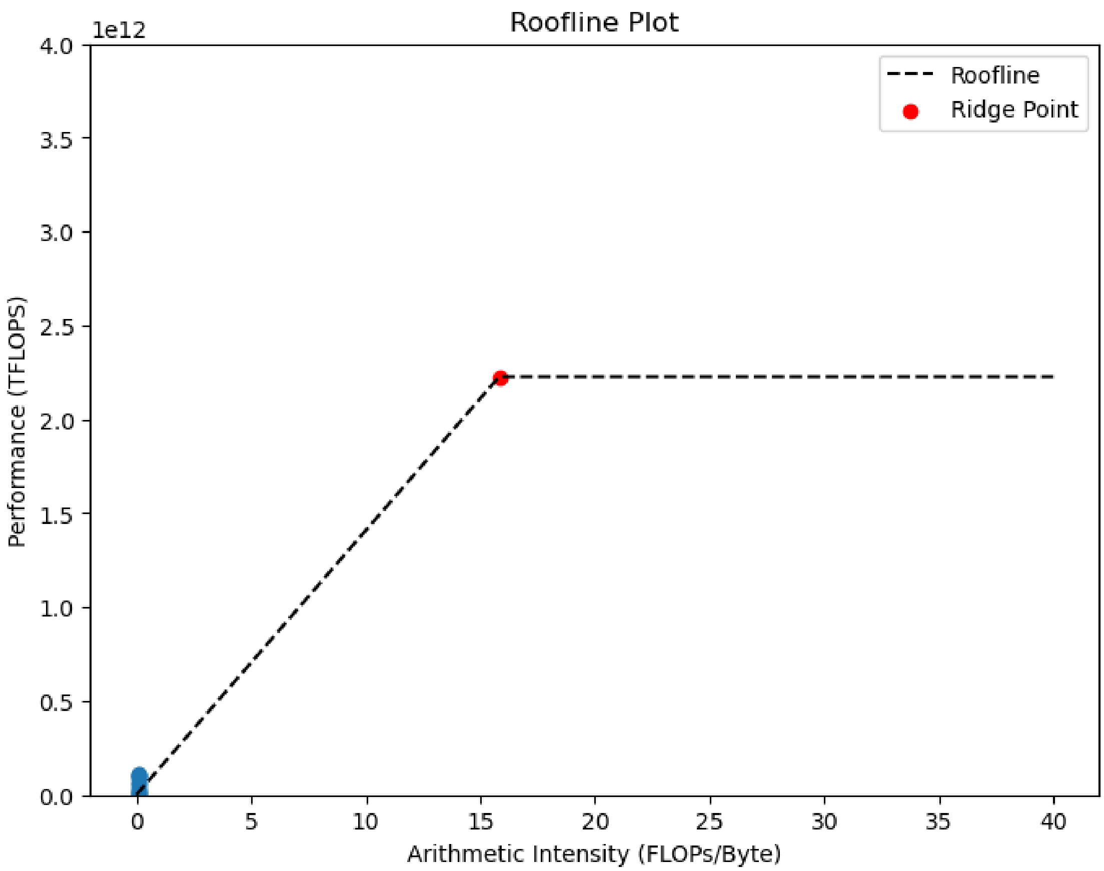
[ ]:
```python
# Compute the roof-line plot for the analysis above.
subset = df[df['allocator'] == 'DisjointMemoryAllocator']
subset = subset[subset['multiplier'] == 'RowColumnMultiplier']
subset = subset[subset['m'] == 10000]
subset = subset[subset['n'] == 100000]

# Plot the roof-line plot
max_flop_rate = 2.227e12 # 7.987 TFLOPS
max_mem_bandwidth = 140.78e9 # 140.78 GB/s
ridge_point = max_flop_rate / max_mem_bandwidth

arithmetic_intensity = np.arange(0, 40, 0.01);
roofline = np.minimum(arithmetic_intensity * max_mem_bandwidth, max_flop_rate)

fig, ax = plt.subplots(figsize=(8, 6))
ax.plot(arithmetic_intensity, roofline, 'k--', label='Roofline')
ax.scatter(ridge_point, ridge_point * max_mem_bandwidth, marker='o', color='r',␣
↪label='Ridge Point')

ax.scatter(1/14 * np.ones(len(subset)), subset["flops"])

ax.set_ylim(0, 4e12)
ax.set_xlabel('Arithmetic Intensity (FLOPs/Byte)')
ax.set_ylabel('Performance (TFLOPS)')
ax.set_title('Roofline Plot')
ax.legend()

subset
```

[ ]:
```
           n      m  threads                allocator           multiplier  \
1904  100000  10000        1  DisjointMemoryAllocator  RowColumnMultiplier
1912  100000  10000        2  DisjointMemoryAllocator  RowColumnMultiplier
1920  100000  10000        4  DisjointMemoryAllocator  RowColumnMultiplier
1928  100000  10000        8  DisjointMemoryAllocator  RowColumnMultiplier
1936  100000  10000       16  DisjointMemoryAllocator  RowColumnMultiplier
1944  100000  10000       32  DisjointMemoryAllocator  RowColumnMultiplier
1952  100000  10000       64  DisjointMemoryAllocator  RowColumnMultiplier
```

1960	100000	10000	1	DisjointMemoryAllocator	RowColumnMultiplier
1968	100000	10000	2	DisjointMemoryAllocator	RowColumnMultiplier
1976	100000	10000	4	DisjointMemoryAllocator	RowColumnMultiplier
1984	100000	10000	8	DisjointMemoryAllocator	RowColumnMultiplier
1992	100000	10000	16	DisjointMemoryAllocator	RowColumnMultiplier
2000	100000	10000	32	DisjointMemoryAllocator	RowColumnMultiplier
2008	100000	10000	64	DisjointMemoryAllocator	RowColumnMultiplier

	iterations	time_us	stdev_us	flops	gflops	\
1904	10	2.730146e+06	0.000000	7.325617e+09	7.325617	
1912	10	1.369082e+06	2231.756348	1.460832e+10	14.608325	
1920	10	6.844341e+05	0.000000	2.922122e+10	29.221220	
1928	10	3.424635e+05	90.509666	5.840038e+10	58.400384	
1936	10	2.091755e+05	45619.386719	9.561349e+10	95.613492	
1944	10	1.816549e+05	11106.904297	1.100989e+11	110.098871	
1952	10	2.033748e+05	5928.901367	9.834060e+10	98.340602	
1960	10	2.730905e+06	0.000000	7.323580e+09	7.323580	
1968	10	1.368980e+06	627.069397	1.460941e+10	14.609412	
1976	10	6.843876e+05	0.000000	2.922321e+10	29.223205	
1984	10	3.422718e+05	156.767349	5.843309e+10	58.433091	
1992	10	2.647591e+05	1641.696655	7.554037e+10	75.540370	
2000	10	1.757544e+05	7530.001953	1.137952e+11	113.795156	
2008	10	1.996775e+05	8169.469238	1.001615e+11	100.161510	

	iops
1904	1.831404e+09
1912	3.652081e+09
1920	7.305305e+09
1928	1.460010e+10
1936	2.390337e+10
1944	2.752472e+10
1952	2.458515e+10
1960	1.830895e+09
1968	3.652353e+09
1976	7.305801e+09
1984	1.460827e+10
1992	1.888509e+10
2000	2.844879e+10
2008	2.504038e+10

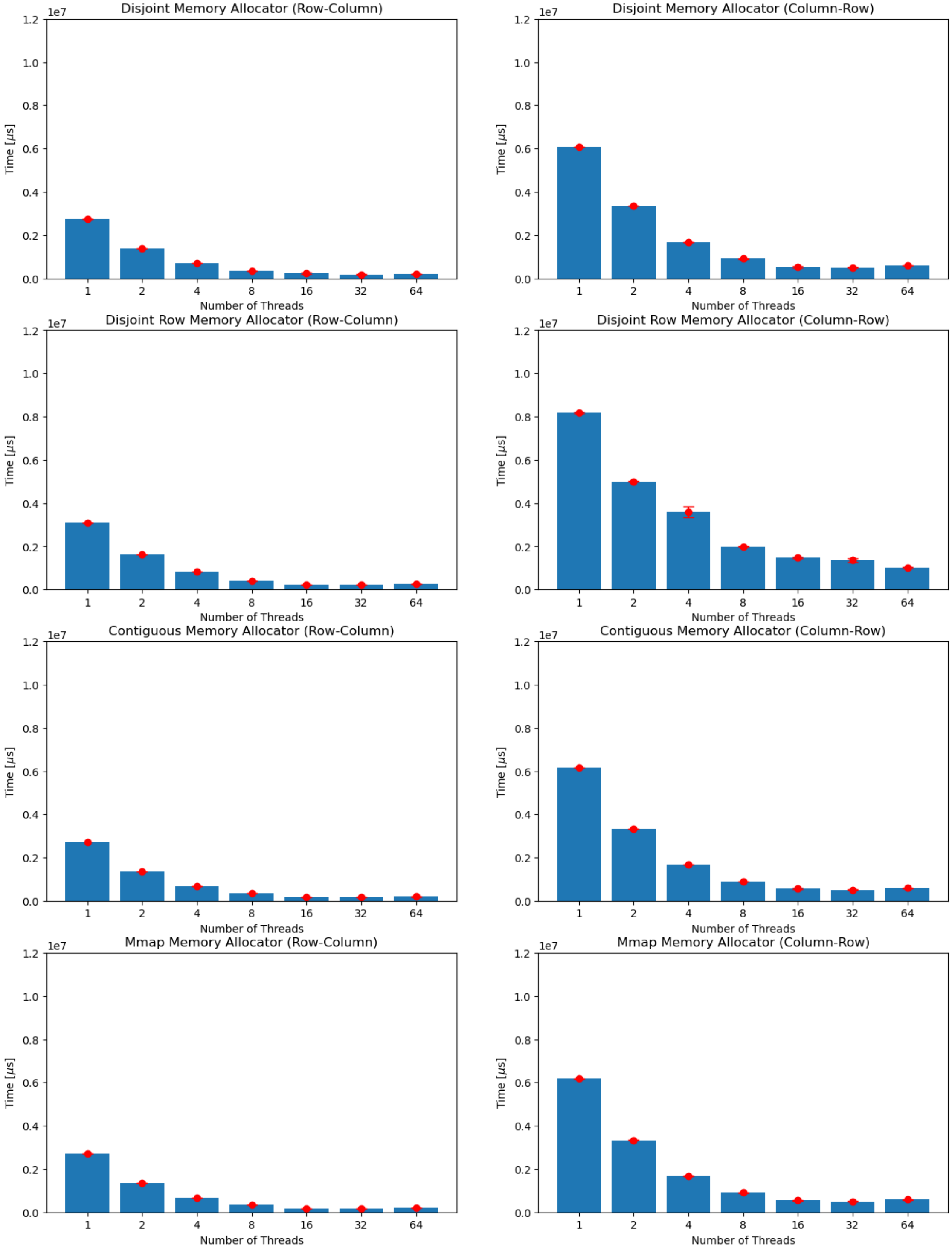


1.2 Performance Analysis

To test each of the 2016 possible configurations we performed an experiment on matrices of varying sizes using a batch script on Oscar. Warmup computations were used prior to the start of the experiments. Each experiment was repeated for 10 iterations and the the mean and standard deviations of each runtime was computed. A sample of the data for the largest set of matrices is shown below.

[ ]:
```
# allocators bar charts for large size over threads and multipliers
def get_allocator_data(name: str):
    subset = df[df['allocator'] == name]
    subset = subset[subset['m'] == 10000]
    subset = subset[subset['n'] == 100000]
    return subset[subset['multiplier'] == 'RowColumnMultiplier'].
  ↪groupby("threads").mean(numeric_only=True), subset[subset['multiplier'] ==␣
  ↪'ColumnRowMultiplier'].groupby("threads").mean(numeric_only=True)


disjoint_row_column, disjoint_column_row =␣
  ↪get_allocator_data('DisjointMemoryAllocator')
```

```python
disjoint_row_row_column, disjoint_row_column_row = get_allocator_data('DisjointRowMemoryAllocator')
contiguous_row_column, contiguous_column_row = get_allocator_data('ContiguousMemoryAllocator')
mmap_row_column, mmap_column_row = get_allocator_data('MmapMemoryAllocator')

bars = [disjoint_row_column, disjoint_column_row, disjoint_row_row_column, disjoint_row_column_row, contiguous_row_column, contiguous_column_row, mmap_row_column, mmap_column_row]
labels = ["Disjoint Memory Allocator (Row-Column)", "Disjoint Memory Allocator (Column-Row)", "Disjoint Row Memory Allocator (Row-Column)", "Disjoint Row Memory Allocator (Column-Row)", "Contiguous Memory Allocator (Row-Column)", "Contiguous Memory Allocator (Column-Row)", "Mmap Memory Allocator (Row-Column)", "Mmap Memory Allocator (Column-Row)"]

fig, axs = plt.subplots(4, 2, figsize=(15, 20))

for i in range(4):
    for j in range(2):
        trace = bars[j + i * 2]["time_us"]
        axs[i, j].bar([str(x) for x in trace.keys()], trace.values)
        axs[i, j].errorbar([str(x) for x in trace.keys()], trace.values, yerr=bars[j + i * 2]["stdev_us"], fmt='o', capsize=5, color="red")
        axs[i, j].set_ylim(0, 12e6)
        axs[i, j].set_title(labels[j + i * 2])
        axs[i, j].set_xlabel("Number of Threads")
        axs[i, j].set_ylabel("Time [$\mu$s]")
```


Disjoint Memory Allocator (Row-Column)
Disjoint Memory Allocator (Column-Row)
Disjoint Row Memory Allocator (Row-Column)
Disjoint Row Memory Allocator (Column-Row)
Contiguous Memory Allocator (Row-Column)
Contiguous Memory Allocator (Column-Row)
Mmap Memory Allocator (Row-Column)
Mmap Memory Allocator (Column-Row)
1e7
Time [μs]
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Number of Threads
1
2
4
8
16
32
64