CSE 210A - HW 3 - Induction

Robert Sato

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Exercise 1

Given:

A: t := x; x := y; y := t
B: t := y; y := x; x := t

Proof by counter example. To show that this assertion is false, find an example where this assertion does not hold. In other words, show an example where A and B are not the same. Choose

x = 1
y = 2
t = 3

Then,

A: t := x = 1; x := y = 2; y := t = 1
A: t = 1; x = 2; y = 1
B: t := y = 2; y := x = 1; x := t = 2
B: t = 2; x = 1; y = 2
A != B

Exercise 2

If:

$$\langle while\ b\ do\ y\ := y - x, s\rangle \Downarrow s'$$

then there exists k such that:

$$s(y) = s'(y) + k \cdot s(x)$$

Case: while-false

$$\frac{\langle b, s\rangle \Downarrow False}{\langle w, s\rangle \Downarrow s}$$

s' = s

k = 0 so s(y) = s'(y) + 0

Therefore, inequality holds for the while-false case

Case: while-true

$$\frac{\langle b, s\rangle \Downarrow True \qquad \langle y\ := s(y) - s(x), s\rangle \Downarrow s''}{\langle w, s\rangle \Downarrow s'}$$

This above tree is the initial execution of the while loop. It then calls this tree where the input state is now s”

$$\frac{\langle b, s''\rangle \Downarrow True \qquad \langle y\ := s''(y) - s(x), s''\rangle \Downarrow s'}{\langle w, s''\rangle \Downarrow s'}$$

By our inductive hypothesis, we know that $\langle w, s''\rangle \Downarrow s'$ i.e. s”(y) = s'(y) + k*s”(x)
Need to show: s(y) = s'(y) + k*s(x)

Have: s” = s[y |-> s(y) - s(x)]

Hence: s(y) = s(y) + k*s(x) for some k

s(y) + k * s(x) = s”(y) for k = 1

s(y) = s”(x)

s(y) = s”(x) + k * s(x)

We have arrived back at our inductive hypothesis. We have shown that this is true for both the while-true and while-false cases. By showing it is true for all cases, we have proved that this is true.