TheoryExercice

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1 Ray-cylinder intersection

1.1 Notations

- $\mathbf{o}$: ray origin
- $\mathbf{d}$: ray direction
- $\mathbf{c}$: cylinder center
- r : cylinder radius
- $\mathbf{a}$: cylinder axis
- h : cylinder height

1.2 Intersection with ∞ length tube

We will first consider the intersection of the ray with a tube considering $h \to \infty$. We identified that the equation of such cylinder might be written as the set of points $\mathbf{x}$ that validate the equation

$$((\mathbf{x}-\mathbf{c})-\mathbf{a}(\mathbf{a}^T(\mathbf{x}-\mathbf{c})))^2 = r^2$$

The left part of this equation is represented as the $\mathbf{r}$ vector on Figure 1. This vector $\mathbf{r}$ is built as the projection of $\mathbf{x}-\mathbf{c}$ (the vector from the cylinder center to a point $\mathbf{x}$) on the vector that is orthogonal to $\mathbf{a}$. Any point $\mathbf{x}$ indeed belongs to the cylinder if and only if $\mathbf{r}$ has length r.

The intersection between the ray and the cylinder happens then at distance t, solutions of

$$((\mathbf{d}t+\mathbf{o}-\mathbf{c})-\mathbf{a}(\mathbf{a}^T(\mathbf{d}t+\mathbf{o}-\mathbf{c})))^2 = r^2$$

After developing this equations, we obtain the following degree 2 polynomial for the variable t:

$$t^2(\mathbf{d}-\mathbf{a}(\mathbf{a}^T\mathbf{d}))^2+2t(\mathbf{d}-\mathbf{a}(\mathbf{a}^T\mathbf{d}))(\mathbf{o}-\mathbf{c}-\mathbf{a}(\mathbf{a}^T(\mathbf{o}-\mathbf{c})))+(\mathbf{o}-\mathbf{c}-\mathbf{a}(\mathbf{a}^T(\mathbf{o}-\mathbf{c})))^2-r^2$$

We then obtain:

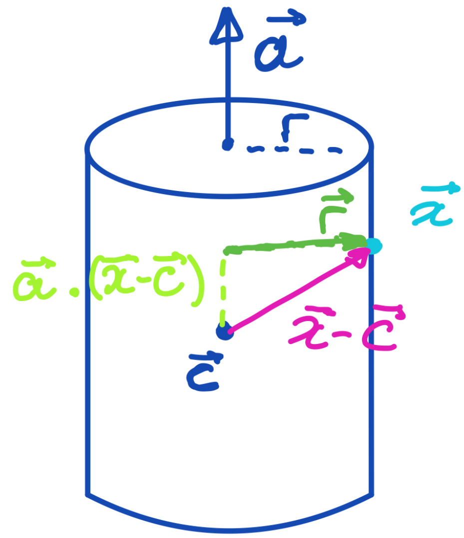

Figure 1: The cylinder representation

- 1 solution if the ray is a tangent of the cylinder (of ∞ length)
- 2 solutions if the ray crosses the cylinder (of ∞ length)
- 0 solutions, in which case we know the ray won't intersect the cylinder (even with restricted length)

1.3 Selection of correct candidates

Now that we computed the potential intersections, we want to only keep those that are between both caps. We will compute this by checking the length of the smallest vector that links the intersection point to the cap's plane. We will use the points $\mathbf{c}_1$ and $\mathbf{c}_2$, the centers of each cap.

$$||\mathbf{c}_1 - \mathbf{x} + \mathbf{r}||^2 \leq h^2 \tag{1}$$

$$||\mathbf{c}_2 - \mathbf{x} + \mathbf{r}||^2 \leq h^2 \tag{2}$$

We define $\mathbf{x}_1 = \mathbf{d}t_1 + \mathbf{o}$ and $\mathbf{x}_2 = \mathbf{d}t_2 + \mathbf{o}$ with $t_1 < t_2$ (supposing that if there is only one solution, no $\mathbf{x}_2$ exists). To take the closest intersection, we take $\mathbf{x}_1$ if it validates 1 and 2, else $\mathbf{x}_2$ if it validates 1 and 2. If none validates the equations, there is no point that intersects our cylinder. The normal to this intersection is the vector $\mathbf{r}$ that we defined earlier. To be towards ray's origin, we want it to have a negative scalar product with $\mathbf{d}$.