Least Squares approximation for humidity sensor

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1 Introduction

In the table below, *hum* stands for the humidity measured by a gehaka measuring device that is calibrated correctly. This is the reference humidity that we are trying to calibrate at. The variable *cycles* is the amount of cycles measured by our device. These are purely fictive numbers, solely for the purpose of providing an example.

Sample number	Cycles	Humidity
0	3497	11.1
1	3994	11.9
2	4511	13.0
3	4913	14.1
...	...	...
n	2900	10.1

We will attempt to make a function of the following form, making the *error* term as small as possible

$$f(cycles) = hum + error \tag{1}$$

2 Linear regression

For linear regression, eq. 1 looks like this:

$$a * cycles + b = hum + error \tag{2}$$

The function that yields the distance between the real and estimated humidities is denoted as follows

$$\begin{aligned} f(a,b) = [hum_0 - (a * cycles_0 + b)]^2 \\ +[hum_1 - (a * cycles_1 + b)]^2 \\ +... + [hum_n - (a * cycles_n + b)]^2 \end{aligned} \tag{3}$$

$$\begin{aligned} f(a,b) = hum_0^2 - 2 * hum_0 * (a * cycles_0 + b) + a^2 * cycles_0^2 + 2 * a * b * cycles_0 + b^2 \\ +hum_1^2 - 2 * hum_1 * (a * cycles_1 + b) + a^2 * cycles_1^2 + 2 * a * b * cycles_1 + b^2 \\ +... + hum_n^2 - 2 * hum_0 * (a * cycles_n + b) + a^2 * cycles_n^2 + 2 * a * b * cycles_n + b^2 \end{aligned} \tag{4}$$

To minimize this distance function, we need to set all first partial derivatives to zero:

$$\frac{\partial}{\partial a} f(a,b) = 0 \tag{5a}$$

$$\frac{\partial}{\partial b} f(a,b) = 0 \tag{5b}$$

$$\begin{aligned}\frac{\partial}{\partial a} f(a,b) = -2 * hum_0 * cycles_0 + 2 * a * cycles_0^2 + 2 * b * cycles_0 \\ -2 * hum_1 * cycles_1 + 2 * a * cycles_1^2 + 2 * b * cycles_1 \\ + \cdots - 2 * hum_n * cycles_n + 2 * a * cycles_n^2 + 2 * b * cycles_n \\ = 0\end{aligned} \tag{6a}$$

$$\begin{aligned}\frac{\partial}{\partial b} f(a,b) = -2 * hum_0 + 2 * a * cylos_0 + 2 * b \\ -2 * hum_1 + 2 * a * cylos_1 + 2 * b \\ + \cdots - 2 * hum_n + 2 * a * cylos_n + 2 * b \\ = 0\end{aligned} \tag{6b}$$

Or equivalently,

$$\frac{\partial}{\partial a} f(a,b) = -2 * \sum_{i=0}^{n} hum_i * cycles_i + 2 * a * \sum_{i=0}^{n} cycles_i^2 + 2 * b * \sum_{i=0}^{n} cycles_i = 0 \tag{7a}$$

$$\frac{\partial}{\partial a} f(a,b) = -2 * \sum_{i=0}^{n} hum_i + 2 * a * \sum_{i=0}^{n} cycles_i + 2 * n * b = 0 \tag{7b}$$

Or equivalently, from 7b,

$$b = \frac{\sum_{i=0}^{n} hum_i}{n} - a * \frac{\sum_{i=0}^{n} cycles_i}{n} \tag{8a}$$

Note that this is also the average *hum* - a times the average *cycles*, which is what we'd expect to see, as we are trying to get to a function of the same form as eq. 2.

Then, from 7a, we can find a:

$$-\sum_{i=0}^{n} hum_i * cycles_i + a * \sum_{i=0}^{n} cycles_i^2 + b * \sum_{i=0}^{n} cycles_i = 0 \tag{9}$$

Substituting b for eq. 8a, we find

$$\begin{aligned}-\sum_{i=0}^{n} hum_i * cycles_i + a * \sum_{i=0}^{n} cycles_i^2 \\ + \left(\frac{\sum_{i=0}^{n} hum_i}{n} - a * \frac{\sum_{i=0}^{n} cycles_i}{n} \right) * \sum_{i=0}^{n} cycles_i = 0\end{aligned} \tag{10}$$

Or

$$
\begin{aligned}
-\sum_{i=0}^{n} hum_i * cycles_i + \frac{\sum_{i=0}^{n} hum_i}{n} * \sum_{i=0}^{n} cycles_i \\
= -a * \sum_{i=0}^{n} cycles_i^2 + a * \frac{(\sum_{i=0}^{n} cycles_i)^2}{n}
\end{aligned} \tag{11}
$$

Which finally yields the equation for a:

$$
a = \frac{-\sum_{i=0}^{n} hum_i * cycles_i + \frac{\sum_{i=0}^{n} hum_i}{n} * \sum_{i=0}^{n} cycles_i}{-\sum_{i=0}^{n} cycles_i^2 + \frac{(\sum_{i=0}^{n} cycles_i)^2}{n}} \tag{12}
$$

3 Quadratic regression

4 Exponential regression

$$
a * e^{b*cycles} + c \tag{13}
$$

$$
f(a,b,c) = \sum_{i=0}^{n} \left(hum_i - (a * e^{b*cycles_i} + c)\right)^2 \tag{14}
$$

$$
f(a,b,c) = \sum_{i=0}^{n} \left(hum_i^2 - 2 * hum_i * (a * e^{b*cycles_i} + c) + a^2 * e^{2*b*cycles} + 2 * a * c * e^{b*cycles} + c^2\right) \tag{15}
$$

$$
\frac{\partial}{\partial a} f = 0 \tag{16a}
$$

$$
\frac{\partial}{\partial b} f = 0 \tag{16b}
$$

$$
\frac{\partial}{\partial c} f = 0 \tag{16c}
$$

$$
\frac{\partial}{\partial a} f = -2 * hum * e^{b*cycl} + 2 * a * e^{2*b*cycles} + 2 * c * e^{b*cycles} = 0 \tag{17a}
$$

$$
\frac{\partial}{\partial b} f = -2*a*hum*cycl*e^{b*cycl} + 2*a^2*cycles*e^{2*b*cycles} + 2*a*cycles*c*e^{b*cycles} = 0 \tag{17b}
$$

$$
\frac{\partial}{\partial c} f = -2 * hum + 2 * a * e^{b*cyxles} + 2 * c = 0 \tag{17c}
$$

5 Conclusion

For linear approximation of the form $a * cycles + b = hum + error$ The coefficients a and b can be determined as follows:

$$
a = \frac{-\sum_{i=0}^{n} hum_i * cycles_i + \frac{\sum_{i=0}^{n} hum_i}{n} * \sum_{i=0}^{n} cycles_i}{-\sum_{i=0}^{n} cycles_i^2 + \frac{(\sum_{i=0}^{n} cycles_i)^2}{n}}
$$

$$
b = \frac{\sum_{i=0}^{n} hum_i}{n} - a * \frac{\sum_{i=0}^{n} cycles_i}{n}
$$