Canonical Variate Analysis (CVA) for Closed-Loop Identification

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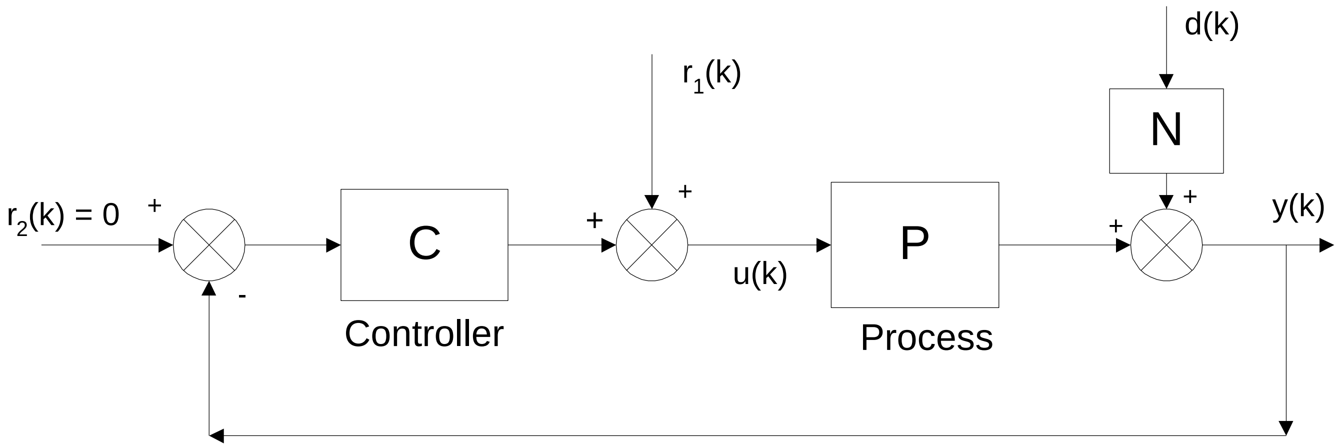
Introduction



Block diagram of the closed-loop configuration

$$y(k) = \frac{P}{1 + CP} r_1(k) + \frac{N}{1 + CP} d(k)$$

$$u(k) = \frac{1}{1 + CP} r_1(k) - \frac{NC}{1 + CP} d(k)$$

$$= \frac{1}{1 + CP} r_1(k) + N^* d(k)$$

Identification Two step method

- Estimate $S = \dfrac{1}{1 + CP}$ by performing $u(k) \longleftrightarrow r_1(k)$

1. $y(k) = \dfrac{P}{1 + CP} r_1(k) + \dfrac{N}{1 + CP} d(k)$

$$\longrightarrow y^f(k) = P\, r_1(k) + N_* \, d(k)$$

Identify between $y^f(k)$ and $r_1(k)$ to estimate P

2. $y(k) = P \left(\dfrac{1}{1 + CP}\right) r_1(k) + \dfrac{N}{1 + CP} d(k)$

$$= P\, r_1^f(k) + N_* \, d(k)$$

Identify between $y(k)$ and $r_1^f(k)$ to estimate P

Parallel Method

$$u(k) = G_1\, r_1(k) + N^* \, d(k)$$

$$y(k) = G_2\, r_1(k) + N_* \, d(k)$$

where $G_1 = \dfrac{1}{1 + CP}$ & $G_2 = \dfrac{P}{1 + CP}$

ID between $u(k)$ and $r_1(k)$ gives G_1

ID between $y(k)$ and $r_1(k)$ gives G_2

$$P = G_2\, G_1^{-1}$$

Issues

Two Step Method

- Need for filtering using the sensitivity function - scope for propagating errors into the second step
- Perfect filtering may not always be possible - for example, the sensitivity function may have unstable zeros

Parallel Method

- Existence of G_1^{-1}. Unstable zeros of G_1 will get transformed into unstable poles of P
- Can’t guarantee that the estimated transfer functions G_1 and G_2 will have the same denominator

The State Space Approach

- Identify a MIMO State Space Model using $r_1(k)$ as the input signal with y(k) and u(k) as the output signals

- This will guarantee that both $\dfrac{y(k)}{r_1(k)}$ and $\dfrac{u(k)}{r_1(k)}$ have the same denominator

- The plant transfer function can be obtained as before by first obtaining the two transfer functions from the MIMO state space model and then taking their ratio

The State Space Model

$$X_{t+1} = \Phi X_t + GU_t + W_t$$

State Noise

$$Y_t = H X_t + AU_t + \underbrace{BW_t + V_t}_{\text{Measurement Noise}}$$

p inputs and *q* outputs

Note : Measurement noise is correlated with state noise

Open Loop Identification with CVA

- *Fully automated and reliable* system identification procedure
- *Identifies correct (or close) model order* even for small sample sizes, low SNR or for any choice of probing signals
- CVA is insensitive to scaling. Other methods are not !
- *Simple logic and computations*

o CVA estimates are as *asymptotically efficient* as the Maximum Likelihood (ML) estimates

The CVA Method

The Key Steps

- *Determine optimal memory length*
- Compute the states using CCA
- *Pick up the optimal number of states* using AIC
- Generate the system matrices and estimates for the noise covariance matrices

Akaike Information Criterion (AIC)

- Desired Model

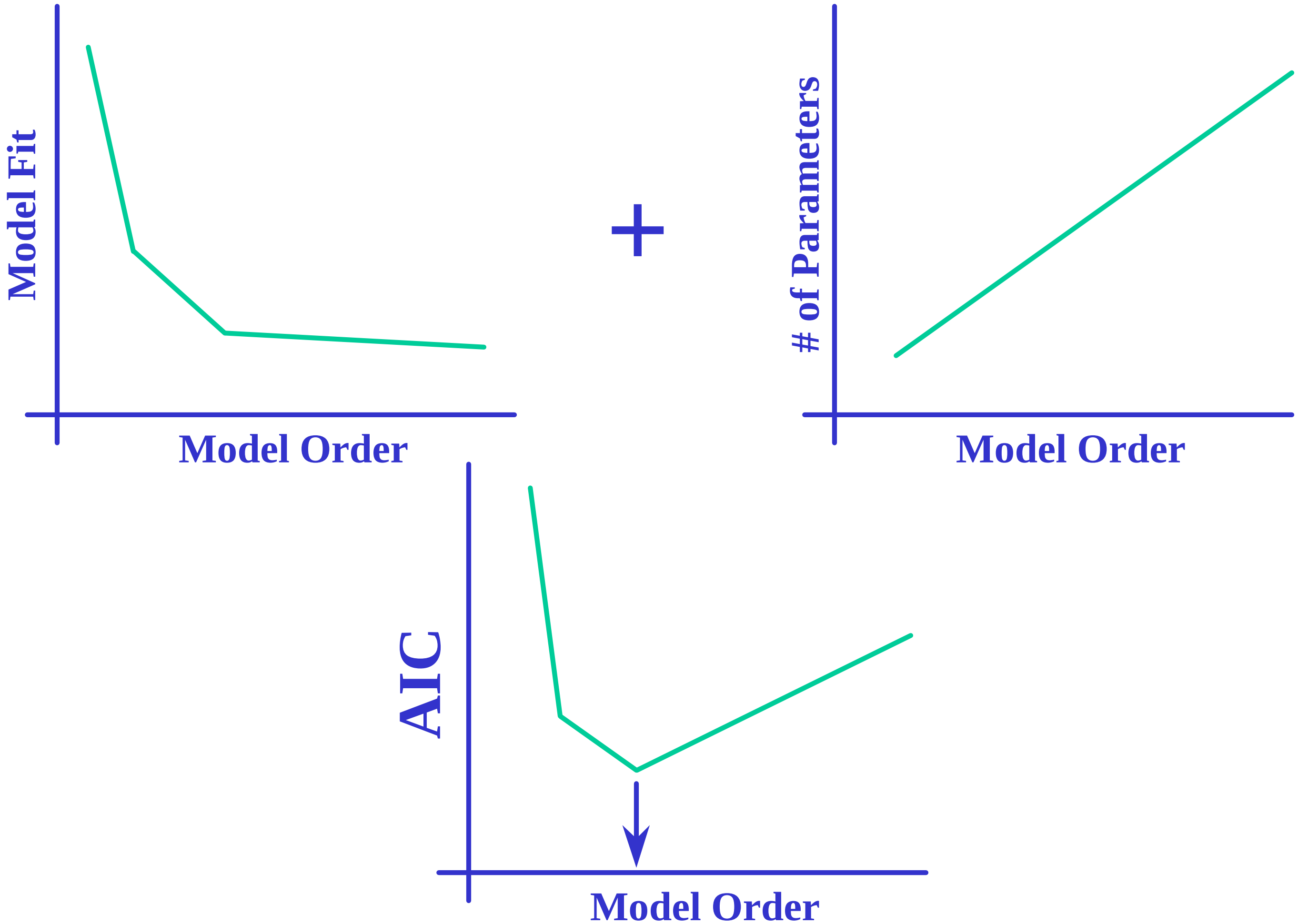
Minimum Information distance from the true system
Minimum Complexity

- Principle of Parsimony

Add more complexity (extra parameters) only when there is significant *payback*

The AIC balances model complexity and model fit

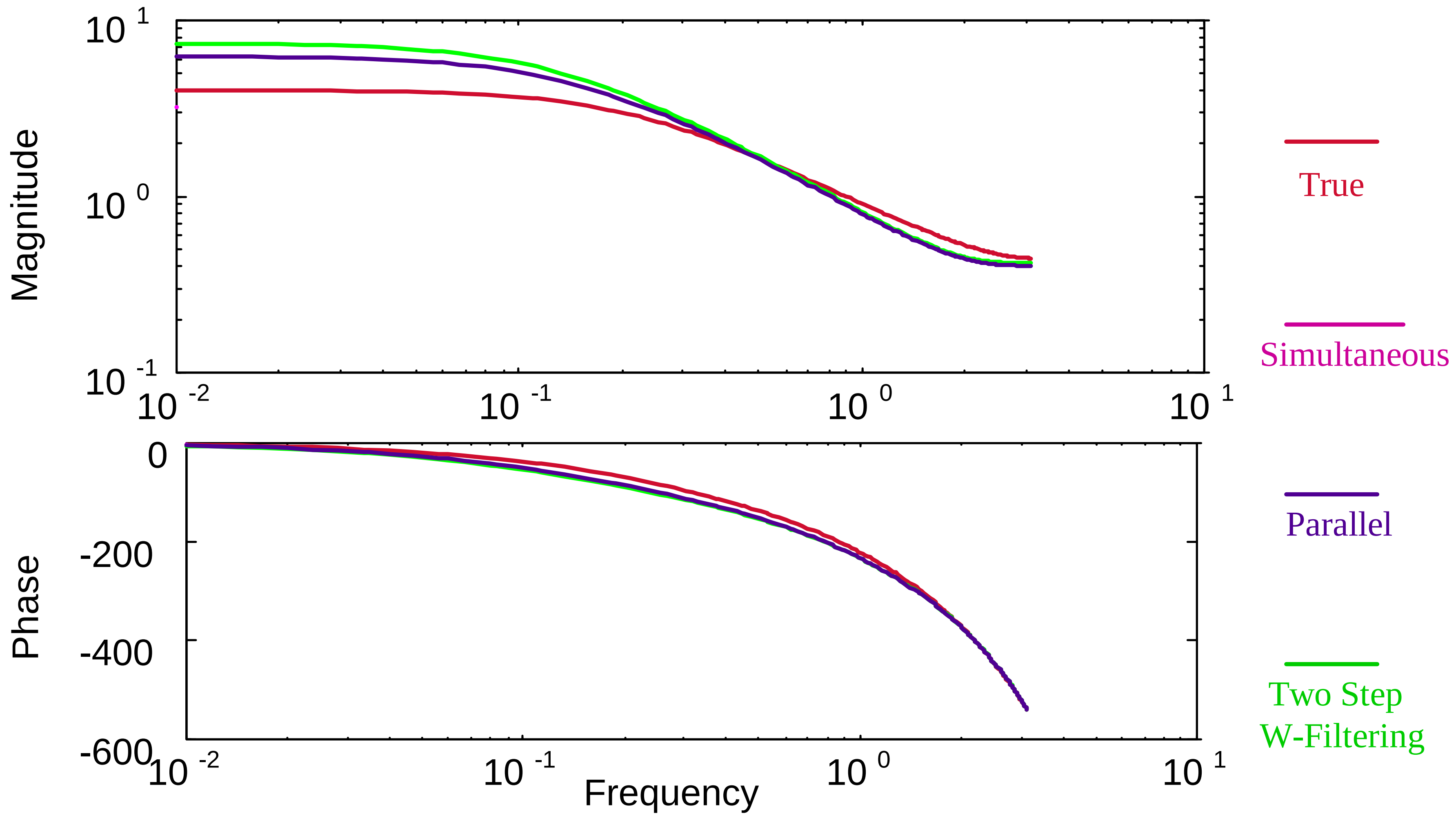
Akaike Information Criterion (AIC)



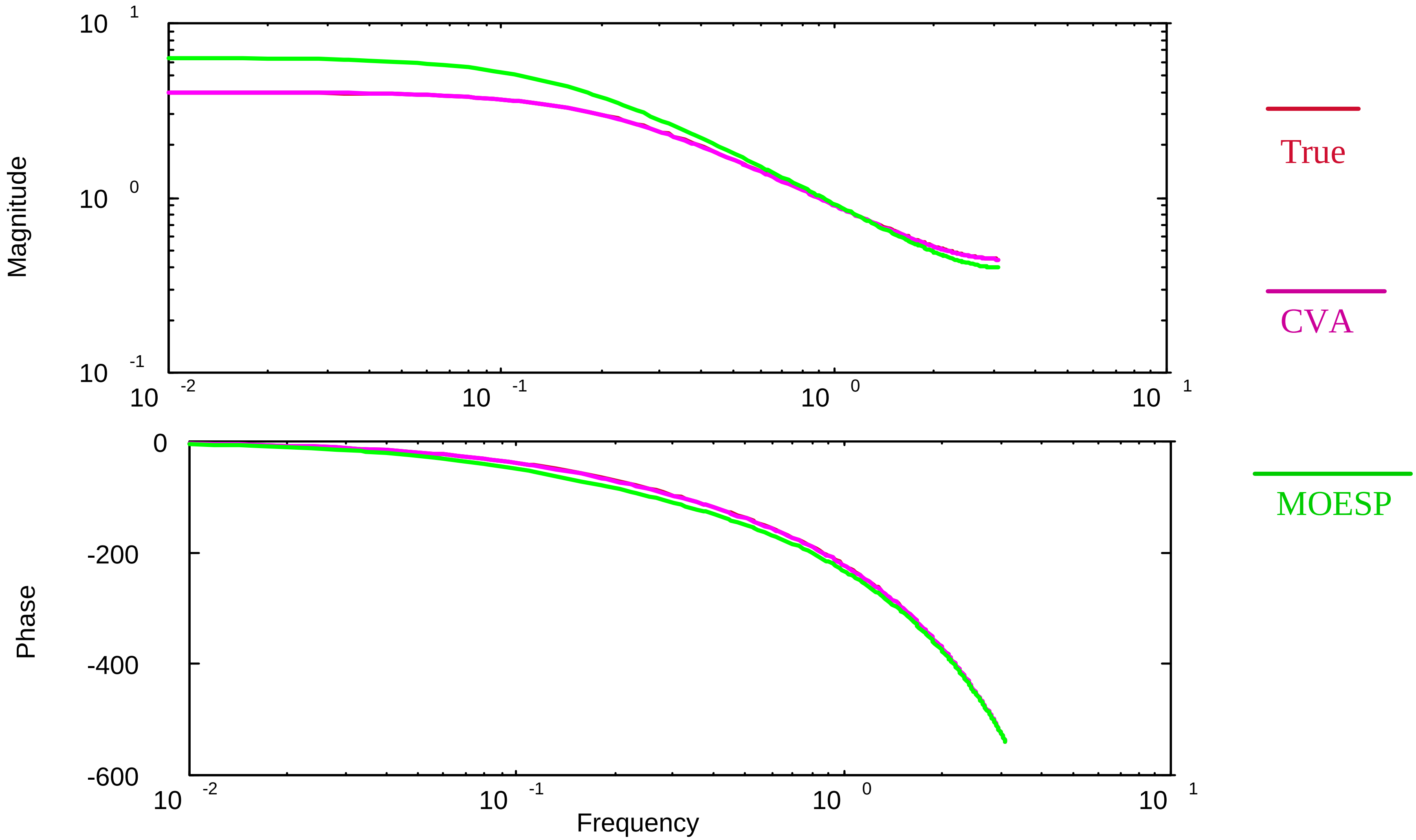
State Space Identification using MOESP

- Multivariable Output-Error State Space
- Constructs the *Extended Observability Matrix* based on a user specified maximum order
- Examines the *Singular Values* of the EOM to identify the optimal state order
- Computes model matrices using Ordinary Least Squares
- Has been used for both open and closed loop identification

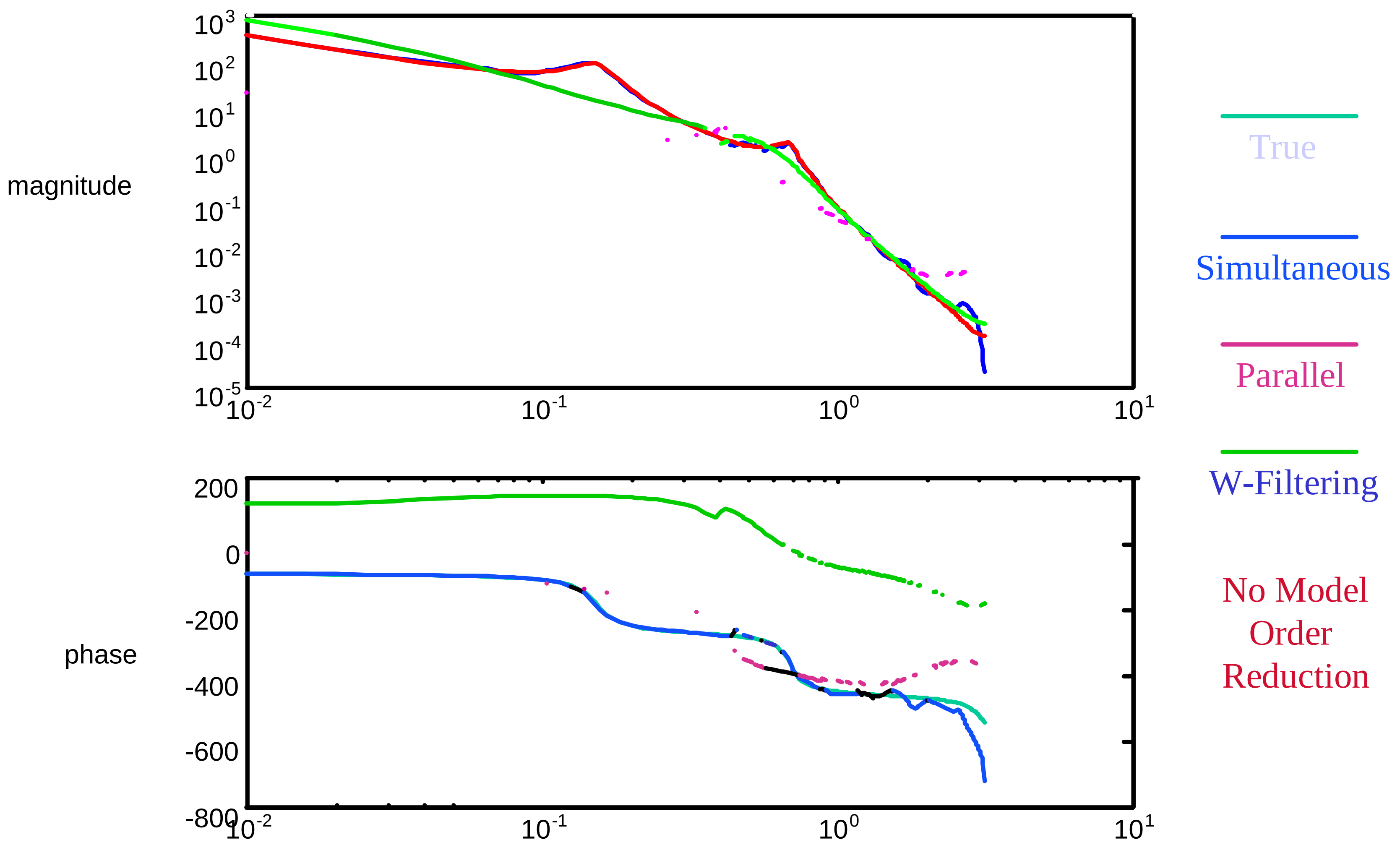
CVA based Identification Results



CVA vs. MOESP (Simultaneous Identification)



Results : Fifth Order Process & 4th Order Controller



Conclusions

- It is necessary to understand the mechanism at a fundamental level.
- CVA with simultaneous identification works well and holds a lot of promise.