Identification of Bilinear Models for Chemical Processes Using Canonical Variate Analysis§

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The identification of nonlinear models for chemical processes solely from experimental data is described in this paper. The canonical variate analysis (CVA) technique that has served well in the identification of empirical linear process models is extended to construct data-based bilinear models in an iterative fashion. Numerous examples involving engineering systems are included to illustrate the practicality of the suggested approach for bilinear model identification. Finally, the use of the identified nonlinear models for control is demonstrated using the example of a simulated paper machine headbox system.

1. Introduction

Most industrial systems are regulated using controllers designed on the basis of linear process models. Irrespective of the process size [single-input single-output (SISO) or multiple-input multiple-output (MIMO)], the control configuration (centralized or decentralized), or the controller type [PID or model predictive control (MPC)], it is common in industrial practice to obtain parsimonious, linear process models using data collected from designed plant tests. Such an approach is acceptable for linear or mildly nonlinear systems. However, suboptimal or unstable control performance can result if the process is expected to operate over a wide range of conditions. Poor operation of industrial units is not favored for economic and safety reasons. The inadequacy of linear models has long been realized with the possible remedies being (1) employment of adaptive control strategies (Seborg et al.[34]), (2) construction and use of multiple linear models (Banerjee et al.[1]), and (3) identification and use of nonlinear models (Zhu and Seborg[38]).

Without delving into the relative merits and drawbacks of the above approaches, we consider the third option. Although the use of first-principles-based nonlinear process models for control purposes is on the increase, synthesis of nonlinear controllers based on simple empirical nonlinear models remains the predominant route at least in the chemical industry. The structures and methods needed to arrive at these data-based nonlinear models are reviewed by Haber and Keviczky,[14] Billings[3], Haber and Unbehauen,[15] Cinar,[4] and Pearson and Ogunnaike.[31] For highly complex nonlinear processes, the use of *rich* nonlinear structures such as NARMAX or neural nets is preferred. Use of such models for identification and/or control can be found in Hernandez and Arkun[17] and Proll and Karim.[32] Neural networks seem to enjoy tremendous popularity for nonlinear modeling applications in the chemical industry. Pearson and Ogunnaike[31] investigated the modeling capability of several nonlinear structures and the kind of phenomena they can capture. For model structure selection, the work of Menold et al.[26] provides a systematic approach based on the concept of suitability measures. In their approach, the structure is determined prior to the actual parameter identification.

Among the rich variety of nonlinear model structures, two classes are important for modeling chemical systems: block-oriented models and bilinear models. Both of these have simple structures and yet are able to explain a wide variety of process characteristics. If only the process gain varies in response to changes in the operating conditions, block-oriented Hammerstein models and Wiener models might be adequate. These structures have proven useful in the modeling of heat exchangers, high-purity distillation columns, and pH neutralization systems. When the gain changes are compensated appropriately (via nonlinear transformation of signals), linear controllers can deliver good performance. The identification and control of such systems has been the subject of many research articles (Eskinat et al.,[9] Zhu and Seborg,[38] Lakshminarayanan et al.,[19] Norquay et al.,[28] and Patwardhan et al.[30]) in the chemical engineering literature.

Bilinear models represent another simple class of structures that is useful for the chemical engineer. These structures arise naturally in chemical engineering systems. In mass or energy balance expressions, terms involving interactions between manipulated variables and disturbance variables or manipulated variables and state variables are usually encountered. Examples include interactions between (1) the feed flow rate and feed composition in a chemical reactor and (2) the flow rate of fresh nutrient and concentration of cells in a continuous mass culture of a bioreactor. Espana and Landau[10] have shown that the detailed fundamental model of a distillation column can be closely approximated using a bilinear model involving a far smaller number of states. The approximated bilinear representation mimics the actual dynamics for large variations of the inputs and of the operating point. Williamson[36] derived a system of bilinear equations that can model the microbial cell growth and product formation of various waste treatment and fermentation systems. Oh and Yeo[29] obtained a multivariable bilinear model for

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a simulated steam-reforming plant and performed adaptive predictive control of the unit with the identified model. Mohler[27] showed that it is possible to transform certain nonlinear systems to the bilinear structure by redefining state variables. These observations imply the utility of bilinear models in modeling a wide range of chemical systems and motivate the development of tools that can identify reliable bilinear models from plant input/output data.

This paper proposes an extension of the subspace-based state space system identification strategies that are well established for linear systems, for nonlinear model identification. Because of the relative superiority of canonical variate analysis method (CVA) over other subspace methods (Favoreel,[11] Favoreel et al.[12]), we use the CVA algorithm as the prime vehicle in this investigation. In our approach, a linear model of the process is first identified using the CVA algorithm of Larimore.[21] Subsequently, bilinear functions are iteratively constructed using the states generated by the CVA algorithm in a previous iteration and the process inputs. Borrowing from the presence of a state and input term combination of bilinear systems, we introduce this additional transformation in the model building step. It is shown that the above bilinear structure iteratively extracts the relevant process nonlinearities from the plant data. The identification algorithm converges quite rapidly and is able to represent the overall nonlinear behavior quite well.

This paper is organized as follows. The CVA technique is briefly reviewed first. An introduction to bilinear models and previous studies concerned with their identification is presented next. Section 3 describes the proposed iterative identification algorithm. Identification examples and control results are presented in section 4, followed by conclusions and plans for future work in the final section.

2. Canonical Variate Analysis

Subspace-based state space system identification (4SID) methods have recently attracted much attention because of their computational simplicity and effectiveness in identifying dynamic linear multivariable systems. The 4SID methods estimate a fairly general linear model and can automatically identify the "optimal" model order by using information theoretic criteria or by analyzing the singular values of a certain "observability" matrix. The most popular 4SID methods are canonical variate analysis (CVA), numerical algorithms for subspace state space system identification (N4SID), and multivariable output error state space identification (MOESP). These methods can be put into a unified framework, and it has been shown that the choice of certain weighting matrices provides the distinction between them. Because of the relative superiority of CVA over N4SID and MOESP (for comparative studies, see Diestler et al.[7] and Favoreel[11]), we use the CVA algorithm as the prime vehicle in this investigation.

The CVA technique was originally developed by Larimore[21] and employs singular value decomposition as the core computational machinery. This makes it computationally stable and accurate. The CVA procedure is scale-independent and has been shown to provide optimal parameter estimates even with small sample sizes and small signal-to-noise ratios and in the presence of unknown feedback mechanisms. In CVA, a multivariate technique named *canonical correlations analysis* (CCA) is used to generate the *states* of the following state space model

$$X_{k+1} = \Phi X_k + GU_k + W_k \tag{1}$$

$$Y_k = HX_k + AU_k + BW_k + V_k \tag{2}$$

With the knowledge of these canonical states and the plant data, the state space matrices Φ, G, H, A, and B and the noise covariance matrices $Q = E(W_k{}^T W_k)$ and $R = E(V_k{}^T V_k)$ can be easily computed via simple linear least-squares regression. In this representation, X_k refers to the states, Y_k refers to the outputs, U_k refers to the manipulated inputs, W_k refers to the state or process noise, and $BW_k + V_k$ represents the measurement noise. Even though W_k and V_k are independent zero-mean white noise sequences, the above model allows for the existence of correlations between the state and measurement noise through the use of a nonzero B matrix. It must be pointed out that the states generated by this identification approach are not the true (physically meaningful) states. Rather, they are optimal linear combinations of the past inputs and outputs of the plant that have mathematical validity for being included in the model. Without resorting to any iterative and nonlinear optimization procedures, CVA has been shown to provide a *near* maximum likelihood estimate of the system parameters (Larimore et al.[22]). Consequently, it has become one of the most popular tools for multivariable linear system identification (Schaper et al.[33]).

The only user-defined parameter in the CVA procedure is the maximum memory length (MML) of the system. The algorithm then identifies the process model via four main steps as outlined below.

Selection of the Optimal Memory Length. Using MML, the optimal memory length (OML) of the process is calculated. In simplistic terms, the OML refers to the number of past inputs and outputs required to predict the output(s) of the process over a finite future horizon. This step can significantly reduce the computational load, particularly for large MIMO processes. If the OML is very close to the user-specified MML, it might be necessary to start the identification procedure all over again by specifying a higher value for the MML.

Determination of Process States. The well-known canonical correlation analysis (CCA) technique is employed to obtain the statistically significant states of the process. This is achieved by the creation of the "past" and "future" spaces from the experimental plant data. For each sample k the past is defined as the inputs and outputs covering samples $k -$ OML to $k - 1$; the future refers to the outputs from sample k to sample $k +$ OML $- 1$. When this is done for each sample in the data record, the past and future matrices are created. CCA is then used to determine linear combinations of the past space that are highly correlated with the linear combinations of the future space. The process states (pseudo-states) are thus the optimal linear combinations (canonical variates) of the past space that are highly predictive of the canonical variates of the future space. In principle, although any multivariate statistical technique such as partial least squares (PLS) can be used, it has been found that the use of CCA results in accurate and parsimonious process representation (Lakshminarayanan[18]).

Determination of Optimal State Order. The above step results in the creation of several pseudo-states for the process. By construction, the first canonical variate

is the strongest in the sense of explaining the future output; the power decreases for each successive canonical variate. This implies that a point of diminishing returns will be reached beyond which the inclusion of an additional state will only increase the model complexity (order) but will be of no utility for the prediction of the future output. It is generally accepted that process identification is also an exercise in model reduction with a goal of minimizing the *information distance* of the model from the plant while maintaining the lowest possible model complexity (Ljung[25]). The Akaike information criterion (AIC) combines these objectives and provides a definite and optimal procedure for the comparison of different models given a fixed set of observations. It does this by using a penalty term, thereby ensuring that a model of increased complexity is chosen only when it offers a significantly better fit of the observed data. The CVA method employs the AIC in determining the optimal state order. For the exact mathematical expression needed to compute the AIC for state space models, the reader is directed to Schaper et al.[33]

Computation of the State Space Model Matrices. Once the states have been computed, with the knowledge of the optimal number of states (via AIC), it is possible to estimate the state space matrices in eqs 1 and 2 via ordinary least squares. Because of space limitations, we refrain from providing the equations here but point the reader to Larimore[21] for a comprehensive mathematical treatment of the CVA method.

In summary, it can be stated that CVA is based on simple and stable numerical computations and relies on statistical information theory for selection of the optimal model complexity. Recently, DeCicco and Cinar[5] modeled a CSTR that exhibited output multiplicity behavior using a combination of a nonparametric approach and the canonical variate analysis technique. CVA is, therefore, easily adaptable for building nonlinear process models from plant data, as this paper will further demonstrate.

3. Bilinear Model Identification

In the Introduction, we emphasized the need for developing strategies to identify models with bilinear structure. In the context of subspace-based state space identification methods for bilinear systems, we will consider identifying models of the following structure from plant input/output data

$$X_{k+1} = \Phi X_k + GU_k + N(X_k \otimes U_k) + W_k \qquad (3)$$

$$Y_k = HX_k + AU_k + BW_k + V_k \qquad (4)$$

The sign $\otimes$ in eq 3 denotes the variables generated by the element-wise product of each computed state with each input variable. For example, if the model has three states (x_1, x_2, and x_3) and two inputs (u_1 and u_2), then $X_k \otimes U_k$ denotes the six variables x_1u_1, x_1u_2, x_2u_1, x_2u_2, x_3u_1, and x_3u_2.

It is easy to observe that the state equation (eq 3) for the bilinear system is different from that of the state equation for the linear system (eq 1) because of the additional term $N(X_k \otimes U_k)$ that is meant to capture the bilinear behavior. The output equations are the same (eqs 2 and 4). In the bilinear system identification method, we need to estimate the system matrices Φ, G, N, H, A, and B and the noise covariance matrices $Q = E(W_k{}^T W_k)$ and $R = E(V_k{}^T V_k)$ from experimental plant data. This primarily involves a modification of the mechanism used to identify the system parameters for linear systems. Favoreel et al.[12] generalized the N4SID algorithm by modification of the Hankel matrices used to estimate the states in their state space model. The Hankel matrices then included not only the plant inputs and outputs but also products between the inputs and the outputs. With this modification, Favoreel et al.[12] were able to propose a noniterative bilinear identification algorithm. When the product terms are dropped, their method results in the identification of the "best" linear model using the N4SID method. Verdult et al.[35] used the separable least-squares technique for the identification of MIMO bilinear state space models.

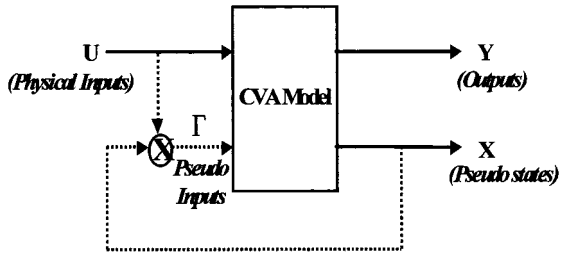

Figure 1. Iterative bilinear model identification method.

Table 1. Outline of Steps in the Bilinear CVA Algorithm

Step 1. Use plant data [U, Y] to construct a CVA model. The result is the linear model represented by matrices [Φ, G, H, A, B, Q, R] and linear states X.
Step 2. Determine whether the model generated in step 1 is adequate. Eyeballing, residual analysis AIC measure, or validation on a new data set can serve this purpose. If the model is adequate, then STOP; otherwise, go to step 3.
Step 3. Compute pseudo-inputs Γ using the computed states and the plant inputs U as $\Gamma = X \otimes U$.
Step 4. Use the CVA algorithm to obtain a model relating the expanded input space [U, Γ] to Y. This results in a model represented by matrices [Φ, G*, H, A, B, Q, R] and new states X. Matrix G* can be partitioned into matrices G and N depending on the number of states X and the number of inputs U.
Step 5. Determine whether the model generated in step 3 is adequate. If the model is adequate, then STOP. Otherwise, recompute the pseudo-inputs Γ using the states generated in step 4 and the plant inputs U. Go back to step 4.
A prespecified number of iterations could also be used as an additional termination criterion.

In this work, we propose an iterative scheme for the identification of bilinear models using the CVA technique. The choice of the iterative technique stems from our past experience in using an iterative technique for the identification of other nonlinear structures using the CVA technique (Lakshminarayanan et al.[19]). Also, for the identification of nonlinear systems modeled with the Hammerstein structure, it has been established that iterative methods are superior to noniterative methods (Gallman[13]). Moreover, with the iterative technique the first step is the identification of a linear model, the results of which can suggest the extent and type of nonlinearity present in the process data. All of these reasons provided the motivation to develop and apply an iterative strategy for identifying bilinear models from process data. It is natural to question the convergence of any iterative technique. However, we have noticed no such problems in all of our studies conducted thus far.

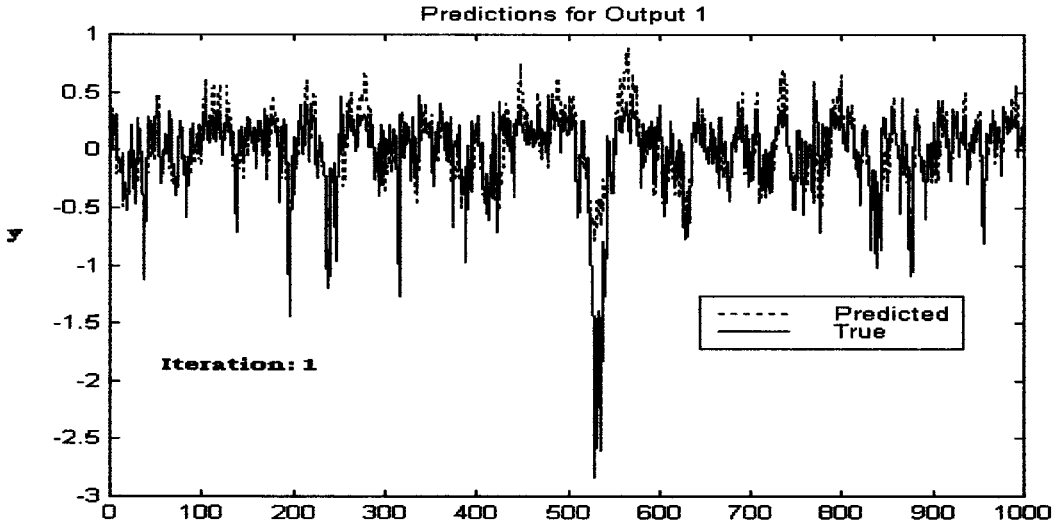

Figure 2. Comparison of linear model predictions with the actual outputs for example 1.

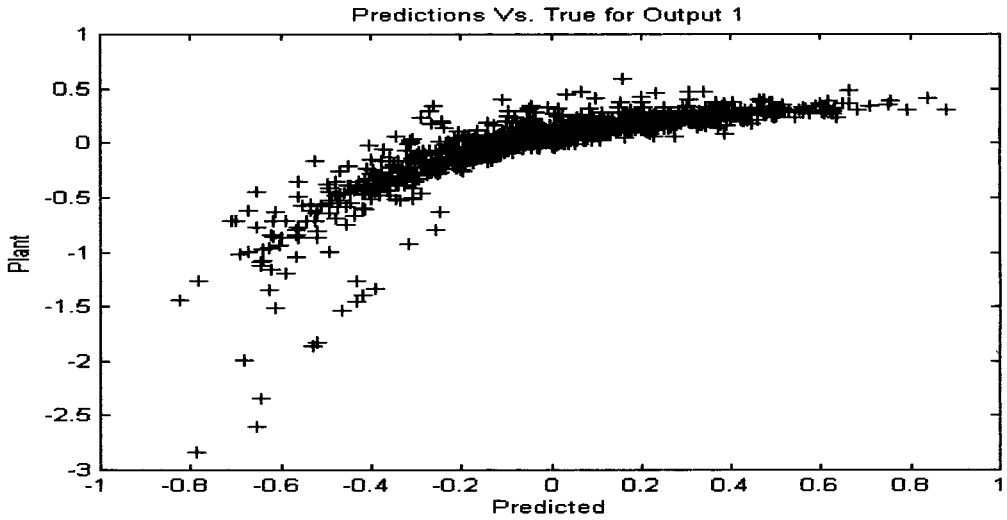

Figure 3. Scatter plot for example 1 (linear model).

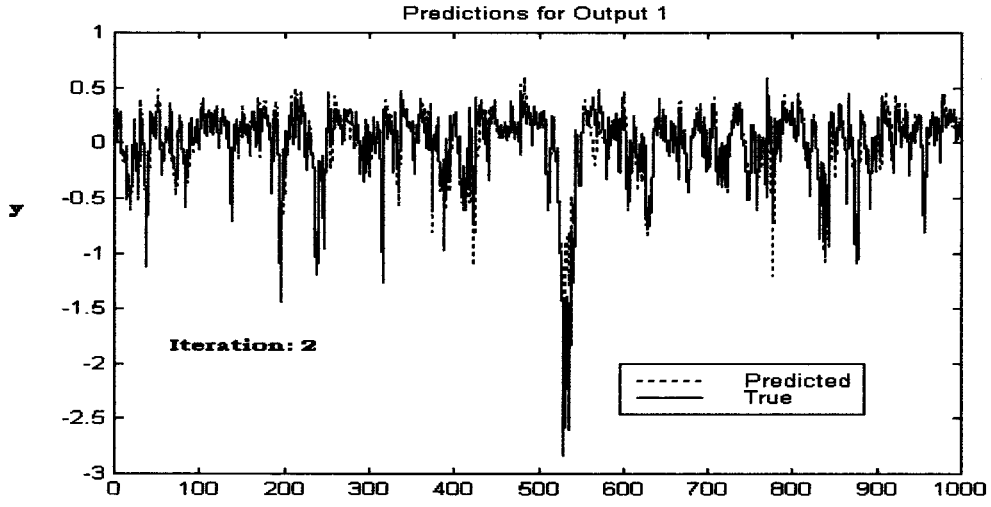

Figure 4. Results of iteration 2 (first bilinear model) for example 1.

As shown in Figure 1, the basic CVA algorithm forms the core machinery of the computations for the identification of the bilinear model. This explains the rather elaborate section on the CVA technique earlier. To begin, a linear model is identified. The appropriateness of the linear model is checked by performing analysis of residuals; alternatively, in most cases, simple visual examination of the actual and fitted trajectories will suffice. If the linear model is deemed insufficient, the pseudo-inputs (Γ) are created from the identified state sequences (obtained from the second step of the CVA procedure) and the input sequences. The CVA algorithm

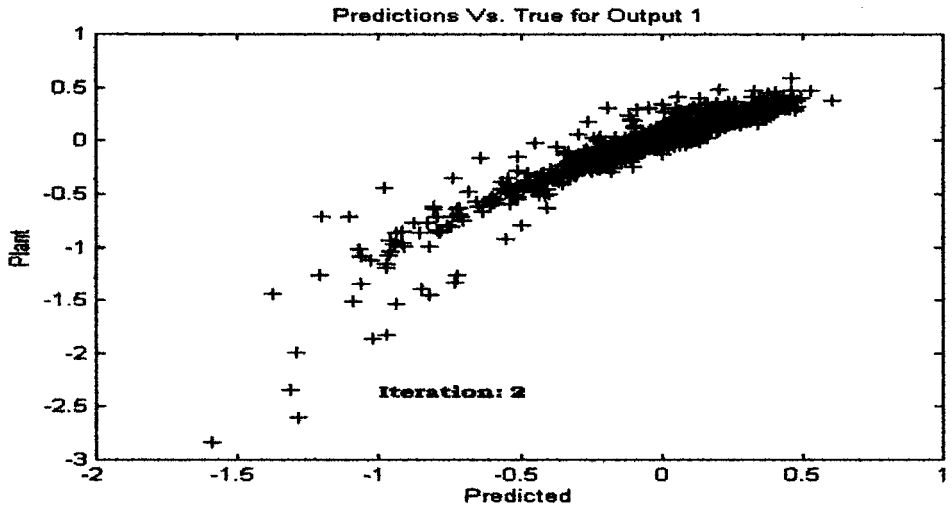

Figure 5. Scatter plot for iteration 2 of example 1.

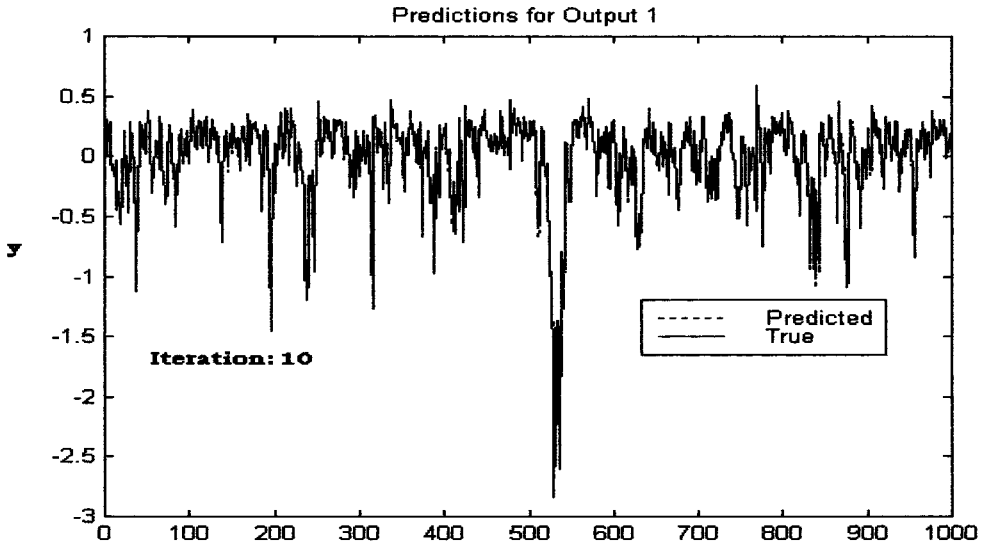

Figure 6. Results of the model building after 10 iterations of example 1.

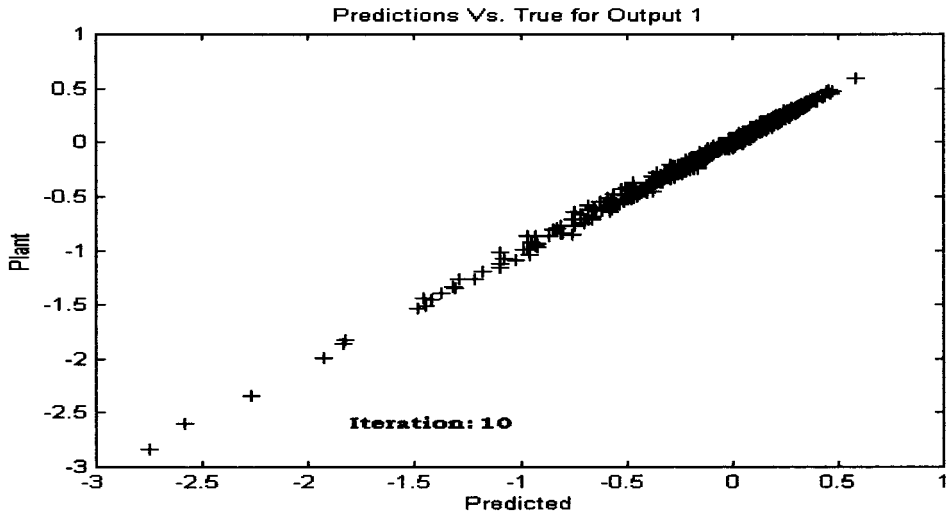

Figure 7. Scatter plots for the output after 10 iterations for example 1.

is executed again, this time with a larger number of inputs (real inputs plus pseudo-inputs). This procedure is continued for a specified number of iterations or until the obtained model is adequate (as determined by residual analysis of the output error/visual inspection). In this work, we used the convergence of the AIC measure of three successive iterations, specifically, the three AIC values must lie within a certain prespecified percentage (1% was employed here) of each other. This strategy for stopping the iterations works quite well in

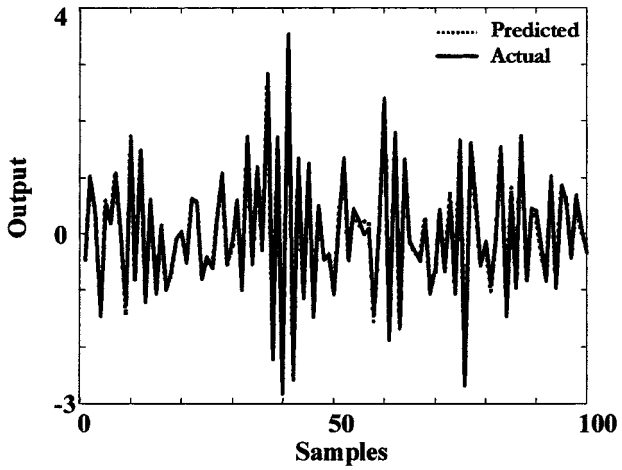

Figure 8. Cross validation of the model identified in example 2.

practice and also requires no extra calculations or analysis. Thus, the bilinear model is obtained by repeated application of the linear CVA algorithm. The bilinear model matrices are obtained via suitable partition of the CVA model matrices. The sequence of the computations in the proposed strategy is presented in Table 1.

A comment on the limitations of the bilinear model is also appropriate at this juncture. The bilinear model presented in eqs 3 and 4 cannot represent nonlinear behavior such as "jump phenomena". In the context of chemical engineering processes, it cannot be used for representing the behavior of systems exhibiting output multiplicity (e.g., the exothermic CSTR). When the physical system of interest is known to exhibit such behavior, the bilinear model is not appropriate, and structures such as the polynomial ARMA (Hernandez and Arkun[17]), multivariable additive NARX (DeCicco and Cinar[6]), or general nonlinear canonical variate analysis approach (Larimore[23]) should be employed. However, if the bilinear system is controlled using linear state feedback controllers, the closed-loop system can exhibit multiple equilibrium points. This aspect has been studied by several researchers, including Benallou et al. [2].

4. Illustrative Examples

We now demonstrate the theory described above using several illustrative examples.

Example 1: Identification of a Simple Bilinear Model. Consider the following bilinear state space model

$$x(k+1) = 0.8x(k) - 0.2u(k) + 0.4x(k)\,u(k) + 0.1w(k)$$

$$y(k) = x(k) + 0.1v(k)$$

In the above equations, $w(k)$ and $v(k)$ are white noise sequences. A normally distributed iid sequence having unit variance was used to excite the process, and a data set consisting of 1500 samples was collected. This data record of input and output values was used to build a model of the process.

Iteration 1. First, a linear model was identified for the process using the linear CVA algorithm. The algorithm identified a state space model with a single state variable with parameters $\Phi = 0.7983$, $G = -0.5584$, $H = 0.3317$, and $A = 0$. Figure 2 compares the model predictions with those obtained from the first linear model. As can be seen, the linear model is not able to capture all of the dynamics. Figure 3 shows the scatter plot for the same data. The distinct curvature in this figure indicates the presence of nonlinearity in the data.

Iteration 2. In the next step, a bilinear combination of the state profile (obtained in the linear step) and the process input is considered as an additional variable along with the process input itself. Thus, this is a multi-input (two inputs) single-output problem for identifica-

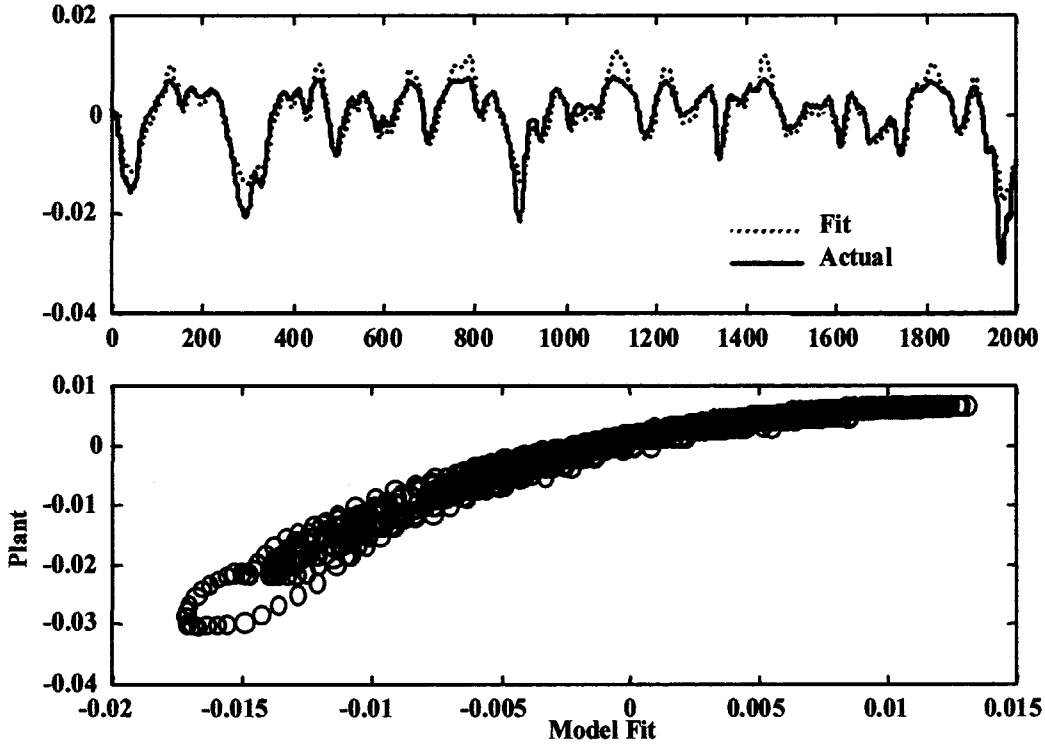

Figure 9. Fit obtained with a linear model for the distillation column data.

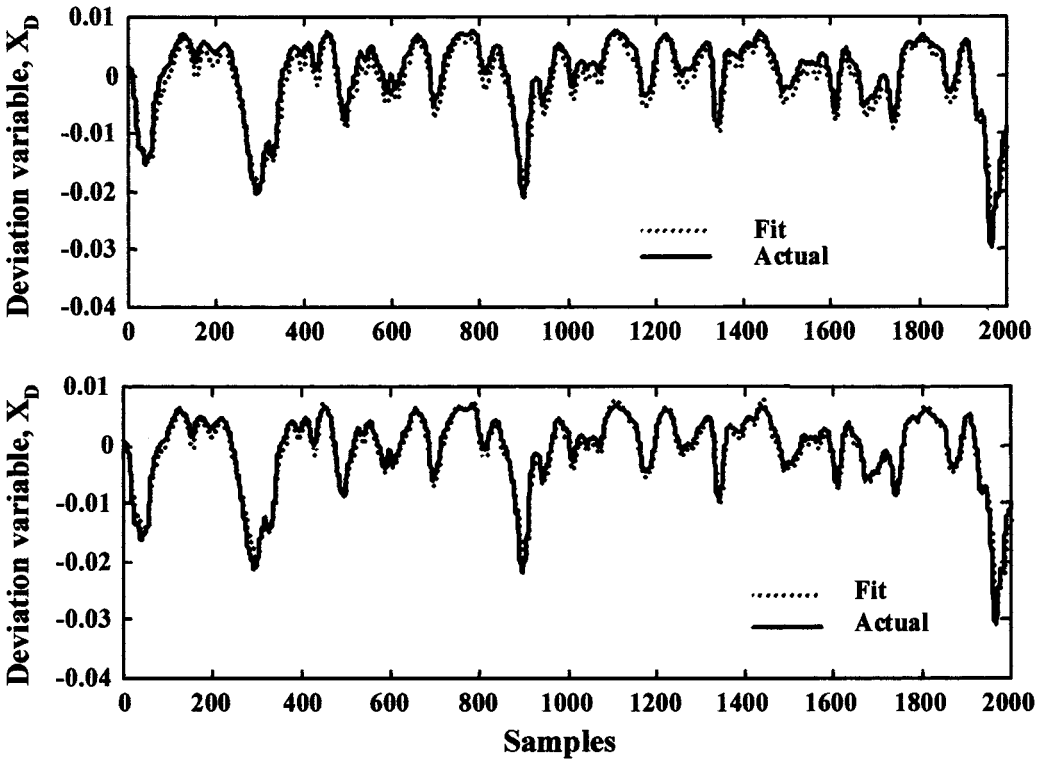

Figure 10. Model fit using bilinear structure. First bilinear model (top) and final bilinear model (bottom).

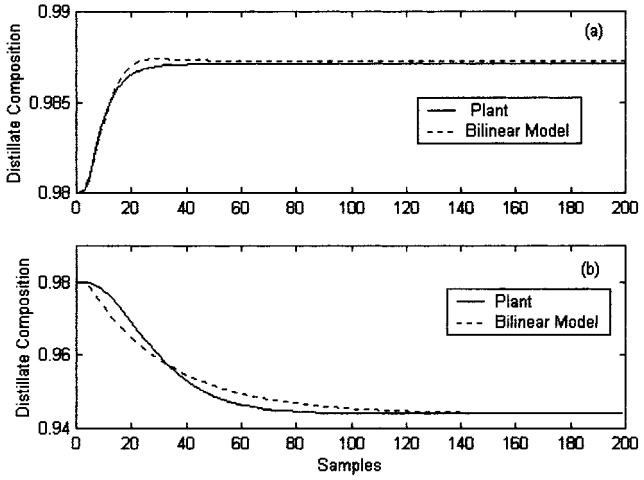

Figure 11. True and model-predicted responses in distillate purity to (a) positive and (b) negative step changes in reflux flow.

tion by CVA. The algorithm again suggested a single-state model with the following state space matrices

$$\Phi = [0.8100] \qquad G^* = [-0.5162 \;\; 0.3449]$$

$$H = [0.3468] \qquad A = [0 \;\; 0]$$

Based on this model, Figures 4 and 5 show the data and the model predictions as time series and scatter plots, respectively. It can be seen that the model predictions are better and the curvature in the scatter plot has significantly diminished, indicating that most of the nonlinearity has been modeled.

Figures 6 and 7 indicate the results at the end of the 10th iteration for this data set. It can be seen here that all of the nonlinearity has been effectively captured and the scatter plot is linear. For this problem, the results from the 3rd iteration onward were almost identical, indicating that the bilinear CVA algorithm estimated the nonlinear model quite rapidly. The final model parameters are

$$\Phi = [0.7990] \qquad G = [-0.2606] \qquad N = [0.4022]$$

$$H = [0.9751] \qquad A = [0 \;\; 0]$$

The identified values agree quite well with the original plant parameters.

Example 2: Identification of a Bilinear System with Time Delay. Simulated data were obtained from the following system and used for identification

$$X_{k+1} = (-0.5X_k) + (0.2X_kU_k) + (0.7U_{k-5})$$

As before, a normally distributed iid sequence having unit variance was used to excite the process. In this case, the state variable (corrupted with Gaussian noise) is the output. Of interest here is the fact that the input can affect the output only after five samples**,** assuming that both X_k and U_k are zero for all negative values of k and that the input U is changed at $k = 0$ ($U_0 \neq 0$).

A maximum lag of 10 samples was specified for the CVA algorithm. The iterative technique automatically (without any user input) identified a sixth-order model (five of the states were used to represent the process delay) by the third iteration. After three iterations, no significant change in the AIC was noticed. The minimum AIC value for the linear model was 419.18, and for the final bilinear model, it turned out to be −1771.9, indicating the huge degree of improvement in going from the linear model to the bilinear model. The process delay of five samples was also correctly identified. Cross validation (using output obtained by perturbing the process with a different input sequence) of this model is shown in Figure 8.

The first two examples have demonstrated the fact that, if the "true" plant has a bilinear structure, then

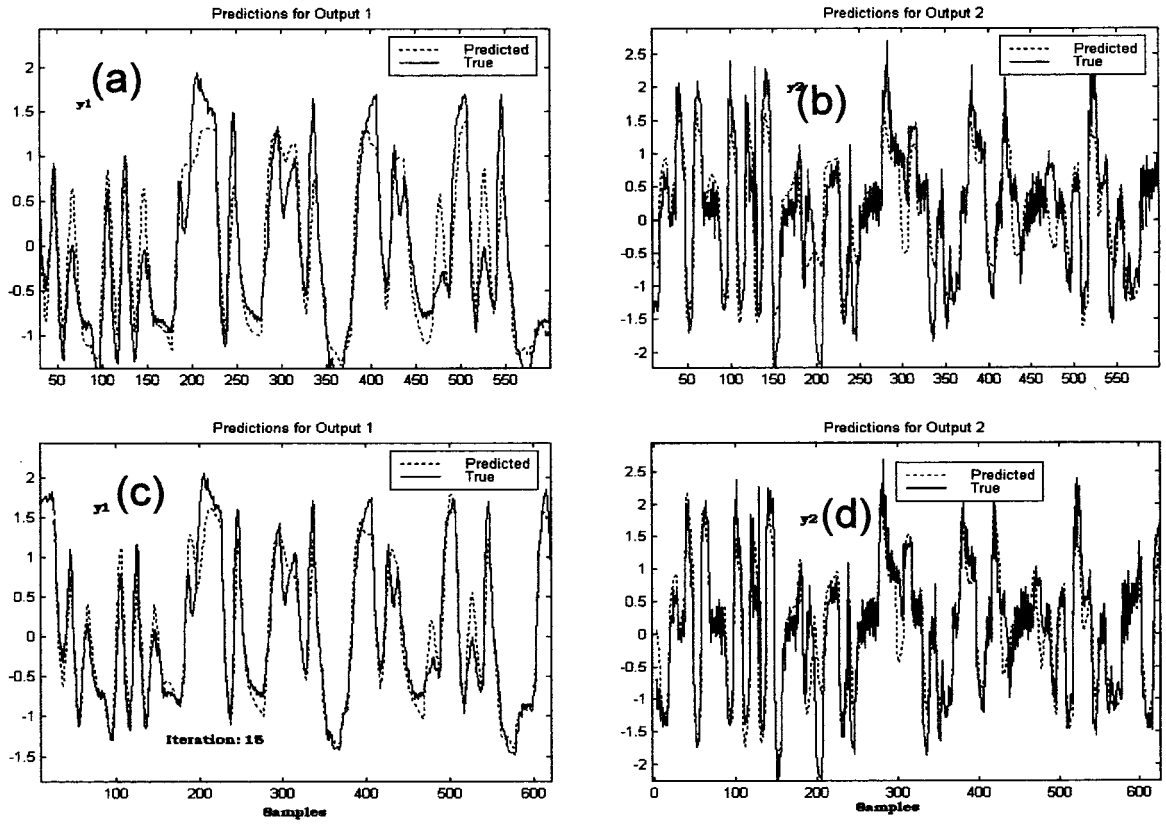

Figure 12. Results from the identification of the industrial winding process.

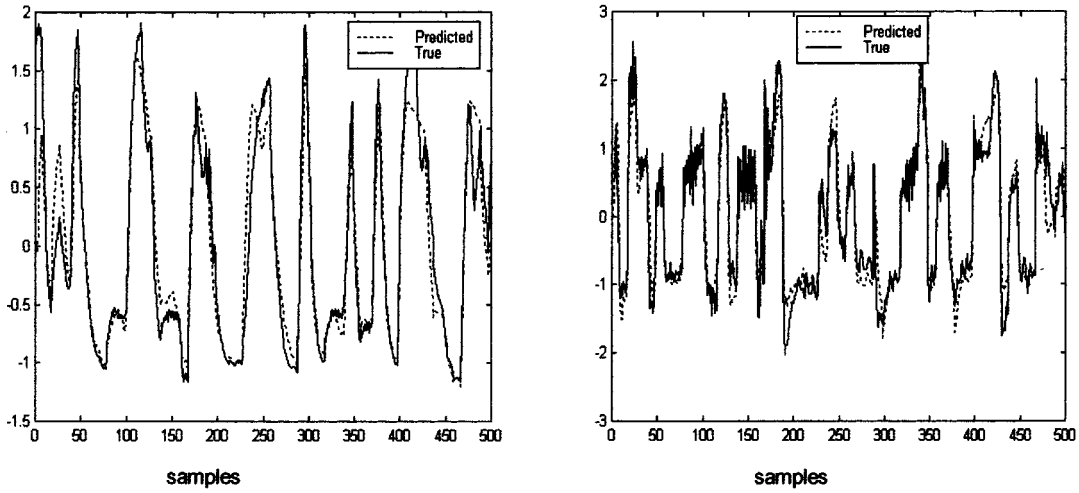

Figure 13. Cross validation of the model for the industrial winding process (left, for output 1; right, for output 2).

the proposed identification procedure can identify the parameters of the model accurately. The real utility of the model structure and the identification method is demonstrated when it can adequately represent and identify models of physical systems. The following examples illustrate the efficacy of the proposed bilinear identification method in modeling general processes.

Example 3: High-Purity Distillation Column. The dynamics of distillation columns is one of the most often studied systems in the context of process identification. Here, we consider an ideal binary distillation column (Luyben[24]) with 20 ideal (100% efficient) trays. Simplifying assumptions such as constant relative volatility (equal to 2), equimolal overflow, and simple tray hydraulics are made. Feed (saturated liquid) enters the 10th tray. In this study, the reflux flow rate to the column is the input variable, and the distillate composition is the output variable.

The dynamics of a distillation column are usually approximated by a linear model (first- or second-order with time delay); however, in high-purity distillation columns such as the one considered here, the process characteristic is nonlinear, and therefore, a linear model would not be sufficient. The common remedy is the use of a logarithmic transformation of composition in both the identification and control tasks. Eskinat et al.[9] used a Hammerstein structure to represent the behavior of high-purity columns. Motivated by the work of Espana and Landau,[10] we try to capture the nonlinear behavior using a bilinear model.

A data set comprising 2000 samples was collected by probing the process using multilevel step signals. Such

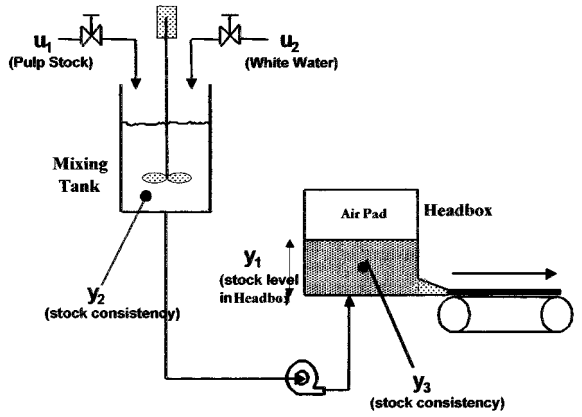

Figure 14. Schematic of a paper machine headbox at a paper mill.

Table 2. Paper Machine Headbox Details

	Inputs
u_1	stock flow rate into the mixing tank
u_2	white water flow rate into the mixing tank
u_3	consistency of the mixing stock in the mixing tank (measured disturbance)
u_4	consistency of the white water in the mixing tank (unmeasured disturbance)
	Outputs
y_1	level of the stock in headbox
y_2	consistency of the stock in the mixing tank
y_3	consistency of the stock in the headbox

multilevel signals facilitate accurate identification of nonlinear systems. First, a linear CVA model was identified. The fit obtained with this model is shown in Figure 9. It is seen that the linear model is not capable of fitting the data satisfactorily. The scatter plot shown at the bottom of Figure 9 indicates the nonlinearity that is present in the data. The top plot in Figure 10 shows the performance of the first bilinear model, and the bottom plot shows the final bilinear model obtained after five iterations. It is seen that even the first bilinear model provides a better approximation of the data, and there is slight improvement in going from the first bilinear model to the final bilinear model in this example. This also points to the fact that the iterative strategy converges to the final identified model very quickly. The state space matrices at the end of the 5th iteration were

$$\Phi = \begin{bmatrix} 0.9983 & 0.0594 & 0.0012 \\ -0.1811 & 0.8694 & -0.1355 \\ 1.0780 & 1.1473 & 0.1592 \end{bmatrix} \qquad \Gamma = \begin{bmatrix} 0.000 \\ 0.3788 \\ -3.6116 \end{bmatrix}$$

$$H = [0.0048 \;\; 0.001 \;\; -0.0001] \qquad A = [0]$$

The step models identified for this process are shown in Figure 11, along with the true system step responses. The responses for positive and negative step changes in the reflux flow are not symmetrical. For a positive step change in the reflux flow, the distillate purity increases; however, in this direction, the steady-state gain diminishes rapidly, approaching zero as the distillate purity increases. The identified model is able to represent this characteristic (unequal gains for positive and negative step responses). Also, the time constants for these responses are quite different. This is typical of the behavior of high-purity distillation columns where the time constant is known to change depending on the extent of deviation (and also on the direction of deviation) from the steady state (see Eskinat et al.[9]). The identified model also exhibits this phenomenon. Together, these results indicate that the bilinear model is able to qualitatively and quantitatively model the high-purity distillation process.

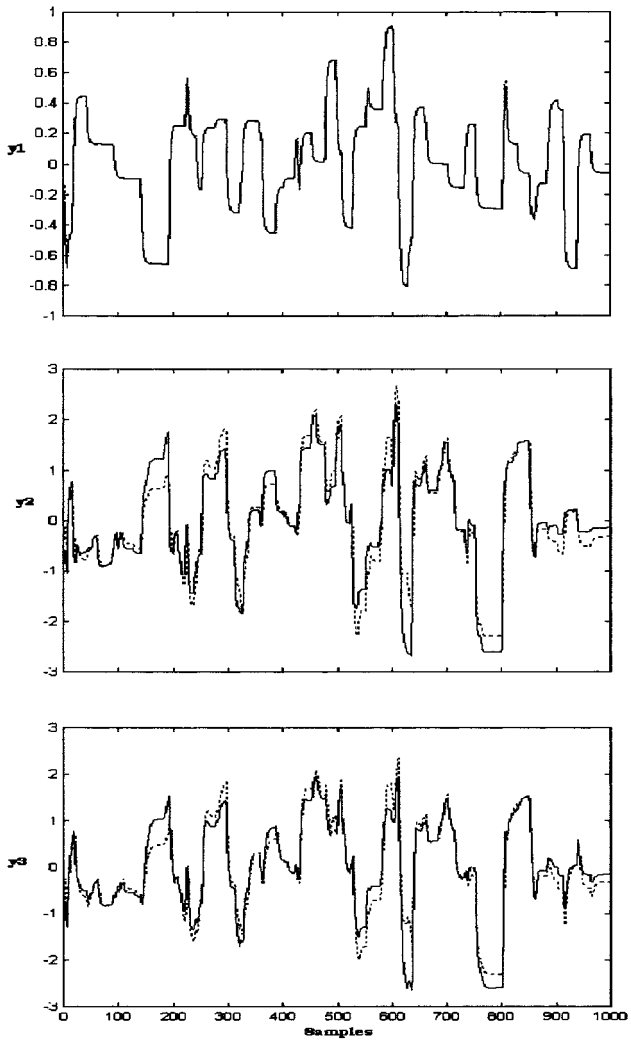

Figure 15. Linear model for the paper machine headbox system data. The top plot is for y_1, the middle plot is for y_2, and the bottom plot is for y_3. The solid line represents the process data, and the dotted line represents the model outputs.

Example 4: Industrial Winding Process. The process considered here is a test setup of an industrial winding process. The main part of the plant is composed of a plastic web that is unwound from the first reel (unwinding reel), goes over the traction reel, and is finally rewound on the rewinding reel. Reels 1 and 3 are coupled with a DC motor that is controlled with input set-point currents $I_{1,sp}$ and $I_{3,sp}$. The angular speeds of the reels (S1, S2, and S3) and the tensions in the webs between reel 1 and 2 (T1) and between reel 2 and 3 (T3) are measured by dynamo tachometers and tension meters. The angular speed of the reels and the set points for the currents are the input variables, and the two tension measurements are the output variables in this identification study. The data samples arrive every 0.1 s. The data set for this five-input two-output identification problem has been taken from the DaISy database (data file code number [97-003]) compiled by De Moor et al.[8] The first 2000 samples of the data set were used for identification, and the last 500 samples were set aside for validation of the identified model.

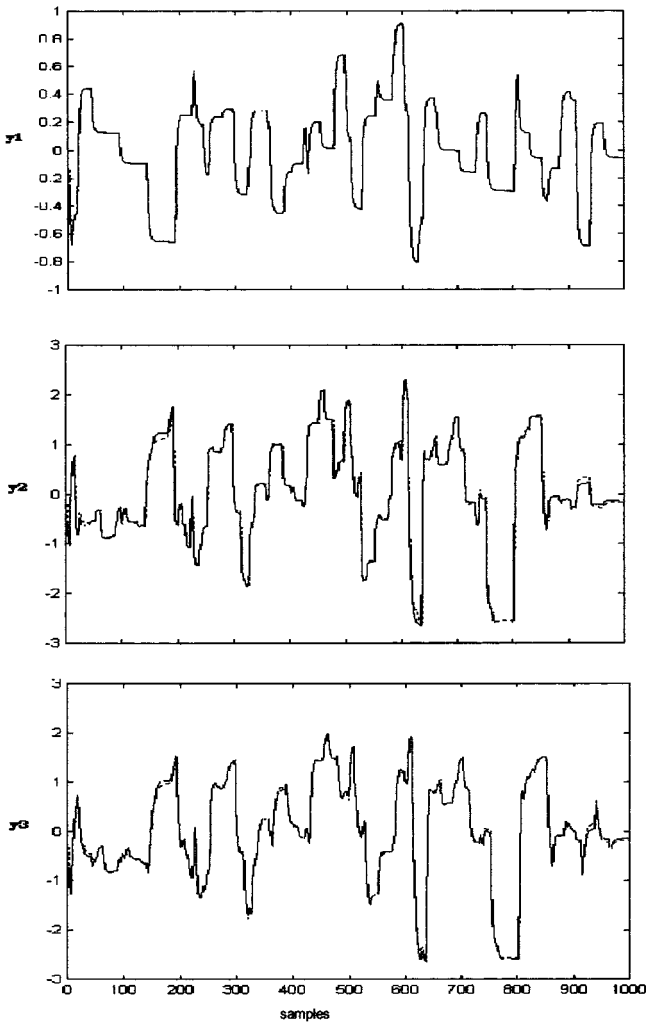

Figure 16. Fit obtained with the bilinear model for the paper machine headbox system. The top plot is for y_1, the middle plot is for y_2, and the bottom plot is for y_3. The solid line represents the process data, and the dotted line represents the model outputs.

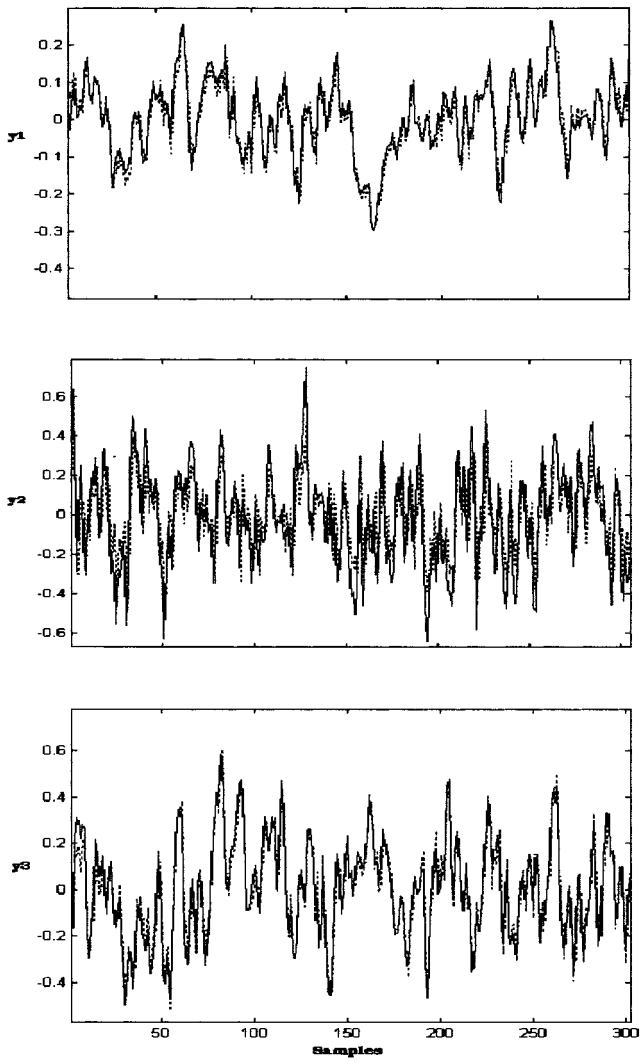

Figure 17. Cross validation results for the paper machine headbox system.

Figure 12 contains the results of the identification exercise. The two sub-plots shown on the top (labeled a and b) correspond to the linear model for the two outputs and the two subplots on the bottom (labeled c and d) correspond to the bilinear model-fitted values after 15 iterations. The RMS prediction errors at the end of the linear model building step were 0.1515 and 0.2614 for the first and second outputs, respectively. They decreased further to 0.0952 and 0.1925 at the end of the ^{5th} iteration and then to 0.0567 and 0.1153 at the end of the 14th iteration. With a view to maintaining clarity, only about 600 samples of the 2000 samples used for identification are shown here. In these plots, the model predictions are shown as dotted lines, and the actual values are shown as solid lines. Substantial improvement in the fit is noticed for output 1 (T1) in going from the linear model to the bilinear model. No significant improvement is noticed for the second output.

The cross-validation result shown in Figure 13 indicates that the quality of the identified model is very good. This example also indicates that not only chemical processes but also mechanical systems can be modeled using the bilinear structure.

Example 5: Identification and Control of a Paper Machine Headbox: A Simulation Case Study. A paper-making machine at a paper mill that produces the super-thin condenser paper is described by Ying et al.[37] The stock from a pulp workshop is pumped into a mixing box, where it is mixed with white water. The mixture goes through the filter, where the dregs in the stock are removed. Then, the mixture is filled into the headbox. The next step is to place the stock onto the forming wire and to remove most of the water from the paper. The paper sheet goes through the paper part and dryer section (cylinder) to remove the remaining water and finally to accomplish the process of production. A schematic of the system is provided in Figure 14.

The headbox system is very important in the paper-making process. The level and consistency of stock in the headbox are the main factors affecting product quality. Details of the input and output variables are provided in Table 2.

The detailed equations describing the plant are described in Ying et al.[37] A square-wave-type input was used to perturb the plant and generate data for the model building step. Although the third input (consistency of the mixing stock) is actually a measured disturbance, it was also treated like the other manipulable inputs so that the disturbance model could be identified. A random Gaussian signal was employed for

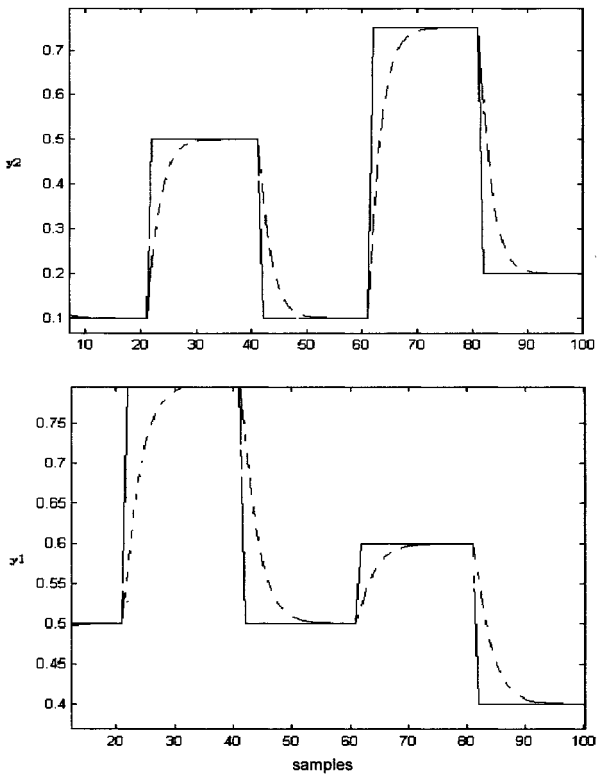

Figure 18. Closed-loop control results for the paper machine headbox system.

u_4. One thousand samples were collected. The linear model identified for this system is shown in Figure 15. The top plot is for y_1, the middle plot for y_2, and the bottom plot for y_3. The solid line represents the process data, and the dotted line represents predictions of the identified model. It is seen that the fit for y_1 is adequate—it is not surprising that a linear model is adequate for the level. However, for the variables describing the consistency of the stocks, the linear model is inaccurate. The fit obtained from the bilinear model after five iterative steps is shown in Figure 16. It is seen that the bilinear model is able to provide an excellent fit of the process dynamics. More importantly, the validation plot shown in Figure 17 confirms the good quality of the obtained model.

For control, a simple predictive control problem was formulated as

$$\min J = \sum_{i=N_1}^{N_2} [(y_d(k+i) - yp(k+i)]^2 + \lambda \sum_{i=1}^{NU} \Delta u(k+i-1)^2$$

In this standard model predictive control formulation, N_1 and N_2 are the initial and final prediction horizons, respectively; λ is the penalty factor for the manipulated input; and NU is the control horizon. $y_d(k+i)$ is the set-point trajectory over the control horizon. $yp(k+i)$ is the ith step ahead prediction of the output(s) using the bilinear model identified using the proposed methodology. Implementation of the control action is done in a receding horizon fashion.

For $N_1 = N_2 = NU = 1$ and $\lambda = 0.1$, Figure 18 shows the closed-loop responses for the two outputs obtained when the system is subject to changes in the set points as well as in the measured disturbance (u_3). The corresponding trajectories of the manipulated variables and disturbances are shown in Figure 19. It can be seen that the set points are tracked well even in the presence of disturbances affecting the process. This indicates that the identified multivariable nonlinear process model represents the true process dynamics adequately and that the identified models are easy to use in advanced feedback control strategies.

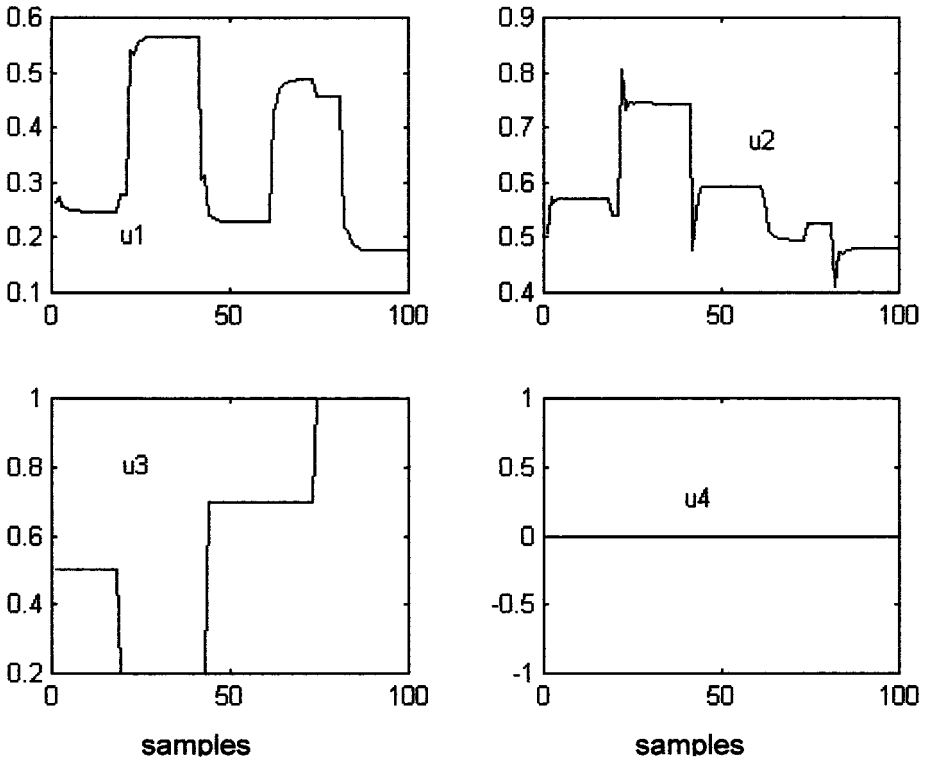

Figure 19. Profiles of the manipulative variables for the closed-loop control of Figure 18. Changes in the measured disturbance variable u_3 are deliberately introduced, whereas the unmeasured disturbance u_4 is maintained constant in the simulations.

5. Conclusions

We have presented an iterative methodology for identifying bilinear models from process data. This method uses one of the most accurate and robust multivariable linear identification methods, namely, CVA, as its basic machinery. The bilinear model identified can therefore be expected to be accurate as well. For a majority of the processes that we have analyzed (and presented here), a bilinear model can be obtained within about five iterations. Several case studies were presented to highlight the capabilities of the proposed algorithm, such as handling of time delays, multivariable systems, etc. Finally, an example was presented where the model predictions were used in a predictive control framework. The proposed method is expected to be beneficial in the empirical modeling of nonlinear processes.

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Nomenclature

G = matrix corresponding to the input in the state equation of the state space description
H = matrix corresponding to the input in the output equation of the state space description.
Q = process noise covariance matrix
R = measurement noise covariance matrix
U = vector of process/model inputs
V = measurement noise vector
W = process noise vector
X = states in the state space description of the process
Y = outputs in the state space description of the process
Φ = state transition matrix in the state space description
Γ = vector of pseudo-inputs (i.e., consisting of true inputs as well as bilinear combinations of inputs) in the bilinear model

Acronyms

CCA = canonical correlation analysis
OML = optimal memory length
PLS = projection to latent structures
AIC = Akaike information criteria

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