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MEng Individual Project

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A Virtual Volumetric Screen

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Abstract

Your abstract.

Acknowledgments

Comment this out if not needed.

Contents

Chapter 1

Introduction

Figure 1.1 is an example of a figure.

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Figure 1.1: Imperial College Logo. It's nice blue, and the font is quite stylish. But you can choose a different one if you don't like it.

Chapter 2

Background

2.1 Nix/NixOS

Figure 2.1.1: the NixOS logo, also know as the Nix Snowflake.

Nix [5] is an open-source, "purely functional package manager" used in unix-like operating systems to provide a functional and reproducible approach to package management. Started in 2003 as research project Nix [4] is widely used in both industry [1] and academia [3] [11] [2], and its associated public package repository `nixpkgs` [8] as of Jan 2024 has over 80,000 unique packages making it the largest up-to-date package repository in the world as can be seen in Fig 2.1.2. Out of Nix has also grown **NixOS** [6] a Linux distribution that is conceived and defined as a deterministic and reproducible entity that is declared functionally and is built using the **Nix** package manager.

Figure 2.1.2: Comparison of all package repositories from Repology.org [12]
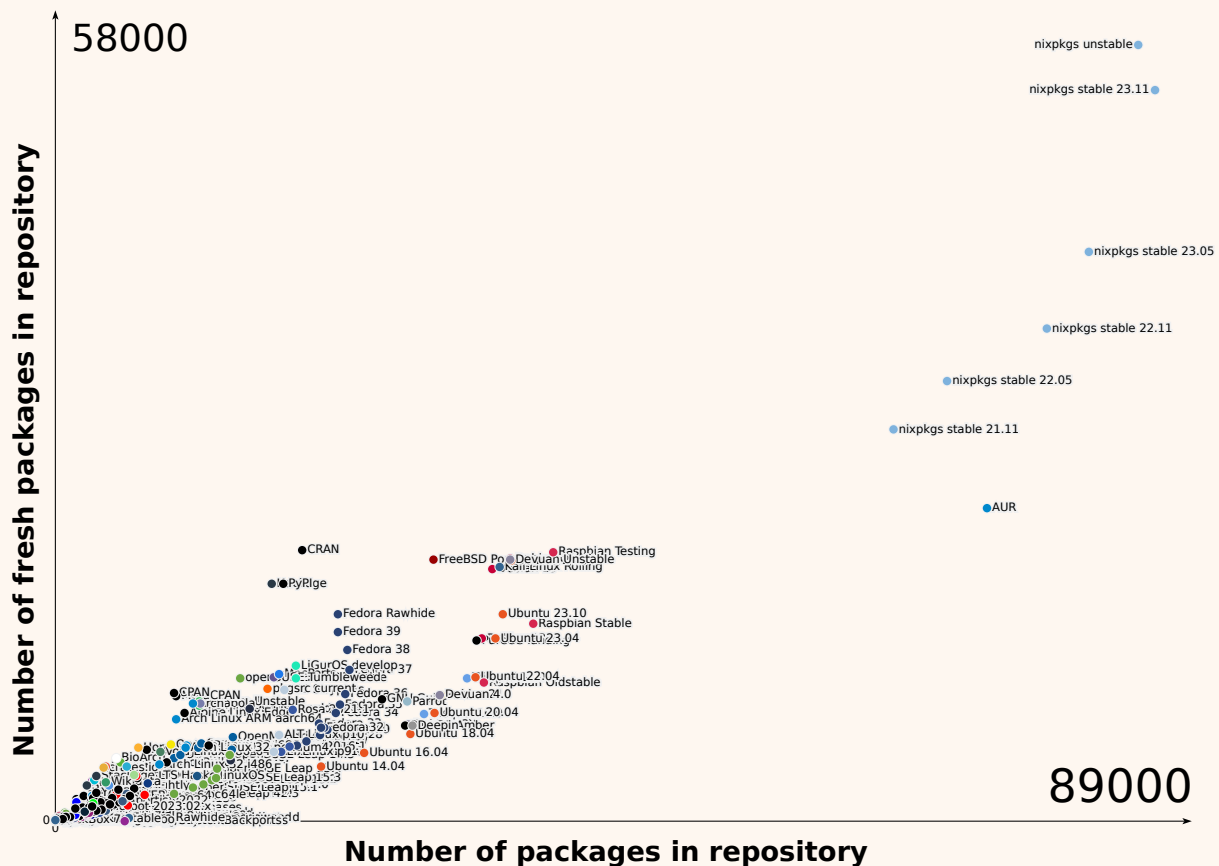
Nix packages are defined in the **Nix Language** a lazy functional programming language where packages are treated like purely functional values that are built by side effect-less functions and once produced are immutable. Packages are built with every dependency all the way down to the ELF interpreter and `libc` (C standard library) defined in nix. All packages are installed in the store directory, typically `/nix/store/` by their unique hash and package name as can be seen in Fig 2.1.3.

Figure 2.1.3: Nix Store Path

```
/nix/store/sbldylj3clbkc0aqvjjzfa6slp4zdvlj-hello-2.12.1
```
Prefix   Hash part   Package name

Source files, like tarballs and patches are downloaded and stored in the store directory to insure all required inputs are always available. As different dependencies result in a different hash and therefore location in the store directory you can have multiple versions or variants of the same package installed while also at the same time avoiding "DLL hell" by making it impossible to accidentally point at the wrong package. Another important result is that upgrading or uninstalling a package cannot ever break other applications. Nix builds packages

in a sandbox to ensure packages are built the same way on every machine by restricting access to non reproducible files and the network [13]. A package can be pinned (and should be) to nix release meaning that once it builds and works today it will continue to work the exact same way in the future, regardless of when and where it is used.

These features provide extremely useful for scientific work, CERN uses Nix to package the LHCb Experiment because it allows software to be stable for long periods of time (longer than ever long term support operating systems) and it means that as the software is reproducible; all the experiments are completely reproducible as all bugs present in the original version stay to ensure the accuracy of the results [2].

To create a package Nix evaluates a **derivation** which is a specification/recipe that defines how a package should be built. It includes all the necessary information and instructions for building a package from its source code, such as the source location, build dependencies, build commands, and post-installation steps. By default, Nix uses binary caching to build packages faster, the default cache is `cache.nixos.org` is open to everyone and is constantly populated by CI systems. You can also specific custom caches. The basic process for building nix packages can be seen in Fig 2.1.4.

> **Figure 2.1.4:** Nix Build Loop
>
> 1. A hash is computed for the derivation and, using that hash, generate a nix store path, e.g `/nix/store/sbldylj3clbkc0aqvjjzfa6slp4zdvlj-hello-2.12.1`.
>
> 2. With the store path in hand, check if the derivation has already been built. First, checks the configured Nix store e.g `/nix/store/` to see if the path e.g `sbldylj3clbkc0aqvjjzfa6slp4zdvlj-hello-2.12.1` already exists. If it does, use that it, if not continue.
>
> 3. Next it checks if the store path exists in a configured binary cache, this is by default `cache.nixos.org`. If it does download from the cache and use that if not continue.
>
> 4. Use Nix to build the derivation from scratch, recursively following all of the steps in this list, using already-realised packages whenever possible and building only what is necessary.

Listing 2.1.1: flake.nix

```nix
{
  description = "A flake for building Hello World";
  inputs.nixpkgs.url = "github:NixOS/nixpkgs/nixos-23.11";

  outputs = { self, nixpkgs }: {
    defaultPackage.x86_64-linux =
      let
        pkgs = nixpkgs.legacyPackages.x86_64-linux;
      in
      pkgs.stdenv.mkDerivation {
        name = "hello-2.12.1";
        src = self;
        # Not strictly necessary as stdenv will add gcc
        buildInputs = [ pkgs.gcc ];
        configurePhase = "echo 'int main() { printf(\"Hello World!\");
        ↪  }' > hello.c";
        buildPhase = "gcc -o hello ./hello.c";
        installPhase = "mkdir -p $out/bin; install -t $out/bin hello";
      };
  };
}
```

Listing 2.1.2: Terminal

```
[shell:~]$ ls
flake.lock   flake.nix

[shell:~]$ nix flake show
└──defaultPackage
    └──x86\_64-linux: package 'hello-2.12.1'

[shell:~]$ nix run .
Hello, world!

[shell:~]$ tree $(nix path-info .)
"\nix\store\sbldylj3clbkc0aqvjjzfa6slp4zdvlj-hello-2.12.1"
└──bin
    └──hello

[shell:~]$ TODO get nix depencies
/nix/store/s2f1sqfsdi4pmh23nfnrh42v17zsvi5y-libunistring-1.1
/nix/store/08n25j4vxyjidjf93fyc15icxwrxm2p8-libidn2-2.3.4
/nix/store/lmidwx4id2q87f4z9aj79xwb03gsmq5j-xgcc-12.3.0-libgcc
/nix/store/qn3ggz5sf3hkjs2c797xf7nan3amdxmp-glibc-2.38-27
/nix/store/sbldylj3clbkc0aqvjjzfa6slp4zdvlj-hello-2.12.1
```

Figure 2.1.5: Dependency graph
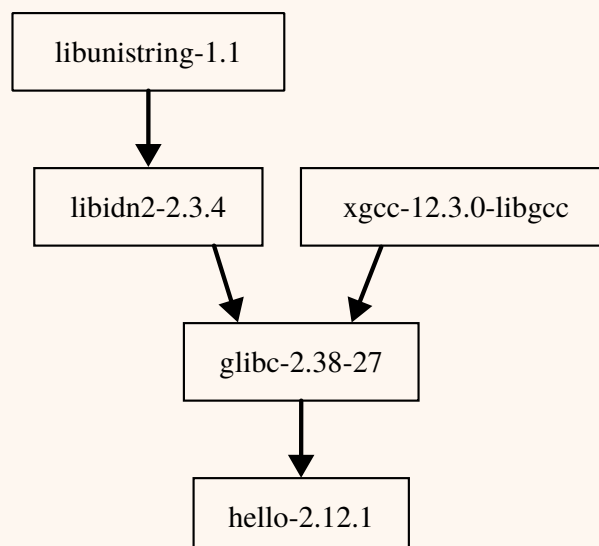
For an example of making our own nix package. I have create a flake in Fig that builds the basic "hello" package also available on `nixpkgs`.

Highlighting particular lines in the `flake.nix`

Line 2: We have specified that we want to build our flake with the stable **nix channel** `nixos-23.11`, the most recent channel at the time of writing. This "channel" is really just a release branch on the nixpkgs github repository. Channels do receive conservative updates such as bug fixes and security patches but no major updates after initial release. The first time I build the hello package from my `flake.nix` a `flake.lock` is automatically generated that pins us to a specific revision of `nixos-23.11`. Our built inputs will not change until we relock our flake to either a different revision of `nixos-23.11` or a new channel entirely.

Line 5: Here we define `outputs` as a function that accepts, `self` (the flake) and `nixpkgs` (the set of packages we just pinned to on the line 2). What nix does is resolves all inputs and then calls the `output` function.

Line 6: Here we specify that we are defining the default package for users on `x86_64-linux`. If we tried to build this package on a different cpu architecture like for example ARM (`aarch64-linux`) the flake would refuse to build the package as it has not been defined for ARM yet. If we desired we could fix this by adding a `defaultPackage.aarch64-linux` definition.

Line 7-9: Here we are just defining a shorthand way to referring to x86 linux packages. This syntax is similar if not identical to Haskell.

Line 10: Here we begin the definition of the derivation which is the instruction set nix uses to build the package.

Line 14: We specify here that we need `gcc` in our sandbox to build our package. `gcc` here is shorthand for `gcc12` but we could specify and c compiler and version of that compiler we liked. If you really wanted to you could compile different parts of the code with different versions of gcc.

Line 15: Here we are slightly abusing the configure phase to generate a hello.c file. Each phase is essentially run as a bash script. Everything inside `mkDerivation` is happening inside a sandbox and will be discarded once the package is built (techically after we garbage collect).

Line 16: Here we actually build our package

Line 17: In this line we copy our executable we have generate which is currently in the sandbox into the actual package we are producing.

2.2   Perspective Projection

Figure 2.2.1: Orthographic and perspective projections
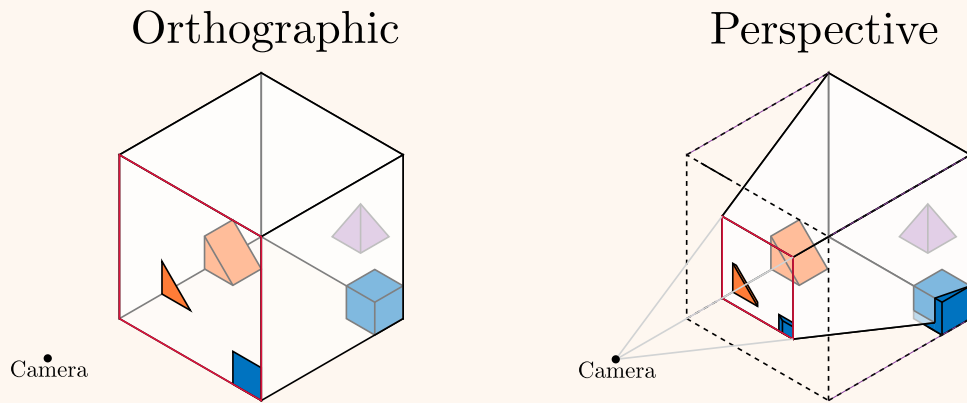
To represent 3D objects on a 2D surface (our screen) OpenGL support two type of projections: perspective and orthographic as seen in Fig 2.2.1. Orthographic features parallel projection lines (orthogonal to the projection plane), which means that it does not depict the effect of perspective. Distances are preserved, making it useful for technical drawings where measurements need to be accurate and unskewed by perspective (All diagrams in this report are in the orthographic perspective). Unlike orthographic projection, perspective projection simulates the way the human eye perceives the world, with objects appearing smaller as they are farther from the viewpoint as the projection lines converge at a vanishing point. To create the illusion of 3D in this project we must use a perspective projection.

Figure 2.2.2: Using frustum to generate a perspective projection
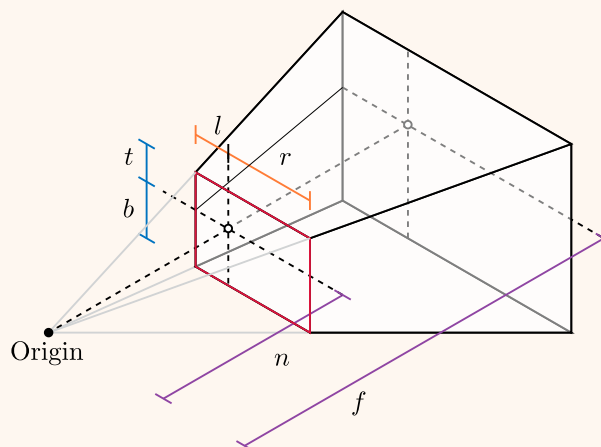
OpenGL provides the `frustum` function as seen in Fig 2.2.2 which can be used to construct a

perspective matrix Fig 2.1 that maps a specified viewing frustum screen-space (with intermediate steps handelled by OpenGL) ToCite . The viewing frustum, is specified by six parameters: f, l, r, b, t, n which represent left, right, bottom, top, near, and far. These parameters define the sides of the near clipping plane, highlighted in red, relative to the origin of the coordinate system. These parameters do not represent distances or magnitudes in a traditional sense but rather define the vectors from the center of the near clipping plane to its edges.

$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Figure 2.1: frustum perspective matrix

The l and r parameters specify the horizontal boundaries of the frustum on the near clipping plane, with left typically being a negative value and right a positive value, defining the extent to which the frustum extends to the left and right of the origin. Similarly, the b and t parameters determine the vertical boundaries, with bottom often negative and top positive, expressing the extent of the frustum below and above the origin.

The n and f parameters are scalar values that specify the distances from the origin to the near and far clipping planes along the view direction. Altering the value of n will change the angles of the lines (or vectors) that connect the corners of the near plane to the eye, effectively changing the "field of view". Changing the value f affects the range of depth that is captured within the scene.

If we are able to track the position of a viewers eye in real time then we can create the illusion of a 3D scene behind and in front of a display using `frustum`. This can be done fairly trivially following Robert Kooima's method he sets out in "Generalised Perspective Projection" to calculate f, l, r, b, t, n as the viewers eye moves [10].

To encode the position and size of the screen we take 3 points, p_a, p_b and p_c which represent the lower-left, lower-right and upper left points of the screen respectively when viewed from the front on. These points are in tracker space, the co-ordinate system of the device we use to track the eyes. These point can be used to generate an orthonormal basis of the screen of s_r, s_u and s_n which represents the directions up, right and normal to the screen respectively as seen in Fig 2.2.3. We can compute these values from the screen corners as follows:

$$s_r = \frac{p_b - p_a}{||p_b - p_a||} \quad s_u = \frac{p_c - p_a}{||p_c - p_a||} \quad s_n = \frac{s_r \times s_u}{||s_r \times s_u||}$$

Figure 2.2.3: Defining a screen in 3D space
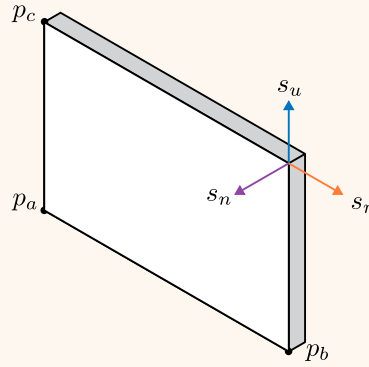
Introducing the viewers eye which we will refer to as p_e. We can draw three vectors v_a, v_b, v_c from the viewers eye p_e to the corners of the screen p_a, p_b, p_c as seen in Fig 2.2.4. In the diagram we also have labelled the components of each of these vectors in the basis of the screen. We can compute these as follows:

$$v_a = p_a - p_e \quad v_b = p_b - p_e \quad v_c = p_c - p_e$$

To calculate the required values for `frustum` we must first find the point where line drawn perpendicular to the plane of the screen that passes through p_e strikes the plane of the screen. We refer to this point as the *screen-space-origin*, it is worth noting that this point can lie outside the screen (the rectangle bounded by p_a, p_b, p_c). We can find the distance of the *screen-space-origin* from the eye p_e by taking its component of any of v_a, v_b, v_c in the screen basis vector s_n, however as s_n is in the opposite direction we must invert it. Similarly we can calculate t by taking the component of v_c in the basis vector s_u, b by v_b in s_u, l by v_c in s_r and lastly r by v_b in s_r. We can compute these as follows:

$$d = -(s_n \cdot v_a) \quad l = (v_c \cdot s_r) \quad r = (v_b \cdot s_r) \quad b = (v_c \cdot s_u) \quad t = (v_b \cdot s_u)$$

Figure 2.2.4: Screen Intersection with view
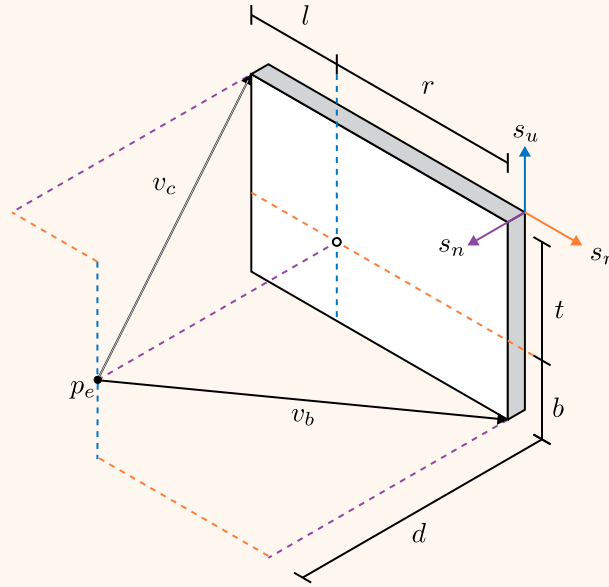
We can now generate a projection matrix by calling `frustum` using d as our nearClipping plane distance n with an arbitrary value for the farClipping plane f. We have now successfully generated our viewing frustum but we still have two problems. Firstly our frustum has been defined in tracker space so is pointed in the direction of our camera not the normal of our screen. We can remerdy this problem by using a rotation matrix M to align our frustum with s_n, s_u and s_r, the basis of our screen as seen in Fig 2.2.5. M is defined as follows:

$$\begin{bmatrix} v_{rx} & v_{ry} & v_{rz} & 0 \\ v_{ux} & v_{uy} & v_{uz} & 0 \\ v_{nx} & v_{ny} & v_{nz} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Figure 2.2.5: Moving the frustum from tracker space to screen space

Tracker aligned   Screen aligned

$\times M$
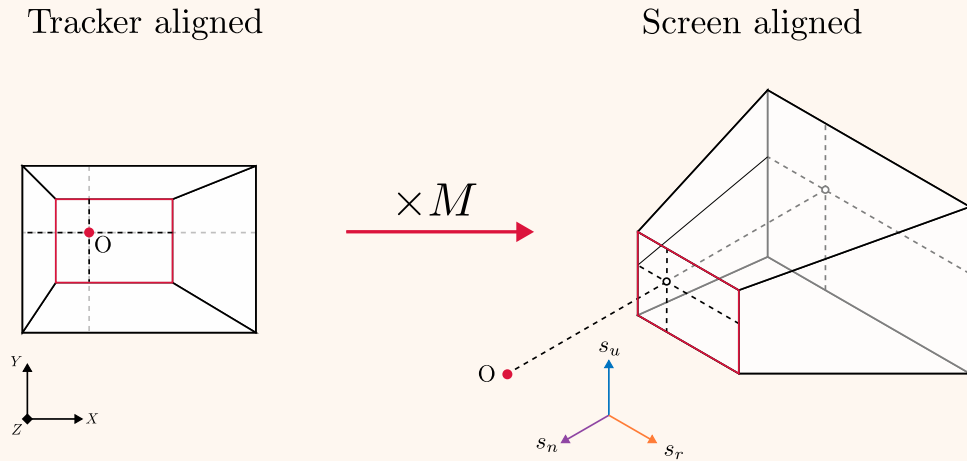
The second problem we have is that we want our projection matrix to move around with the viewers eye however the mathematics of perspective projection disallow this, with the camera forever trapped at the origin. To translate our viewing frustum to our eye position we must instead translate our eye position (and the whole world) to the apex/origin of our frustum. This can be done with a translation matrix T as seen in Fig 2.2.6. T T can be generated with the OpenGL function `translate` where we want to offset it by the vector from our Origin to the viewers eye p_e. T is defined as follows:

$$\begin{bmatrix} 1 & 0 & 0 & -p_{ex} \\ 0 & 1 & 0 & -p_{ey} \\ 0 & 0 & 1 & -p_{ez} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Figure 2.2.6: Translating the viewing frustum to sit inside the screen
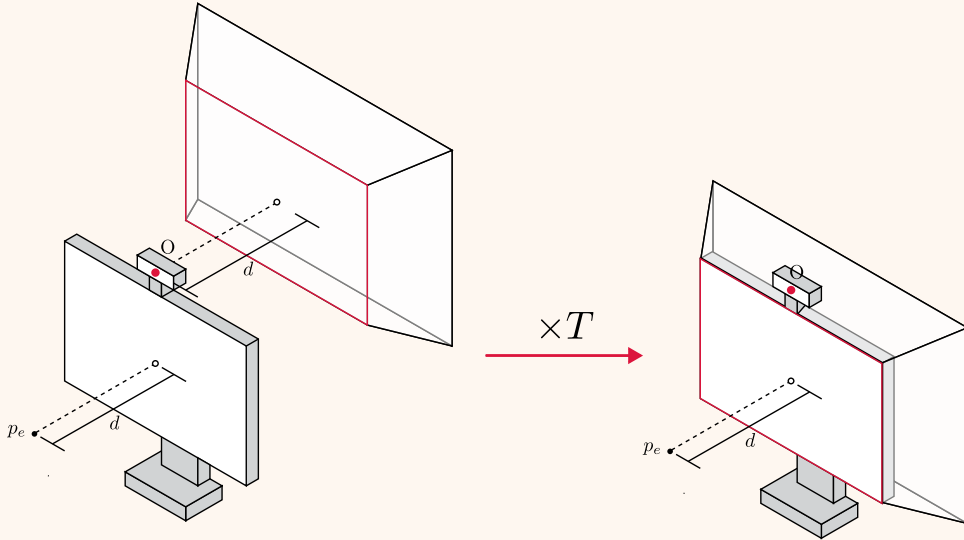
We now have a working method for projecting virtual objects behind our screen onto our screen however it is also possible if we desire to project objects in front of the screen onto the screen as well as long as they lie within the pyramid formed between the edges of the screen and the viewers eye. We can use similar triangles to scale near clipping plane from the plane of the screen to a small distance n from our eye as seen in Fig 2.2.7. We now have scaled down values of t, b l and r we can use for our new viewing frustum which we call t_n, b_n l_n and r_n. They are defined as follows:

$$l_n = (v_c \cdot s_r)\frac{n}{d} \quad r_n = (v_b \cdot s_r)\frac{n}{d} \quad b_n = (v_c \cdot s_u)\frac{n}{d} \quad t_n = (v_b \cdot s_u)\frac{n}{d}$$

So our final viewing frustum takes in frustum extents t_n, b_n l_n and r_n and n and f defining the distances to the near and far clipping plane.

Figure 2.2.7: Extending the near plane to not clip out objects in front of the screen
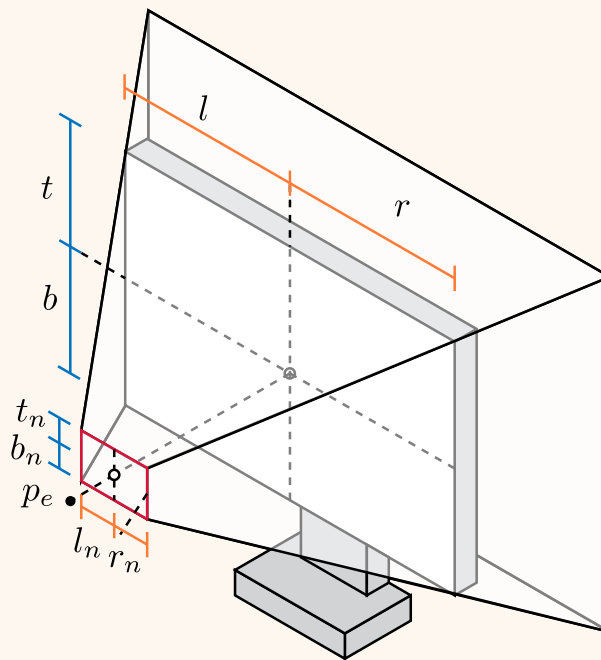
Below I have attached some sample code of a function implementing the process we just described.

Listing 2.2.1: projection.cpp, Sample code for creating the 3D illusion projection

```cpp
#include <glad/gl.h>
#include <glm/glm.hpp>
#include <glm/gtc/matrix_transform.hpp>

using namespace glm;

mat4 projectionToEye(vec3 pa, vec3 pb, vec3 pc, vec3 eye, GLfloat n,
↪   GLfloat f)
{
    // Orthonormal basis of the screen
    vec3 sr = normalize(pb - pa);
    vec3 su = normalize(pc - pa);
    vec3 sn = normalize(cross(sr, su));

    // Vectors from eye to opposite screen corners
    vec3 vb = pb - eye;
    vec3 vc = pc - eye;

    // Distance from eye to screen
    GLfloat d = -dot(sn, vc);

    // Frustum extents (scaled to the near clipping plane)
    GLfloat l = dot(sr, vc) * n / d;
    GLfloat r = dot(sr, vb) * n / d;
    GLfloat b = dot(su, vb) * n / d;
    GLfloat t = dot(su, vc) * n / d;

    // Create the projection matrix
    mat4 projMatrix = frustum(l, r, b, t, n, f);

    // Rotate the projection to be aligned with screen basis.
    mat4 rotMatrix(1.0f);
    rotMatrix[0] = vec4(sr, 0);
    rotMatrix[1] = vec4(su, 0);
    rotMatrix[2] = vec4(sn, 0);

    // Translate the world so the eye is at the origin of the viewing
    ↪   frustum
    mat4 transMatrix = translate(mat4(1.0f), -eye);

    return projMatrix * rotMatrix * transMatrix;
}
```

Figure 2.2: c++ Sample code for creating the 3D illusion projection

2.3 3D displays

2.3.1 Volumetric Displays

Volumetric displays [7] are a promising technology that offers a captivating three-dimensional viewing experience. By emitting light for each voxel, or volume element, in a 3D space, these displays transcend the limitations of traditional 2D planes, providing a truly immersive 3D effect. This innovative approach enables the accurate representation of virtual 3D objects, including focal depth, motion parallax, and vergence, which refers to the rotation of a viewer's eye to fixate on the same point they are focusing on . Moreover, volumetric displays allow multiple users to view the same display from different angles, providing unique perspectives of the same object.
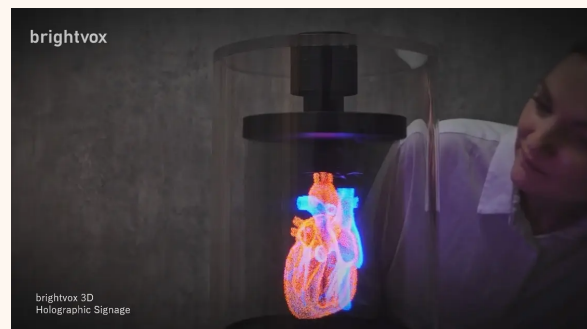
Swept Volume Displays

Swept volume displays represent one category of volumetric displays. They employ a moving 2D display to create a 3D image. This is achieved by moving the 2D display through a 3D space and emitting light from the display at each point. Common techniques for achieving this includes using a rotating mirror, emitting screen typically an LED [9], or transparent surface that can be projected onto. There currently exist commercial products that implement this technology, Two examples can be seen in Fig 2.3.1.

Figure 2.3.1: Two different types of swept volume display

a) The VXR4612 3D Volumetric Display, a projector based persistence of vision display produced by Voxon Photonics.

b) A Volumetric Display / Holographic Signage, a LED based persistence of vision display produced by Brightvox Inc.

Static Volume Displays

Static volume displays are another category of volumetric displays. They employ a static 3D display to create a 3D image. This is achieved by emitting light from the display at each point in a 3D space. Common techniques for achieving this includes using a 3D array of LEDs, lasers, or a transparent medium that can be projected onto. Because they do not reuse the same 2D display, static volume displays are often lower resolution than swept volume displays.

However, they are also more versatile, as they can be used to display a wider range of 3D objects.

Chapter 3

Contribution

Chapter 4

Experimental Results

Chapter 5

Conclusion

Bibliography

[1]   URL: https://nixos.wiki/wiki/Nix_Community (visited on 01/09/2024).

[2]   Burr, Chris, Clemencic, Marco, and Couturier, Ben. "Software packaging and distribution for LHCb using Nix". In: *EPJ Web Conf.* 214 (2019), p. 05005. DOI: 10.1051/epjconf/201921405005. URL: https://doi.org/10.1051/epjconf/201921405005.

[3]   Bruno Bzeznik et al. "Nix as HPC Package Management System". In: *Proceedings of the Fourth International Workshop on HPC User Support Tools*. HUST'17. Denver, CO, USA: Association for Computing Machinery, 2017. ISBN: 9781450351300. DOI: 10.1145/3152493.3152556. URL: https://doi.org/10.1145/3152493.3152556.

[4]   Eelco Dolstra. *The purely functional software deployment model*. Utrecht University, 2006.

[5]   Eelco Dolstra, Merijn De Jonge, Eelco Visser, et al. "Nix: A Safe and Policy-Free System for Software Deployment." In: *LISA*. Vol. 4. 2004, pp. 79–92.

[6]   Eelco Dolstra and Andres Löh. "NixOS: A Purely Functional Linux Distribution". In: *Proceedings of the 13th ACM SIGPLAN International Conference on Functional Programming*. ICFP '08. Victoria, BC, Canada: Association for Computing Machinery, 2008, pp. 367–378. ISBN: 9781595939197. DOI: 10.1145/1411204.1411255. URL: https://doi.org/10.1145/1411204.1411255.

[7]   G.E. Favalora. "Volumetric 3D displays and application infrastructure". In: *Computer* 38.8 (2005), pp. 37–44. DOI: 10.1109/MC.2005.276.

[8]   NixOS Foundation. URL: https://github.com/NixOS/nixpkgs (visited on 01/09/2024).

[9]   Matthew Gately et al. "A Three-Dimensional Swept Volume Display Based on LED Arrays". In: *J. Display Technol.* 7.9 (Sept. 2011), pp. 503–514. URL: https://opg.optica.org/jdt/abstract.cfm?URI=jdt-7-9-503.

[10]   Robert Kooima. "Generalized perspective projection". In: (2009).

[11]   Markus Kowalewski and Phillip Seeber. "Sustainable packaging of quantum chemistry software with the Nix package manager". In: *International Journal of Quantum Chemistry* 122.9 (2022), e26872. DOI: https://doi.org/10.1002/qua.26872. eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1002/qua.26872. URL: https://onlinelibrary.wiley.com/doi/abs/10.1002/qua.26872.

[12]   Dmitry Marakasov. Jan. 2024. URL: https://repology.org/repositories/graphs (visited on 01/09/2024).

[13]   Jörg Thalheim. *About Nix sandboxes and breakpoints (NixCon 2018)*. Youtube. 2018. URL: https://www.youtube.com/watch?v=ULqoCjANK-I (visited on 01/09/2024).