Greedy Problem 9

4 September 2016

1 Problem 9:

1.1 A

The algorithm whereby the skier and ski whose height difference is minimized gets assigned first and then the process is repeated is incorrect.

Consider the following counter example:

$$\text{Skiers } = \{1, 2, 3\}$$

$$\text{Skis } = \{2.5, 2.6, 3.6\}$$

This greedy algorithm will produce the following pairings:

$$\text{(Skiers, Skis) } = \{(1, 3.6), (2, 2.5), (3, 2.6)\}$$

This has an average height difference of 3.5/3.

An optimal pairing would be:

$$\text{(Skiers, Skis) } = \{(1, 2.5), (2, 2.6), (3, 3.6)\}$$

This has an average height difference of 2.7/3.

1.2 A

This greedy algorithm can be shown to be correct using an exchange argument.

Let Alg be the process by which the greedy algorithm operates. Assume there exists some input I such that $Alg(I)$ is incorrect.

Let $Opt(I)$ be the optimal output for input I that agrees with $Alg(I)$ for the greatest number of steps.

Since $Opt(I)$ cannot equal $Alg(I)$, there must be an earliest step of disagreement. Label this step i.