Homework 2

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Question 4 Part A

Let S be the set of all gas stations. Let $G(A)$ be the output of the greedy algorithm on input A. Assume $G_A(A)$ is not optimal for proof by contradiction.

Let *Opt* be an optimal solution.

There are two possible scenarios: Either $G(A)$ and *Opt* select the same gas stations to stop at or they do not. First, we address the former before tackling the latter.

Assume that $G(A)$ and *Opt* select all of the same gas stations. Then, for an arbitrary gas station p, let the amount of time $G(A)$ spends filling up be x. Now, since *Opt* must also stop at that gas station, and $Opt \neq G(A)$, *Opt* must spend more time filling up than $G(A)$, since $G(A)$ fills up the minimum at every stop. Thus, *Opt* stops for some $y : x \leq y$. Since *Opt* and $G(A)$ stop at all the same gas stations, total time t is

$$t_{Opt} = \sum_{p \in S} y_p$$

for *Opt*, and

$$t_{G(A)} = \sum_{p \in S} x_p$$

for $G(A)$. By the properties of addition, $t_{G(A)} \leq t_{Opt}$, and thus is an optimal solution. $\perp$. Since we have resolved a contradiction, our assumption must be false. Thus, $G(A)$ is optimal and a solution, in the case where $G(A)$ and Opt stop at all of the same gas stations.

Now, we address the case where Opt and $G(A)$ do not stop at the same gas stations. Since $\forall p \in S.G(A)$ stops at p., then $\exists p \in S.Opt$ does not stop at p.

Let S be the gas station before p.

Let $Opt' = Opt$ such that Opt stops at S and fills the distance necessary to reach p, just as $G(A)$ does, and subtract that amount from the amount filled up at p.

Opt' is at least as optimal as Opt since the total amount of time spent filling is the same. It is obvious that Opt' is more like $G(A)$ than Opt, which violates our assumption. $\perp$.

Thus, $G(A)$ is an optimal solution.

Question 4 Part B

This algorithm does not work. Consider the following counterexample:

Let $F = 1\text{liter}/95\text{kilo}$. Let $C = 1\text{liter}$. Let $r = 1\text{liter}/95\text{minutes}$. Let the distance between A and B be 100kilos. Let there be a single gas station 95 miles from A and 5 miles from B (that is, at the 95th mile marker).

The greedy algorithm specified will stop at the gas station and fill the tank 1 liter. Filling the tank 1 liter takes 95 minutes. An optimal solution is to stop at the gas station and fill the tank 1/19 liters, which will only take 5 minutes.

Question 5 Part A

Question 5 Part B