Homework 1

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Question 1

Proof by contradiction:

Assume that there is an input J on which the greedy algorithm is incorrect.

Let $Alg(J)$ = the output on input J. Let $Opt(J)$ be the optimal solution on input J.

Let i be index of the output sequence generated by the picking rule for each algorithm, and let Alg_i be the i element in $Alg(J)$ output sequence, and Opt_i be the ith element in the $Opt(J)$ output sequence.

Let k be the first index in the output sequence where $Alg_k \neq Opt_k$.

Since Alg and Opt picked different elements, Alg_k overlaps with fewer elements than Opt_k. This means that for the next selection $k + 1$, Alg has more elements in the pool of selectable elements than Opt. Since there are at least as many elements available in the set of selectable elements, Alg can always select another element if Opt can select another element. This proves that Alg will always have the same cardinality as Opt when selecting element $k+1$. Since Alg and Opt have the same cardinality, Alg is correct. $\perp$. Thus, our assumption is wrong, and Alg must be a correct solution to the problem.

Question 2 Part A

We believe that the algorithm is a solution.

We believe this because, as noted, the algorithm cannot have a number

of colors less than s, which is fairly obvious. It also cannot have a number of colors more than s because each of the colors are necessarily disjoint. The colors are disjoint because if they were not, that would violate the picking criteria which eliminates intervals that are not disjoint. Since they are disjoint, recoloring an interval would result in a collision.

Question 2 Part B

We believe that the algorithm is a solution.

We believe this because, as noted, the algorithm cannot have a number of colors less than s, which is fairly obvious; otherwise, there would necessarily be a conflict. It also cannot have a number of colors more than s because before coloring an interval with a new color, we look to see whether or not it could be colored with a pre-existing color. The only time where it would conflict is if all other rooms had an interval colored already. This is, by definition, s.

Question 3 Part A

The shillings greedy algorithm is not correct. As a counter example, consider 40 shillings. The greedy algorithm would occasional would select 25, then 10, then 5 to make 40. However, an optimal solution would be to take 20 twice.

Question 3 Part B

Proof by contradiction:

Assume that there is an input I on which the greedy algorithm is incorrect. Let $Opt(I)$ be a solution. Let $G(I)$ be the output of the greedy algorithm.

Let i be the first coin on which Opt and G disagree. Since G always takes the highest valued coin, that means that $G_i > Opt_i$. Thus, Opt must select from the remaining coins a selection which sums to G_i. Then, Opt has to take at least 2 of a single coin from the remaining coins, because 1 of each remaining coin is necessarily less than the value of G_i i.e. $2^n > 2^{n-1} + 2^{n-2} ... + 2^0$.

If an algorithm were to select a single coin twice, then it would be able to select that same amount with fewer coins by taking the least-valued coin greater than the selected coin once e.g. $2 * 2^k = 2^{k+1}$. Thus, any algorithm that takes two of a single coin is not a solution, since there exists an algorithm that could take one fewer coin by exchanging the duplicates for a single higher valued coin. Since Opt takes 2 coins, it is not a solution. $\perp$.

Thus, our assumption is false, and the Greedy algorithm is correct.