

- the maximum possible speed and the corresponding kinetic energy. (d) Differentiate the equation for u with respect to time to obtain an equation describing the acceleration of a particle as a function of its kinetic energy and the power input to the particle. (e) Observe that for a nonrelativistic particle we have $u = (2K/m)^{1/2}$ and that differentiating this equation with respect to time gives $a = P/(2mK)^{1/2}$. State the limiting form of the expression in part (d) at low energy. State how it compares with the nonrelativistic expression. (f) State the limiting form of the expression in part (d) at high energy. (g) Consider a particle with constant input power. Explain how the answer to part (f) helps account for the answer to part (c).
- 75.** Consider the astronaut planning the trip to Andromeda in Problem 71. (a) To three significant figures, what is the value for γ for the speed found in part (a) of Problem 71? (b) Just as the astronaut leaves on his constant-speed trip, a light beam is also sent in the direction of Andromeda. According to the Earth observer, how much later does the astronaut arrive at Andromeda after the arrival of the light beam?
- 76.** An object disintegrates into two fragments. One fragment has mass $1.00 \text{ MeV}/c^2$ and momentum $1.75 \text{ MeV}/c$ in the positive x direction, and the other has mass $1.50 \text{ MeV}/c^2$ and momentum $2.00 \text{ MeV}/c$ in the positive y direction. Find (a) the mass and (b) the speed of the original object.
- 77.** The cosmic rays of highest energy are protons that have kinetic energy on the order of 10^{13} MeV . (a) As measured in the proton's frame, what time interval would a proton of this energy require to travel across the Milky Way galaxy, which has a proper diameter $\sim 10^5 \text{ ly}$? (b) From the point of view of the proton, how many kilometers across is the galaxy?
- 78.** **M** Spacecraft I, containing students taking a physics exam, approaches the Earth with a speed of $0.600c$ (relative to the Earth), while spacecraft II, containing professors proctoring the exam, moves at $0.280c$ (relative to the Earth) directly toward the students. If the professors stop the exam after 50.0 min have passed on their clock, for what time interval does the exam last as measured by (a) the students and (b) an observer on the Earth?
- 79. Review.** Around the core of a nuclear reactor shielded by a large pool of water, Cerenkov radiation appears as a blue glow. (See Fig. P17.38 on page 528.) Cerenkov radiation occurs when a particle travels faster through a medium than the speed of light in that medium. It is the electromagnetic equivalent of a bow wave or a sonic boom. An electron is traveling through water at a speed 10.0% faster than the speed of light in water. Determine the electron's (a) total energy, (b) kinetic energy, and (c) momentum. (d) Find the angle between the shock wave and the electron's direction of motion.
- 80.** The motion of a transparent medium influences the speed of light. This effect was first observed by Fizeau in 1851. Consider a light beam in water. The water moves with speed v in a horizontal pipe. Assume the light travels in the same direction as the water moves. The speed of light with respect to the water is c/n , where $n = 1.33$ is the index of refraction of water. (a) Use the velocity transformation equation to show that the speed of the light measured in the laboratory frame is
- $$u = \frac{c}{n} \left(\frac{1 + nv/c}{1 + v/nc} \right)$$
- (b) Show that for $v \ll c$, the expression from part (a) becomes, to a good approximation,
- $$u \approx \frac{c}{n} + v - \frac{v}{n^2}$$
- (c) Argue for or against the view that we should expect the result to be $u = (c/n) + v$ according to the Galilean transformation and that the presence of the term $-v/n^2$ represents a relativistic effect appearing even at "nonrelativistic" speeds. (d) Evaluate u in the limit as the speed of the water approaches c .
- 81.** Imagine that the entire Sun, of mass M_S , collapses to a sphere of radius R_g such that the work required to remove a small mass m from the surface would be equal to its rest energy mc^2 . This radius is called the *gravitational radius* for the Sun. (a) Use this approach to show that $R_g = GM_S/c^2$. (b) Find a numerical value for R_g .
- 82.** *Why is the following situation impossible?* An experimenter is accelerating electrons for use in probing a material. She finds that when she accelerates them through a potential difference of 84.0 kV, the electrons have half the speed she wishes. She quadruples the potential difference to 336 kV, and the electrons accelerated through this potential difference have her desired speed.
- 83.** An alien spaceship traveling at $0.600c$ toward the Earth launches a landing craft. The landing craft travels in the same direction with a speed of $0.800c$ relative to the mother ship. As measured on the Earth, the spaceship is 0.200 ly from the Earth when the landing craft is launched. (a) What speed do the Earth-based observers measure for the approaching landing craft? (b) What is the distance to the Earth at the moment of the landing craft's launch as measured by the aliens? (c) What travel time is required for the landing craft to reach the Earth as measured by the aliens on the mother ship? (d) If the landing craft has a mass of $4.00 \times 10^5 \text{ kg}$, what is its kinetic energy as measured in the Earth reference frame?
- 84.** (a) Prepare a graph of the relativistic kinetic energy and the classical kinetic energy, both as a function of speed, for an object with a mass of your choice. (b) At what speed does the classical kinetic energy underestimate the experimental value by 1%? (c) By 5%? (d) By 50%?
- 85.** **AMT** An observer in a coasting spacecraft moves toward a mirror at speed $v = 0.650c$ relative to the reference frame labeled S in Figure P39.85 (page 1232). The mirror is stationary with respect to S. A light pulse emitted by the spacecraft travels toward the mirror and is

reflected back to the spacecraft. The spacecraft is a distance $d = 5.66 \times 10^{10}$ m from the mirror (as measured by observers in S) at the moment the light pulse leaves the spacecraft. What is the total travel time of the pulse as measured by observers in (a) the S frame and (b) the spacecraft?

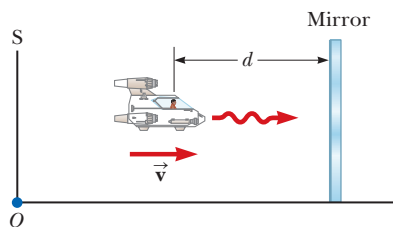


Figure P39.85 Problems 85 and 86.

86. An observer in a coasting spacecraft moves toward a mirror at speed v relative to the reference frame labeled S in Figure P39.85. The mirror is stationary with respect to S. A light pulse emitted by the spacecraft travels toward the mirror and is reflected back to the spacecraft. The spacecraft is a distance d from the mirror (as measured by observers in S) at the moment the light pulse leaves the spacecraft. What is the total travel time of the pulse as measured by observers in (a) the S frame and (b) the spacecraft?
87. A ^{57}Fe nucleus at rest emits a 14.0-keV photon. Use conservation of energy and momentum to find the kinetic energy of the recoiling nucleus in electron volts. Use $Mc^2 = 8.60 \times 10^{-9}$ J for the final state of the ^{57}Fe nucleus.

Challenge Problems

88. A particle with electric charge q moves along a straight line in a uniform electric field \vec{E} with speed u . The electric force exerted on the charge is $q\vec{E}$. The velocity of the particle and the electric field are both in the x direction. (a) Show that the acceleration of the particle in the x direction is given by

$$a = \frac{du}{dt} = \frac{qE}{m} \left(1 - \frac{u^2}{c^2} \right)^{3/2}$$

- (b) Discuss the significance of the dependence of the acceleration on the speed. (c) **What If?** If the particle starts from rest at $x = 0$ at $t = 0$, how would you proceed to find the speed of the particle and its position at time t ?
89. The creation and study of new and very massive elementary particles is an important part of contemporary physics. To create a particle of mass M requires an energy Mc^2 . With enough energy, an exotic particle can be created by allowing a fast-moving proton to collide with a similar target particle. Consider a perfectly inelastic collision between two protons: an incident proton with mass m_p , kinetic energy K , and momentum magnitude p joins with an originally stationary target proton to form a single product particle of mass M . Not all the kinetic energy of the incoming proton is available to create the product particle because conservation of

momentum requires that the system as a whole still must have some kinetic energy after the collision. Therefore, only a fraction of the energy of the incident particle is available to create a new particle. (a) Show that the energy available to create a product particle is given by

$$Mc^2 = 2m_p c^2 \sqrt{1 + \frac{K}{2m_p c^2}}$$

This result shows that when the kinetic energy K of the incident proton is large compared with its rest energy $m_p c^2$, then M approaches $(2m_p K)^{1/2}/c$. Therefore, if the energy of the incoming proton is increased by a factor of 9, the mass you can create increases only by a factor of 3, not by a factor of 9 as would be expected. (b) This problem can be alleviated by using *colliding beams* as is the case in most modern accelerators. Here the total momentum of a pair of interacting particles can be zero. The center of mass can be at rest after the collision, so, in principle, all the initial kinetic energy can be used for particle creation. Show that

$$Mc^2 = 2mc^2 \left(1 + \frac{K}{mc^2} \right)$$

where K is the kinetic energy of each of the two identical colliding particles. Here, if $K \gg mc^2$, we have M directly proportional to K as we would desire.

90. Suppose our Sun is about to explode. In an effort to escape, we depart in a spacecraft at $v = 0.800c$ and head toward the star Tau Ceti, 12.0 ly away. When we reach the midpoint of our journey from the Earth, we see our Sun explode, and, unfortunately, at the same instant, we see Tau Ceti explode as well. (a) In the spacecraft's frame of reference, should we conclude that the two explosions occurred simultaneously? If not, which occurred first? (b) **What If?** In a frame of reference in which the Sun and Tau Ceti are at rest, did they explode simultaneously? If not, which exploded first?
91. Owen and Dina are at rest in frame S' , which is moving at $0.600c$ with respect to frame S. They play a game of catch while Ed, at rest in frame S, watches the action (Fig. P39.91). Owen throws the ball to Dina at $0.800c$ (according to Owen), and their separation (measured in S') is equal to 1.80×10^{12} m. (a) According to Dina, how fast is the ball moving? (b) According to Dina, what time interval is required for the ball to reach her? According to Ed, (c) how far apart are Owen and Dina, (d) how fast is the ball moving, and (e) what time interval is required for the ball to reach Dina?

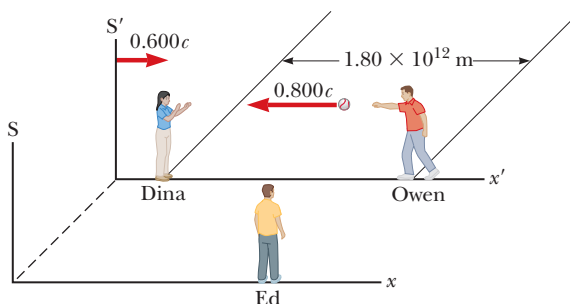
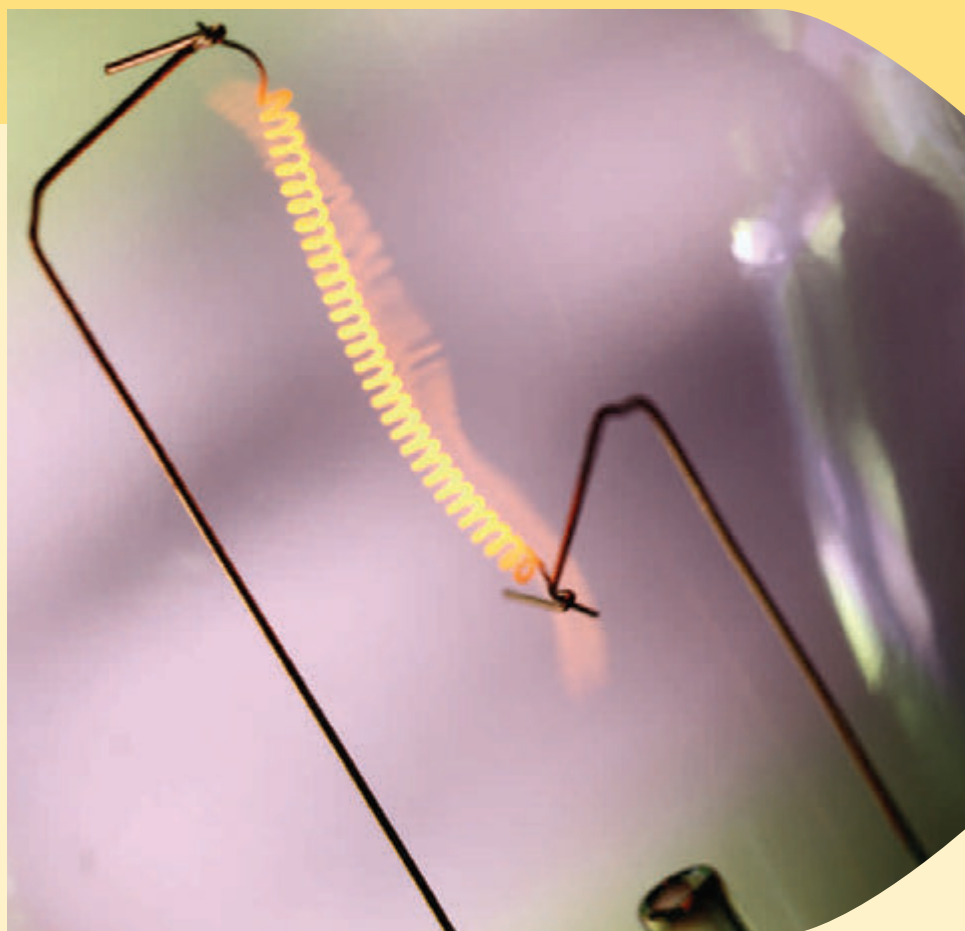


Figure P39.91

Introduction to Quantum Physics

C H A P T E R

40



- 40.1 Blackbody Radiation and Planck's Hypothesis
- 40.2 The Photoelectric Effect
- 40.3 The Compton Effect
- 40.4 The Nature of Electromagnetic Waves
- 40.5 The Wave Properties of Particles
- 40.6 A New Model: The Quantum Particle
- 40.7 The Double-Slit Experiment Revisited
- 40.8 The Uncertainty Principle

In Chapter 39, we discussed that Newtonian mechanics must be replaced by Einstein's special theory of relativity when dealing with particle speeds comparable to the speed of light. As the 20th century progressed, many experimental and theoretical problems were resolved by the special theory of relativity. For many other problems, however, neither relativity nor classical physics could provide a theoretical answer. Attempts to apply the laws of classical physics to explain the behavior of matter on the atomic scale were consistently unsuccessful. For example, the emission of discrete wavelengths of light from atoms in a high-temperature gas could not be explained within the framework of classical physics.

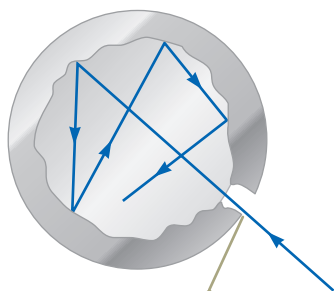
As physicists sought new ways to solve these puzzles, another revolution took place in physics between 1900 and 1930. A new theory called *quantum mechanics* was highly successful in explaining the behavior of particles of microscopic size. Like the special theory of relativity, the quantum theory requires a modification of our ideas concerning the physical world.

The first explanation of a phenomenon using quantum theory was introduced by Max Planck. Many subsequent mathematical developments and interpretations were made by a number of distinguished physicists, including Einstein, Bohr, de Broglie, Schrödinger, and

This lightbulb filament glows with an orange color. Why? Classical physics is unable to explain the experimentally observed wavelength distribution of electromagnetic radiation from a hot object. A theory proposed in 1900 and describing the radiation from such objects represents the dawn of quantum physics. (Steve Cole/Getty Images)

Pitfall Prevention 40.1

Expect to Be Challenged If the discussions of quantum physics in this and subsequent chapters seem strange and confusing to you, it's because your whole life experience has taken place in the macroscopic world, where quantum effects are not evident.



The opening to a cavity inside a hollow object is a good approximation of a black body: the hole acts as a perfect absorber.

Figure 40.1 A physical model of a black body.



Figure 40.2 The glow emanating from the spaces between these hot charcoal briquettes is, to a close approximation, blackbody radiation. The color of the light depends only on the temperature of the briquettes.

Heisenberg. Despite the great success of the quantum theory, Einstein frequently played the role of its critic, especially with regard to the manner in which the theory was interpreted.

Because an extensive study of quantum theory is beyond the scope of this book, this chapter is simply an introduction to its underlying principles.

40.1 Blackbody Radiation and Planck's Hypothesis

An object at any temperature emits electromagnetic waves in the form of **thermal radiation** from its surface as discussed in Section 20.7. The characteristics of this radiation depend on the temperature and properties of the object's surface. Careful study shows that the radiation consists of a continuous distribution of wavelengths from all portions of the electromagnetic spectrum. If the object is at room temperature, the wavelengths of thermal radiation are mainly in the infrared region and hence the radiation is not detected by the human eye. As the surface temperature of the object increases, the object eventually begins to glow visibly red, like the coils of a toaster. At sufficiently high temperatures, the glowing object appears white, as in the hot tungsten filament of an incandescent lightbulb.

From a classical viewpoint, thermal radiation originates from accelerated charged particles in the atoms near the surface of the object; those charged particles emit radiation much as small antennas do. The thermally agitated particles can have a distribution of energies, which accounts for the continuous spectrum of radiation emitted by the object. By the end of the 19th century, however, it became apparent that the classical theory of thermal radiation was inadequate. The basic problem was in understanding the observed distribution of wavelengths in the radiation emitted by a black body. As defined in Section 20.7, a **black body** is an ideal system that absorbs all radiation incident on it. The electromagnetic radiation emitted by the black body is called **blackbody radiation**.

A good approximation of a black body is a small hole leading to the inside of a hollow object as shown in Figure 40.1. Any radiation incident on the hole from outside the cavity enters the hole and is reflected a number of times on the interior walls of the cavity; hence, the hole acts as a perfect absorber. The nature of the radiation leaving the cavity through the hole depends only on the temperature of the cavity walls and not on the material of which the walls are made. The spaces between lumps of hot charcoal (Fig. 40.2) emit light that is very much like blackbody radiation.

The radiation emitted by oscillators in the cavity walls in Figure 40.1 experiences boundary conditions and can be analyzed using the waves under boundary conditions analysis model. As the radiation reflects from the cavity's walls, standing electromagnetic waves are established within the three-dimensional interior of the cavity. Many standing-wave modes are possible, and the distribution of the energy in the cavity among these modes determines the wavelength distribution of the radiation leaving the cavity through the hole.

The wavelength distribution of radiation from cavities was studied experimentally in the late 19th century. Figure 40.3 shows how the intensity of blackbody radiation varies with temperature and wavelength. The following two consistent experimental findings were seen as especially significant:

1. The total power of the emitted radiation increases with temperature.

We discussed this behavior briefly in Chapter 20, where we introduced **Stefan's law**:

Stefan's law ►

$$P = \sigma A \epsilon T^4 \quad (40.1)$$

where P is the power in watts radiated at all wavelengths from the surface of an object, $\sigma = 5.670 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$ is the Stefan–Boltzmann constant, A is the surface area of the object in square meters, ϵ is the emissivity of the

surface, and T is the surface temperature in kelvins. For a black body, the emissivity is $e = 1$ exactly.

2. **The peak of the wavelength distribution shifts to shorter wavelengths as the temperature increases.** This behavior is described by the following relationship, called **Wien's displacement law**:

$$\lambda_{\text{max}} T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K} \quad (40.2)$$

where λ_{max} is the wavelength at which the curve peaks and T is the absolute temperature of the surface of the object emitting the radiation. The wavelength at the curve's peak is inversely proportional to the absolute temperature; that is, as the temperature increases, the peak is "displaced" to shorter wavelengths (Fig. 40.3).

Wien's displacement law is consistent with the behavior of the object mentioned at the beginning of this section. At room temperature, the object does not appear to glow because the peak is in the infrared region of the electromagnetic spectrum. At higher temperatures, it glows red because the peak is in the near infrared with some radiation at the red end of the visible spectrum, and at still higher temperatures, it glows white because the peak is in the visible so that all colors are emitted.

- Quick Quiz 40.1** Figure 40.4 shows two stars in the constellation Orion. Betelgeuse appears to glow red, whereas Rigel looks blue in color. Which star has a higher surface temperature? (a) Betelgeuse (b) Rigel (c) both the same (d) impossible to determine



John Chumack/Photo Researchers, Inc.

Figure 40.4 (Quick Quiz 40.1) Which star is hotter, Betelgeuse or Rigel?

A successful theory for blackbody radiation must predict the shape of the curves in Figure 40.3, the temperature dependence expressed in Stefan's law, and the shift of the peak with temperature described by Wien's displacement law. Early attempts to use classical ideas to explain the shapes of the curves in Figure 40.3 failed.

Let's consider one of these early attempts. To describe the distribution of energy from a black body, we define $I(\lambda, T) d\lambda$ to be the intensity, or power per unit area, emitted in the wavelength interval $d\lambda$. The result of a calculation based on a classical theory of blackbody radiation known as the **Rayleigh-Jeans law** is

$$I(\lambda, T) = \frac{2\pi c k_B T}{\lambda^4} \quad (40.3)$$

where k_B is Boltzmann's constant. The black body is modeled as the hole leading into a cavity (Fig. 40.1), resulting in many modes of oscillation of the electromagnetic field caused by accelerated charges in the cavity walls and the emission of electromagnetic waves at all wavelengths. In the classical theory used to derive

Wien's displacement law

The 4 000-K curve has a peak near the visible range. This curve represents an object that would glow with a yellowish-white appearance.

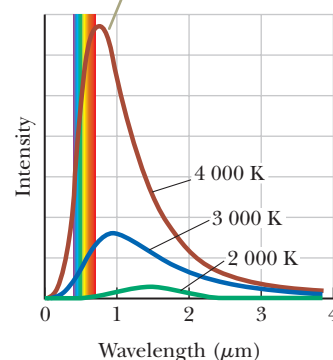


Figure 40.3 Intensity of blackbody radiation versus wavelength at three temperatures. The visible range of wavelengths is between $0.4 \mu\text{m}$ and $0.7 \mu\text{m}$. At approximately 6 000 K, the peak is in the center of the visible wavelengths and the object appears white.

Rayleigh-Jeans law

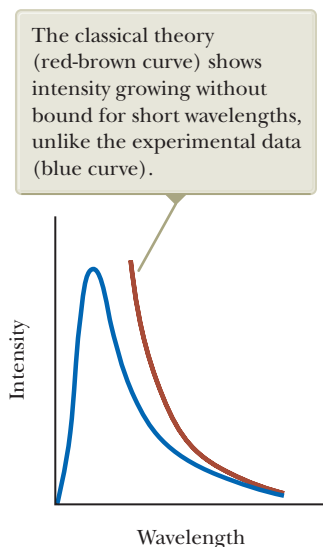


Figure 40.5 Comparison of experimental results and the curve predicted by the Rayleigh–Jeans law for the distribution of blackbody radiation.



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Max Planck

German Physicist (1858–1947)

Planck introduced the concept of “quantum of action” (Planck’s constant, h) in an attempt to explain the spectral distribution of blackbody radiation, which laid the foundations for quantum theory. In 1918, he was awarded the Nobel Prize in Physics for this discovery of the quantized nature of energy.

Equation 40.3, the average energy for each wavelength of the standing-wave modes is assumed to be proportional to $k_B T$, based on the theorem of equipartition of energy discussed in Section 21.1.

An experimental plot of the blackbody radiation spectrum, together with the theoretical prediction of the Rayleigh–Jeans law, is shown in Figure 40.5. At long wavelengths, the Rayleigh–Jeans law is in reasonable agreement with experimental data, but at short wavelengths, major disagreement is apparent.

As λ approaches zero, the function $I(\lambda, T)$ given by Equation 40.3 approaches infinity. Hence, according to classical theory, not only should short wavelengths predominate in a blackbody spectrum, but also the energy emitted by any black body should become infinite in the limit of zero wavelength. In contrast to this prediction, the experimental data plotted in Figure 40.5 show that as λ approaches zero, $I(\lambda, T)$ also approaches zero. This mismatch of theory and experiment was so disconcerting that scientists called it the *ultraviolet catastrophe*. (This “catastrophe”—infinite energy—occurs as the wavelength approaches zero; the word *ultra-violet* was applied because ultraviolet wavelengths are short.)

In 1900, Max Planck developed a theory of blackbody radiation that leads to an equation for $I(\lambda, T)$ that is in complete agreement with experimental results at all wavelengths. In discussing this theory, we use the outline of properties of structural models introduced in Chapter 21:

1. *Physical components:*

Planck assumed the cavity radiation came from atomic oscillators in the cavity walls in Figure 40.1.

2. *Behavior of the components:*

(a) The energy of an oscillator can have only certain *discrete* values E_n :

$$E_n = nhf \quad (40.4)$$

where n is a positive integer called a **quantum number**,¹ f is the oscillator’s frequency, and h is a parameter Planck introduced that is now called **Planck’s constant**. Because the energy of each oscillator can have only discrete values given by Equation 40.4, we say the energy is **quantized**. Each discrete energy value corresponds to a different **quantum state**, represented by the quantum number n . When the oscillator is in the $n = 1$ quantum state, its energy is hf ; when it is in the $n = 2$ quantum state, its energy is $2hf$; and so on.

(b) The oscillators emit or absorb energy when making a transition from one quantum state to another. The entire energy difference between the initial and final states in the transition is emitted or absorbed as a single quantum of radiation. If the transition is from one state to a lower adjacent state—say, from the $n = 3$ state to the $n = 2$ state—Equation 40.4 shows that the amount of energy emitted by the oscillator and carried by the quantum of radiation is

$$E = hf \quad (40.5)$$

According to property 2(b), an oscillator emits or absorbs energy only when it changes quantum states. If it remains in one quantum state, no energy is absorbed or emitted. Figure 40.6 is an **energy-level diagram** showing the quantized energy levels and allowed transitions proposed by Planck. This important semigraphical representation is used often in quantum physics.² The vertical axis is linear in energy, and the allowed energy levels are represented as horizontal lines. The quantized system can have only the energies represented by the horizontal lines.

¹A quantum number is generally an integer (although half-integer quantum numbers can occur) that describes an allowed state of a system, such as the values of n describing the normal modes of oscillation of a string fixed at both ends, as discussed in Section 18.3.

²We first saw an energy-level diagram in Section 21.3.

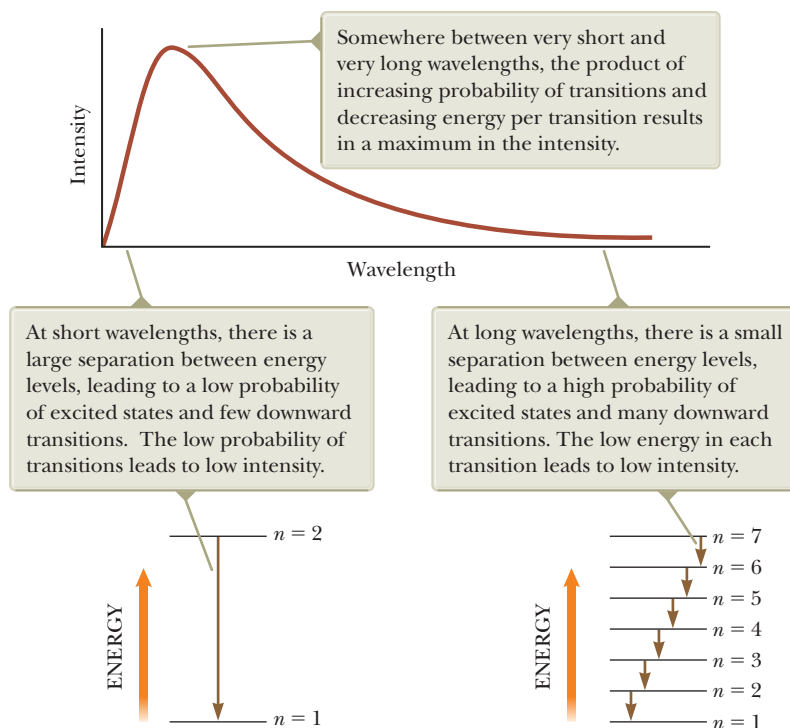
The key point in Planck's theory is the radical assumption of quantized energy states. This development—a clear deviation from classical physics—marked the birth of the quantum theory.

In the Rayleigh–Jeans model, the average energy associated with a particular wavelength of standing waves in the cavity is the same for all wavelengths and is equal to $k_B T$. Planck used the same classical ideas as in the Rayleigh–Jeans model to arrive at the energy density as a product of constants and the average energy for a given wavelength, but the average energy is not given by the equipartition theorem. A wave's average energy is the average energy difference between levels of the oscillator, *weighted according to the probability of the wave being emitted*. This weighting is based on the occupation of higher-energy states as described by the Boltzmann distribution law, which was discussed in Section 21.5. According to this law, the probability of a state being occupied is proportional to the factor $e^{-E/k_B T}$, where E is the energy of the state.

At low frequencies (long wavelengths), according to property 2(a), the energy levels are close together as on the right in Figure 40.7, and many of the energy states are excited because the Boltzmann factor $e^{-E/k_B T}$ is relatively large for these states. Therefore, there are many contributions to the outgoing radiation, although each contribution has very low energy. Now, consider high-frequency radiation, that is, radiation with short wavelength. To obtain this radiation, the allowed energies are very far apart as on the left in Figure 40.7. The probability of thermal agitation exciting these high energy levels is small because of the small value of the Boltzmann factor for large values of E . At high frequencies, the low probability of excitation results in very little contribution to the total energy, even though each quantum is of large energy. This low probability “turns the curve over” and brings it down to zero again at short wavelengths.

Using this approach, Planck generated a theoretical expression for the wavelength distribution that agreed remarkably well with the experimental curves in Figure 40.3:

$$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5 (e^{hc/\lambda k_B T} - 1)} \quad (40.6)$$



Pitfall Prevention 40.2

n Is Again an Integer In the preceding chapters on optics, we used the symbol n for the index of refraction, which was not an integer. Here we are again using n as we did in Chapter 18 to indicate the standing-wave mode on a string or in an air column. In quantum physics, n is often used as an integer quantum number to identify a particular quantum state of a system.

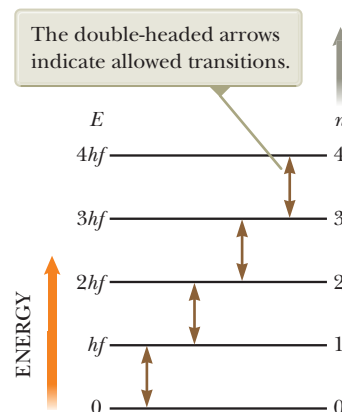


Figure 40.6 Allowed energy levels for an oscillator with frequency f .

Planck's wavelength distribution function

Figure 40.7 In Planck's model, the average energy associated with a given wavelength is the product of the energy of a transition and a factor related to the probability of the transition occurring.

This function includes the parameter h , which Planck adjusted so that his curve matched the experimental data at all wavelengths. The value of this parameter is found to be independent of the material of which the black body is made and independent of the temperature; it is a fundamental constant of nature. The value of h , Planck's constant, which was first introduced in Chapter 35, is

Planck's constant ►

$$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} \quad (40.7)$$

At long wavelengths, Equation 40.6 reduces to the Rayleigh–Jeans expression, Equation 40.3 (see Problem 14), and at short wavelengths, it predicts an exponential decrease in $I(\lambda, T)$ with decreasing wavelength, in agreement with experimental results.

When Planck presented his theory, most scientists (including Planck!) did not consider the quantum concept to be realistic. They believed it was a mathematical trick that happened to predict the correct results. Hence, Planck and others continued to search for a more “rational” explanation of blackbody radiation. Subsequent developments, however, showed that a theory based on the quantum concept (rather than on classical concepts) had to be used to explain not only blackbody radiation but also a number of other phenomena at the atomic level.

In 1905, Einstein rederived Planck's results by assuming the oscillations of the electromagnetic field were themselves quantized. In other words, he proposed that quantization is a fundamental property of light and other electromagnetic radiation, which led to the concept of photons as shall be discussed in Section 40.2. Critical to the success of the quantum or photon theory was the relation between energy and frequency, which classical theory completely failed to predict.

You may have had your body temperature measured at the doctor's office by an *ear thermometer*, which can read your temperature very quickly (Fig. 40.8). In a fraction of a second, this type of thermometer measures the amount of infrared radiation emitted by the eardrum. It then converts the amount of radiation into a temperature reading. This thermometer is very sensitive because temperature is raised to the fourth power in Stefan's law. Suppose you have a fever 1°C above normal. Because absolute temperatures are found by adding 273 to Celsius temperatures, the ratio of your fever temperature to normal body temperature of 37°C is

$$\frac{T_{\text{fever}}}{T_{\text{normal}}} = \frac{38^\circ\text{C} + 273^\circ\text{C}}{37^\circ\text{C} + 273^\circ\text{C}} = 1.0032$$

which is only a 0.32% increase in temperature. The increase in radiated power, however, is proportional to the fourth power of temperature, so

$$\frac{P_{\text{fever}}}{P_{\text{normal}}} = \left(\frac{38^\circ\text{C} + 273^\circ\text{C}}{37^\circ\text{C} + 273^\circ\text{C}} \right)^4 = 1.013$$

The result is a 1.3% increase in radiated power, which is easily measured by modern infrared radiation sensors.



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Figure 40.8 An ear thermometer measures a patient's temperature by detecting the intensity of infrared radiation leaving the eardrum.

Example 40.1 Thermal Radiation from Different Objects

(A) Find the peak wavelength of the blackbody radiation emitted by the human body when the skin temperature is 35°C .

SOLUTION

Conceptualize Thermal radiation is emitted from the surface of any object. The peak wavelength is related to the surface temperature through Wien's displacement law (Eq. 40.2).

Categorize We evaluate results using an equation developed in this section, so we categorize this example as a substitution problem.

▶ 40.1 continued

Solve Equation 40.2 for λ_{\max} :

$$(1) \quad \lambda_{\max} = \frac{2.898 \times 10^{-3} \text{ m} \cdot \text{K}}{T}$$

Substitute the surface temperature:

$$\lambda_{\max} = \frac{2.898 \times 10^{-3} \text{ m} \cdot \text{K}}{308 \text{ K}} = 9.41 \mu\text{m}$$

This radiation is in the infrared region of the spectrum and is invisible to the human eye. Some animals (pit vipers, for instance) are able to detect radiation of this wavelength and therefore can locate warm-blooded prey even in the dark.

(B) Find the peak wavelength of the blackbody radiation emitted by the tungsten filament of a lightbulb, which operates at 2 000 K.

SOLUTION

Substitute the filament temperature into Equation (1):

$$\lambda_{\max} = \frac{2.898 \times 10^{-3} \text{ m} \cdot \text{K}}{2\,000 \text{ K}} = 1.45 \mu\text{m}$$

This radiation is also in the infrared, meaning that most of the energy emitted by a lightbulb is not visible to us.

(C) Find the peak wavelength of the blackbody radiation emitted by the Sun, which has a surface temperature of approximately 5 800 K.

SOLUTION

Substitute the surface temperature into Equation (1):

$$\lambda_{\max} = \frac{2.898 \times 10^{-3} \text{ m} \cdot \text{K}}{5\,800 \text{ K}} = 0.500 \mu\text{m}$$

This radiation is near the center of the visible spectrum, near the color of a yellow-green tennis ball. Because it is the most prevalent color in sunlight, our eyes have evolved to be most sensitive to light of approximately this wavelength.

Example 40.2 The Quantized Oscillator **AM**

A 2.00-kg block is attached to a massless spring that has a force constant of $k = 25.0 \text{ N/m}$. The spring is stretched 0.400 m from its equilibrium position and released from rest.

(A) Find the total energy of the system and the frequency of oscillation according to classical calculations.

SOLUTION

Conceptualize We understand the details of the block's motion from our study of simple harmonic motion in Chapter 15. Review that material if you need to.

Categorize The phrase "according to classical calculations" tells us to categorize this part of the problem as a classical analysis of the oscillator. We model the block as a *particle in simple harmonic motion*.

Analyze Based on the way the block is set into motion, its amplitude is 0.400 m.

Evaluate the total energy of the block–spring system using Equation 15.21:

$$E = \frac{1}{2}kA^2 = \frac{1}{2}(25.0 \text{ N/m})(0.400 \text{ m})^2 = 2.00 \text{ J}$$

Evaluate the frequency of oscillation from Equation 15.14:

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{25.0 \text{ N/m}}{2.00 \text{ kg}}} = 0.563 \text{ Hz}$$

(B) Assuming the energy of the oscillator is quantized, find the quantum number n for the system oscillating with this amplitude.

continued

40.2 continued

SOLUTION

Categorize This part of the problem is categorized as a quantum analysis of the oscillator. We model the block–spring system as a Planck oscillator.

Analyze Solve Equation 40.4 for the quantum number n :

$$n = \frac{E_n}{hf}$$

Substitute numerical values:

$$n = \frac{2.00 \text{ J}}{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(0.563 \text{ Hz})} = 5.36 \times 10^{33}$$

Finalize Notice that 5.36×10^{33} is a very large quantum number, which is typical for macroscopic systems. Changes between quantum states for the oscillator are explored next.

WHAT IF? Suppose the oscillator makes a transition from the $n = 5.36 \times 10^{33}$ state to the state corresponding to $n = 5.36 \times 10^{33} - 1$. By how much does the energy of the oscillator change in this one-quantum change?

Answer From Equation 40.5 and the result to part (A), the energy carried away due to the transition between states differing in n by 1 is

$$E = hf = (6.626 \times 10^{-34} \text{ J} \cdot \text{s})(0.563 \text{ Hz}) = 3.73 \times 10^{-34} \text{ J}$$

This energy change due to a one-quantum change is fractionally equal to $3.73 \times 10^{-34} \text{ J} / 2.00 \text{ J}$, or on the order of one part in 10^{34} ! It is such a small fraction of the total energy of the oscillator that it cannot be detected. Therefore, even though the energy of a macroscopic block–spring system is quantized and does indeed decrease by small quantum jumps, our senses perceive the decrease as continuous. Quantum effects become important and detectable only on the submicroscopic level of atoms and molecules.

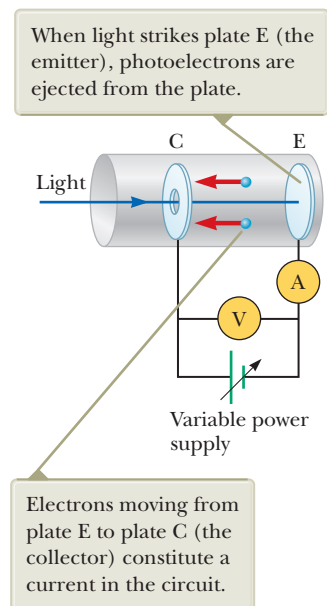


Figure 40.9 A circuit diagram for studying the photoelectric effect.

40.2 The Photoelectric Effect

Blackbody radiation was the first phenomenon to be explained with a quantum model. In the latter part of the 19th century, at the same time that data were taken on thermal radiation, experiments showed that light incident on certain metallic surfaces causes electrons to be emitted from those surfaces. This phenomenon, which was first discussed in Section 35.1, is known as the **photoelectric effect**, and the emitted electrons are called **photoelectrons**.³

Figure 40.9 is a diagram of an apparatus for studying the photoelectric effect. An evacuated glass or quartz tube contains a metallic plate E (the emitter) connected to the negative terminal of a battery and another metallic plate C (the collector) that is connected to the positive terminal of the battery. When the tube is kept in the dark, the ammeter reads zero, indicating no current in the circuit. However, when plate E is illuminated by light having an appropriate wavelength, a current is detected by the ammeter, indicating a flow of charges across the gap between plates E and C. This current arises from photoelectrons emitted from plate E and collected at plate C.

Figure 40.10 is a plot of photoelectric current versus potential difference ΔV applied between plates E and C for two light intensities. At large values of ΔV , the current reaches a maximum value; all the electrons emitted from E are collected at C, and the current cannot increase further. In addition, the maximum current increases as the intensity of the incident light increases, as you might expect,

³Photoelectrons are not different from other electrons. They are given this name solely because of their ejection from a metal by light in the photoelectric effect.

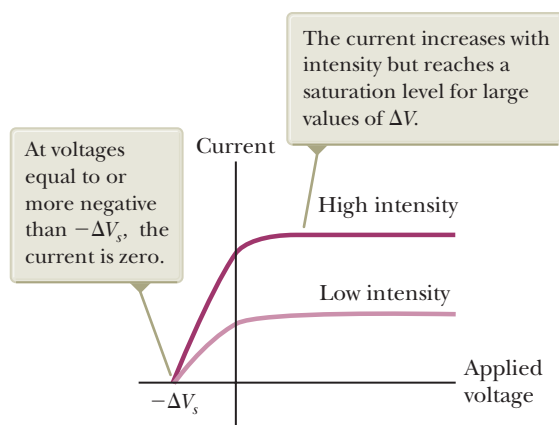


Figure 40.10 Photoelectric current versus applied potential difference for two light intensities.

because more electrons are ejected by the higher-intensity light. Finally, when ΔV is negative—that is, when the battery in the circuit is reversed to make plate E positive and plate C negative—the current drops because many of the photoelectrons emitted from E are repelled by the now negative plate C. In this situation, only those photoelectrons having a kinetic energy greater than $e|\Delta V|$ reach plate C, where e is the magnitude of the charge on the electron. When ΔV is equal to or more negative than $-\Delta V_s$, where ΔV_s is the **stopping potential**, no photoelectrons reach C and the current is zero.

Let's model the combination of the electric field between the plates and an electron ejected from plate E as an isolated system. Suppose this electron stops just as it reaches plate C. Because the system is isolated, the appropriate reduction of Equation 8.2 is

$$\Delta K + \Delta U = 0$$

where the initial configuration is at the instant the electron leaves the metal with kinetic energy K_i and the final configuration is when the electron stops just before touching plate C. If we define the electric potential energy of the system in the initial configuration to be zero, we have

$$(0 - K_f) + [(q)(\Delta V) - 0] = 0 \rightarrow K_i = q\Delta V = -e\Delta V$$

Now suppose the potential difference ΔV is increased in the negative direction just until the current is zero at $\Delta V = -\Delta V_s$. In this case, the electron that stops immediately before reaching plate C has the maximum possible kinetic energy upon leaving the metal surface. The previous equation can then be written as

$$K_{\max} = e \Delta V_s \quad (40.8)$$

This equation allows us to measure K_{\max} experimentally by determining the magnitude of the voltage ΔV_s at which the current drops to zero.

Several features of the photoelectric effect are listed below. For each feature, we compare the predictions made by a classical approach, using the wave model for light, with the experimental results.

1. Dependence of photoelectron kinetic energy on light intensity

Classical prediction: Electrons should absorb energy continuously from the electromagnetic waves. As the light intensity incident on a metal is increased, energy should be transferred into the metal at a higher rate and the electrons should be ejected with more kinetic energy.

Experimental result: The maximum kinetic energy of photoelectrons is *independent* of light intensity as shown in Figure 40.10 with both curves falling to zero at the *same* negative voltage. (According to Equation 40.8, the maximum kinetic energy is proportional to the stopping potential.)

2. Time interval between incidence of light and ejection of photoelectrons

Classical prediction: At low light intensities, a measurable time interval should pass between the instant the light is turned on and the time an electron is ejected from the metal. This time interval is required for the electron to absorb the incident radiation before it acquires enough energy to escape from the metal.

Experimental result: Electrons are emitted from the surface of the metal almost *instantaneously* (less than 10^{-9} s after the surface is illuminated), even at very low light intensities.

3. Dependence of ejection of electrons on light frequency

Classical prediction: Electrons should be ejected from the metal at any incident light frequency, as long as the light intensity is high enough, because energy is transferred to the metal regardless of the incident light frequency.

Experimental result: No electrons are emitted if the incident light frequency falls below some **cutoff frequency** f_c , whose value is characteristic of the material being illuminated. No electrons are ejected below this cutoff frequency *regardless* of the light intensity.

4. Dependence of photoelectron kinetic energy on light frequency

Classical prediction: There should be *no* relationship between the frequency of the light and the electron kinetic energy. The kinetic energy should be related to the intensity of the light.

Experimental result: The maximum kinetic energy of the photoelectrons increases with increasing light frequency.

For these features, experimental results contradict *all four* classical predictions. A successful explanation of the photoelectric effect was given by Einstein in 1905, the same year he published his special theory of relativity. As part of a general paper on electromagnetic radiation, for which he received a Nobel Prize in Physics in 1921, Einstein extended Planck's concept of quantization to electromagnetic waves as mentioned in Section 40.1. Einstein assumed light (or any other electromagnetic wave) of frequency f from *any* source can be considered a stream of quanta. Today we call these quanta **photons**. Each photon has an energy E given by Equation 40.5, $E = hf$, and each moves in a vacuum at the speed of light c , where $c = 3.00 \times 10^8$ m/s.

- Quick Quiz 40.2** While standing outdoors one evening, you are exposed to the following four types of electromagnetic radiation: yellow light from a sodium street lamp, radio waves from an AM radio station, radio waves from an FM radio station, and microwaves from an antenna of a communications system.
- Rank these types of waves in terms of photon energy from highest to lowest.

Let us organize Einstein's model for the photoelectric effect using the properties of structural models:

1. *Physical components:*

We imagine the system to consist of two physical components: (1) an electron that is to be ejected by an incoming photon and (2) the remainder of the metal.

2. *Behavior of the components:*

- (a) In Einstein's model, a photon of the incident light gives *all* its energy hf to a *single* electron in the metal. Therefore, the absorption of energy by the electrons is not a continuous process as envisioned in the wave model, but rather a discontinuous process in which energy is delivered to the electrons in bundles. The energy transfer is accomplished via a one-photon/one-electron event.⁴

⁴In principle, two photons could combine to provide an electron with their combined energy. That is highly improbable, however, without the high intensity of radiation available from very strong lasers.

- (b) We can describe the time evolution of the system by applying the non-isolated system model for energy over a time interval that includes the absorption of one photon and the ejection of the corresponding electron. Energy is transferred into the system by electromagnetic radiation, the photon. The system has two types of energy: the potential energy of the metal–electron system and the kinetic energy of the ejected electron. Therefore, we can write the conservation of energy equation (Eq. 8.2) as

$$\Delta K + \Delta U = T_{\text{ER}} \quad (40.9)$$

The energy transfer into the system is that of the photon, $T_{\text{ER}} = hf$. During the process, the kinetic energy of the electron increases from zero to its final value, which we assume to be the maximum possible value K_{max} . The potential energy of the system increases because the electron is pulled away from the metal to which it is attracted. We define the potential energy of the system when the electron is outside the metal as zero. The potential energy of the system when the electron is in the metal is $U = -\phi$, where ϕ is called the **work function** of the metal. The work function represents the minimum energy with which an electron is bound in the metal and is on the order of a few electron volts. Table 40.1 lists selected values. The increase in potential energy of the system when the electron is removed from the metal is the work function ϕ . Substituting these energies into Equation 40.9, we have

$$(K_{\text{max}} - 0) + [0 - (-\phi)] = hf$$

$$K_{\text{max}} + \phi = hf \quad (40.10)$$

If the electron makes collisions with other electrons or metal ions as it is being ejected, some of the incoming energy is transferred to the metal and the electron is ejected with less kinetic energy than K_{max} .

The prediction made by Einstein is an equation for the maximum kinetic energy of an ejected electron as a function of frequency of the illuminating radiation. This equation can be found by rearranging Equation 40.10:

$$K_{\text{max}} = hf - \phi \quad (40.11)$$

With Einstein's structural model, one can explain the observed features of the photoelectric effect that cannot be understood using classical concepts:

1. Dependence of photoelectron kinetic energy on light intensity

Equation 40.11 shows that K_{max} is independent of the light intensity. The maximum kinetic energy of any one electron, which equals $hf - \phi$, depends only on the light frequency and the work function. If the light intensity is doubled, the number of photons arriving per unit time is doubled, which doubles the rate at which photoelectrons are emitted. The maximum kinetic energy of any one photoelectron, however, is unchanged.

2. Time interval between incidence of light and ejection of photoelectrons

Near-instantaneous emission of electrons is consistent with the photon model of light. The incident energy appears in small packets, and there is a one-to-one interaction between photons and electrons. If the incident light has very low intensity, there are very few photons arriving per unit time interval; each photon, however, can have sufficient energy to eject an electron immediately.

3. Dependence of ejection of electrons on light frequency

Because the photon must have energy greater than the work function ϕ to eject an electron, the photoelectric effect cannot be observed below a

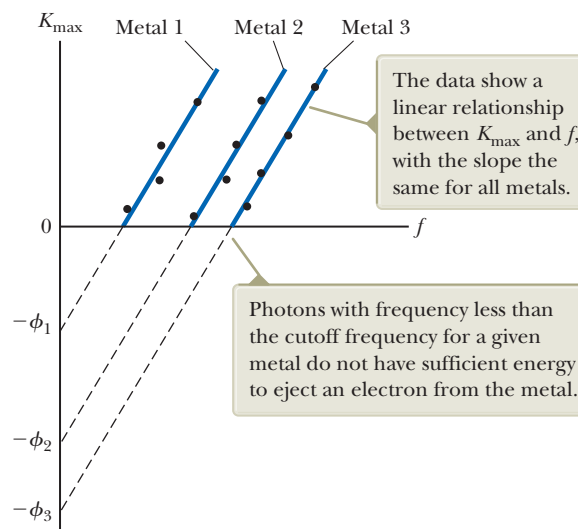
Table 40.1 Work Functions of Selected Metals

Metal	ϕ (eV)
Na	2.46
Al	4.08
Fe	4.50
Cu	4.70
Zn	4.31
Ag	4.73
Pt	6.35
Pb	4.14

Note: Values are typical for metals listed. Actual values may vary depending on whether the metal is a single crystal or polycrystalline. Values may also depend on the face from which electrons are ejected from crystalline metals. Furthermore, different experimental procedures may produce differing values.

◀ Photoelectric effect equation

Figure 40.11 A plot of K_{\max} for photoelectrons versus frequency of incident light in a typical photoelectric effect experiment.



certain cutoff frequency. If the energy of an incoming photon does not satisfy this requirement, an electron cannot be ejected from the surface, even though many photons per unit time are incident on the metal in a very intense light beam.

4. Dependence of photoelectron kinetic energy on light frequency

A photon of higher frequency carries more energy and therefore ejects a photoelectron with more kinetic energy than does a photon of lower frequency.

Einstein's model predicts a linear relationship (Eq. 40.11) between the maximum electron kinetic energy K_{\max} and the light frequency f . Experimental observation of a linear relationship between K_{\max} and f would be a final confirmation of Einstein's theory. Indeed, such a linear relationship was observed experimentally within a few years of Einstein's theory and is sketched in Figure 40.11. The slope of the lines in such a plot is Planck's constant h . The intercept on the horizontal axis gives the cutoff frequency below which no photoelectrons are emitted. The cutoff frequency is related to the work function through the relationship $f_c = \phi/h$. The cutoff frequency corresponds to a **cutoff wavelength** λ_c , where

Cutoff wavelength ►

$$\lambda_c = \frac{c}{f_c} = \frac{c}{\phi/h} = \frac{hc}{\phi} \quad (40.12)$$

and c is the speed of light. Wavelengths greater than λ_c incident on a material having a work function ϕ do not result in the emission of photoelectrons.

The combination hc in Equation 40.12 often occurs when relating a photon's energy to its wavelength. A common shortcut when solving problems is to express this combination in useful units according to the following approximation:

$$hc = 1\,240 \text{ eV} \cdot \text{nm}$$

One of the first practical uses of the photoelectric effect was as the detector in a camera's light meter. Light reflected from the object to be photographed strikes a photoelectric surface in the meter, causing it to emit photoelectrons that then pass through a sensitive ammeter. The magnitude of the current in the ammeter depends on the light intensity.

The phototube, another early application of the photoelectric effect, acts much like a switch in an electric circuit. It produces a current in the circuit when light of sufficiently high frequency falls on a metal plate in the phototube, but produces no current in the dark. Phototubes were used in burglar alarms and in the detection of the soundtrack on motion picture film. Modern semiconductor devices have now replaced older devices based on the photoelectric effect.

Today, the photoelectric effect is used in the operation of photomultiplier tubes. Figure 40.12 shows the structure of such a device. A photon striking the photocathode ejects an electron by means of the photoelectric effect. This electron accelerates across the potential difference between the photocathode and the first *dynode*, shown as being at +200 V relative to the photocathode in Figure 40.12. This high-energy electron strikes the dynode and ejects several more electrons. The same process is repeated through a series of dynodes at ever higher potentials until an electrical pulse is produced as millions of electrons strike the last dynode. The tube is therefore called a *multiplier*: one photon at the input has resulted in millions of electrons at the output.

The photomultiplier tube is used in nuclear detectors to detect photons produced by the interaction of energetic charged particles or gamma rays with certain materials. It is also used in astronomy in a technique called *photoelectric photometry*. In that technique, the light collected by a telescope from a single star is allowed to fall on a photomultiplier tube for a time interval. The tube measures the total energy transferred by light during the time interval, which can then be converted to a luminosity of the star.

The photomultiplier tube is being replaced in many astronomical observations with a *charge-coupled device* (CCD), which is the same device used in a digital camera (Section 36.6). Half of the 2009 Nobel Prize in Physics was awarded to Willard S. Boyle (b. 1924) and George E. Smith (b. 1930) for their 1969 invention of the charge-coupled device. In a CCD, an array of pixels is formed on the silicon surface of an integrated circuit (Section 43.7). When the surface is exposed to light from an astronomical scene through a telescope or a terrestrial scene through a digital camera, electrons generated by the photoelectric effect are caught in “traps” beneath the surface. The number of electrons is related to the intensity of the light striking the surface. A signal processor measures the number of electrons associated with each pixel and converts this information into a digital code that a computer can use to reconstruct and display the scene.

The *electron bombardment CCD camera* allows higher sensitivity than a conventional CCD. In this device, electrons ejected from a photocathode by the photoelectric effect are accelerated through a high voltage before striking a CCD array. The higher energy of the electrons results in a very sensitive detector of low-intensity radiation.

Quick Quiz 40.3 Consider one of the curves in Figure 40.10. Suppose the intensity of the incident light is held fixed but its frequency is increased. Does the stopping potential in Figure 40.10 (a) remain fixed, (b) move to the right, or (c) move to the left?

Quick Quiz 40.4 Suppose classical physicists had the idea of plotting K_{\max} versus f as in Figure 40.11. Draw a graph of what the expected plot would look like, based on the wave model for light.

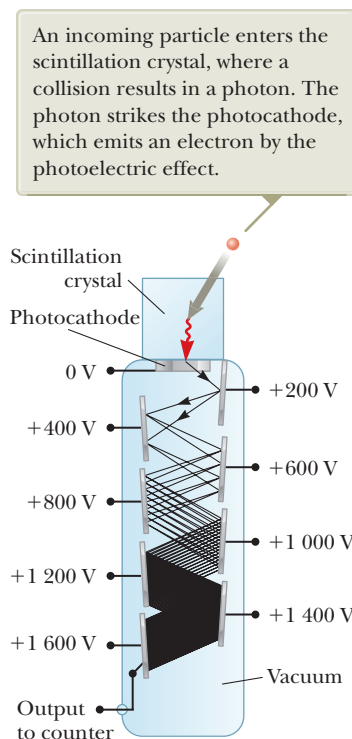


Figure 40.12 The multiplication of electrons in a photomultiplier tube.

Example 40.3 The Photoelectric Effect for Sodium

A sodium surface is illuminated with light having a wavelength of 300 nm. As indicated in Table 40.1, the work function for sodium metal is 2.46 eV.

(A) Find the maximum kinetic energy of the ejected photoelectrons.

SOLUTION

Conceptualize Imagine a photon striking the metal surface and ejecting an electron. The electron with the maximum energy is one near the surface that experiences no interactions with other particles in the metal that would reduce its energy on its way out of the metal.

continued

40.3 continued

Categorize We evaluate the results using equations developed in this section, so we categorize this example as a substitution problem.

Find the energy of each photon in the illuminating light beam from Equation 40.5:

$$E = hf = \frac{hc}{\lambda}$$

From Equation 40.11, find the maximum kinetic energy of an electron:

$$K_{\max} = \frac{hc}{\lambda} - \phi = \frac{1\,240\text{ eV} \cdot \text{nm}}{300\text{ nm}} - 2.46\text{ eV} = 1.67\text{ eV}$$

(B) Find the cutoff wavelength λ_c for sodium.

SOLUTION

Calculate λ_c using Equation 40.12:

$$\lambda_c = \frac{hc}{\phi} = \frac{1\,240\text{ eV} \cdot \text{nm}}{2.46\text{ eV}} = 504\text{ nm}$$



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Arthur Holly Compton

American Physicist (1892–1962)

Compton was born in Wooster, Ohio, and attended Wooster College and Princeton University. He became the director of the laboratory at the University of Chicago, where experimental work concerned with sustained nuclear chain reactions was conducted. This work was of central importance to the construction of the first nuclear weapon. His discovery of the Compton effect led to his sharing of the 1927 Nobel Prize in Physics with Charles Wilson.

40.3 The Compton Effect

In 1919, Einstein concluded that a photon of energy E travels in a single direction and carries a momentum equal to $E/c = hf/c$. In 1923, Arthur Holly Compton (1892–1962) and Peter Debye (1884–1966) independently carried Einstein's idea of photon momentum further.

Prior to 1922, Compton and his coworkers had accumulated evidence showing that the classical wave theory of light failed to explain the scattering of x-rays from electrons. According to classical theory, electromagnetic waves of frequency f incident on electrons should have two effects: (1) radiation pressure (see Section 34.5) should cause the electrons to accelerate in the direction of propagation of the waves, and (2) the oscillating electric field of the incident radiation should set the electrons into oscillation at the apparent frequency f' , where f' is the frequency in the frame of the moving electrons. This apparent frequency is different from the frequency f of the incident radiation because of the Doppler effect (see Section 17.4). Each electron first absorbs radiation as a moving particle and then reradiates as a moving particle, thereby exhibiting two Doppler shifts in the frequency of radiation.

Because different electrons move at different speeds after the interaction, depending on the amount of energy absorbed from the electromagnetic waves, the scattered wave frequency at a given angle to the incoming radiation should show a distribution of Doppler-shifted values. Contrary to this prediction, Compton's experiments showed that at a given angle only *one* frequency of radiation is observed. Compton and his coworkers explained these experiments by treating photons not as waves but rather as point-like particles having energy hf and momentum hf/c and by assuming the energy and momentum of the isolated system of the colliding photon–electron pair are conserved. Compton adopted a particle model for something that was well known as a wave, and today this scattering phenomenon is known as the **Compton effect**. Figure 40.13 shows the quantum picture of the collision between an individual x-ray photon of frequency f_0 and an electron. In the quantum model, the electron is scattered through an angle ϕ with respect to this direction as in a billiard-ball type of collision. (The symbol ϕ used here is an angle and is not to be confused with the work function, which was discussed in the preceding section.) Compare Figure 40.13 with the two-dimensional collision shown in Figure 9.11.

Figure 40.14 is a schematic diagram of the apparatus used by Compton. The x-rays, scattered from a carbon target, were diffracted by a rotating crystal spectrometer, and the intensity was measured with an ionization chamber that generated a current proportional to the intensity. The incident beam consisted of monochromatic x-rays of wavelength $\lambda_0 = 0.071\text{ nm}$. The experimental intensity-

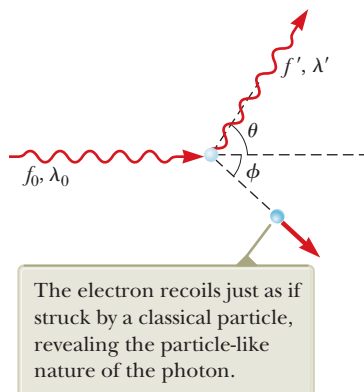


Figure 40.13 The quantum model for x-ray scattering from an electron.

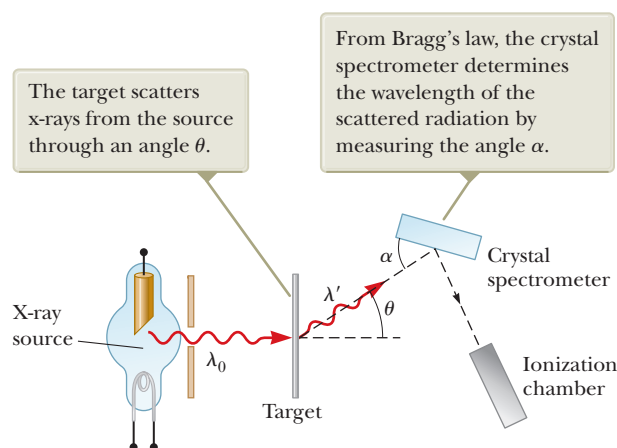


Figure 40.14 Schematic diagram of Compton's apparatus.

versus-wavelength plots observed by Compton for four scattering angles (corresponding to θ in Fig. 40.13) are shown in Figure 40.15. The graphs for the three nonzero angles show two peaks, one at λ_0 and one at $\lambda' > \lambda_0$. The shifted peak at λ' is caused by the scattering of x-rays from free electrons, which was predicted by Compton to depend on scattering angle as

$$\lambda' - \lambda_0 = \frac{h}{m_e c} (1 - \cos \theta) \quad (40.13)$$

◀ **Compton shift equation**

where m_e is the mass of the electron. This expression is known as the **Compton shift equation** and correctly describes the positions of the peaks in Figure 40.15. The factor $h/m_e c$, called the **Compton wavelength** of the electron, has a currently accepted value of

$$\lambda_C = \frac{h}{m_e c} = 0.002\,43\text{ nm}$$

The unshifted peak at λ_0 in Figure 40.15 is caused by x-rays scattered from electrons tightly bound to the target atoms. This unshifted peak also is predicted by Equation 40.13 if the electron mass is replaced with the mass of a carbon atom, which is approximately 23 000 times the mass of the electron. Therefore, there is a wavelength shift for scattering from an electron bound to an atom, but it is so small that it was undetectable in Compton's experiment.

Compton's measurements were in excellent agreement with the predictions of Equation 40.13. These results were the first to convince many physicists of the fundamental validity of quantum theory.

Quick Quiz 40.5 For any given scattering angle θ , Equation 40.13 gives the same value for the Compton shift for any wavelength. Keeping that in mind, for which of the following types of radiation is the fractional shift in wavelength at a given scattering angle the largest? (a) radio waves (b) microwaves (c) visible light (d) x-rays

Derivation of the Compton Shift Equation

We can derive the Compton shift equation by assuming the photon behaves like a particle and collides elastically with a free electron initially at rest as shown in Figure 40.13. The photon is treated as a particle having energy $E = hf = hc/\lambda$ and zero rest energy. We apply the isolated system analysis models for energy and momentum to the photon and the electron. In the scattering process, the total energy and total linear momentum of the system are conserved. Applying the isolated system model for energy to this process gives

$$\Delta K_{\text{photon}} + \Delta K_e = 0 \rightarrow \frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + K_e$$

◀ Compton wavelength

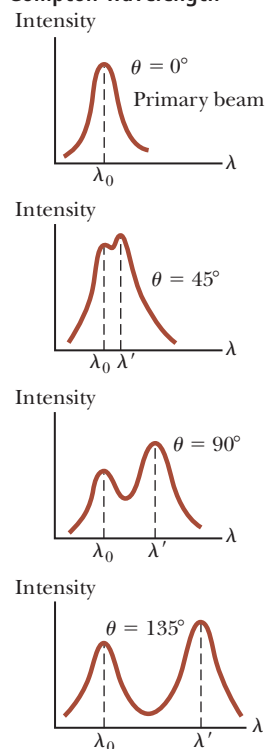


Figure 40.15 Scattered x-ray intensity versus wavelength for Compton scattering at $\theta = 0^\circ$, 45° , 90° , and 135° .

where hc/λ_0 is the energy of the incident photon, hc/λ' is the energy of the scattered photon, and K_e is the kinetic energy of the recoiling electron. Because the electron may recoil at a speed comparable to that of light, we must use the relativistic expression $K_e = (\gamma - 1)m_e c^2$ (Eq. 39.23). Therefore,

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + (\gamma - 1)m_e c^2 \quad (40.14)$$

where $\gamma = 1/\sqrt{1 - (u^2/c^2)}$ and u is the speed of the electron.

Next, let's apply the isolated system model for momentum to this collision, noting that the x and y components of momentum are each conserved independently. Equation 39.28 shows that the momentum of a photon has a magnitude $p = E/c$, and we know from Equation 40.5 that $E = hf$. Therefore, $p = hf/c$. Substituting λf for c (Eq. 34.20) in this expression gives $p = h/\lambda$. Because the relativistic expression for the momentum of the recoiling electron is $p_e = \gamma m_e u$ (Eq. 39.19), we obtain the following expressions for the x and y components of linear momentum, where the angles are as described in Figure 40.13:

$$x \text{ component: } \frac{h}{\lambda_0} = \frac{h}{\lambda'} \cos \theta + \gamma m_e u \cos \phi \quad (40.15)$$

$$y \text{ component: } 0 = \frac{h}{\lambda'} \sin \theta - \gamma m_e u \sin \phi \quad (40.16)$$

Eliminating u and ϕ from Equations 40.14 through 40.16 gives a single expression that relates the remaining three variables (λ' , λ_0 , and θ). After some algebra (see Problem 64), we obtain Equation 40.13.

Example 40.4 Compton Scattering at 45°

X-rays of wavelength $\lambda_0 = 0.200\,000\text{ nm}$ are scattered from a block of material. The scattered x-rays are observed at an angle of 45.0° to the incident beam. Calculate their wavelength.

SOLUTION

Conceptualize Imagine the process in Figure 40.13, with the photon scattered at 45° to its original direction.

Categorize We evaluate the result using an equation developed in this section, so we categorize this example as a substitution problem.

Solve Equation 40.13 for the wavelength of the scattered x-ray:

$$(1) \quad \lambda' = \lambda_0 + \frac{h(1 - \cos \theta)}{m_e c}$$

Substitute numerical values:

$$\begin{aligned} \lambda' &= 0.200\,000 \times 10^{-9} \text{ m} + \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(1 - \cos 45.0^\circ)}{(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})} \\ &= 0.200\,000 \times 10^{-9} \text{ m} + 7.10 \times 10^{-13} \text{ m} = \mathbf{0.200\,710 \text{ nm}} \end{aligned}$$

WHAT IF? What if the detector is moved so that scattered x-rays are detected at an angle larger than 45° ? Does the wavelength of the scattered x-rays increase or decrease as the angle θ increases?

Answer In Equation (1), if the angle θ increases, $\cos \theta$ decreases. Consequently, the factor $(1 - \cos \theta)$ increases. Therefore, the scattered wavelength increases.

We could also apply an energy argument to achieve this same result. As the scattering angle increases, more energy is transferred from the incident photon to the electron. As a result, the energy of the scattered photon decreases with increasing scattering angle. Because $E = hf$, the frequency of the scattered photon decreases, and because $\lambda = c/f$, the wavelength increases.

40.4 The Nature of Electromagnetic Waves

In Section 35.1, we introduced the notion of competing models of light: particles and waves. Let's expand on that earlier discussion. Phenomena such as the photoelectric effect and the Compton effect offer ironclad evidence that when light (or other forms of electromagnetic radiation) and matter interact, the light behaves as if it were composed of particles having energy hf and momentum h/λ . How can light be considered a photon (in other words, a particle) when we know it is a wave? On the one hand, we describe light in terms of photons having energy and momentum. On the other hand, light and other electromagnetic waves exhibit interference and diffraction effects, which are consistent only with a wave interpretation.

Which model is correct? Is light a wave or a particle? The answer depends on the phenomenon being observed. Some experiments can be explained either better or solely with the photon model, whereas others are explained either better or solely with the wave model. We must accept both models and admit that the true nature of light is not describable in terms of any single classical picture. The same light beam that can eject photoelectrons from a metal (meaning that the beam consists of photons) can also be diffracted by a grating (meaning that the beam is a wave). In other words, the particle model and the wave model of light complement each other.

The success of the particle model of light in explaining the photoelectric effect and the Compton effect raises many other questions. If light is a particle, what is the meaning of the “frequency” and “wavelength” of the particle, and which of these two properties determines its energy and momentum? Is light *simultaneously* a wave and a particle? Although photons have no rest energy (a nonobservable quantity because a photon cannot be at rest), is there a simple expression for the *effective mass* of a moving photon? If photons have effective mass, do they experience gravitational attraction? What is the spatial extent of a photon, and how does an electron absorb or scatter one photon? Some of these questions can be answered, but others demand a view of atomic processes that is too pictorial and literal. Many of them stem from classical analogies such as colliding billiard balls and ocean waves breaking on a seashore. Quantum mechanics gives light a more flexible nature by treating the particle model and the wave model of light as both necessary and complementary. Neither model can be used exclusively to describe all properties of light. A complete understanding of the observed behavior of light can be attained only if the two models are combined in a complementary manner.

40.5 The Wave Properties of Particles

Students introduced to the dual nature of light often find the concept difficult to accept. In the world around us, we are accustomed to regarding such things as baseballs solely as particles and other things such as sound waves solely as forms of wave motion. Every large-scale observation can be interpreted by considering either a wave explanation or a particle explanation, but in the world of photons and electrons, such distinctions are not as sharply drawn.

Even more disconcerting is that, under certain conditions, the things we unambiguously call “particles” exhibit wave characteristics. In his 1923 doctoral dissertation, Louis de Broglie postulated that because photons have both wave and particle characteristics, perhaps all forms of matter have both properties. This highly revolutionary idea had no experimental confirmation at the time. According to de Broglie, electrons, just like light, have a dual particle–wave nature.

In Section 40.3, we found that the momentum of a photon can be expressed as

$$p = \frac{h}{\lambda}$$



SPL/Getty Images

Louis de Broglie

French Physicist (1892–1987)

De Broglie was born in Dieppe, France. At the Sorbonne in Paris, he studied history in preparation for what he hoped would be a career in the diplomatic service. The world of science is lucky he changed his career path to become a theoretical physicist. De Broglie was awarded the Nobel Prize in Physics in 1929 for his prediction of the wave nature of electrons.

This equation shows that the photon wavelength can be specified by its momentum: $\lambda = h/p$. De Broglie suggested that material particles of momentum p have a characteristic wavelength that is given by the *same expression*. Because the magnitude of the momentum of a particle of mass m and speed u is $p = mu$, the **de Broglie wavelength** of that particle is⁵

$$\lambda = \frac{h}{p} = \frac{h}{mu} \quad (40.17)$$

Furthermore, in analogy with photons, de Broglie postulated that particles obey the Einstein relation $E = hf$, where E is the total energy of the particle. The frequency of a particle is then

$$f = \frac{E}{h} \quad (40.18)$$

The dual nature of matter is apparent in Equations 40.17 and 40.18 because each contains both particle quantities (p and E) and wave quantities (λ and f).

The problem of understanding the dual nature of matter and radiation is conceptually difficult because the two models seem to contradict each other. This problem as it applies to light was discussed earlier. The **principle of complementarity** states that

the wave and particle models of either matter or radiation complement each other.

Neither model can be used exclusively to describe matter or radiation adequately. Because humans tend to generate mental images based on their experiences from the everyday world, we use both descriptions in a complementary manner to explain any given set of data from the quantum world.

The Davisson–Germer Experiment

De Broglie's 1923 proposal that matter exhibits both wave and particle properties was regarded as pure speculation. If particles such as electrons had wave properties, under the correct conditions they should exhibit diffraction effects. Only three years later, C. J. Davisson (1881–1958) and L. H. Germer (1896–1971) succeeded in observing electron diffraction and measuring the wavelength of electrons. Their important discovery provided the first experimental confirmation of the waves proposed by de Broglie.

Interestingly, the intent of the initial Davisson–Germer experiment was not to confirm the de Broglie hypothesis. In fact, their discovery was made by accident (as is often the case). The experiment involved the scattering of low-energy electrons (approximately 54 eV) from a nickel target in a vacuum. During one experiment, the nickel surface was badly oxidized because of an accidental break in the vacuum system. After the target was heated in a flowing stream of hydrogen to remove the oxide coating, electrons scattered by it exhibited intensity maxima and minima at specific angles. The experimenters finally realized that the nickel had formed large crystalline regions upon heating and that the regularly spaced planes of atoms in these regions served as a diffraction grating for electrons. (See the discussion of diffraction of x-rays by crystals in Section 38.5.)

Shortly thereafter, Davisson and Germer performed more extensive diffraction measurements on electrons scattered from single-crystal targets. Their results showed conclusively the wave nature of electrons and confirmed the de Broglie relationship $p = h/\lambda$. In the same year, G. P. Thomson (1892–1975) of Scotland also observed electron diffraction patterns by passing electrons through very thin gold

Pitfall Prevention 40.3

What's Waving? If particles have wave properties, what's waving? You are familiar with waves on strings, which are very concrete. Sound waves are more abstract, but you are likely comfortable with them. Electromagnetic waves are even more abstract, but at least they can be described in terms of physical variables and electric and magnetic fields. In contrast, waves associated with particles are completely abstract and cannot be associated with a physical variable. In Chapter 41, we describe the wave associated with a particle in terms of probability.

⁵The de Broglie wavelength for a particle moving at any speed u is $\lambda = h/\gamma mu$, where $\gamma = [1 - (u^2/c^2)]^{-1/2}$.

foils. Diffraction patterns were subsequently observed in the scattering of helium atoms, hydrogen atoms, and neutrons. Hence, the wave nature of particles has been established in various ways.

- Quick Quiz 40.6** An electron and a proton both moving at nonrelativistic speeds have the same de Broglie wavelength. Which of the following quantities are also the same for the two particles? (a) speed (b) kinetic energy (c) momentum (d) frequency

Example 40.5 Wavelengths for Microscopic and Macroscopic Objects

(A) Calculate the de Broglie wavelength for an electron ($m_e = 9.11 \times 10^{-31}$ kg) moving at 1.00×10^7 m/s.

SOLUTION

Conceptualize Imagine the electron moving through space. From a classical viewpoint, it is a particle under constant velocity. From the quantum viewpoint, the electron has a wavelength associated with it.

Categorize We evaluate the result using an equation developed in this section, so we categorize this example as a substitution problem.

Evaluate the de Broglie wavelength using Equation 40.17:

$$\lambda = \frac{h}{m_e u} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(1.00 \times 10^7 \text{ m/s})} = 7.27 \times 10^{-11} \text{ m}$$

The wave nature of this electron could be detected by diffraction techniques such as those in the Davisson–Germer experiment.

(B) A rock of mass 50 g is thrown with a speed of 40 m/s. What is its de Broglie wavelength?

SOLUTION

Evaluate the de Broglie wavelength using Equation 40.17:

$$\lambda = \frac{h}{mu} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(50 \times 10^{-3} \text{ kg})(40 \text{ m/s})} = 3.3 \times 10^{-34} \text{ m}$$

This wavelength is much smaller than any aperture through which the rock could possibly pass. Hence, we could not observe diffraction effects, and as a result, the wave properties of large-scale objects cannot be observed.

The Electron Microscope

A practical device that relies on the wave characteristics of electrons is the **electron microscope**. A *transmission* electron microscope, used for viewing flat, thin samples, is shown in Figure 40.16 on page 1252. In many respects, it is similar to an optical microscope; the electron microscope, however, has a much greater resolving power because it can accelerate electrons to very high kinetic energies, giving them very short wavelengths. No microscope can resolve details that are significantly smaller than the wavelength of the waves used to illuminate the object. The shorter wavelengths of electrons gives an electron microscope a resolution that can be 1 000 times better than that from the visible light used in optical microscopes. As a result, an electron microscope with ideal lenses would be able to distinguish details approximately 1 000 times smaller than those distinguished by an optical microscope. (Electromagnetic radiation of the same wavelength as the electrons in an electron microscope is in the x-ray region of the spectrum.)

The electron beam in an electron microscope is controlled by electrostatic or magnetic deflection, which acts on the electrons to focus the beam and form an image. Rather than examining the image through an eyepiece as in an optical microscope, the viewer looks at an image formed on a monitor or other type of

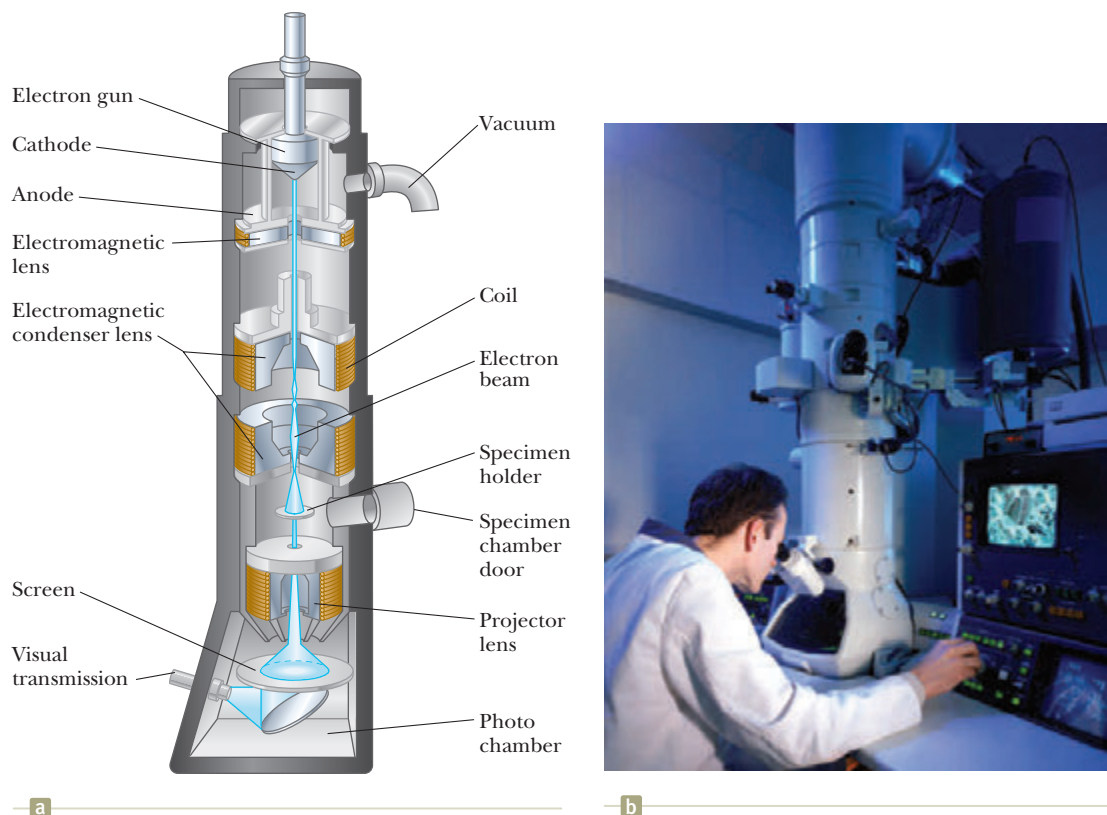


Figure 40.16 (a) Diagram of a transmission electron microscope for viewing a thinly sectioned sample. The “lenses” that control the electron beam are magnetic deflection coils. (b) An electron microscope in use.



Figure 40.17 A scanning electron microscope photograph shows significant detail of a cheese mite, *Tyrolichus casei*. The mite is so small, with a maximum length of 0.70 mm, that ordinary microscopes do not reveal minute anatomical details.

display screen. Figure 40.17 shows the amazing detail available with an electron microscope.

40.6 A New Model: The Quantum Particle

Because in the past we considered the particle and wave models to be distinct, the discussions presented in previous sections may be quite disturbing. The notion that both light and material particles have both particle and wave properties does not fit with this distinction. Experimental evidence shows, however, that this conclusion is exactly what we must accept. The recognition of this dual nature leads to a new model, the **quantum particle**, which is a combination of the particle model introduced in Chapter 2 and the wave model discussed in Chapter 16. In this new model, entities have both particle and wave characteristics, and we must choose one appropriate behavior—particle or wave—to understand a particular phenomenon.

In this section, we shall explore this model in a way that might make you more comfortable with this idea. We shall do so by demonstrating that an entity that exhibits properties of a particle can be constructed from waves.

Let's first recall some characteristics of ideal particles and ideal waves. An ideal particle has zero size. Therefore, an essential feature of a particle is that it is *localized* in space. An ideal wave has a single frequency and is infinitely long as suggested by Figure 40.18a. Therefore, an ideal wave is *unlocalized* in space. A localized entity can be built from infinitely long waves as follows. Imagine drawing one wave along the x axis, with one of its crests located at $x = 0$, as at the top of Figure 40.18b. Now draw a second wave, of the same amplitude but a different frequency, with one of its

crests also at $x = 0$. As a result of the superposition of these two waves, *beats* exist as the waves are alternately in phase and out of phase. (Beats were discussed in Section 18.7.) The bottom curve in Figure 40.18b shows the results of superposing these two waves.

Notice that we have already introduced some localization by superposing the two waves. A single wave has the same amplitude everywhere in space; no point in space is any different from any other point. By adding a second wave, however, there is something different about the in-phase points compared with the out-of-phase points.

Now imagine that more and more waves are added to our original two, each new wave having a new frequency. Each new wave is added so that one of its crests is at $x = 0$ with the result that all the waves add constructively at $x = 0$. When we add a large number of waves, the probability of a positive value of a wave function at any point $x \neq 0$ is equal to the probability of a negative value, and there is destructive interference *everywhere* except near $x = 0$, where all the crests are superposed. The result is shown in Figure 40.19. The small region of constructive interference is called a **wave packet**. This localized region of space is different from all other regions. We can identify the wave packet as a particle because it has the localized nature of a particle! The location of the wave packet corresponds to the particle's position.

The localized nature of this entity is the *only* characteristic of a particle that was generated with this process. We have not addressed how the wave packet might achieve such particle characteristics as mass, electric charge, and spin. Therefore, you may not be completely convinced that we have built a particle. As further evidence that the wave packet can represent the particle, let's show that the wave packet has another characteristic of a particle.

To simplify the mathematical representation, we return to our combination of two waves. Consider two waves with equal amplitudes but different angular frequencies ω_1 and ω_2 . We can represent the waves mathematically as

$$y_1 = A \cos(k_1x - \omega_1t) \quad \text{and} \quad y_2 = A \cos(k_2x - \omega_2t)$$

where, as in Chapter 16, $k = 2\pi/\lambda$ and $\omega = 2\pi f$. Using the superposition principle, let's add the waves:

$$y = y_1 + y_2 = A \cos(k_1x - \omega_1t) + A \cos(k_2x - \omega_2t)$$

It is convenient to write this expression in a form that uses the trigonometric identity

$$\cos a + \cos b = 2 \cos\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right)$$

Letting $a = k_1x - \omega_1t$ and $b = k_2x - \omega_2t$ gives

$$\begin{aligned} y &= 2A \cos\left[\frac{(k_1x - \omega_1t) - (k_2x - \omega_2t)}{2}\right] \cos\left[\frac{(k_1x - \omega_1t) + (k_2x - \omega_2t)}{2}\right] \\ y &= \left[2A \cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)\right] \cos\left(\frac{k_1 + k_2}{2}x - \frac{\omega_1 + \omega_2}{2}t\right) \end{aligned} \quad (40.19)$$

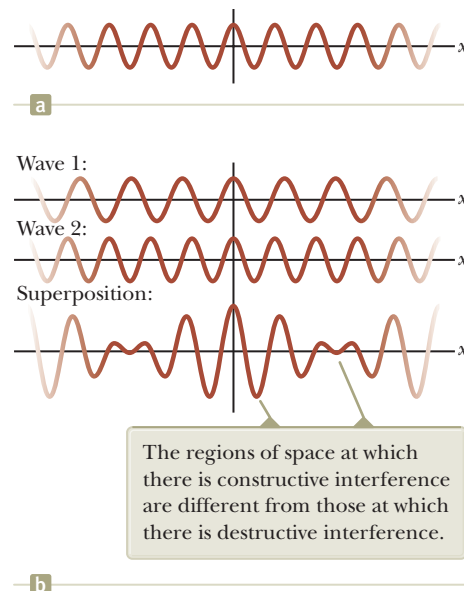
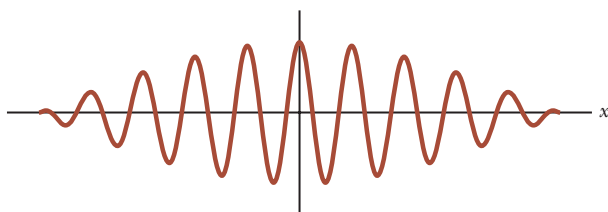
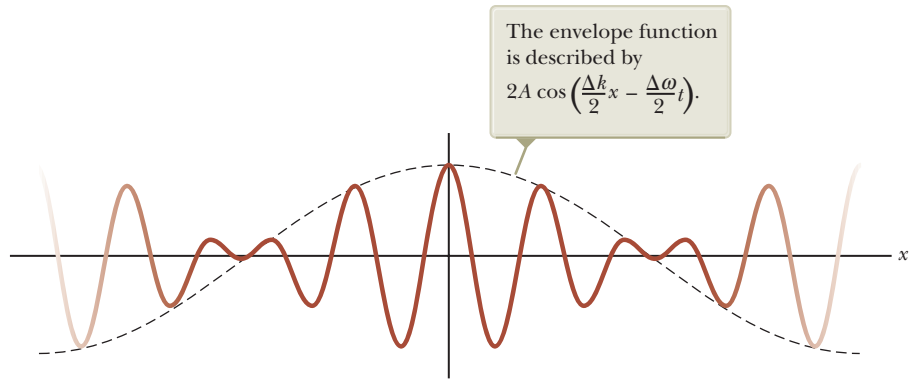


Figure 40.18 (a) An idealized wave of an exact single frequency is the same throughout space and time. (b) If two ideal waves with slightly different frequencies are combined, beats result (Section 18.7).

Figure 40.19 If a large number of waves are combined, the result is a wave packet, which represents a particle.

Figure 40.20 The beat pattern of Figure 40.18b, with an envelope function (dashed curve) superimposed.



where $\Delta k = k_1 - k_2$ and $\Delta\omega = \omega_1 - \omega_2$. The second cosine factor represents a wave with a wave number and frequency that are equal to the averages of the values for the individual waves.

In Equation 40.19, the factor in square brackets represents the envelope of the wave as shown by the dashed curve in Figure 40.20. This factor also has the mathematical form of a wave. This envelope of the combination can travel through space with a different speed than the individual waves. As an extreme example of this possibility, imagine combining two identical waves moving in opposite directions. The two waves move with the same speed, but the envelope has a speed of *zero* because we have built a standing wave, which we studied in Section 18.2.

For an individual wave, the speed is given by Equation 16.11,

$$v_{\text{phase}} = \frac{\omega}{k} \quad (40.20)$$

Phase speed of a wave
in a wave packet

This speed is called the **phase speed** because it is the rate of advance of a crest on a single wave, which is a point of fixed phase. Equation 40.20 can be interpreted as follows: the phase speed of a wave is the ratio of the coefficient of the time variable t to the coefficient of the space variable x in the equation representing the wave, $y = A \cos(kx - \omega t)$.

The factor in brackets in Equation 40.19 is of the form of a wave, so it moves with a speed given by this same ratio:

$$v_g = \frac{\text{coefficient of time variable } t}{\text{coefficient of space variable } x} = \frac{(\Delta\omega/2)}{(\Delta k/2)} = \frac{\Delta\omega}{\Delta k}$$

The subscript g on the speed indicates that it is commonly called the **group speed**, or the speed of the wave packet (the *group* of waves) we have built. We have generated this expression for a simple addition of two waves. When a large number of waves are superposed to form a wave packet, this ratio becomes a derivative:

Group speed of a wave packet

$$v_g = \frac{d\omega}{dk} \quad (40.21)$$

Multiplying the numerator and the denominator by \hbar , where $\hbar = h/2\pi$, gives

$$v_g = \frac{\hbar d\omega}{\hbar dk} = \frac{d(\hbar\omega)}{d(\hbar k)} \quad (40.22)$$

Let's look at the terms in the parentheses of Equation 40.22 separately. For the numerator,

$$\hbar\omega = \frac{h}{2\pi}(2\pi f) = hf = E$$

For the denominator,

$$\hbar k = \frac{h}{2\pi}\left(\frac{2\pi}{\lambda}\right) = \frac{h}{\lambda} = p$$

Therefore, Equation 40.22 can be written as

$$v_g = \frac{d(\hbar\omega)}{d(\hbar k)} = \frac{dE}{dp} \quad (40.23)$$

Because we are exploring the possibility that the envelope of the combined waves represents the particle, consider a free particle moving with a speed u that is small compared with the speed of light. The energy of the particle is its kinetic energy:

$$E = \frac{1}{2}mu^2 = \frac{p^2}{2m}$$

Differentiating this equation with respect to p gives

$$v_g = \frac{dE}{dp} = \frac{d}{dp} \left(\frac{p^2}{2m} \right) = \frac{1}{2m} (2p) = u \quad (40.24)$$

Therefore, the group speed of the wave packet is identical to the speed of the particle that it is modeled to represent, giving us further confidence that the wave packet is a reasonable way to build a particle.

Quick Quiz 40.7 As an analogy to wave packets, consider an “automobile packet” that occurs near the scene of an accident on a freeway. The phase speed is analogous to the speed of individual automobiles as they move through the backup caused by the accident. The group speed can be identified as the speed of the leading edge of the packet of cars. For the automobile packet, is the group speed (a) the same as the phase speed, (b) less than the phase speed, or (c) greater than the phase speed?

40.7 The Double-Slit Experiment Revisited

Wave-particle duality is now a firmly accepted concept reinforced by experimental results, including those of the Davisson–Germer experiment. As with the postulates of special relativity, however, this concept often leads to clashes with familiar thought patterns we hold from everyday experience.

One way to crystallize our ideas about the electron’s wave-particle duality is through an experiment in which electrons are fired at a double slit. Consider a parallel beam of mono-energetic electrons incident on a double slit as in Figure 40.21. Let’s assume the slit widths are small compared with the electron wavelength so that we need not worry about diffraction maxima and minima as discussed for light in Section 38.2. An electron detector screen is positioned far from the slits at a distance much greater than d , the separation distance of the slits. If the detector screen collects electrons for a long enough time, we find a typical wave interference pattern for the counts per minute, or probability of arrival of electrons. Such an interference

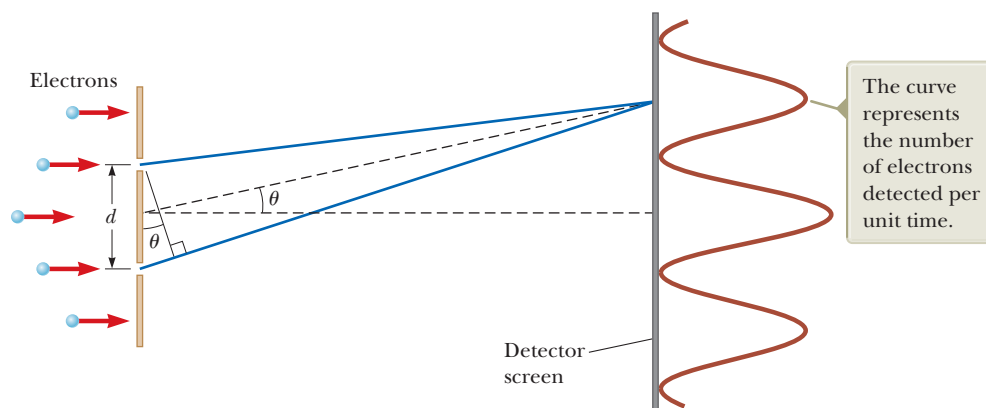


Figure 40.21 Electron interference. The slit separation d is much greater than the individual slit widths and much less than the distance between the slit and the detector screen.

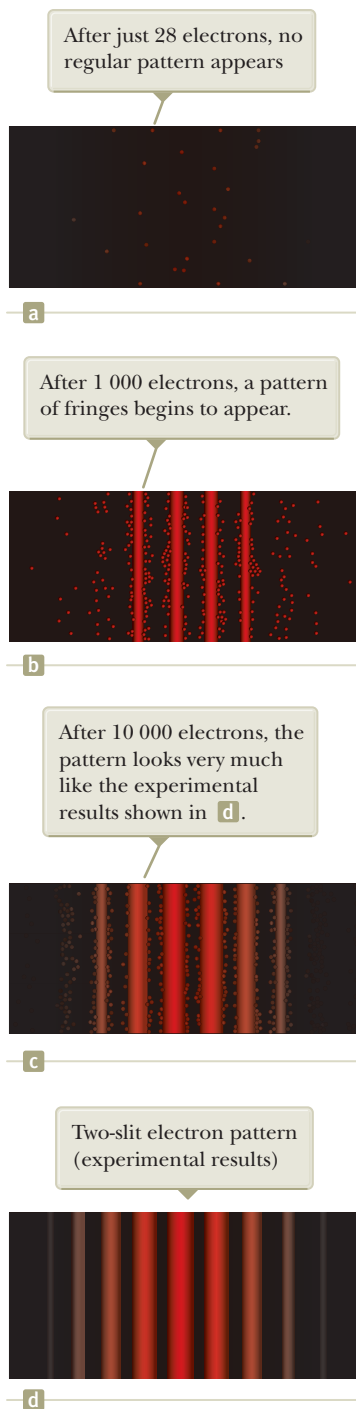


Figure 40.22 (a)–(c) Computer-simulated interference patterns for a small number of electrons incident on a double slit. (d) Computer simulation of a double-slit interference pattern produced by many electrons.

pattern would not be expected if the electrons behaved as classical particles, giving clear evidence that electrons are interfering, a distinct wave-like behavior.

If we measure the angles θ at which the maximum intensity of electrons arrives at the detector screen in Figure 40.21, we find they are described by exactly the same equation as that for light, $d \sin \theta = m\lambda$ (Eq. 37.2), where m is the order number and λ is the electron wavelength. Therefore, the dual nature of the electron is clearly shown in this experiment: the electrons are detected as particles at a localized spot on the detector screen at some instant of time, but the probability of arrival at that spot is determined by finding the intensity of two interfering waves.

Now imagine that we lower the beam intensity so that one electron at a time arrives at the double slit. It is tempting to assume the electron goes through either slit 1 or slit 2. You might argue that there are no interference effects because there is not a second electron going through the other slit to interfere with the first. This assumption places too much emphasis on the particle model of the electron, however. The interference pattern is still observed if the time interval for the measurement is sufficiently long for many electrons to pass one at a time through the slits and arrive at the detector screen! This situation is illustrated by the computer-simulated patterns in Figure 40.22 where the interference pattern becomes clearer as the number of electrons reaching the detector screen increases. Hence, our assumption that the electron is localized and goes through only one slit when both slits are open must be wrong (a painful conclusion!).

To interpret these results, we are forced to conclude that an electron interacts with both slits *simultaneously*. If you try to determine experimentally which slit the electron goes through, the act of measuring destroys the interference pattern. It is impossible to determine which slit the electron goes through. In effect, we can say only that the electron passes through *both* slits! The same arguments apply to photons.

If we restrict ourselves to a pure particle model, it is an uncomfortable notion that the electron can be present at both slits at once. From the quantum particle model, however, the particle can be considered to be built from waves that exist throughout space. Therefore, the wave components of the electron are present at both slits at the same time, and this model leads to a more comfortable interpretation of this experiment.

40.8 The Uncertainty Principle

Whenever one measures the position or velocity of a particle at any instant, experimental uncertainties are built into the measurements. According to classical mechanics, there is no fundamental barrier to an ultimate refinement of the apparatus or experimental procedures. In other words, it is possible, in principle, to make such measurements with arbitrarily small uncertainty. Quantum theory predicts, however, that it is fundamentally impossible to make simultaneous measurements of a particle's position and momentum with infinite accuracy.

In 1927, Werner Heisenberg (1901–1976) introduced this notion, which is now known as the **Heisenberg uncertainty principle**:

If a measurement of the position of a particle is made with uncertainty Δx and a simultaneous measurement of its x component of momentum is made with uncertainty Δp_x , the product of the two uncertainties can never be smaller than $\hbar/2$:

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \quad (40.25)$$

That is, it is physically impossible to measure simultaneously the exact position and exact momentum of a particle. Heisenberg was careful to point out that the inescapable uncertainties Δx and Δp_x do not arise from imperfections in practical measuring instruments. Rather, the uncertainties arise from the quantum structure of matter.

To understand the uncertainty principle, imagine that a particle has a single wavelength that is known *exactly*. According to the de Broglie relation, $\lambda = h/p$, we would therefore know the momentum to be precisely $p = h/\lambda$. In reality, a single-wavelength wave would exist throughout space. Any region along this wave is the same as any other region (Fig. 40.18a). Suppose we ask, Where is the particle this wave represents? No special location in space along the wave could be identified with the particle; all points along the wave are the same. Therefore, we have *infinite* uncertainty in the position of the particle, and we know nothing about its location. Perfect knowledge of the particle's momentum has cost us all information about its location.

In comparison, now consider a particle whose momentum is uncertain so that it has a range of possible values of momentum. According to the de Broglie relation, the result is a range of wavelengths. Therefore, the particle is not represented by a single wavelength, but rather by a combination of wavelengths within this range. This combination forms a wave packet as we discussed in Section 40.6 and illustrated in Figure 40.19. If you were asked to determine the location of the particle, you could only say that it is somewhere in the region defined by the wave packet because there is a distinct difference between this region and the rest of space. Therefore, by losing some information about the momentum of the particle, we have gained information about its position.

If you were to lose *all* information about the momentum, you would be adding together waves of all possible wavelengths, resulting in a wave packet of zero length. Therefore, if you know nothing about the momentum, you know exactly where the particle is.

The mathematical form of the uncertainty principle states that the product of the uncertainties in position and momentum is always larger than some minimum value. This value can be calculated from the types of arguments discussed above, and the result is the value of $\hbar/2$ in Equation 40.25.

Another form of the uncertainty principle can be generated by reconsidering Figure 40.19. Imagine that the horizontal axis is time rather than spatial position x . We can then make the same arguments that were made about knowledge of wavelength and position in the time domain. The corresponding variables would be frequency and time. Because frequency is related to the energy of the particle by $E = hf$, the uncertainty principle in this form is

$$\Delta E \Delta t \geq \frac{\hbar}{2} \quad (40.26)$$

The form of the uncertainty principle given in Equation 40.26 suggests that energy conservation can appear to be violated by an amount ΔE as long as it is only for a short time interval Δt consistent with that equation. We shall use this notion to estimate the rest energies of particles in Chapter 46.

- Quick Quiz 40.8** A particle's location is measured and specified as being exactly at $x = 0$, with *zero* uncertainty in the x direction. How does that location affect the uncertainty of its velocity component in the y direction? (a) It does not affect it. (b) It makes it infinite. (c) It makes it zero.



Werner Heisenberg

German Theoretical Physicist
(1901–1976)

Heisenberg obtained his Ph.D. in 1923 at the University of Munich. While other physicists tried to develop physical models of quantum phenomena, Heisenberg developed an abstract mathematical model called *matrix mechanics*. The more widely accepted physical models were shown to be equivalent to matrix mechanics. Heisenberg made many other significant contributions to physics, including his famous uncertainty principle for which he received a Nobel Prize in Physics in 1932, the prediction of two forms of molecular hydrogen, and theoretical models of the nucleus.

Pitfall Prevention 40.4

The Uncertainty Principle Some students incorrectly interpret the uncertainty principle as meaning that a measurement interferes with the system. For example, if an electron is observed in a hypothetical experiment using an optical microscope, the photon used to see the electron collides with it and makes it move, giving it an uncertainty in momentum. This scenario does *not* represent the basis of the uncertainty principle. The uncertainty principle is independent of the measurement process and is based on the wave nature of matter.

Example 40.6 Locating an Electron

The speed of an electron is measured to be 5.00×10^3 m/s to an accuracy of 0.003 00%. Find the minimum uncertainty in determining the position of this electron.

SOLUTION

Conceptualize The fractional value given for the accuracy of the electron's speed can be interpreted as the fractional uncertainty in its momentum. This uncertainty corresponds to a minimum uncertainty in the electron's position through the uncertainty principle.

continued

▶ 40.6 continued

Categorize We evaluate the result using concepts developed in this section, so we categorize this example as a substitution problem.

Assume the electron is moving along the x axis and find the uncertainty in p_x , letting f represent the accuracy of the measurement of its speed:

$$\Delta p_x = m \Delta v_x = m f v_x$$

Solve Equation 40.25 for the uncertainty in the electron's position and substitute numerical values:

$$\begin{aligned} \Delta x &\geq \frac{\hbar}{2 \Delta p_x} = \frac{\hbar}{2 m f v_x} = \frac{1.055 \times 10^{-34} \text{ J} \cdot \text{s}}{2(9.11 \times 10^{-31} \text{ kg})(0.000\,030\,0)(5.00 \times 10^3 \text{ m/s})} \\ &= 3.86 \times 10^{-4} \text{ m} = \mathbf{0.386 \text{ mm}} \end{aligned}$$

Example 40.7 The Line Width of Atomic Emissions

Atoms have quantized energy levels similar to those of Planck's oscillators, although the energy levels of an atom are usually not evenly spaced. When an atom makes a transition between states separated in energy by ΔE , energy is emitted in the form of a photon of frequency $f = \Delta E/h$. Although an excited atom can radiate at any time from $t = 0$ to $t = \infty$, the average time interval after excitation during which an atom radiates is called the **lifetime** τ . If $\tau = 1.0 \times 10^{-8} \text{ s}$, use the uncertainty principle to compute the line width Δf produced by this finite lifetime.

SOLUTION

Conceptualize The lifetime τ given for the excited state can be interpreted as the uncertainty Δt in the time at which the transition occurs. This uncertainty corresponds to a minimum uncertainty in the frequency of the radiated photon through the uncertainty principle.

Categorize We evaluate the result using concepts developed in this section, so we categorize this example as a substitution problem.

Use Equation 40.5 to relate the uncertainty in the photon's frequency to the uncertainty in its energy:

$$E = hf \rightarrow \Delta E = h \Delta f \rightarrow \Delta f = \frac{\Delta E}{h}$$

Use Equation 40.26 to substitute for the uncertainty in the photon's energy, giving the minimum value of Δf :

$$\Delta f \geq \frac{1}{h} \frac{\hbar}{2 \Delta t} = \frac{1}{h} \frac{h/2\pi}{2 \Delta t} = \frac{1}{4\pi \Delta t} = \frac{1}{4\pi \tau}$$

Substitute for the lifetime of the excited state:

$$\Delta f \geq \frac{1}{4\pi(1.0 \times 10^{-8} \text{ s})} = \mathbf{8.0 \times 10^6 \text{ Hz}}$$

WHAT IF? What if this same lifetime were associated with a transition that emits a radio wave rather than a visible light wave from an atom? Is the fractional line width $\Delta f/f$ larger or smaller than for the visible light?

Answer Because we are assuming the same lifetime for both transitions, Δf is independent of the frequency of radiation. Radio waves have lower frequencies than light waves, so the ratio $\Delta f/f$ will be larger for the radio waves. Assuming a light-wave frequency f of $6.00 \times 10^{14} \text{ Hz}$, the fractional line width is

$$\frac{\Delta f}{f} = \frac{8.0 \times 10^6 \text{ Hz}}{6.00 \times 10^{14} \text{ Hz}} = 1.3 \times 10^{-8}$$

This narrow fractional line width can be measured with a sensitive interferometer. Usually, however, temperature and pressure effects overshadow the natural line width and broaden the line through mechanisms associated with the Doppler effect and collisions.

Assuming a radio-wave frequency f of $94.7 \times 10^6 \text{ Hz}$, the fractional line width is

$$\frac{\Delta f}{f} = \frac{8.0 \times 10^6 \text{ Hz}}{94.7 \times 10^6 \text{ Hz}} = 8.4 \times 10^{-2}$$

Therefore, for the radio wave, this same absolute line width corresponds to a fractional line width of more than 8%.

Summary

Concepts and Principles

The characteristics of **blackbody radiation** cannot be explained using classical concepts. Planck introduced the quantum concept and Planck's constant h when he assumed atomic oscillators existing only in discrete energy states were responsible for this radiation. In Planck's model, radiation is emitted in single quantized packets whenever an oscillator makes a transition between discrete energy states. The energy of a packet is

$$E = hf \quad (40.5)$$

where f is the frequency of the oscillator. Einstein successfully extended Planck's quantum hypothesis to the standing waves of electromagnetic radiation in a cavity used in the blackbody radiation model.

The **photoelectric effect** is a process whereby electrons are ejected from a metal surface when light is incident on that surface. In Einstein's model, light is viewed as a stream of particles, or **photons**, each having energy $E = hf$, where h is Planck's constant and f is the frequency. The maximum kinetic energy of the ejected photoelectron is

$$K_{\max} = hf - \phi \quad (40.11)$$

where ϕ is the **work function** of the metal.

X-rays are scattered at various angles by electrons in a target. In such a scattering event, a shift in wavelength is observed for the scattered x-rays, a phenomenon known as the **Compton effect**. Classical physics does not predict the correct behavior in this effect. If the x-ray is treated as a photon, conservation of energy and linear momentum applied to the photon–electron collisions yields, for the Compton shift,

$$\lambda' - \lambda_0 = \frac{h}{m_e c} (1 - \cos \theta) \quad (40.13)$$

where m_e is the mass of the electron, c is the speed of light, and θ is the scattering angle.

Light has a dual nature in that it has both wave and particle characteristics. Some experiments can be explained either better or solely by the particle model, whereas others can be explained either better or solely by the wave model.

Every object of mass m and momentum $p = mu$ has wave properties, with a **de Broglie wavelength** given by

$$\lambda = \frac{h}{p} = \frac{h}{mu} \quad (40.17)$$

By combining a large number of waves, a single region of constructive interference, called a **wave packet**, can be created. The wave packet carries the characteristic of localization like a particle does, but it has wave properties because it is built from waves. For an individual wave in the wave packet, the **phase speed** is

$$v_{\text{phase}} = \frac{\omega}{k} \quad (40.20)$$

For the wave packet as a whole, the **group speed** is

$$v_g = \frac{d\omega}{dk} \quad (40.21)$$

For a wave packet representing a particle, the group speed can be shown to be the same as the speed of the particle.

The **Heisenberg uncertainty principle** states that if a measurement of the position of a particle is made with uncertainty Δx and a simultaneous measurement of its linear momentum is made with uncertainty Δp_x , the product of the two uncertainties is restricted to

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \quad (40.25)$$

Another form of the uncertainty principle relates measurements of energy and time:

$$\Delta E \Delta t \geq \frac{\hbar}{2} \quad (40.26)$$

Objective Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. Rank the wavelengths of the following quantum particles from the largest to the smallest. If any have equal

wavelengths, display the equality in your ranking. (a) a photon with energy 3 eV (b) an electron with