Mecánica Intermedia (LFIS 312)

Licenciatura en Física

Profesor: J. R. Villanueva Semestre I 2025

Nombre: Carlos Pincheira P. *RUT:* 20.155.517-5

Prueba 1: P1: 3.5 P2: 2.5 P3: 4.5 P4: 3.0 NF: 3.4

1. Una partícula se mueve en la dirección positiva del eje x de modo que su velocidad varía según la ley $v = \alpha\sqrt{x}$, donde α es una constante positiva. Teniendo en cuenta que en el momento $t = 0$ se encontraba en el punto $x = 0$, determinar:

   (a) la dependencia de la velocidad y de la aceleración respecto del tiempo;

   (b) la velocidad media de la partícula en el tiempo, en el transcurso del cual recorre los primeros s metros.

2. Considere una partícula moviéndose en la órbita

$$r(\phi) = a(1 + \cos\phi), \qquad (a > 0).$$

   (a) ¿Cuál es el potencial que provoca esta órbita?

   (b) Determine la aceleración radial y tangencial de la partícula.

3. Una masa puntual m desliza sin fricción en el interior de la superficie de revolución $z = \alpha \sin(s/R)$ cuyo eje de simetría está a lo largo de la dirección de un campo gravitacional $\vec{g}$. Considere $0 < s/R < \pi/2$.

   (a) Construya el Lagrangiano $L(s, \phi, \dot{s}, \dot{\phi})$, y calcule las ecuaciones de movimiento para las coordenadas generalizadas s y ϕ.

   (b) ¿Existen las órbitas circulares estacionarias horizontales?

   (c) ¿Cuáles de estas órbitas son estables bajo pequeños impulsos a lo largo de la superficie transversal a la dirección?

   (d) Si la órbita es estable, ¿cuál es la frecuencia de oscilación en torno a la órbita de equilibrio?

4. Una partícula de masa m se mueve bajo la acción de la gravedad sobre la superficie de un cilindro horizontal.

   (a) Obtener las ecuaciones de movimiento de Lagrange para la partícula.

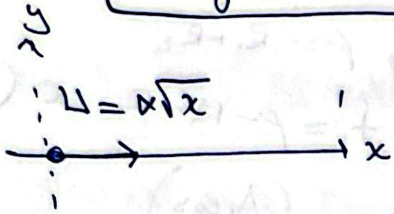
   (b) Si la partícula se desliza en un plano vertical habiendo partido de la parte superior del cilindro con una velocidad muy pequeña, hállese la fuerza de reacción en función de la posición.

   (c) ¿En qué punto se separará la partícula del cilindro?

Pregunta 1

Carlos Pincheira P.

$v = \alpha\sqrt{x}$ $\quad t_0 = 0 \rightarrow x_0 = 0 \rightarrow v_0 = 0$ $\quad x = \frac{\alpha^2}{4}t^2$



(a) $v = \alpha\sqrt{x} \rightarrow \frac{dx}{dt} = \alpha\sqrt{x} \Rightarrow \frac{dx}{\sqrt{x}} = \alpha dt \Big/ \int$

$\Rightarrow \int_{x_0}^{x} \frac{dx}{\sqrt{x}} = \alpha \int_{t_0}^{t} dt$ $\quad x^{-1/2} = \frac{x^{1/2}}{-1/2+1} = \cancel{\frac{1}{2}} 2 x^{1/2}$

$\rightarrow \frac{1}{2} x^{1/2}\Big|_{x_0}^{x} = \alpha(t - t_0) \rightarrow \cancel{\frac{1}{2}} 2\sqrt{x} = \alpha t \Rightarrow \sqrt{x} = \cancel{2} \frac{1}{2}\alpha t$

$\Rightarrow \boxed{t = \frac{2\sqrt{x}}{\cancel{\alpha}\ \alpha}}$

$\Rightarrow \boxed{v = \cancel{2} \frac{1}{2} \alpha^2 t} \rightarrow \frac{dv}{dt} = \boxed{a = 2\alpha^2}$

(b) $\overline{v}$ en s metros, $\overline{v} = \frac{s}{t} = \frac{s}{\frac{\sqrt{s}}{2\alpha}} = 2\alpha\sqrt{s}$

$\Rightarrow \boxed{\overline{v} = 2\alpha\sqrt{s}}$

$t = \frac{\sqrt{x}}{2\alpha}$ $\quad \overline{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{s}{\frac{2}{\alpha}\sqrt{s}} = \frac{\alpha}{2}\sqrt{s}$

Pregunta 2 | Carlos Pincheira P.

Partícula mov. en órbita $\boxed{r(\phi) = a(1+\cos\phi)}$ $u = \dfrac{1}{a(1+\cos\phi)}$

a) Potencial que provoca esta órbita

* Teorema de Binet $\longrightarrow$ * $\boxed{r = r(\phi) = \frac{1}{u}} \Rightarrow u = \frac{1}{r(\phi)}$

  * $F(1/u) = -\frac{L^2}{m}(u^2 u'' + u^3)$

* ~~Fuerza~~

$\Rightarrow \vec{F} = f(r)\hat{r} = -\vec{\nabla} U = \frac{dU}{dr}\hat{r} \Rightarrow \int f(r)dr = \int \frac{dU}{dr}$

$\Rightarrow \int f(r)dr = U\Big|_a^b$

$\therefore$ si $r(\phi) = a(1+\cos\phi) \longrightarrow u = \frac{1}{a(1+\cos\phi)}$; $u' = u^2 a \sin\phi$

$u' = \frac{+a\cancel{\dot\phi}\sin\phi}{a^2(1+\cos\phi)^2} \Longrightarrow u' = \frac{\cancel{\dot\phi}\sin\phi}{a(1+\cos\phi)^2} = \frac{a\cancel{\dot\phi}\sin\phi}{u^2}$ (*)

$\Rightarrow u'' = \frac{a\ddot\phi\sin\phi}{u^2} + \frac{a\dot\phi^2\cos\phi}{u^2} + \frac{a\dot\phi\sin\phi(-2u')}{u^3}$; $\left(\frac{1}{u^2}\right)' = -2u^{-3}\cdot u'$

$= \frac{a\ddot\phi\sin\phi}{u^2} + \frac{a\dot\phi^2\cos\phi}{u^2} - \frac{2a\dot\phi\sin\phi}{u^3}\cdot\frac{a\dot\phi\sin\phi}{u^2}$

$= \frac{a\ddot\phi\sin\phi}{u^2} + \frac{a\dot\phi^2\cos\phi}{u^2} - \frac{2a^2\dot\phi^2\sin^2\phi}{u^5}$

$\Rightarrow u^2 u'' = a\ddot\phi\sin\phi + a\dot\phi^2\cos\phi - \frac{2a^2\dot\phi^2\sin^2\phi}{u^3}$

+ ni comino $\sin^2\phi = 1-\cos^2\phi \Rightarrow -\frac{2a^2\dot\phi^2(1-\cos^2\phi)}{u^3} = -\frac{\cancel{2a^2\dot\phi^2 + 2a^2\dot\phi^2\cos^2\phi}}{u^3}$

$\Rightarrow$

$=$

Pregunta 3

Carlos Pinchuira

$$z = \alpha \sin\left(\frac{r}{R}\right) \longrightarrow \dot{z} = \frac{\alpha \dot{r}\cos\left(\frac{r}{R}\right)}{R}$$

$$x = r\cos\phi \longrightarrow \dot{x} = \dot{r}\cos\phi - r\sin\phi\dot{\phi}$$

$$y = r\sin\phi \longrightarrow \dot{y} = \dot{r}\sin\phi + r\cos\phi\dot{\phi}$$

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

$$U = mgz = mg\alpha\sin\left(\frac{r}{R}\right)$$

$$\dot{x}^2 + \dot{y}^2 = (\dot{r}\cos\phi - r\sin\phi\dot{\phi})^2 + (\dot{r}\sin\phi + r\cos\phi\dot{\phi})^2$$

$$= \dot{r}^2\cos^2\phi - \cancel{2\dot{r}\dot{\phi}r\cos\phi\sin\phi} + r^2\dot{\phi}^2\sin^2\phi + \dot{r}^2\sin^2\phi + \cancel{2\dot{r}\dot{\phi}r\sin\phi\cos\phi} + r^2\cos^2\phi\dot{\phi}^2 = \dot{r}^2 + r^2\dot{\phi}^2$$

$$\therefore \mathcal{L} = T - U = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\phi}^2 + \frac{\alpha^2}{R^2}\dot{r}^2\cos^2\left(\frac{r}{R}\right)\right) - mg\alpha\sin\left(\frac{r}{R}\right)$$

$$\boxed{\mathcal{L} = \frac{1}{2}m\left(\dot{r}^2\left\{1 + \frac{\alpha^2}{R^2}\cos^2\left(\frac{r}{R}\right)\right\} + r^2\dot{\phi}^2\right) - mg\alpha\sin\left(\frac{r}{R}\right)}$$

$s = r$
$\dot{s} = \dot{r}$

Ecs. MOV $\Longrightarrow \frac{d}{dt}\partial_{\dot{q}}\mathcal{L} - \partial_q\mathcal{L} = 0$; $q = \{r, \phi\}$

$\boxed{r}$ $$\frac{d}{dt}\left[m\dot{r}\left\{1 + \frac{\alpha^2}{R^2}\cos^2\left(\frac{r}{R}\right)\right\}\right] + \frac{\alpha^2}{R^3}m\dot{r}^2\sin\left(\frac{r}{R}\right)\cos\left(\frac{r}{R}\right) - mr\dot{\phi}^2 + \frac{\alpha}{R}mg\cos\left(\frac{r}{R}\right) = 0$$

$$\rightarrow \underbrace{\frac{d}{dt}\left[m\dot{r}\left\{1 + \frac{\alpha^2}{R^2}\cos^2\left(\frac{r}{R}\right)\right\}\right]}_{(I)} + \frac{2\alpha^2}{R^3}m\dot{r}^2\sin\left(\frac{2r}{R}\right) - mr\dot{\phi}^2 + \frac{\alpha}{R}mg\cos\left(\frac{r}{R}\right) = 0$$

$$\Rightarrow (I) \rightarrow \frac{d}{dt}\left[m\dot{r}\left\{1 + \frac{\alpha^2}{R^2}\cos^2\left(\frac{r}{R}\right)\right\}\right] = m\ddot{r}\left\{1 + \frac{\alpha^2}{R^2}\cos^2\left(\frac{r}{R}\right)\right\} - m\dot{r}\frac{\alpha^2}{R^3}\dot{r}\cos\left(\frac{r}{R}\right)\sin\left(\frac{r}{R}\right)$$

$$= m\ddot{r}\left\{1 + \frac{\alpha^2}{R^2}\cos^2\left(\frac{r}{R}\right)\right\} - 2m\dot{r}^2\frac{\alpha^2}{R^3}\sin\left(\frac{2r}{R}\right)$$

$$\therefore m\ddot{r}\left\{1 + \frac{\alpha^2}{R^2}\cos^2\left(\frac{r}{R}\right)\right\} - \cancel{2m\dot{r}^2\frac{\alpha^2}{R^3}\sin\left(\frac{2r}{R}\right)} + \cancel{2m\dot{r}^2\frac{\alpha^2}{R^3}\sin\left(\frac{2r}{R}\right)} - mr\dot{\phi}^2 + \frac{\alpha}{R}mg\cos\left(\frac{r}{R}\right) = 0$$

$$\Longrightarrow \boxed{m\ddot{r}\left\{1 + \frac{\alpha^2}{R^2}\cos^2\left(\frac{r}{R}\right)\right\} - mr\dot{\phi}^2 + \frac{\alpha}{R}mg\cos\left(\frac{r}{R}\right) = 0}$$

Ec. MOV para r. ($\blacktriangle$)

$\boxed{\phi}$ $\frac{d}{dt}\partial_{\dot{\phi}}\mathcal{L} - \partial_\phi\mathcal{L} = 0$; $\partial_\phi\mathcal{L} = 0$; $\frac{d}{dt}\partial_{\dot{\phi}}\mathcal{L} = 0$

$\therefore \partial_{\dot{\phi}}\mathcal{L} = \text{cte} = L \Rightarrow \boxed{L = mr^2\dot{\phi} = \text{cte}}$ Conservación momentum ∡

---

b) órbita circular $\longrightarrow \dot{r} = \ddot{r} = 0$

$\therefore (\blacktriangle)\ -mr\dot{\phi}^2 + \frac{\alpha}{R}mg\cos\left(\frac{r}{R}\right) = 0$; $\dot{\phi} = \frac{L}{mr^2}$

$$\Rightarrow \frac{mrL^2}{m^2r^4} = \frac{\alpha}{R}mg\cos\left(\frac{r}{R}\right) \Rightarrow r^3 = \frac{L^2/R}{\alpha m^2 g\cos\left(\frac{r}{R}\right)}$$

$$\therefore \boxed{r = \left(\frac{L^2/R}{\alpha m^2 g \cos\left(\frac{r}{R}\right)} \right)^{1/3}}$$

radio de órbita circular

$\therefore \exists$ órbitas circulares estacionarios horizontales

---

(c) órbitas estables bajo pequeños impulsos $\Rightarrow r = r_0 + \epsilon(t)$

$$\Rightarrow \dot{r} = \dot{\epsilon} \Rightarrow \ddot{r} = \ddot{\epsilon}$$

EC. MOV. $r \Rightarrow m\ddot{r}\left\{1 + \frac{\alpha^2}{R^2}\cos^2\left(\frac{r}{R}\right)\right\} - \frac{L^2}{mr^3} + \frac{\alpha}{R} mg\cos\left(\frac{r}{R}\right) = 0$

Reemplazando $r = r_0 + \epsilon$ $\rightarrow m\ddot{\epsilon}\left\{1 + \frac{\alpha^2}{R^2}\cos^2\left(\frac{r_0+\epsilon}{R}\right)\right\} - \frac{L^2}{m(r_0+\epsilon)^3} + \frac{\alpha}{R} mg\cos\left(\frac{r_0+\epsilon}{R}\right) = 0$

Solo hasta 1er orden

$$* \frac{L^2}{m(r_0+\epsilon)^3} = \frac{L^2}{mr_0^3}\left(1 + \frac{\epsilon}{r_0}\right)^{-3} \simeq \frac{L^2}{mr_0^3}\left(1 - \frac{3\epsilon}{r_0}\right) \simeq \frac{L^2}{mr_0^3} - \frac{3L^2}{mr_0^4}\epsilon$$

$$* \cos^2\left(\frac{r_0+\epsilon}{R}\right) \simeq \cos\left(\frac{r_0}{R}\right) - \frac{2\epsilon}{R}\sin\left(\frac{r_0}{R}\right)$$

$$* \cos\left(\frac{r_0+\epsilon}{R}\right) \simeq \cos\left(\frac{r_0}{R}\right) - \frac{\epsilon}{R}\sin\left(\frac{r_0}{R}\right)$$

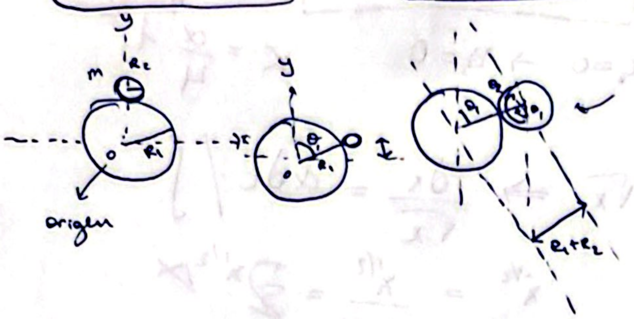
$$m\ddot{\epsilon}\left\{1 + \frac{\alpha^2}{R^2}\cos^2\left(\frac{r_0}{R}\right)\right\} - \cancel{\frac{L^2}{mr_0^3}} + \frac{3L^2}{mr_0^4}\epsilon + \frac{\alpha}{R} mg\cos\left(\frac{r_0}{R}\right) = 0$$

orden 0

$$\Rightarrow \ddot{\epsilon} + \frac{\frac{3L^2}{mr_0^4}}{1 + \frac{\alpha^2}{R^2}\cos^2\left(\frac{r_0}{R}\right)}\epsilon = 0$$

$$\Rightarrow \boxed{\omega^2 = \frac{3L^2/r_0^4}{m\left(1 + \frac{\alpha^2}{R^2}\cos^2\left(\frac{r_0}{R}\right)\right)}}$$

$$\Rightarrow \omega = \sqrt{\frac{3L^2/r_0^4}{m\left(1 + \frac{\alpha^2}{R^2}\cos^2\left(\frac{r_0}{R}\right)\right)}}$$



ligaduras $\Rightarrow$ $\rho = R_1 + R_2$

$f_1 = \rho - R_1 - R_2 = 0 \quad (1)$

$S_1 = S_2$ (Arcos)

$\Rightarrow \theta_1 R_1 = (\theta_2 - \theta_1) R_2$

$\Rightarrow f_2 = \theta_1 R_1 - (\theta_2 - \theta_1) R_2 = 0 \quad (2)$

$\therefore T = \frac{1}{2} m(\dot{\rho}^2 + \rho^2 \dot{\theta}_1^2 + R_2^2 \dot{\theta}_2^2)$

$U = m g \rho \cos\theta_1 \quad \rightarrow \boxed{\mathcal{L} = \frac{1}{2} m(\dot{\rho}^2 + \rho^2 \dot{\theta}_1^2 + R_2^2 \dot{\theta}_2^2) - m g \rho \cos\theta_1}$ (3)

$q = \{\rho, \theta_1, \theta_2\}$

$\boxed{\rho}$ $\frac{d}{dt}\partial_{\dot{\rho}}\mathcal{L} - \partial_\rho \mathcal{L} = \lambda_1 \partial_\rho f_1 \equiv Q_r$; $\frac{d}{dt}[m\dot{\rho}] - m\rho\dot{\theta}_1^2 + m g \cos\theta_1 = \lambda_1 \equiv Q_r$

$\partial_\rho f_1 = 1$, $\partial_{\theta_1} f_2 = (R_1 + R_2)$

$\rightarrow \boxed{m\ddot{\rho} - m\rho\dot{\theta}_1^2 + m g \cos\theta_1 = \lambda_1 \equiv Q_r}$ (4)

$\boxed{\theta_1}$ $\frac{d}{dt}\partial_{\dot{\theta}_1}\mathcal{L} - \partial_{\theta_1}\mathcal{L} = \lambda_2 \partial_{\theta_1} f_2 \equiv Q_1$; $\frac{d}{dt}[m\rho^2\dot{\theta}_1] - m g \rho \sin\theta_1 =$

$\rightarrow \boxed{2m\rho\dot{\rho}\dot{\theta}_1 + m\rho^2\ddot{\theta}_1 - m g \rho \sin\theta_1 = \lambda_2(R_1 + R_2) \equiv Q_1}$ (5)

$\boxed{\theta_2}$ $\frac{d}{dt}\partial_{\dot{\theta}_2}\mathcal{L} - \partial_{\theta_2}\mathcal{L} = \lambda_2 \partial_{\theta_2} f_2 \equiv Q_2$; $\frac{d}{dt}[m R_2^2 \dot{\theta}_2] = -\lambda_2 R_2$

$\Rightarrow \boxed{m R_2^2 \ddot{\theta}_2 = -\lambda_2 R_2 \equiv Q_2}$ (6) Ecuaciones de Mov.

---

b) Fuerza reacción Q_r , $\delta W = Q_r \delta\rho + Q_1 \delta\theta_1 + Q_2 \delta\theta_2$ $\quad \boxed{\lambda_1 = N}$

$\hookrightarrow$ Normal

Cond. Contacto $\Longrightarrow$ $\delta\rho \neq 0$, $\delta\theta_1 = \delta\theta_2 = 0 \Rightarrow \delta W = \lambda_1 \delta\rho = N \delta\rho$

Ligaduras $\Longrightarrow$ $\dot{\rho} = \ddot{\rho} = 0$, $(R_1 + R_2)\dot{\theta}_1 = R_2\dot{\theta}_2 \Rightarrow (R_1 + R_2)\ddot{\theta}_1 = R_2\ddot{\theta}_2$

$\therefore (4) \Rightarrow -m(R_1 + R_2)\dot{\theta}_1^2 + m g \cos\theta_1 = \lambda_1 \equiv N$ *

$(5) \Rightarrow m(R_1 + R_2)^{\cancel{2}}\ddot{\theta}_1 - m g \cancel{(R_1 + R_2)} \sin\theta_1 = \lambda_2 \cancel{(R_1 + R_2)}$

$\rightarrow m(R_1 + R_2)\ddot{\theta}_1 - m g \sin\theta_1 = \lambda_2$

$(6) \Rightarrow -m R_2 \ddot{\theta}_2 = \lambda_2 = -m(R_1 + R_2)\ddot{\theta}_1$

$\lambda_2 = \lambda_2$

$\rightarrow m(R_1 + R_2)\ddot{\theta}_1 - m g \sin\theta_1 = -m(R_1 + R_2)\ddot{\theta}_1 \Rightarrow 2m(R_1 + R_2)\ddot{\theta}_1 - m g \sin\theta_1 = 0$

$\ddot{\theta}_1 = \frac{d\dot{\theta}_1}{dt} = \frac{d\theta_1}{dt}\frac{d\dot{\theta}_1}{d\theta_1} = \dot{\theta}_1 \frac{d\dot{\theta}_1}{d\theta_1}$ $\rightarrow 2m(R_1 + R_2)\dot{\theta}_1 d\dot{\theta}_1 = m g \sin\theta_1 d\theta_1$

$\rightarrow 2m(R_1 + R_2)\int \dot{\theta}_1 d\dot{\theta}_1 = m g \int \sin\theta_1 d\theta_1 \rightarrow m(R_1 + R_2)\dot{\theta}_1^2 = -m g \cos\theta_1 + C$

P. 4.

Carlos HJ

$$\Rightarrow \cancel{m}(R_1+R_2)\dot{\theta}_1^2 = -\cancel{m}g\cos\theta_1 + C \quad ; \text{ si } \theta_1 = 0 \rightarrow \dot{\theta}_1 = 0$$

$$\Rightarrow 0 = -g + C \Rightarrow C = g$$

$$\therefore (R_1+R_2)\dot{\theta}_1^2 = -g\cos\theta_1 + g = g(1-\cos\theta_1)$$

$$\Rightarrow (R_1+R_2)\dot{\theta}_1^2 = g(1-\cos\theta_1) \longrightarrow (4) -m(R_1+R_2)\dot{\theta}_1^2 + mg\cos\theta_1 = \lambda_1 \equiv N$$

$$\therefore -mg(1-\cos\theta_1) + mg\cos\theta_1 = N$$

$$\Rightarrow -mg + mg\cos\theta_1 + mg\cos\theta_1 = N$$

$$\Rightarrow -mg + 2mg\cos\theta_1 = N \Rightarrow \boxed{mg(1-2\cos\theta_1) = N}$$

$\rightarrow$ Para encontrar K $\rightarrow$ necesario que $N=0$

$$\therefore \cos\theta_1 = \frac{1}{2} \Rightarrow \boxed{\theta_1 = 60^\circ}$$

Mecánica Intermedia (LFIS 312)

Licenciatura en Física

Profesor: J. R. Villanueva Semestre I 2025

Nombre: Carlos Pincheira P *RUT:* 20.155.517-5

Prueba 2 (R): P1: 7.0 P2: 3.0 P3: 5.5 P4:____ NF: 5.2

1. Un pión cargado (π^+ o π^-) tiene una energía cinética (no-relativista) T. Un núcleo masivo tiene una carga $Q = Ze$ y un radio efectivo b. Clásicamente, el pión golpea el núcleo si la distancia de máxima aproximación es b o menor. Despreciando el retroceso del núcleo (y los efectos electrón–núcleo), muestre que la sección eficaz de scattering para estos piones es:

$$\sigma = \begin{cases} \frac{\pi b^2 (T-V)}{T}, & \text{para } \pi^+ \\ \frac{\pi b^2 (T+V)}{T}, & \text{para } \pi^-, \end{cases}$$

donde

$$V = \frac{Ze^2}{b}.$$

2. Un aro delgado de radio R y masa M oscila en su propio plano con un punto del aro fijo. Unido al aro hay una masa puntual m obligada a moverse sin fricción a lo largo del aro. El sistema está en un campo gravitacional $\vec{g}$. Considere sólo pequeñas oscilaciones.

   (a) Encuentre las frecuencias de modos normales
   (b) Encuentre los autovectores de modos normales. Dibuje su movimiento.
   (c) Construya la matriz modal.
   (d) Encuentre las coordenadas normales y muestre que ellas diagonalizan el Lagrangiano.

3. Un flujo de partículas con energía E son dispersadas por un potencial central atractivo

$$V(r) = \begin{cases} 0 & r > a \\ -V_0, & r < a. \end{cases}$$

Muestre que la órbita de las partículas es idéntica con la de los rayos de luz refractados por una esfera de radio a e índice de refracción $n = [(E + V_0)/E]^{1/2}$. Pruebe que la sección eficaz elástica diferencial para $\cos\frac{1}{2}\theta > n^{-1}$ es

$$\left(\frac{d\sigma}{d\Omega}\right)_{el} = \frac{n^2 a^2}{4\cos\frac{1}{2}\theta} \frac{\left[n\cos\left(\frac{1}{2}\theta\right) - 1\right]\left(n - \cos\frac{1}{2}\theta\right)}{\left(1 + n^2 - 2n\cos\frac{1}{2}\theta\right)^2} \quad (1)$$

¿Cuál es la sección eficaz total? ?

Pregunta 1 | Carlos Pinchueira P.

$$\sigma = \begin{cases} \frac{\pi b^2 (T-V)}{T} & \pi^+ \\ \frac{\pi b^2 (T+V)}{T} & \pi^- \end{cases} \quad ; \quad \sigma_T = \pi b_{max}^2 = \pi s^2 \to ? \quad , \quad V = \frac{Ze^2}{b} \quad s^2 = ?$$

Conservación energía $E = T + V(r) \xrightarrow{r\to\infty} E = T$; $E = \frac{1}{2}mv^2 + V($

Conservación momento ∡ $l = mvb$, $l = mvs$; $T = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2T}{m}}$

$E = \frac{L^2}{2mb^2} + V$; $L = ms\sqrt{\frac{2T}{m}} \,/^2 \Rightarrow L^2 = 2m^2 s^2 \frac{T}{m} = 2ms^2T$ (*)

$$\Rightarrow E = \frac{2ms^2T}{2mb^2} + V \xrightarrow[E=T]{\text{Conservación Energía}} T = T\frac{s^2}{b^2} + V \Rightarrow T\left(1 - \frac{s^2}{b^2}\right) = V$$

$$\Rightarrow 1 - \frac{s^2}{b^2} = \frac{V}{T} \Rightarrow \frac{s^2}{b^2} = 1 - \frac{V}{T} = \frac{s^2}{b^2} \Rightarrow s^2 = b^2\left(\frac{T-V}{T}\right)$$

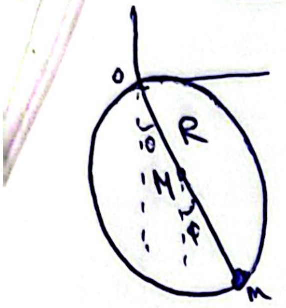
$\therefore$ para $\pi^+ \Rightarrow \sigma = \frac{\pi b^2 (T-V)}{T}$

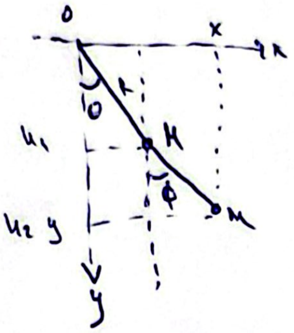
$$\therefore \boxed{\sigma = \begin{cases} \frac{\pi b^2 (T-V)}{T} & \text{para } \pi^+ \\ \frac{\pi b^2 (T+V)}{T} & \text{para } \pi^- \end{cases}}$$

$$E = \frac{L^2}{2mb^2} - V \Rightarrow E = \frac{2ms^2T}{2mb^2} - V \Rightarrow \frac{E+V}{T} = \frac{s^2}{b^2} \xrightarrow{E=T} s^2 = \frac{b^2(T+V)}{T}$$

$$\Rightarrow \sigma = \frac{\pi b^2 (T+V)}{T} \quad \text{para } \pi^-$$

Pregunta 2 | Carlos Pincheira





Centro de masa, anillo, masa

$$I = MR^2 + mR^2 = (M+m)R^2 \to T_1 = \frac{1}{2}(M+m)R^2\dot{\theta}^2$$

$$\cos x \approx 1 - \frac{x^2}{2}$$

$$h_1 = R - R\cos\theta = R(1-\cos\theta)$$

$$h_2 = R - R\cos\theta - R\cos\phi = R(1-\cos\theta-\cos\phi)$$

$$\to h_1 \approx R\left(\cancel{1}-\cancel{1}+\frac{\theta^2}{2}\right) \approx \frac{R\theta^2}{2}$$

$$\to h_2 \approx R\left(\cancel{1}-\cancel{1}+\frac{\theta^2}{2}-1+\frac{\phi^2}{2}\right) \approx R\left(-1+\frac{\theta^2}{2}+\frac{\phi^2}{2}\right) \approx \frac{R}{2}(\theta^2+\phi^2)$$

Masa puntual m

$$\therefore\ x = R\sin\theta + R\sin\phi \to \dot{x} = R(\dot{\theta}\cos\theta + \dot{\phi}\cos\phi) \to \dot{x}^2 = R^2(\dot{\theta}^2\cos^2\theta + 2\dot{\theta}\dot{\phi}\cos\theta\cos\phi + \dot{\phi}^2\cos^2\phi)$$

$$y = -R\cos\theta - R\cos\phi \to \dot{y} = R(\dot{\theta}\sin\theta + \dot{\phi}\sin\phi) \to \dot{y}^2 = R^2(\dot{\theta}^2\sin^2\theta + 2\dot{\theta}\dot{\phi}\sin\theta\sin\phi + \dot{\phi}^2\sin^2\phi)$$

$$\Rightarrow \dot{x}^2+\dot{y}^2 = R^2(\dot{\theta}^2 + 2\dot{\theta}\dot{\phi}\cos(\theta-\phi) + \dot{\phi}^2),\quad \theta,\phi \ll 1 \Rightarrow \cos(\theta-\phi) \Rightarrow \dot{x}^2+\dot{y}^2 = R^2(\dot{\theta}^2+2\dot{\theta}\dot{\phi}+\dot{\phi}^2)$$

$$T_2 = \frac{1}{2}mR^2(\dot{\theta}^2+2\dot{\theta}\dot{\phi}+\dot{\phi}^2)\ ;\ V_1 = \frac{MgR}{2}(2\theta^2+\phi^2)\ ;\ y = -R(\cos\theta+\cos\phi) \approx -R\left(1-\frac{\theta^2}{2}+1-\frac{\phi^2}{2}\right)$$

$$= -R\left(2-\frac{(\theta^2+\phi^2)}{2}\right) = \frac{R}{2}(\theta^2+\phi^2)$$

$$V_2 = \frac{mgR}{2}(\theta^2+\phi^2)$$

$$\therefore\ T = T_1+T_2 = \frac{1}{2}(M+m)R^2\dot{\theta}^2 + \frac{1}{2}mR^2(\dot{\theta}^2+2\dot{\theta}\dot{\phi}+\dot{\phi}^2)$$

$$V = V_1+V_2 = \frac{MgR}{2}(2\theta^2+\phi^2) + \frac{mgR}{2}(\dot{\theta}^2+\phi^2)$$

$$T = \frac{1}{2}MR^2\dot{\theta}^2 + \frac{1}{2}mR^2\dot{\theta}^2 + \frac{1}{2}mR^2(\dot{\theta}^2+2\dot{\theta}\dot{\phi}+\dot{\phi}^2) = \frac{1}{2}R^2(M\dot{\theta}^2+m\dot{\theta}^2+m\dot{\theta}^2+2m\dot{\theta}\dot{\phi}+m\dot{\phi}^2)$$

$$= \frac{1}{2}R^2((2m+M)\dot{\theta}^2+2m\dot{\theta}\dot{\phi}+m\dot{\phi}^2) = \frac{1}{2}((2m+M)R^2\dot{\theta}^2+2mR^2\dot{\theta}\dot{\phi}+mR^2\dot{\phi}^2)$$

$$V = MgR\theta^2+\frac{1}{2}MgR\phi^2+\frac{1}{2}mgR\theta^2+\frac{1}{2}mgR\phi^2 = gR\left(M+\frac{1}{2}m\right)\theta^2+gR\left(\frac{1}{2}M+\frac{1}{2}m\right)\phi^2$$

$$= \frac{1}{2}gR(2M+m)\theta^2+\frac{1}{2}gR(M+m)\phi^2$$

$$T = \frac{1}{2}\dot{q}^T \mathbb{M}\dot{q} = \frac{1}{2}(\dot{\theta}\ \dot{\phi})\begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} \dot{\theta} \\ \dot{\phi} \end{pmatrix} = \frac{1}{2}(A\dot{\Phi}^2+B\dot{\theta}\dot{\phi}+C\dot{\theta}\dot{\phi}+D\dot{\phi}^2)$$

$$\mathbb{M} = \begin{pmatrix} (2m+M)R^2 & mR^2 \\ mR^2 & mR^2 \end{pmatrix}\ ;\ \mathbb{K} = \begin{pmatrix} gR(2M+m) & 0 \\ 0 & gR(M+m) \end{pmatrix}$$

$$(\mathbb{K}-\omega^2\mathbb{M})$$

Pregunta 3 | Carlos Pinheiro

$$V = \begin{cases} 0 & r>a \to E=\frac{1}{2} m v_0^2 \to v_0=\sqrt{\frac{2E}{m}} \\ -V_0 & r<a \to E=\frac{1}{2} m v^2 - V_0 \to v=\sqrt{\frac{2}{m}(E+V_0)}\end{cases}$$

Ley de Snell → $n=\frac{v}{v_0} = \sqrt{\frac{2}{m}(E+V_0)}\sqrt{\frac{m}{2E}} = \sqrt{\frac{E+V_0}{E}} \to n=\sqrt{\frac{E+V_0}{E}}$

Scattering de Snell → $b=a\sin\alpha$, $n=\frac{\sin\alpha}{\sin\beta}$, $\theta/2=\beta-\alpha$

∴ $\sin\alpha=\frac{b}{a}$, $\sin\beta=\frac{b}{na}$, $\theta/2=\beta-\alpha$ /cos → $\cos(\theta/2)=\cos(\beta-\alpha)=\psi$

$\alpha=\beta-\theta/2$ /sin ⇒ $\sin\alpha=\sin(\beta-\theta/2)$; $\cos^2\theta/2+\sin^2\theta/2=1 \Rightarrow \sin\theta/2=\sqrt{1-\psi^2}$

∴ $b=a\sin\alpha=a\sin(\beta-\theta/2)=a\left[\sin\beta\cos\theta/2-\sin\theta/2\cos\beta\right]$ | $\sin^2\beta+\cos^2\beta=1\to\cos\beta=\sqrt{1-\left(\frac{b}{na}\right)^2}$ ⇒ $\cos\beta=\sqrt{\frac{n^2a^2-b^2}{n^2a^2}}$

$$=a\left[\frac{b}{na}\psi-\sqrt{1-\psi^2}\sqrt{1-\left(\frac{b}{na}\right)^2}\right] = \cancel{a}\left[\frac{b}{n\cancel{a}}\psi - \frac{1}{n\cancel{a}}\sqrt{1-\psi^2}\sqrt{n^2a^2-b^2}\right]$$

$$=\frac{1}{n}\left[b\psi-\sqrt{1-\psi^2}\sqrt{n^2a^2-b^2}\right] \to b^2(\psi-n)^2=(1-\psi^2)(n^2a^2-b^2)=n^2a^2-b^2-n^2a^2\psi^2+b^2\psi^2$$

$$\Rightarrow b^2(\psi-n)^2+b^2-b^2\psi^2=n^2a^2-n^2a^2\psi^2 \Rightarrow b^2\left(1-\psi^2+(\psi-n)^2\right)=n^2a^2-n^2a^2\psi^2$$

$* \; 1-\cancel{\psi^2}+\cancel{\psi^2}-2n\psi+n^2=1-2n\psi+n^2$ ∴ $b^2(1-2n\psi+n^2)=n^2a^2(1-\psi^2) \Rightarrow \boxed{b=\frac{na\sqrt{1-\psi^2}}{\sqrt{1-2n\psi+n^2}}}$ (*)

∴ $\frac{db}{d\psi}=na\left\{\frac{-\psi}{\sqrt{1-\psi^2}\sqrt{1-2n\psi+n^2}}+\frac{n\sqrt{1-\psi^2}}{(1-2n\psi+n^2)^{3/2}}\right\}$

$\sqrt{1-\psi^2}$ $\quad (1-\psi$ $\quad \frac{1}{2}\frac{-2\psi}{\sqrt{1-\psi^2}}$ $\quad -\frac{1}{2}\frac{-2n}{(1-2n\psi+n^2)^{-3/2}}$

$$=na\left\{\frac{-\psi(1-2n\psi+n^2)+n\sqrt{1-\psi^2}\sqrt{1-\psi^2}}{\sqrt{1-\psi^2}(1-2n\psi+n^2)^{3/2}}\right\} = na\left\{\frac{-\psi+2n\psi^2-n^2\psi+n(1-\psi^2)}{\sqrt{1-\psi^2}(1-2n\psi+n^2)^{3/2}}\right\}$$

$$=na\left\{\frac{-\psi+\cancel{2}n\psi^2-n^2\psi+n-\cancel{n\psi^2}}{\sqrt{1-\psi^2}(1-2n\psi+n^2)^{3/2}}\right\} = na\left\{\frac{-\psi+n\psi^2-n^2\psi+n}{\sqrt{1-\psi^2}(1-2n\psi+n^2)^{3/2}}\right\}$$; $-\psi+n\psi^2-n^2\psi+n=-(n\psi-1)(n-$ (Δ)

$$=-na\left\{\frac{(n\psi-1)(n-\psi)}{\sqrt{1-\psi^2}(1-2n\psi+n^2)^{3/2}}\right\} \Rightarrow \boxed{\left|\frac{db}{d\psi}\right| = \frac{na(n\psi-1)(n-\psi)}{\sqrt{1-\psi^2}(1-2n\psi+n^2)^{3/2}}}$$; $* \left(\frac{d\sigma}{d\Omega}\right)=\frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$

Si $\sin\theta/2=\sqrt{1-\psi^2}$ ⇒ $\cancel{\sin\theta=2\cos\theta/2\sin\theta/2=2\psi\sqrt{1-\psi^2}}$ $\quad 3/2+1/2=2$

$$b\frac{db}{d\psi}=\frac{na\cancel{\sqrt{1-\psi^2}}}{\sqrt{1-2n\psi+n^2}}\cdot\frac{na(n\psi-1)(n-\psi)}{\cancel{\sqrt{1-\psi^2}}(1-2n\psi+n^2)^{3/2}} = \frac{n^2a^2(n\psi-1)(n-\psi)}{(1-2n\psi+n^2)^2}$$

$$\to \frac{n^2a^2(n\cos\frac{1}{2}\theta-1)(n-\cos\frac{1}{2}\theta)}{(1-2n\cos\frac{1}{2}\theta+n^2)^2} \quad ; \quad \sin\theta=4\cos\theta/2$$

$$\Rightarrow \boxed{\left(\frac{d\sigma}{d\Omega}\right)=\frac{n^2a^2(n\cos\frac{1}{2}\theta-1)(n-\cos\frac{1}{2}\theta)}{4\cos\theta/2\,(1+n^2-2n\cos\frac{1}{2}\theta)^2}}$$

Mecánica Intermedia (LFIS 312)

Licenciatura en Física

Profesor: J. R. Villanueva Semestre I 2025

Nombre: Carlos Puchiera P. *RUT:* 20.155.517-5

Prueba 3: P1: 7.0 P2: 7.0 P3: 4.0 P4: 4.0 NF: 5.5

1. Una partícula de masa m se mueve en un campo de fuerza cuyo potencial en coordenadas esféricas es
$$V(r,\theta) = -\frac{k\cos\theta}{r^2}, \tag{1}$$
donde k es una constante. Obtenga las ecuaciones de movimiento canónicas.

2. (a) Determine los paréntesis de Poisson formado por las componentes cartesianas del momentum $\vec{p}$ y del momentum angular $\vec{L} = \vec{r} \times \vec{p}$.
   (b) Determine los paréntesis de Poisson formados a partir de las componentes del momentum angular $\vec{L}$

3. (a) Pruebe que la siguiente transformación es canónica:
$$Q = p\cot q, \quad P = \log\left(\frac{\sin q}{p}\right)$$
   (b) Encuentre la función generatriz $F_1(q, Q)$ para esta transformación.

4. Considere el sistema físico descrito por la siguiente energía cinética T y la energía potencial V,
$$T = \frac{1}{2}\left(\dot{q}_1^2 + \dot{q}_2^2\right)\left(q_1^2 + q_2^2\right), \quad V = \left(q_1^2 + q_2^2\right)^{-1}$$
donde q_1 y q_2 son coordenadas generalizadas. ¿Cuál es la ecuación de Hamilton-Jacobi para este sistema? Encuentre la función principal de Hamilton S. De ahí deducir el movimiento dinámico del sistema (no necesita evaluar ninguna integral definida)

Pregunta 1

Carlos Pincheira P

Coor. esféricas , Campo cuyo potencial $V(r,\theta) = -\frac{K\cos\theta}{r^2}$

$T = \frac{1}{2}mv^2$, $v^2 = \dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2$. $\mathcal{L} = T - V$

$\Rightarrow \mathcal{L} = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2) + \frac{K\cos\theta}{r^2}$; $H = \sum_j p_j\dot{q}_j - \mathcal{L}$

$\Rightarrow H = \dot{r}p_r + \dot{\theta}p_\theta + \dot{\phi}p_\phi - \frac{1}{2}m\dot{r}^2 - \frac{1}{2}mr^2\dot{\theta}^2 - \frac{1}{2}mr^2\sin^2\theta\dot{\phi}^2 + \frac{K\cos\theta}{r^2}$

$= \dot{r}(p_r - \frac{1}{2}m\dot{r}) + \dot{\theta}(p_\theta - \frac{1}{2}mr^2\dot{\theta}) + \dot{\phi}(p_\phi - \frac{1}{2}mr^2\sin^2\theta\dot{\phi}) + \frac{K\cos\theta}{r^2}$

<u>Ecs. mov. canon.</u> : $\partial_{\dot{q}_j}\mathcal{L} = p_j$, $\dot{p}_j = -\partial_{q_j}H$, $\dot{q}_j = \partial_{p_j}H$

$\Rightarrow p_r = \partial_{\dot{r}}\mathcal{L} = m\dot{r} \Rightarrow p_r = m\dot{r} \Rightarrow \dot{r} = \frac{p_r}{m}$

$\Rightarrow p_\theta = \partial_{\dot{\theta}}\mathcal{L} = mr^2\dot{\theta} \Rightarrow p_\theta = mr^2\dot{\theta} \Rightarrow \dot{\theta} = \frac{p_\theta}{mr^2}$

$\Rightarrow p_\phi = \partial_{\dot{\phi}}\mathcal{L} = mr^2\sin^2\theta\dot{\phi} \Rightarrow p_\phi = mr^2\sin^2\theta\dot{\phi} \Rightarrow \dot{\phi} = \frac{p_\phi}{mr^2\sin^2\theta}$

$\Rightarrow H = \frac{p_r}{m}(p_r - \frac{1}{2}p_r) + \frac{p_\theta}{mr^2}(p_\theta - \frac{1}{2}p_\theta) + \frac{p_\phi}{mr^2\sin^2\theta}(p_\phi - \frac{1}{2}p_\phi) + \frac{K\cos\theta}{r^2}$

$\Rightarrow \boxed{H = \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} + \frac{p_\phi^2}{2mr^2\sin^2\theta} + \frac{K\cos\theta}{r^2}}$

$\therefore \boxed{\dot{p}_r = -\partial_r H = \frac{p_\theta^2}{mr^3} + \frac{p_\phi^2}{mr^3\sin^2\theta} + \frac{2K\cos\theta}{r^3}}$; $\boxed{\dot{r} = \partial_{p_r}H = \frac{p_r}{m}}$

$\boxed{\dot{p}_\phi = -\partial_\phi H = 0}$; $\boxed{\dot{\phi} = \partial_{p_\phi}H = \frac{p_\phi}{mr^2\sin^2\theta}}$

$\boxed{\dot{p}_\theta = -\partial_\theta H = \frac{p_\phi^2\cos\theta}{mr^2\sin^3\theta} + \frac{K\cos\theta}{r^2}}$; $\boxed{\dot{\theta} = \partial_{p_\theta}H = \frac{p_\theta}{mr^2}}$

2— (a) $\vec{L} = \vec{r}\times\vec{p} = \hat{i}(yp_z - zp_y) + \hat{j}(zp_x - xp_z) + \hat{k}(xp_y - yp_x)$

$* \{q_i, p_j\} = \delta_{ij}$ $\quad \vec{p} = (p_x, p_y, p_z)$; $* \{q_i, q_j\} = \{p_i, p_j\} = 0$

$\therefore \{L_x, p_x\} = \{yp_z - zp_y, p_x\} = \underbrace{\{yp_z, p_x\}}_{①} - \underbrace{\{zp_y, p_x\}}_{②}$

① $\{yp_z, p_x\} = y\underbrace{\{p_z, p_x\}}_{0} + p_z\underbrace{\{y, p_x\}}_{\delta_{yx}\,=\,0} = 0$

② $\{zp_y, p_x\} = z\underbrace{\{p_y, p_x\}}_{0} + p_y\underbrace{\{z, p_x\}}_{\delta_{zx}\,=\,0} = 0$

$\boxed{\{L_x, p_x\} = 0} \rightarrow \boxed{\{L_y, p_y\} = \{L_z, p_z\} = 0}$

$\cdot \{L_x, p_y\} = \{yp_z - zp_y, p_y\} = \underbrace{\{yp_z, p_y\}}_{①} - \underbrace{\{zp_y, p_y\}}_{②}$

① $\{yp_z, p_y\} = y\underbrace{\{p_z, p_y\}}_{0} + p_z\underbrace{\{y, p_y\}}_{\delta_{yy}} = p_z$

② $\{zp_y, p_y\} = z\underbrace{\{p_y, p_y\}}_{0} + p_y\underbrace{\{z, p_y\}}_{0} = 0$

$\boxed{\{L_x, p_y\} = p_z}$

$\cdot \{L_x, p_z\} = \{yp_z - zp_y, p_z\} = \underbrace{\{yp_z, p_z\}}_{①} - \underbrace{\{zp_y, p_z\}}_{②}$

① $\{yp_z, p_z\} = y\underbrace{\{p_z, p_z\}}_{0} + p_z\underbrace{\{y, p_z\}}_{\delta_{yz}\,=\,0} = 0$

② $\{zp_y, p_z\} = z\underbrace{\{p_y, p_z\}}_{0} + p_y\underbrace{\{z, p_z\}}_{\delta_{zz}} = p_y$

$\boxed{\{L_x, p_z\} = -p_y}$

$\{L_y, p_x\} = \{zp_x - xp_z, p_x\} = \underbrace{\{zp_x, p_x\}}_{①} - \underbrace{\{xp_z, p_x\}}_{②}$

① $\{zp_x, p_x\} = z\underbrace{\{p_x, p_x\}}_{0} + p_x\underbrace{\{z, p_x\}}_{\delta_{zx}\,=\,0} = 0$

② $\{xp_z, p_x\} = x\underbrace{\{p_z, p_x\}}_{0} + p_z\underbrace{\{x, p_x\}}_{\delta_{xx}} = p_z$

$\boxed{\{L_y, p_x\} = -p_z}$

$\{L_y, p_z\} = \{zp_x - xp_z, p_z\} = \underbrace{\{zp_x, p_z\}}_{①} - \{xp_z, p_z\}$

① $\{zp_x, p_z\} = z\underbrace{\{p_x, p_z\}}_{0} + p_x\underbrace{\{z, p_z\}}_{\delta_{zz}} = p_x$

② $\{xp_z, p_z\} = x\underbrace{\{p_z, p_z\}}_{0} + p_z\underbrace{\{x, p_z\}}_{\delta_{xz}\,=\,0} = 0$

$\boxed{\{L_y, p_z\} = p_x}$

Por lo tanto, se observa que el comportamiento de el momentum lineal y para los corchetes de Poisson es de la forma

$$\boxed{\{L_i, p_j\} = \epsilon_{ijk}\, p_k}$$

Pregunta 3 | Carlos Pincheira

$u = \frac{\sin q}{p}$

$\partial_p P = \partial_u \log[u] \partial_p u$

$\partial_q P = \partial_u \log[u] \partial_q u$

$$Q = p\cot q \quad , \quad P = \log\left(\frac{\sin q}{p}\right)$$

$$* \{f,g\} = \sum_{j=1}^{n} \left[\partial_{q_j} f \partial_{p_j} g - \partial_{q_j} g \partial_{p_j} f\right] \Rightarrow \{Q,P\} = \partial_q Q \partial_p P - \partial_p Q \partial_q P$$

$$\Rightarrow \partial_q Q = -p\csc^2 q \quad , \quad \partial_p P = \frac{\cancel{p}}{\cancel{\sin q}}\left(\frac{\cancel{\sin q}}{p}\right)'_p = -\frac{1}{p}$$

$$\Rightarrow \partial_p Q = \cot q \quad , \quad \partial_q P = \frac{\cancel{p}}{\sin q}\left(\frac{\sin q}{\cancel{p}}\right)'_q = \frac{\cos q}{\sin q} = \cot q$$

$\therefore \{Q,P\} = 1$ son canónicas $\quad 1 + \cot^2\theta = \csc^2\theta$

$$\Rightarrow (-\cancel{p}\csc^2 q)\left(-\frac{1}{\cancel{p}}\right) - \cot q \cot q = \csc^2 q - \cot^2 q = 1$$

$\Rightarrow$ $\boxed{Q \text{ y } P \text{ son canónicas}}$

b) $F_1(q,Q) \Rightarrow p = \partial_q F_1$, $P = \log\left(\frac{\sin q}{p}\right)$

$P = -\partial_Q F_1$

$Q = p\cot q \rightarrow p = Q\tan q$

$$\Rightarrow \partial_q F_1(q,Q) = Q\tan q \quad / \int dq$$

$$\Rightarrow F_1(q,Q) = Q\int \tan q \, dq \quad *$$

$$\Rightarrow P = -\partial_Q F_1 = \log\left(\frac{\sin q}{p}\right) \quad / \int$$

$$A - T = \frac{1}{2}(\dot{q}_1^2 + \dot{q}_2^2)(q_1^2 + q_2^2), \; V = (q_1^2 + q_2^2)^{-1}$$ Carlosh1

$$\Rightarrow \mathcal{L} = \frac{1}{2}(\dot{q}_1^2 + \dot{q}_2^2)(q_1^2 + q_2^2) - \frac{1}{q_1^2 + q_2^2}$$

Momento generalizado : $\partial_{\dot{q}_j} \mathcal{L} = p_i$

$; \; q \{ q_1, q_2 \}$

$\dot{q} \{ \dot{q}_1, \dot{q}_2 \}$

$\frac{1}{2}(\dot{q}_1^2 q_1^2 + \dot{q}_1^2$

$\Rightarrow \frac{1}{2}\dot{q}_1^2 (q_1^2 + q_2^2)$

$+ \frac{1}{2}\dot{q}_2^2 (q_1^2 + q_2^2)$

$$\Rightarrow p_1 = \partial_{\dot{q}_1} \mathcal{L} = \dot{q}_1 (q_1^2 + q_2^2)$$

$$\Rightarrow p_2 = \partial_{\dot{q}_2} \mathcal{L} = \dot{q}_2 (q_1^2 + q_2^2)$$

$$\therefore \; \dot{q}_1 = \frac{p_1}{q_1^2 + q_2^2} \wedge \dot{q}_2 = \frac{p_2}{q_1^2 + q_2^2}$$

$$H = \dot{q}_1 p_1 + \dot{q}_2 p_2 - \mathcal{L}$$

$$\Rightarrow H = \dot{q}_1 p_1 + \dot{q}_2 p_2 - \frac{1}{2}(\dot{q}_1^2 + \dot{q}_2^2)(q_1^2 + q_2^2) + \frac{1}{q_1^2 + q_2^2}$$

$$= \dot{q}_1 \left(p_1 - \frac{1}{2}\dot{q}_1 (q_1^2 + q_2^2)\right) + \dot{q}_2 \left(p_2 - \frac{1}{2}\dot{q}_2 (q_1^2 + q_2^2)\right) + \frac{1}{q_1^2 + q_2^2}$$

$$= \frac{p_1}{q_1^2 + q_2^2}\left(p_1 - \frac{1}{2}\frac{\cancel{(q_1^2 + q_2^2)}\, p_1}{\cancel{(q_1^2 + q_2^2)}}\right) + \frac{p_2}{q_1^2 + q_2^2}\left(p_2 - \frac{1}{2}\frac{\cancel{(q_1^2 + q_2^2)}\, p_2}{\cancel{(q_1^2 + q_2^2)}}\right) + \frac{1}{q_1^2 + q_2^2}$$

$$= \frac{p_1}{q_1^2 + q_2^2}\left(p_1 - \frac{1}{2}p_1\right) + \frac{p_2}{q_1^2 + q_2^2}\left(p_2 - \frac{1}{2}p_2\right) + \frac{1}{q_1^2 + q_2^2}$$

$$\Rightarrow H = \frac{p_1^2}{2(q_1^2 + q_2^2)} + \frac{p_2^2}{2(q_1^2 + q_2^2)} + \frac{1}{q_1^2 + q_2^2}$$

$$\Rightarrow \boxed{H = \frac{1}{(q_1^2 + q_2^2)}\left\{\frac{p_1^2}{2} + \frac{p_2^2}{2} + 1\right\}}$$

Hamilton – Jacobi $\Rightarrow H + \partial_t F = K \Rightarrow H + \partial_t S = K$

$$\Rightarrow \frac{1}{(q_1^2 + q_2^2)}\left\{\frac{p_1^2}{2} + \frac{p_2^2}{2} + 1\right\} + \partial_t S = K$$

$$\Rightarrow \boxed{\partial_t S = K - \frac{1}{(q_1^2 + q_2^2)}\left\{\frac{p_1^2}{2} + \frac{p_2^2}{2} + 1\right\}}$$