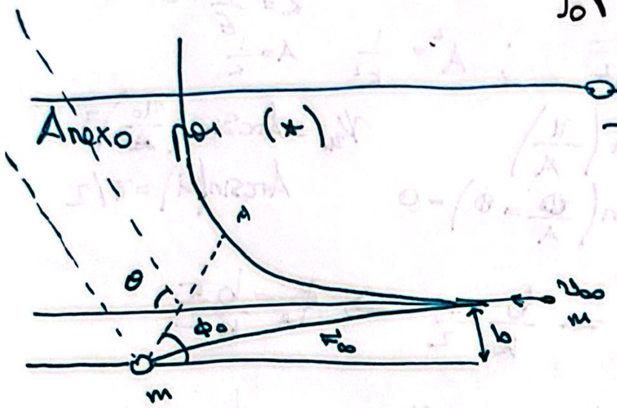
Scattering

$$\left(\frac{d\sigma}{d\Omega}\right)_{cl} = \frac{\gamma \pi^2}{E\sin\theta}\,\frac{\pi-\theta}{\theta^2(2\pi-\theta)^2} \qquad V(r)=\frac{\gamma}{r^2}$$

$$* \ \theta = \pi - 2\phi_0 \quad ; \quad \phi_0 = b\int_0^{u_m}\frac{b\,du}{\sqrt{1-\frac{V}{E}-b^2u^2}} \quad (*)$$



Anexo για (*)

$$\vec{L} = m b u_\infty \hat{k} \quad , \quad \theta = \pi - 2\phi_0$$

$$E = \frac{1}{2}mu^2 + U(r) = \frac{1}{2}m(\dot r^2 + r^2\dot\phi^2) + U(r)$$

$$\dot r^2 + r^2\dot\phi^2 = \frac{2}{m}(E - U(r))$$

$$\dot r^2 = \frac{2}{m}(E-U(r)) - r^2\dot\phi^2 \quad ; \quad \dot\phi = \frac{l}{mr^2}$$

$$\dot r = \frac{dr}{dt} = \frac{d\phi}{dt}\frac{dr}{d\phi} = \frac{l}{mr^2}\frac{dr}{d\phi} \qquad \dot r = \sqrt{\frac{2}{m}(E-U(r)) - r^2\dot\phi^2}$$

$$= \sqrt{\frac{2}{m}(E-U(r)) - \frac{l^2}{m^2r^2}}$$

$$\Rightarrow \frac{l}{mr^2}\frac{dr}{d\phi} = \sqrt{\frac{2}{m}(E-U(r)) - \frac{l^2}{m^2r^2}}$$

$$\Rightarrow d\phi = \frac{l}{mr^2}\frac{dr}{\sqrt{\frac{2}{m}(E-U(r))-\frac{l^2}{m^2r^2}}} = \frac{l/r^2\,dr}{\sqrt{2m(E-U(r)) - l^2/r^2}}$$

$$\Rightarrow \phi_0 = \int_{r_{min}}^{\infty}\frac{l/r^2\,dr}{\sqrt{\underbrace{2m(E-U)-l^2/r^2}_{p(r)}}} \quad \Big\{ \ u=\frac{1}{r} \rightarrow du = -\frac{1}{r^2}dr$$

$$\cancel{p(r) = 2m(E-U) - l^2/r^2}$$

$$= \frac{1}{\sqrt{2m}}\int_{r_{min}}^{\infty}\frac{l/r^2\,dr}{\sqrt{E - \underbrace{(U + l^2/2mr^2)}_{U_{ef}}}} \qquad \xrightarrow{\text{parameter impacto}} \quad l = mbu_\infty \qquad \frac{l}{r^2} = \frac{mbu_\infty}{r^2}$$

$$E - U_{ef} = \frac{1}{2}mu_\infty^2 - \frac{1}{2mr^2}mbu_\infty - U(r) = \frac{1}{2}mu_\infty^2\left(1 - \frac{b^2}{r^2} - \frac{U}{E}\right)$$

$$\sqrt{E-U_{ef}} = \sqrt{\frac{m}{2}}u_\infty\sqrt{1-\frac{b^2}{r^2}-\frac{U}{E}} \quad \Rightarrow \quad \phi_0 = \frac{1}{\sqrt{2m}}\int_{r_{min}}^{\infty}\frac{mu_\infty b/r^2\,dr}{\sqrt{\frac{m}{2}}u_\infty\sqrt{1-\frac{b^2}{r^2}-\frac{U}{E}}}$$

$$\Rightarrow \phi_0 = \int_{r_{min}}^{\infty}\frac{b/r^2\,dr}{\sqrt{1-b^2/r^2-\frac{U}{E}}} \quad \Big\{ \begin{array}{l} u = 1/r \\ du = -\frac{1}{r^2}dr \end{array} \quad \rightarrow \quad \begin{array}{l} u_{min} = 1/r_{min} \\ r\to\infty \Rightarrow u \to 0 \end{array}$$

$$\phi_0 = -\int_{u_{min}}^{0} \frac{b\,du}{\sqrt{1-b^2u^2-\frac{U}{E}}} = \int_0^{u_{min}} \frac{b\,du}{\sqrt{1-b^2u^2-\frac{U}{E}}}$$

$\therefore$ Si $V(r) = \frac{\gamma}{r^2}$; $\phi_0 = \int_0^{u_{min}=u_0} \frac{b\,du}{\sqrt{1-b^2u^2-\frac{\gamma u^2}{E}}}$; $1-b^2u^2-\frac{\gamma u^2}{E} = 1-\left(\frac{\gamma}{E}+b^2\right)u^2$

$= 1-u^2k^2$

$k^2 = \frac{\gamma}{E}+b^2$

$$\Rightarrow \phi_0 = b\int_0^{u_0} \frac{du}{\sqrt{1-u^2k^2}} = \frac{b}{k}\int_0^{u_0} \frac{du}{\sqrt{A^2-u^2}} \quad ; \quad A^2 = \frac{1}{k^2} \qquad A = \frac{1}{k}$$

$$\begin{cases} u = A\sin\psi \\ du = A\cos\psi\, d\psi \end{cases} \quad \Bigg| \quad \begin{matrix} 1-u_0^2k^2 = 0 \\ u_0 = \frac{1}{k} \end{matrix} \quad \Bigg| \quad \begin{matrix} \psi = \operatorname{Arcsin}\left(\frac{u}{A}\right) \\ \psi_0 = \operatorname{Arcsin}\left(\frac{0}{A} = 0\right) - 0 \end{matrix} \qquad \begin{matrix} \psi_{u_0} = \operatorname{ArcSin}\left(\frac{u_0}{A}\right) = \\ \operatorname{Arcsin}(1) = \pi/2 \end{matrix}$$

$$\Rightarrow \phi_0 = \frac{b}{k}\int_0^{\pi/2} \frac{A\cos\psi\, d\psi}{\sqrt{A^2-A^2\sin^2\psi}} = \frac{b}{k}\int_0^{\pi/2} d\psi = \frac{b}{k}\frac{\pi}{2} \rightarrow \phi_0 = \frac{b}{k}\frac{\pi}{2}$$

$$* \; k = \sqrt{\frac{\gamma}{E}+b^2} \Longrightarrow \phi_0 = \frac{\pi}{2}\frac{b}{\sqrt{\frac{\gamma}{E}+b^2}} \overset{\alpha = \gamma/E}{=} \frac{\pi}{2}\frac{b}{\sqrt{\alpha+b^2}}$$

$$\therefore \boxed{\Theta = \pi\left(1-\frac{b}{\sqrt{\alpha+b^2}}\right)} \Longrightarrow \frac{\Theta}{\pi}-1 = -\frac{b}{\sqrt{\alpha+b^2}} \Rightarrow \frac{\pi-\Theta}{\pi} = \frac{b}{\sqrt{\alpha+b^2}}$$

$$\Rightarrow \frac{(\pi-\theta)^2}{\pi^2} = \frac{b^2}{\alpha+b^2} \Rightarrow b^2 = (\alpha+b^2)\frac{(\pi-\theta)^2}{\pi^2}$$

$$= \alpha\frac{(\pi-\theta)^2}{\pi^2} + b^2\frac{(\pi-\theta)^2}{\pi^2}$$

$$\Rightarrow b^2\left(1-\frac{(\pi-\theta)^2}{\pi^2}\right) = \alpha\frac{(\pi-\theta)^2}{\pi^2}$$

$$\Rightarrow b^2\left(\frac{\cancel{\pi^2}-\cancel{\pi^2}+2\pi\theta-\theta^2}{\cancel{\pi^2}}\right) = \alpha\frac{(\pi^2-\theta)^2}{\cancel{\pi^2}} \Rightarrow b^2(2\pi\theta-\theta^2) = \alpha(\pi^2-\theta)^2$$

$$\Rightarrow b^2 = \frac{\alpha(\pi-\theta)^2}{2\pi\theta-\theta^2} \longrightarrow b = (\pi-\theta)\sqrt{\frac{\alpha}{2\pi\theta-\theta^2}}$$

$$\rightarrow \boxed{b = \pi\sqrt{\frac{\alpha}{2\pi\theta-\theta^2}} - \theta\sqrt{\frac{\alpha}{2\pi\theta-\theta^2}}}$$

$$\rightarrow b = \pi\sqrt{\alpha}\,(2\pi\theta-\theta^2)^{-1/2} - \theta\sqrt{\alpha}\,(2\pi\theta-\theta^2)^{-1/2} \quad ; \quad \frac{db}{d\theta}$$

$$\frac{db}{d\theta} = -\pi\sqrt{\alpha}\,\frac{1}{\cancel{2}}(2\pi\theta-\theta^2)^{-3/2}(\cancel{2}\pi-\cancel{2}\theta) - \sqrt{\alpha}\,(2\pi\theta-\theta^2)^{-1/2} + \theta\sqrt{\alpha}\,\frac{1}{\cancel{2}}(2\pi\theta-\theta^2)^{-3/2}(\cancel{2}\pi-\cancel{2}\theta)$$

$$= -\frac{\pi\sqrt{\alpha}(\pi-\theta)}{(2\pi\theta-\theta^2)^{3/2}} - \frac{\sqrt{\alpha}}{(2\pi\theta-\theta^2)^{1/2}} + \frac{\theta\sqrt{\alpha}(\pi-\theta)}{(2\pi\theta-\theta^2)^{3/2}}$$

$$\boxed{\frac{db}{d\theta}} = \frac{-\pi\sqrt{\alpha}(\pi-\theta) - \sqrt{\alpha}(2\pi\theta-\theta^2) + \theta\sqrt{\alpha}(\pi-\theta)}{(2\pi\theta-\theta^2)^{3/2}}$$

$$= \frac{-\pi\sqrt{\alpha}\pi + \cancel{\pi\theta\sqrt{\alpha}} - \cancel{\sqrt{\alpha}2\pi\theta} + \cancel{\sqrt{\alpha}\theta^2} + \cancel{\theta\sqrt{\alpha}\pi} - \cancel{\sqrt{\alpha}\theta^2}}{(2\pi\theta-\theta^2)^{3/2}}$$

$$= -\frac{\pi^2\sqrt{\alpha}}{(2\pi\theta-\theta^2)^{3/2}} \Rightarrow \left|\frac{db}{d\theta}\right| = \frac{\pi^2\sqrt{\alpha}}{(2\pi\theta-\theta^2)^{3/2}}$$

$$b = (\pi-\theta)\sqrt{\frac{\alpha}{2\pi\theta-\theta^2}}$$

$$= \frac{\pi^2\sqrt{\alpha}}{(2\pi\theta-\theta^2)\sqrt{2\pi\theta-\theta^2}}$$

$$\Rightarrow b\left|\frac{db}{d\theta}\right| = \frac{(\pi-\theta)\sqrt{\alpha}}{\sqrt{2\pi\theta-\theta^2}} \frac{\pi^2\sqrt{\alpha}}{(2\pi\theta-\theta^2)\sqrt{2\pi\theta-\theta^2}}$$

$$= \frac{\alpha\pi^2(\pi-\theta)}{(2\pi\theta-\theta^2)^2} = \frac{\alpha\pi^2(\pi-\theta)}{\theta^2(2\pi-\theta)^2}$$

$$d\sigma = 2\pi b\left|\frac{db}{d\theta}\right|d\theta \quad , \quad d\Omega = 2\pi\sin\theta\, d\theta \Rightarrow \frac{d\Omega}{\sin\theta} = 2\pi\, d\theta$$

$$\Rightarrow d\sigma = \frac{d\Omega}{\sin\theta}\frac{\alpha\pi^2(\pi-\theta)}{\theta^2(2\pi-\theta)^2} \Rightarrow \boxed{\left(\frac{d\sigma}{d\Omega}\right) = \frac{\alpha\pi^2(\pi-\theta)}{\sin\theta\,\theta^2(2\pi-\theta)^2}}$$

$$\alpha = \frac{\gamma}{E}$$

$$V(r) = \begin{cases} 0 & r > a \\ -V_0 & r < a \end{cases}$$

Muestre que la órbita de las partículas es idéntica con la de los rayos de luz refractados por una esfera de radio a e índice de refracción $n = [(E+V_0)/E]^{1/2}$

Pruebe que la sección eficaz elástica diferencial para $\cos\frac{1}{2}\theta > n^{-1}$ es

$$\left(\frac{d\sigma}{d\Omega}\right)_{el} = \frac{n^2a^2}{4\cos\frac{1}{2}\theta} \frac{(n\cos\frac{1}{2}\theta)(n-\cos\frac{1}{2}\theta)}{(1+n^2-2n\cos\frac{1}{2}\theta)^2}$$

$$V(r) = \begin{cases} 0 & r > a & \longrightarrow E = \frac{1}{2}mu_0^2 \Rightarrow u_0 = \sqrt{\frac{2E}{m}} \\ -V_0 & r < a & \longrightarrow E = \frac{1}{2}mu^2 - \quad \rightarrow u = \sqrt{\frac{2}{m}(E+V_0)} \end{cases}$$

Ley de Snell $\Longrightarrow$ $n = \frac{u}{u_0} = \sqrt{\frac{\cancel{2}}{\cancel{m}}(E+V_0)}\sqrt{\frac{\cancel{m}}{\cancel{2}E}} = \sqrt{\frac{E+V_0}{E}}$

$$\boxed{n = \sqrt{\frac{E+V_0}{E}}}$$

Snell scattering : $b = a\sin\alpha$, $n = \frac{\sin\alpha}{\sin\beta}$, $\frac{\theta}{2} = \beta - \alpha$ / cos

$\rightarrow \sin\alpha = \frac{b}{a}$; $\sin\beta = \frac{\sin\alpha}{n} = \frac{b}{na}$; $\cos(\theta/2) = \cos(\beta-\alpha) = \psi$

* $\theta/2 = \beta - \alpha \rightarrow \alpha = \beta - \theta/2$ / sin U $\Rightarrow \sin\alpha = \sin(\beta - \theta/2)$

$\therefore \sin(\beta-\theta/2) = \frac{b}{a} \rightarrow b = a\sin(\beta - \theta/2)$

* $\cos^2\theta/2 + \sin^2\theta/2 = 1 \Rightarrow \sin\theta/2 = \sqrt{1-\psi^2}$
* $\cos^2\beta + \sin^2\beta = 1 \Rightarrow \cos\beta = \sqrt{1-(\frac{b}{na})^2} = \sqrt{\frac{n^2a^2-b^2}{n^2a^2}}$

$\therefore \; b = a\sin\alpha = a\sin(\beta-\theta/2) = a[\sin\beta\cos\theta/2 - \sin\theta/2\cos\beta]$

$= a\left[\frac{b}{na}\psi - \sqrt{1-\psi^2}\sqrt{\frac{n^2a^2-b^2}{n^2a^2}}\right] = \frac{1}{n}\left[b\psi - \sqrt{1-\psi^2}\sqrt{n^2a^2-b^2}\right]$

$\rightarrow nb = b\psi - \sqrt{1-\psi^2}\sqrt{n^2a^2-b^2} \rightarrow b^2(\psi - n)^2 = (1-\psi^2)(n^2a^2-b^2)$

$$b^2(\psi-n)^2 = (1-\psi^2)(n^2a^2-b^2) = n^2a^2 - \underline{b^2} - \psi^2 n^2 a^2 + \underline{b^2\psi^2}$$

$$\Rightarrow b^2(\psi-n)^2 + b^2 - b^2\psi^2 = n^2a^2 - \psi^2 n^2 a^2$$

$$\Rightarrow b^2\big(\underbrace{1-\psi^2+(\psi-n)^2}_{1-\cancel{\psi^2}+\cancel{\psi^2}-2\psi n+n^2}\big) = n^2a^2(1-\psi^2) \Rightarrow b^2 = \frac{n^2a^2(1-\psi^2)}{1-2\psi n+n^2}$$

$$\psi = \cos\theta/2$$

$$\Rightarrow \boxed{b = \frac{na\sqrt{1-\psi^2}}{\sqrt{1-2\psi n+n^2}}} \quad ; \quad \frac{db}{d\theta} = \frac{d\psi}{d\theta}\frac{db}{d\psi} \quad ; \quad \frac{d\psi}{d\theta} = -\frac{1}{2}\sin\theta/2$$

$$\frac{db}{d\psi} = an\left\{\frac{1}{\sqrt{1-2\psi n+n^2}}\frac{-\psi}{\sqrt{1-\psi^2}} + \frac{\sqrt{1-\psi^2}}{\sqrt{1-2n\psi+n^2}}\frac{n}{(1-2n\psi+n^2)}\right\}$$

$$= an\left\{\frac{-\psi(1-2n\psi+n^2) + \sqrt{1-\psi^2}\sqrt{1-\psi^2}\,n}{\sqrt{1-2\psi n+n^2}\sqrt{1-\psi^2}\,(1-2n\psi+n^2)}\right\}$$

$$= an\left\{\frac{-\psi+2n\psi^2-n^2\psi + n(1-\psi^2)}{\sqrt{1-2\psi n+n^2}\sqrt{1-\psi^2}\,(1-2n\psi+n^2)}\right\}$$

$$-\psi+2n\psi^2-n^2\psi+n-n\psi^2$$

$$-\psi+n\psi^2-n^2\psi+n$$

$$= -(n\psi-1)(n-\psi)$$

$$= -na\left\{\frac{(n\psi-1)(n-\psi)}{\sqrt{1-\psi^2}\,(1-2\psi n+n^2)^{3/2}}\right\}$$

$$\sin\theta/2 = \sqrt{1-\psi^2}$$

$$\sin\theta = 2\sin\theta/2\cos\theta/2$$

$$\therefore \frac{db}{d\theta} = \frac{d\psi}{d\theta}\frac{db}{d\psi} = \frac{\cancel{\sin\theta/2}}{2}na\left\{\frac{(n\psi-1)(n-\psi)}{\cancel{\sqrt{1-\psi^2}}\,(1-2\psi n+n^2)^{3/2}}\right\} \quad = 2\psi\sqrt{1-\psi^2}$$

$$\frac{db}{d\theta} = \frac{na}{2}\left\{\frac{(n\psi-1)(n-\psi)}{(1-2\psi n+n^2)^{3/2}}\right\}$$

$$* \quad \frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| = \frac{na\cancel{\sqrt{1-\psi^2}}}{\sqrt{1-2\psi n+n^2}}\;\frac{1}{2\psi\cancel{\sqrt{1-\psi^2}}}\;\frac{na}{2}\left\{\frac{(n\psi-1)(n-\psi)}{(1-2\psi n+n^2)^{3/2}}\right\}$$

$$= \frac{n^2a^2}{4\psi}\;\frac{(n\psi-1)(n-\psi)}{(1-2\psi n+n^2)^2}$$

$$\therefore \boxed{\left(\frac{d\sigma}{d\Omega}\right) = \frac{n^2a^2}{4\cos\frac{1}{2}\theta}\;\frac{(n\cos\frac{1}{2}\theta-1)(n-\cos\frac{1}{2}\theta)}{(1-2n\cos\frac{1}{2}\theta+n^2)^2}}$$