Ejercicio 1. *Para discutir el caso donde la ecuación secular de autovalores tiene raíces múltiples y los correspondientes autovalores son degenerados considere dos grados de libertad con*

$$\boxed{\bar{v} \equiv \begin{pmatrix} v & v_{12} \\ v_{12} & v \end{pmatrix}, \quad \bar{m} \equiv \begin{pmatrix} m & m_{12} \\ m_{12} & m \end{pmatrix}}$$

(a) *Encuentre los autovalores y autovectores. Muestre que en el límite* $(m_{12}, v_{12}) \to 0$ *o* $(m, v) \to 0$ *los autovalores se vuelven degenerados con* $\omega_1^2 = \omega_2^2$.

Ecuación generalizada de autovectores:

$$(\bar{v} - \omega^2 \bar{m})\vec{\rho} = 0 \; *$$

$$* \sum_{\lambda=1}^{n} (v_{\sigma\lambda} - \omega^2 m_{\sigma\lambda})\dot{x}_\lambda = 0$$

$$\therefore \; \underline{\bar{v} - \omega^2 \bar{m}} = \begin{pmatrix} v & v_{12} \\ v_{12} & v \end{pmatrix} - \omega^2 \begin{pmatrix} m & m_{12} \\ m_{12} & m \end{pmatrix}$$

$$= \begin{pmatrix} \underbrace{v-\omega^2 m}_{A} & \underbrace{v_{12}-\omega^2 m_{12}}_{B} \\ \underbrace{v_{12}-\omega^2 m_{12}}_{B} & \underbrace{v-\omega^2 m}_{A} \end{pmatrix} \; *$$

$$\det(\bar{v} - \omega^2 \bar{m}) = 0$$

$$= (v-\omega^2 m)^2 - (v_{12} - \omega^2 m_{12})^2 = 0$$

$$A = v - \omega^2 m, \quad B = v_{12} - \omega^2 m_{12}$$

$$A^2 - B^2 = 0 \Rightarrow (A-B)(A+B) = 0$$

$$\underbrace{\left[(U-\omega^2 m) - (U_{12} - \omega^2 m_{12})\right]}_{\mathrm{I}} \times \underbrace{\left[(U-\omega^2 m) + (U_{12} - \omega^2 m_{12})\right]}_{\mathrm{II}} = 0 \quad (\blacktriangle)$$

I) $U - \omega_1^2 m - U_{12} + \omega_1^2 m_{12} = 0$

$$U - U_{12} - \omega_1^2 (m - m_{12}) = 0$$

$$\Rightarrow \boxed{\omega_1^2 = \frac{U - U_{12}}{m - m_{12}}} \checkmark$$

II) $U + U_{12} - \omega_2^2 (m + m_{12}) = 0$

$$\boxed{\omega_2^2 = \frac{U + U_{12}}{m + m_{12}}} \checkmark$$

$$(\blacktriangle) \xrightarrow{A,B} \underbrace{(A-B)}_{\mathrm{I}}\underbrace{(A+B)}_{\mathrm{II}} = 0$$

$$A - B = 0 \quad (\omega_1)^{\mathrm{I}}$$

$A+B=0 \quad (\omega_2)$

$$\begin{pmatrix} A & B \\ B & A \end{pmatrix}\begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}=0 \quad \left\{\begin{array}{l} A\rho_1+B\rho_2=0 \\ B\rho_1+A\rho_2=0 \end{array}\right. \quad (1)$$

Si $\underline{A-B=0} \Rightarrow A=B$

$(1) \quad A\rho_1+B\rho_2=A(\rho_1+\rho_2)=0$

$\Rightarrow \rho_1+\rho_2=0 \rightarrow \boxed{\rho_1=-\rho_2}$

$$\boxed{\bar{\rho}^{(1)}=\begin{pmatrix} 1 \\ -1 \end{pmatrix}} \checkmark$$

Si $A+B=0 \Rightarrow A=-B$

$\Rightarrow A\rho_1+B\rho_2=A(\rho_1-\rho_2)=0$

$\Rightarrow \boxed{\rho_1=\rho_2}$

$$\Rightarrow \boxed{\bar{\rho}^{(2)}=\begin{pmatrix} 1 \\ 1 \end{pmatrix}} \checkmark$$

límites:

$(m_{12}, \nu_{12}) \xrightarrow{(*)} 0$ y $(m, \nu) \xrightarrow{(**)} 0$

$\omega_1^2 = \omega_2^2$ autovalores deg.

$$\omega_1^2 = \frac{\nu - \nu_{12}}{m - m_{12}} \overset{(*)}{=} \frac{\nu}{m}$$

$$\omega_2^2 = \frac{\nu + \nu_{12}}{m + m_{12}} \overset{(*)}{=} \frac{\nu}{m}$$

$$\therefore \boxed{\omega_1^2 = \omega_2^2}$$

$$\omega_1^2 \overset{(**)}{=} \frac{\nu_{12}}{m_{12}} \; ; \; \omega_2^2 \overset{(**)}{=} \frac{\nu_{12}}{m_{12}}$$

$$\boxed{\omega_1^2 = \omega_2^2}$$

(b) *Muestre que en estos límites se pierde información concerniente a los correspondientes autovectores y que uno siempre puede encontrar dos soluciones linealmente independientes de la forma*

$z_\sigma^{(s)} = e^{i\varphi_s}\rho_\sigma^{(s)} \quad con \quad s = 1,2 \quad y \quad \rho_\sigma^{(s)}$ *real.*

Casos límite

$$(M_{12}, U_{12}) \xrightarrow{(*)} 0 \; ; \; (M, U) \xrightarrow{(*)} 0$$

$$\begin{array}{cc} 1 & 2 \end{array}$$

$$\overline{U} \equiv \begin{bmatrix} U & U_{12} \\ U_{12} & U \end{bmatrix} \; ; \; \overline{M} \equiv \begin{bmatrix} M & M_{12} \\ M_{12} & M \end{bmatrix}$$

$$\overline{U} \overset{(*)}{=} U I \; ; \; \overline{M} \overset{(*)}{=} M I$$

$$\omega^2 = \frac{U}{M} \; ; \; (\overline{U} - \omega^2 \overline{M})\rho = 0$$

$$\Rightarrow (U I - \omega^2 M I)\rho = 0$$

$$\left(U I - \frac{U}{\cancel{M}} \cancel{M} I\right)\rho = 0$$

$$\Rightarrow \boxed{\rho = 0}$$

Cualquier vector ρ satisface

Se pierde inf. sobre los autovectores

↳ la ecuación ya no selecciona una única de autovectores, selecciona un subespacio infinitas sol.

* Cualquier combinación válida

$\rho = (\rho_1, \rho_2)^T$, $(1,0)^T$
$(0,1)^T$
$(1,1)^T$, etc

* Podemos elegir 2 vectores $\mathbb{R}$ e ind.

$\bar{\rho}^{(1)}, \bar{\rho}^{(2)} \in \mathbb{R}^2$ *

fase multiplica $\mathbb{C}$ $\{e^{i\phi}\}$

$z_r^{(s)} = e^{i\phi_s} \bar{\rho}_r^{(s)}$; sol. $\mathbb{C}$

$(\bar{T} - \omega^2 \bar{m}) z = 0$ *

* Físicamente: 2 modos de la misma frecuencia pero con desfaces arbitrarios

$$z^{(1)} = e^{i\phi_1}\begin{pmatrix}1\\1\end{pmatrix} \quad \omega_1$$

$$z^{(2)} = e^{i\phi_2}\begin{pmatrix}1\\-1\end{pmatrix} \quad \omega_2$$

$\phi = \omega t$

(c) *Muestre que estas soluciones pueden ser ortonormales de acuerdo a la ecuación*

$$\sum_\lambda \sum_\sigma \rho(t)_\sigma m_{\sigma\lambda} \rho_\lambda^{(s)} = \delta_{st} \qquad (\neq 1$$

escogiendo

$$\underbrace{\rho_\sigma^{(1)} \equiv C_1 \bar{\rho}_\sigma^{(1)}}_{C_1}, \quad \underbrace{\rho_\sigma^{(2)} \equiv C_2 \left(\bar{\rho}_\sigma^{(2)} - \alpha \bar{\rho}_\sigma^{(1)}\right)}_{C_2},$$

con

$$\alpha \equiv \frac{\sum_\lambda \sum_\sigma \bar{\rho}_\sigma^{(2)} m_{\sigma\lambda} \bar{\rho}_\lambda^{(1)}}{\sum_\lambda \sum_\sigma \bar{\rho}_\sigma^{(1)} m_{\sigma\lambda} \bar{\rho}_\lambda^{(1)}}.$$

¿Qué son C_1 y C_2? Este es un ejemplo de la ortogonalización de Gram-Schmidt.

1) Probar ortogonalidad de

$\bar{\rho}_\sigma^{(1)}$ y $\bar{\rho}_\sigma^{(2)} - \alpha \bar{\rho}_\sigma^{(1)}$ con

$\bar{M}$

2) C_1 y C_2 (normalizar)

$$\sum_\lambda \sum_\sigma \rho_\sigma^{(t)} M_{\sigma\lambda} \rho_\lambda^{(s)} = \delta_{st}$$

$$\langle \rho^{(t)} | \hat{M} | \rho^{(s)} \rangle = \delta_{st}$$

$$|\rho^{(1)}\rangle = \underline{C_1 |\bar{\rho}^{(1)}\rangle} \propto |\tilde{\rho}$$

$$|\tilde{\rho}^{(2)}\rangle = |\bar{\rho}^{(2)}\rangle - \alpha |\bar{\rho}^{(1)}\rangle$$

$$\therefore \langle \rho^{(1)} | \hat{M} | \tilde{\rho}^{(2)} \rangle =$$

$$\langle \rho^{(1)} | \hat{M} | \bar{\rho}^{(2)} \rangle - \alpha \langle \bar{\rho}^{(1)} | \hat{M} | \bar{\rho}^{(1)} \rangle$$

$$\alpha = \frac{\langle \bar{\rho}^{(1)} | \hat{M} | \bar{\rho}^{(2)} \rangle}{\langle \bar{\rho}^{(1)} | \hat{M} | \bar{\rho}^{(1)} \rangle}$$

$$\langle \rho^{(1)}|\hat{m}|\rho^{(2)}\rangle = \langle \rho^{(1)}|\hat{m}|\rho^{(2)}\rangle$$
$$-\langle \bar{\rho}^{(1)}|\hat{m}|\bar{\rho}^{(2)}\rangle$$

$$= 0$$

$$\langle \rho^{(1)}|\hat{m}|\tilde{\rho}^{(2)}\rangle = 0$$

$$|\rho^{(2)}\rangle = C_2|\tilde{\rho}^{(2)}\rangle \; *$$

Normalización

$\underline{C_1}$ $\langle \rho^{(1)}|\hat{m}|\rho^{(1)}\rangle$

$$= C_1^2\langle \bar{\rho}^{(1)}|\hat{m}|\bar{\rho}^{(1)}\rangle = 1$$

$$\rightarrow C_1 = \frac{1}{\sqrt{\langle \bar{\rho}^{(1)}|\hat{m}|\bar{\rho}^{(1)}\rangle}}$$

$$\boxed{C_1 = \left(\sum_\sigma \sum_\lambda \bar{\rho}_\sigma^{(1)} m_{\sigma\lambda} \bar{\rho}_\lambda^{(1)}\right)^{-1/2}}$$

$\underline{C_2}$ $\langle \rho^{(2)}|\hat{m}|\rho^{(2)}\rangle = 1$

$$|\rho^{(2)}\rangle = C_2\left(|\bar{\rho}^{(2)}\rangle - \alpha|\bar{\rho}^{(1)}\rangle\right)$$

$$|\tilde{\rho}^{(2)}\rangle = C_2\left(|\bar{\rho}^{(2)}\rangle - \alpha|\bar{\rho}^{(1)}\rangle\right)$$

$$C_2^2\langle\tilde{\rho}^{(2)}|\hat{m}|\tilde{\rho}^{(2)}\rangle = 1$$

$$\Rightarrow \boxed{C_2^2 = \frac{1}{\sqrt{\langle\tilde{\rho}^{(2)}|\hat{m}|\tilde{\rho}^{(2)}\rangle}}}$$

$$\langle\tilde{\rho}^{(2)}|\hat{m}|\tilde{\rho}^{(2)}\rangle = \left(\langle\bar{\rho}^{(2)}| - \alpha^*\langle\bar{\rho}^{(1)}|\right)\hat{m}|\bar{\rho}$$

$$\langle\bar{\rho}^{(2)}|\hat{m}|\bar{\rho}^{(2)}\rangle - \underline{\alpha}\langle\bar{\rho}^{(2)}|\hat{m}|\bar{\rho}^{(1)}\rangle$$

$$- \underline{\alpha^*}\langle\bar{\rho}^{(1)}|\hat{m}|\bar{\rho}^{(2)}\rangle$$

$$+ |\alpha|^2\langle\bar{\rho}^{(1)}|\hat{m}|\bar{\rho}^{(1)}\rangle$$

$$\alpha = \frac{\langle\bar{\rho}^{(1)}|\hat{m}|\bar{\rho}^{(2)}\rangle}{\langle\bar{\rho}^{(1)}|\hat{m}|\bar{\rho}^{(1)}\rangle}$$

$$\alpha^* = \frac{\langle\bar{\rho}^{(2)}|\hat{m}|\bar{\rho}^{(1)}\rangle}{\langle\bar{\rho}^{(1)}|\hat{m}|\bar{\rho}^{(1)}\rangle}$$

$$\langle \tilde{\rho}^{(2)} | \hat{m} | \tilde{\rho}^{(2)} \rangle =$$

$$\langle \bar{\rho}^{(2)} | \hat{m} | \bar{\rho}^{(2)} \rangle - \frac{\langle \bar{\rho}^{(1)} | \hat{m} | \bar{\rho}^{(2)} \rangle \langle \bar{\rho}^{(2)} | \hat{m} | \bar{\rho}^{(1)} \rangle}{\langle \bar{\rho}^{(1)} | \hat{m} | \bar{\rho}^{(1)} \rangle}$$

$$\boxed{C_2 = \frac{1}{\sqrt{\langle \bar{\rho}^{(2)} | \hat{m} | \bar{\rho}^{(2)} \rangle - \frac{|\langle \bar{\rho}^{(1)} | \hat{m} | \bar{\rho}^{(2)} \rangle|^2}{\langle \bar{\rho}^{(1)} | \hat{m} | \bar{\rho}^{(1)} \rangle}}}}$$

Gram-schmidt

$$|\rho^{(1)}\rangle = \frac{|\bar{\rho}^{(1)}\rangle}{\sqrt{\langle \bar{\rho}^{(1)} | \hat{m} | \bar{\rho}^{(1)} \rangle}}$$

$$|\rho^{(2)}\rangle = \frac{|\rho^{(2)}\rangle - \frac{\langle \rho^{(1)} | \hat{m} | \bar{\rho}^{(2)} \rangle |\rho^{(1)}\rangle}{\langle \rho^{(1)} | \hat{m} | \bar{\rho}^{(2)} \rangle}}{\sqrt{\quad}}$$

$$\sqrt{\langle \bar{\rho}^{(2)} | \bar{M} | \bar{\rho}^{(2)} \rangle}$$

$$C_2^2 = \left(\sum_\sigma \sum_\lambda \left(\bar{\rho}_\sigma - \alpha \bar{\rho}_\sigma^{(1)} \right) M_{\sigma\lambda} \times \left(\bar{\rho}_\lambda^{(2)} - \alpha \bar{\rho}_\lambda^{(1)} \right) \right)^{-1/2}$$

Ejercicio 2. *Un aro delgado de radio R y masa M oscila en su propio plano con un punto del aro fijo. Unido al aro hay una masa puntual m obligada a moverse sin fricción a lo largo del aro. El sistema está en un campo gravitacional $\vec{g}$. Considere sólo pequeñas oscilaciones.*

(a) *Muestre que las frecuencias de los modos normales son*

$$\omega_1 = \frac{1}{2}\sqrt{\frac{2g}{R}}, \quad \omega_2 = \sqrt{\frac{2g}{R}}$$

(b) *Encuentre los autovectores de los modos normales. ~~Dibuje su movimiento~~.*

(c) *Construya la matriz modal.*

(d) *Encuentre las coordenadas normales y muestre que ellas diagonalizan el Lagrangiano.*

$$\det(\bar{D} - \omega^2 \bar{M}) = 0$$

$$\omega_1 = \frac{1}{2}\sqrt{\frac{2g}{R}}$$

$$\omega_2 = \sqrt{\frac{2g}{R}} \rightarrow \bar{K}$$

$\bar{D}$, $\bar{m}$; $\bar{D} \rightarrow V$

$\bar{m} \rightarrow T$

$\phantom{\bar{m}} \rightarrow M$

$$T = \frac{1}{2}\dot{q}^T M \dot{q}$$

$$V = \frac{1}{2} q^T K q$$

$$q = \begin{pmatrix} \theta \\ \phi \end{pmatrix} \quad q^T = (\theta \ \phi)$$

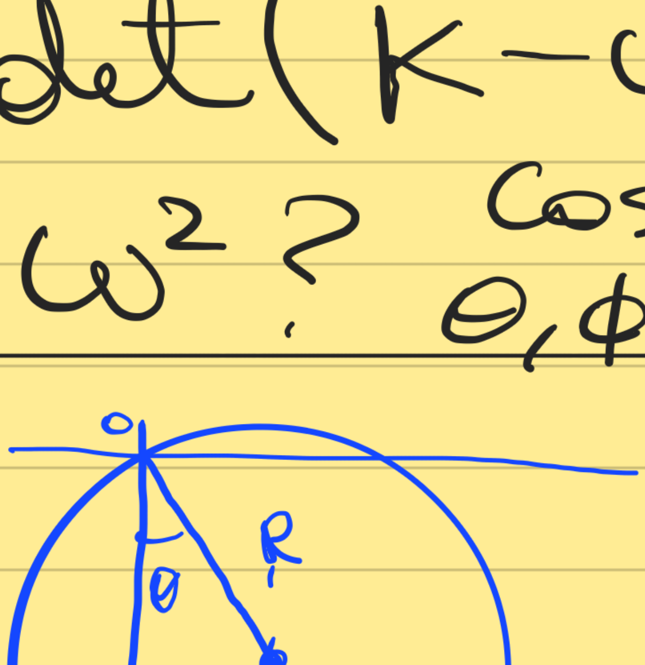
$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} *$$

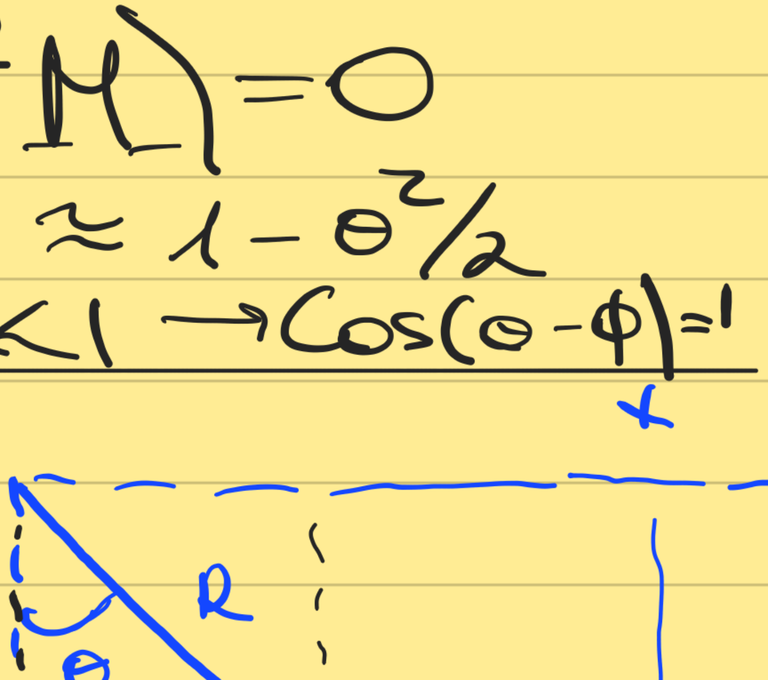
$$K = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} *$$

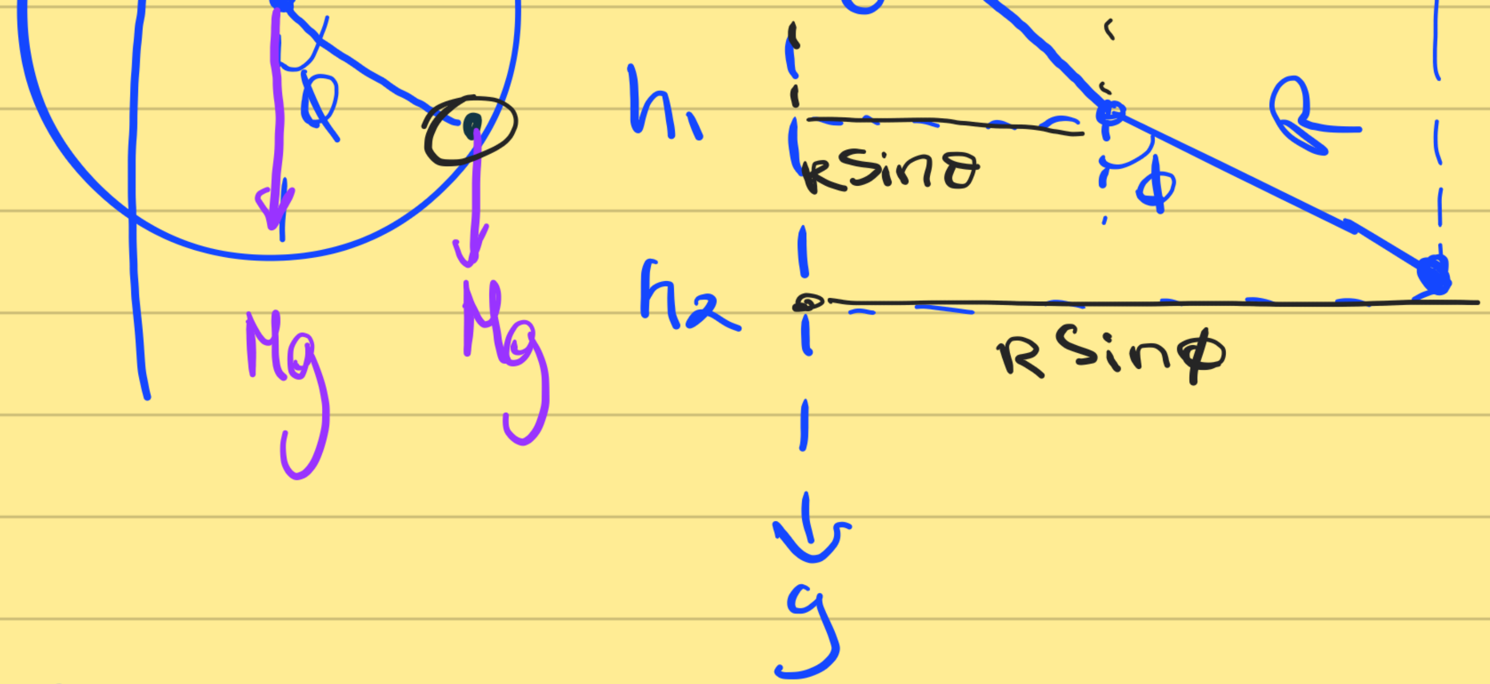
$$\det(K - \omega^2 M) = 0$$

ω^2 ? $\cos\theta \approx 1 - \theta^2/2$

$\theta, \phi \ll 1 \rightarrow \cos(\theta - \phi) = 1$







1) Energía cinética y potencial

$$h_1 = R - R\cos\theta = R(1-\cos\theta)$$

$$\overset{(p.o.)}{=} R\left(1 - 1 + \theta^2/2\right) = R\theta^2/2$$

$$h_2 = R - R\cos\theta - R\cos\phi$$

$$= R\left(1 - \cos\theta - \cos\phi\right)$$

$$\overset{(p.o.)}{=} R\left(-1 + \theta^2/2 + \phi^2/2\right)$$

Masa puntual:

$$x = R(\sin\theta + \sin\phi)$$

$$y = -R(\cos\theta + \cos\phi)$$

$$\dot{x} = R(\dot{\theta}\cos\theta + \dot{\phi}\cos\phi)$$

$$\dot{y} = R(\dot{\theta}\sin\theta + \dot{\phi}\sin\phi)$$

$$\dot{x}^2 + \dot{y}^2 = R^2\left(\dot{\theta}^2 + 2\dot{\theta}\dot{\phi}\cos(\theta-\phi) + \dot{\phi}^2\right)$$

pequeñas oscilaciones

$$\theta, \phi \ll 1 \Rightarrow \cos(\theta - \phi) = 1$$

$$\dot{x}^2 + \dot{y}^2 = R^2(\dot{\theta}^2 + 2\dot{\theta}\dot{\phi} + \dot{\phi}^2)$$

Para CM

$$I = I_{aro} + I_{cm}$$

$$= MR^2 + MR^2$$

$$= 2MR^2$$

$$T = T_1 + T_2$$

$$T_1 = \frac{1}{2} I \dot{\theta}^2 = MR^2 \dot{\theta}^2$$

$$T_2 = \frac{1}{2} MR^2 \left(\dot{\theta}^2 + 2\dot{\theta}\dot{\phi} + \dot{\phi}^2 \right)$$

$$T = \underline{\frac{1}{2} MR^2} \left(\underline{3}\dot{\theta}^2 + 2\dot{\theta}\dot{\phi} + \dot{\phi}^2 \right) *$$

$$(\bar{\Delta})$$

$$V = V_1 + V_2$$

$$V_1 = \frac{1}{2} MgR \dot{\theta}^2$$

$$V_2 = \frac{1}{2} MgR \left(\theta^2 + \phi^2 \right)$$

$$V = \underline{\frac{MgR}{2} \left(2\theta^2 + \phi^2 \right)} *$$

$\bar{T}$ y $\bar{V}$

$$\bar{T} = \frac{1}{2} \dot{q}^T M \dot{q} \; ; \; q = \begin{pmatrix} \theta \\ \phi \end{pmatrix}$$

$$q^T = \begin{pmatrix} \theta & \phi \end{pmatrix}$$

$$\dot{q} = \begin{pmatrix} \dot{\theta} \\ \dot{\phi} \end{pmatrix} \quad \dot{q}^T = \begin{pmatrix} \dot{\theta} & \dot{\phi} \end{pmatrix}$$

$+ \ (\phi) \ + -(\theta \ \phi)$

$$\overline{T} = \frac{1}{2}(\dot{\theta}\ \dot{\phi})\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}\begin{pmatrix} \dot{\theta} \\ \dot{\phi} \end{pmatrix}$$

$$= \frac{1}{2}\left(M_{11}\dot{\theta}^2 + M_{12}\dot{\phi}\dot{\theta} + M_{21}\dot{\phi}\dot{\theta} + M_{22}\dot{\phi}^2\right) \quad (\Delta)$$

$$(\overline{\Delta}) = (\Delta) \Longrightarrow \boxed{M = MR^2\begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}}$$

$$\overline{V} = \frac{1}{2}(\theta\ \phi)\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}\begin{pmatrix} \theta \\ \phi \end{pmatrix}$$

$$K = MgR\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

2) Autovalores $\underbrace{(K - \omega^2 M)}\,|D\rangle = 0$

$$\Longrightarrow \begin{pmatrix} 2MgR - 3\omega^2 MR^2 & -\omega^2 MR^2 \end{pmatrix}$$

$$-\omega^2 MR^2 \qquad MgR - \omega^2 MR^2 \Big)$$

$$\Big/ \frac{1}{MR^2}$$

$$\Rightarrow \begin{pmatrix} 2\frac{g}{R} - 3\omega^2 & -\omega^2 \\ -\omega^2 & \frac{g}{R} - \omega^2 \end{pmatrix} \quad \omega_0^2 = \frac{g}{R}$$

$$\Rightarrow \det(K - \omega^2 M) = 0$$

$$\begin{vmatrix} 2\omega_0^2 - 3\omega^2 & -\omega^2 \\ -\omega^2 & \omega_0^2 - \omega^2 \end{vmatrix} = 0$$

$$= (2\omega_0^2 - 3\omega^2)(\omega_0^2 - \omega^2) - \omega^4 = 0$$

$$x = \omega^2$$

$$(2\omega_0^2 - 3x)(\omega_0^2 - x) - x^2 = 0$$

$$2x^2 - 5\omega_0^2 x + 2\omega_0^4 = 0$$

$$x = \frac{5\omega_0^2 \pm \sqrt{25\omega_0^4 - 16\omega_0^4}}{4}$$

$$= \frac{5\omega_0^2 \pm 3\omega_0^2}{4}$$

$$x_1 = 2\omega_0^2 \Rightarrow \omega_1^2 = 2\frac{g}{R}$$

$$\boxed{\omega_1 = \sqrt{\frac{2g}{R}}}$$

$$x_2 = \frac{1}{2}\omega_0^2 \Rightarrow \boxed{\omega_2 = \sqrt{\frac{g}{2R}}}$$

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b) dutovectores. Dibuje mov.

$$\omega_1 = \sqrt{\frac{2g}{R}}, \quad \omega_2 = \sqrt{\frac{g}{2R}}$$

$M = MR^2\begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}; K = MgR\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

$$(K - \omega_n^2 M)|\vec{v}_n\rangle = 0$$

$n = 1$

$$\underbrace{(K - \omega_1^2 M)}|\vec{v}_1\rangle = 0$$

$$\begin{pmatrix} 2\frac{g}{R} - 3\omega_1^2 & -\omega_1^2 \\ -\omega_1^2 & \frac{g}{R} - \omega_1^2 \end{pmatrix}_{\omega_1^2 = \frac{2g}{R}}$$

$\frac{R}{2g}$

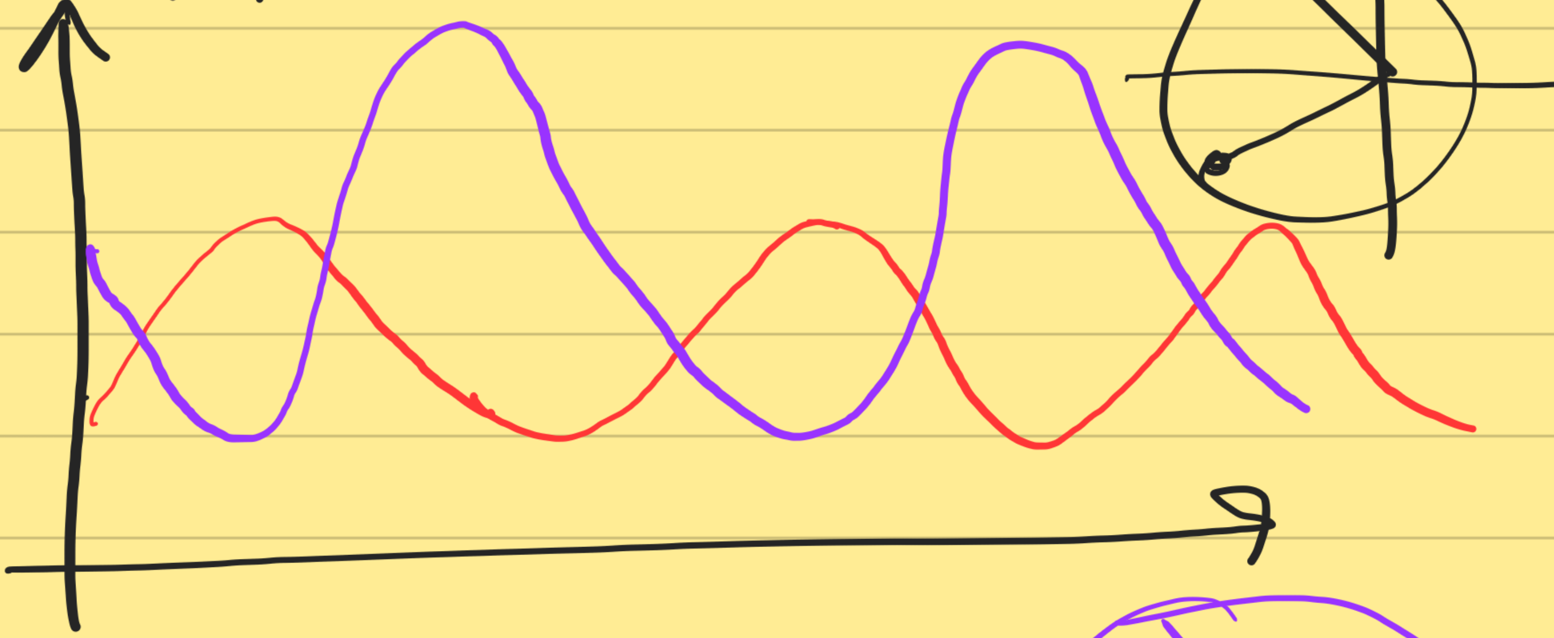
$$= \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} = 0$$

$$\left\{\begin{array}{l} 4A + 2B = 0 \\ 2A + B = 0 \end{array}\right\} \underline{2A = -B}$$

$|\vec{v}_1\rangle = (\overset{\theta}{1}, \overset{\phi}{-2})^T$

$n=2$ | $|\vec{v}_2\rangle = (1,1)^T$

$n=1$



$n=2$ |