PAUTA TALLER EVALUADO Nº3

1. Resolver el PVI:

$$ty'' + (t+2)y' + y = t^2 \ ;\ y(0) = 2\,;\, y'(0) = -1 \quad /\mathcal{L}$$

$$\mathcal{L}\{ty'' + (t+2)y' + y\} = \mathcal{L}\{t^2\}$$

$$-\tfrac{d}{ds}\left(\mathcal{L}\{ty''\}\right) + \mathcal{L}\{ty'\} + 2\mathcal{L}\{y'\} + \mathcal{L}\{y\} = \tfrac{2}{s^3}$$

$$-\tfrac{d}{ds}\left(\mathcal{L}\{y''\}\right) - \tfrac{d}{ds}\mathcal{L}\{y'\} + 2\mathcal{L}\{y'\} + \mathcal{L}\{y\} = \tfrac{2}{s^3}$$

$$-\tfrac{d}{ds}(s^2\mathcal{L}\{y\} - sy(0) - y'(0)) - \tfrac{d}{ds}\{s\mathcal{L}\{y\} - y(0)\} + 2(s\mathcal{L}\{y\} - y(0)) + \mathcal{L}\{y\} = \tfrac{2}{s^3}$$

$$-\tfrac{d}{ds}(s^2Y - 2s + 1) - \tfrac{d}{ds}\{sY + 1\} + 2(sY + 1) + Y = \tfrac{2}{s^3}$$

$$-2sY - s^2Y' - 2 - Y - sY' + 2sY + 2 + Y = \tfrac{2}{s^3}$$

$$-(s^2 + s)Y' = \tfrac{2}{s^3} + 2$$

$$Y' = -2\tfrac{1+s^3}{(s^2+s)s^3} = -2\tfrac{(s+1)(s^2+s+1)}{s(s+1)s^3}$$

$$Y' = -2\tfrac{(s+1)(s^2-s+1)}{(s+1)s^4}$$

$$Y' = -2\tfrac{(s^2-s+1)}{s^4} = -2\left(\tfrac{1}{s^2} - \tfrac{1}{s^3} + \tfrac{1}{s^4}\right) \qquad /\int \cdot\, ds$$

$$Y = -2\left(-\tfrac{1}{s} + \tfrac{1}{2}\tfrac{1}{s^2} - \tfrac{1}{3}\tfrac{1}{s^3}\right) \quad /\mathcal{L}^{-1}$$

$$y = -2\left(-1 + \tfrac{1}{2}t - \tfrac{1}{6}t^2\right)$$

$$y = 2 - t + \tfrac{1}{3}t^2$$

2. Resolver el PVI: $y'' + 36y = f(t)$; $y(0) = y'(0) = 0$

$$\text{donde: } f(t) = \begin{cases} -1 & \text{si } t < 1 \\ -3 & \text{si } 1 \leq t \end{cases}$$

$$y'' + 36y = -1 - 2u(t-1) \qquad /\mathcal{L}$$

$$s^2\mathcal{L}\{y\}\text{-}sy(0)\text{-}y'(0) + 36\mathcal{L}\{y\} = -\tfrac{1}{s} - 2e^{-s}$$

$$(s^2+36)\mathcal{L}\{y\} = -\tfrac{1}{s} - \tfrac{2}{s}e^{-s}$$

$$\mathcal{L}\{y\} = -\tfrac{1}{s(s^2+36)} - \tfrac{2}{s(s^2+36)}e^{-s}$$

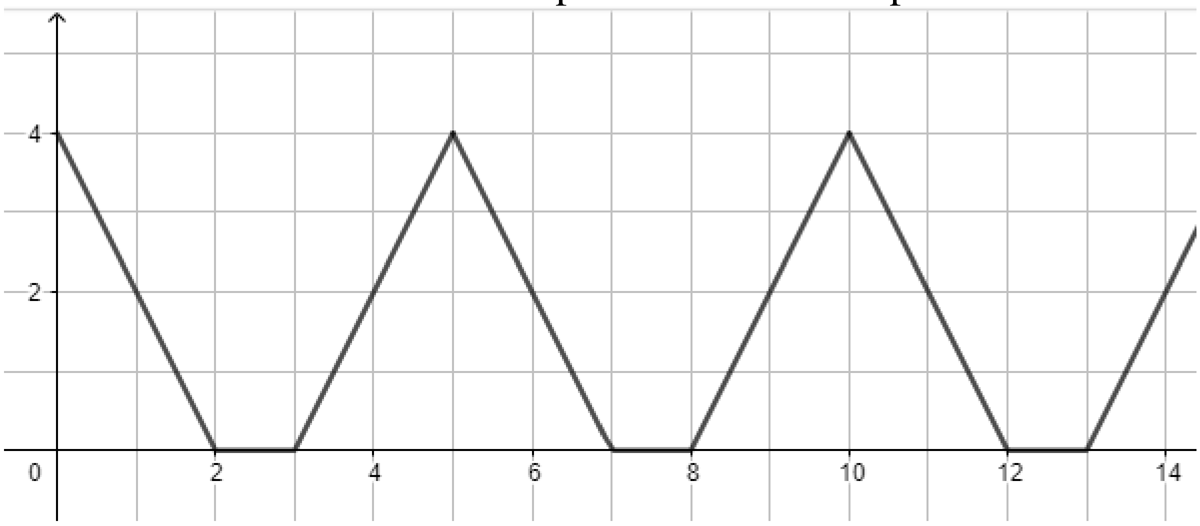
$$\mathcal{L}\{y\} = \tfrac{1}{36}\left[\tfrac{s}{s^2+36} - \tfrac{1}{s}\right] + \tfrac{2}{36}\left[\tfrac{s}{s^2+36} - \tfrac{1}{s}\right]e^{-s} \;/\mathcal{L}^{-1}$$

$$y = \tfrac{1}{36}\cos(6t) - \tfrac{1}{36} - \tfrac{1}{18}\mathcal{L}^{-1}\left\{\left[\tfrac{s}{s^2+36} - \tfrac{1}{s}\right]e^{-s}\right\}$$

$$y = \tfrac{1}{36}(\cos(6t) - 1) - \tfrac{1}{18}u(t-1)\mathcal{L}^{-1}\left\{\tfrac{s}{s^2+36} - \tfrac{1}{s}\right\}(t-1)$$

$$y = \tfrac{1}{36}(\cos(6t) - 1) - \tfrac{1}{18}u(t-1)\text{sen}\big(6(t-1) - 1\big)$$

3. Halle la transformada de Laplace de la función periódica:



$$\overline{f}(t) = \begin{cases} -2t+4 & \text{si } t<2 \\ 0 & \text{si } 2<t<3 \\ 2t-6 & \text{si } 3<t<5 \\ 0 & \text{si } 5<t \end{cases}$$

$$\mathcal{L}\{f(t)\} = \tfrac{1}{1-e^{-5s}}\mathcal{L}\left\{\overline{f}(t)\right\}$$

$$= \tfrac{1}{1-e^{-5s}}\mathcal{L}\{4-2t-(4-2t)u(t-2)+(2t-6)u(t-3)-(2t-6)u(t-5)\}$$

$$= \tfrac{1}{1-e^{-5s}}\mathcal{L}\{4-2t+2(t-2)u(t-2)+2(t-3)u(t-3)-[2(t-5)-2]u(t-5)\}$$

$$= \tfrac{1}{1-e^{-5s}}\left(\tfrac{4}{s}-\tfrac{2}{s^2}+2e^{-2s}\cdot\tfrac{1}{s^2}+2e^{-3s}\cdot\tfrac{1}{s^2}-e^{-5s}\left[\tfrac{2}{s^2}-\tfrac{2}{s}\right]\right)$$

4. Resolver la ecuación:

$$y' - y + \int_0^t (t-u)y'(u)du - \int_0^t y(u)du = t\delta(t-1)$$
$$y(0) = 0$$

$$y' - y + t{*}y' - 1{*}y = t\delta(t-1) \qquad /\mathcal{L}$$

$$s\mathcal{L}\{y\} - \mathcal{L}\{y\} + \mathcal{L}\{t\}\cdot\mathcal{L}\{y'\} - \mathcal{L}\{1\}\cdot\mathcal{L}\{y\} = \mathcal{L}\{t\delta(t-1)\}$$

$$s\mathcal{L}\{y\} - \mathcal{L}\{y\} + \tfrac{1}{s^2}\cdot s\mathcal{L}\{y\} - \tfrac{1}{s}\cdot\mathcal{L}\{y\} = -\tfrac{d}{ds}\mathcal{L}\{\delta(t-1)\}$$
$$s\mathcal{L}\{y\} - \mathcal{L}\{y\} = -\tfrac{d}{ds}(e^{-s})$$

$$\mathcal{L}\{y\}(s-1) = e^{-s}$$

$$\mathcal{L}\{y\} = e^{-s}\tfrac{1}{(s-1)} \qquad /\mathcal{L}^{-1}$$

$$y = \mathcal{L}^{-1}\left\{e^{-s}\tfrac{1}{(s-1)}\right\}$$

$$y = u(t-1)\mathcal{L}^{-1}\{\tfrac{1}{s-1}\}(t-1)$$

$$y = u(t-1)\left[e^{-(t-1)}\right]$$