1. Resolver: $y' = \frac{4x+2y-16}{2x-y-4}$

$$\left.\begin{array}{l} 4x+2y=16 \\ 2x-y=4 \end{array}\right| \quad x = \frac{\begin{vmatrix} 16 & 2 \\ 4 & -1 \end{vmatrix}}{\begin{vmatrix} 4 & 2 \\ 2 & -1 \end{vmatrix}} = \frac{-24}{-8} = 3 \qquad y = \frac{\begin{vmatrix} 4 & 16 \\ 2 & 4 \end{vmatrix}}{-8} = \frac{-16}{-8} = 2$$

$$\bar{x} = x-3 \;;\; \bar{y} = y-2$$

$$\bar{y}' = \frac{4\bar{x}+2\bar{y}}{2\bar{x}-\bar{y}} = \frac{4+2\frac{\bar{y}}{\bar{x}}}{2-\frac{\bar{y}}{\bar{x}}} = \frac{4+2v}{2-v} = g(v)$$

$$v = \frac{\bar{y}}{\bar{x}}$$

$$\int \frac{dv}{\frac{4+2v}{2-v} - v} = \ln|\bar{x}| + c$$

$$-\int \frac{v-2}{v^2+4}\,dv = \ln|\bar{x}|+c$$

$$-\left[\frac{1}{2}\int \frac{2v\,dv}{v^2+4} - 2\int \frac{1}{v^2+4}\,dv\right] = \ln|\bar{x}|+c$$

$$-\frac{1}{2}\ln(v^2+4) + \operatorname{arctg}\left(\frac{v}{2}\right) = \ln|\bar{x}|+c$$

$$-\frac{1}{2}\ln\left(\left(\frac{y-2}{x-3}\right)^2+4\right) + \operatorname{arctg}\left(\frac{y-2}{2(x-3)}\right) = \ln(x-3)+c$$

2. Resolver:

$$\left(x^2y + 3y^3x^2 + y^4\right)dx + (2y^2x^3 + 3y^3x + ye^y)dy = 0$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = \frac{x^2 + 9x^2y^2 + 4y^3 - 6x^2y^2 - 3y^3}{x^2y + 3x^2y^3 + y^4}$$

$$= \frac{x^2 + 3x^2y^2 + y^3}{y(x^2 + 3x^2y^2 + y^3)} = \frac{1}{y}$$

$$\mu(y) = e^{-\int \frac{1}{y}dy} = e^{-\ln(y)} = \frac{1}{y}$$

$$\left(x^2y + 3y^3x^2 + y^4\right)dx + \left(2y^2x^3 + 3y^3x + ye^y\right)dy = 0 \quad / \cdot \frac{1}{y}$$

$$(x^2 + 3y^2x^2 + y^3)dx + (2yx^3 + 3xy^2 + e^y)\,dy = 0$$

$$x^2dx + 3x^2y^2dx + 2x^3y\,dy + y^3dx + 3xy^2dy + e^ydy = 0$$

$$d\left(\frac{x^3}{3}\right) + d(x^3y^2) + d(xy^3) + d(e^y) = 0$$

$$\frac{x^3}{3} + x^3y^2 + xy^3 + e^y = c$$

3. Resolver: $xy' + y^2 + 2y - 4x^2 - 2x = 0$

si una solución particular es de la forma $y_p = ax + b$

$y_p = ax+b \Rightarrow y_p' = a$; reemplazando:

$$xa + (ax+b)^2 + 2(ax+b) - 4x^2 - 2x = 0$$

$$x = 0 \Rightarrow b^2 + 2b = 0 \Rightarrow (b=0 \lor b=-2)$$

$$x = 1 \Rightarrow a + (a+b)^2 + 2(a+b) - 6 = 0$$

si $b=0$ entonces: $a + a^2 + 2a - 6 = 0$

$$a^2 + 3a - 6 = 0$$

$$a = \frac{-3 \pm \sqrt{9+24}}{2}$$

$$a = \frac{-3 \pm \sqrt{23}}{2}$$

si $b=-2$ entonces:

$$a + (a-2)^2 + 2(a-2) - 6 = 0$$

$$a + a^2 - 4a + 4 + 2a - 4 - 6 = 0$$

$$a^2 - a - 6 = 0$$

$$(a-3)(a+2) = 0$$

$$a = 3 \lor a = -2$$

Posibles soluciones:

$$y_1 = \frac{-3+\sqrt{23}}{2}x \; ; \; y_2 = \frac{-3-\sqrt{23}}{2}x \; ; \; y_3 = 3x-2 \; ; \; y_4 = -2x-2$$

Probando se tiene que $y = -2x-2$ es solución.

Cambio de variables: $y = -2x - 2 + \frac{1}{z} \Rightarrow y' = -2 - \frac{z'}{z^2}$

$$x\left(-2 - \frac{z'}{z^2}\right) + \left(-2x-2+\frac{1}{z}\right)^2 + 2\left(-2x-2+\frac{1}{z}\right) - 4x^2 - 2x = 0$$

$$-2x - \frac{z'}{z^2}x + (2x+2)^2 - 2(2x+2)\frac{1}{z} + \frac{1}{z^2} - 4x - 4 + \frac{2}{z} - 4x^2 - 2x = 0$$

$$-\frac{z'}{z^2}x + \cancel{4x^2} + \cancel{8x} + \cancel{4} - \frac{4x}{z} - \frac{4}{z} + \frac{1}{z^2} - \cancel{4x} - \cancel{4} + \frac{2}{z} - \cancel{4x^2} - \cancel{4x} = 0$$

$$-\frac{z'}{z^2}x - \frac{4x}{z} - \frac{2}{z} + \frac{1}{z^2} = 0 \quad / \cdot \left(-\frac{z^2}{x}\right)$$

$$z' + 4z + \frac{2}{x}z - \frac{1}{x} = 0$$

$$z' + \left(4 + \frac{2}{x}\right)z = \frac{1}{x}$$

$$\mu(x) = e^{\int (4 + \frac{2}{x})dx} = e^{4x + 2\ln(x)} = x^2 e^{4x}$$

$$x^2 e^{4x} z' + (4x^2 e^{4x} + 2x e^{4x})z = x e^{4x}$$

$$(x^2 e^{4x} z)' = x e^{4x} \quad / \int$$

$$x^2 e^{4x} z = \frac{1}{4} x e^{4x} - \frac{1}{16} e^{4x} + c$$

$$z = \frac{1}{4x} - \frac{1}{16x^2} + \frac{c}{x^2} e^{-4x}$$

$$y = -2x - 2 + \frac{1}{z} \Rightarrow z = \frac{-1}{2x+2+y}$$

$$-\frac{1}{2x+2+y} = \frac{1}{4x} - \frac{1}{16x^2} + \frac{c}{x^2} e^{-4x}$$

4. Resolver:

Un tanque contiene inicialmente una solución de 480 lts. que contiene 1440 grs. de sal. Si al tanque entra líquido a razón de 3 lts/min con una concentración de sal de 3 gramos/litro y la solución sale del tanque (homogéneamente mezclada) a razón de 6 lts/min.

Determine:

a) La ecuación diferencial, con condición inicial, para $x(t)$ (PVI), donde $x(t)$ es la cantidad de gramos de sal dentro del tanque en el instante t.
b) La solución general de la ecuación diferencial.
c) La solución particular del PVI.
d) En cuántos minutos el tanque contendrá 882 grs. de sal.

a) $$x' = 3\times 3 - \frac{6x}{480+(3-6)t}$$

$$x' + \frac{6}{480-3t}x = 9$$

b) $$\mu(t) = e^{\int \frac{6}{480-3t}dt} = e^{\int \frac{-2}{t-160}dt}$$

$$= (t-160)^{-2}$$

$$(t-160)^{-2}x' + 6\frac{(t-160)^{-2}}{-3(t-160)}x = 9(t-160)^{-2}$$

$$(t-160)^{-2}x' - 2(t-160)^{-3}x = 9(t-160)^{-2}$$

$$[(t-160)^{-2}x]' = 9(t-160)^{-2} \quad / \int \cdot\, dt$$

$$(t-160)^{-2}x = -9(t-160)^{-1} + c$$

$$x = -9(t-160) + c(t-160)^2$$

c) $x(0) = 1440 \Rightarrow$

$$\cancel{1440} = \cancel{1440} + 25600c$$

$$c = 0$$

$$x_p(t) = -9(t-160)$$

d) $x_p(t) = 882$

$$-9t + 1440 = 882$$

$$-9t = -558$$

$$t = 62$$

En 62 min tendrá 882 gr de sal.