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A comparative artificial neural networks for Schwarzschild black hole (SBH) radius

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ABSTRACT

It is consensus among researchers that the data for the black holes is complicated and extremely non-linear in nature. Therefore, it remains a challenging task for them to predict the key characteristics of concerned black holes accurately. The present work offers artificial neural networks assistance in the context of a choice of training functions for the prediction of astrophysical phenomena like the event horizon and radius of black holes. To be more specific, we considered the Schwarzschild black hole as the simplest solution of Einstein's field equations. The Schwarzschild radius and masses are chosen in the last and first layers of the neural networks model, respectively. Two various training functions namely Levenberg-Marquardt training algorithm (LMTA) and Scaled Conjugate Gradient training algorithms (SCGTA) are used. We have observed that the LMTA achieved significantly lower error rates, suggesting a better fit and stronger learning capabilities from the solar masses of black holes. Furthermore, the close alignment between the ANN-predicted and actual Schwarzschild black hole radius demonstrates the LMTA model holds the ability to generalize effectively across unseen masses of black holes.

1. Introduction

The most basic kind of black hole is called a Schwarzschild black hole, which is defined by its spherical symmetry, absence of rotation, and electric charge. The Schwarzschild solution, which was found by German physicist Karl Schwarzschild in 1916 and was the first exact solution to Einstein's field equations of general relativity, describes it [1]. The Schwarzschild solution describes how spacetime is curved around the black hole. As objects approach the event horizon, time slows down relative to a distant observer (time dilation), and the path of light becomes more distorted due to gravitational lensing. In their discussion of gravitational collapse, Oppenheimer and Snyder [2] offered a physical explanation for how black holes form. Their calculations supported the Schwarzschild solution and provided a physical explanation for the formation of black holes by showing how stars with enough mass may collapse under their gravity. Through his investigation of gravitational singularities and their implications for physical laws, Wheeler [3] provided an advanced theory of black holes. He defined "black hole" and discussed the ramifications of Schwarzschild's solution, demonstrating how black holes may be thought of as hairless gravitational objects that can only be explained by mass, charge, and angular momentum. The work of Bardeen et al. [4] was a significant turning point in the advancement of black hole thermodynamics. They developed the principles of black hole mechanics, drawing comparisons between thermodynamic constants like entropy and temperature and black hole characteristics like mass and surface area. Their principles also shed light on thermodynamic behavior in the Schwarzschild situation, even though they were mostly relevant to revolving black holes. Hawking radiation was discovered as a result of Hawking [5] applying quantum mechanics to the study of black holes. The fact that this finding demonstrated that Schwarzschild black holes are not entirely black but rather leak radiation and have the ability to eventually evaporate transformed our understanding of them. Through this process, the traditional understanding of black holes as complete "information traps" was contested, and the thermodynamic characteristics of Schwarzschild's black holes were expanded. By examining the consequences of black holes, especially Schwarzschild black holes, for general relativity and astrophysics, Thorne [6] book makes important contributions. It emphasized the significance of Schwarzschild black holes about gravitational waves and spacetime curvature while offering a more approachable understanding of black hole physics for a broad audience. One of the most thorough books on black hole theory, including

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Schwarzschild's black holes, was written by Chandrasekhar and Thorne [7]. They conducted a thorough investigation into the mathematical characteristics of Schwarzschild black holes, providing a profound understanding of their stability and interactions with radiation and matter. The credit of foundation for the mathematical understanding of black hole physics goes to Chandrasekhar's work. Jacobson [8] proposed a profound connection between temperature, entropy, and spacetime geometry by deriving Einstein's field equations from thermodynamic principles. His work extended the implications of Schwarzschild black holes to spacetime itself and proposed a fundamental relationship between gravity and thermodynamics, building upon previous discoveries regarding black hole thermodynamics. Potential solutions to the information paradox for black holes were explored by Ashtekar and Bojowald [9], especially in light of Hawking radiation and quantum gravity. Their work bridged the gap between general relativity and quantum physics by providing potential explanations for how information could escape from Schwarzschild black holes that are vanishing. According to Hawking, thermal radiation from black holes is released spontaneously through a quantum process. Weinfurtner et al. [10] used the parallel between surface waves on moving water and the propagation of fields surrounding black holes to address this problem empirically. Surface wave horizons can be included in the area of high velocity that is formed over an obstacle by inserting a streamlined object into an open channel flow. Short (deep-water) waves replaced long waves that were moving upstream toward this area. Their measurements of the amplitudes of the converted waves show the thermal nature of the conversion process for this system. This was the analogue of the stimulated emission by hole. Their results confirm the generality of the Hawking process, given the strong link between stimulated and spontaneous emission. Using a semi-regular black hole model, Lee and Yeom [11] proposed an assertion and came to the conclusion that the firewall, if it exists, should have an impact on the asymptotic observer. Furthermore, the argument that the horizon should act as a firewall to prevent anyone from penetrating it for consistency will be made if any opinion ignores the results of the duplication experiment and the huge N rescaling is challenging to accept. Since then, various attempts [12–14] have been carried out to examine the characteristics of black physics, particularly Schwarzschild black holes. The recent developments in this regard can be accessed in Refs. [15–19].

Neural networks are very adaptable to a wide range of applications [20] due to their capacity to learn complicated, non-linear correlations from data. This is the driving force behind their use in numerous domains for quantity prediction. Large datasets [21] with numerous variables are no problem for neural networks, which are adept at seeing patterns that more conventional statistical techniques would miss. Numerous interrelated aspects are involved in the prediction of quantities such as stock prices [22], patient outcomes [23], energy consumption [24], or weather patterns [25], in the domains of finance [26], healthcare [27], engineering [28], and climate science [29,30]. These complex relationships can be modeled by neural networks thanks to their deep structures, which eliminate the need for explicit programming of every link. Their capacity to significantly increase performance by utilizing sophisticated training algorithms and optimization approaches like backpropagation strengthens their predicting powers. Neural networks are also quite flexible; they may be used for tasks like regression [31,32] and classification as well as time-series forecasting. Because of their scalability, they may be used for a wide range of data volumes and types, allowing for real-time prediction in fields like predictive maintenance [33] and autonomous systems [34].

It is believed that the non-linear nature of the underlying physics makes it difficult to forecast Schwarzschild black hole essentials like the event horizon and radius with a high degree of accuracy. Instead, advanced computer techniques are needed. A key factor in replicating these events with artificial neural networks is the selection of training functions. This study looks into how different training functions affect the prediction of the Schwarzschild radius, which is derived from the Schwarzschild metric. The key novelty of the paper includes the assessment of the efficacy of LMTA and SCGTA through a comparison analysis, identifying the most appropriate ones for correctly predicting the event horizon in a Schwarzschild black hole. The article is designed as follows: a motivational literature review on early studies on black holes and the use of ANN in various fields are reported in Section-1 while mathematical features of Schwarzschild black hole are reviewed in Section-2. The fundamentals for training algorithms are debated in Section-3 while graphical outcomes are discussed in Section-4. The limitations and implications are itemized in Section-5. The ultimate outcomes are summarized in Section-6.

2. Black hole mathematical features

The foundation of general relativity lies in Einstein's field equations, which relate the curvature of spacetime through the Einstein tensor $G_{\mu\nu}$ to the energy and momentum of matter that is described by the stress-energy tensor $T_{\mu\nu}$. The $G_{\mu\nu}$ is defined as:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{1}$$

here G, and c represents the gravitational constant and speed of light respectively. For a vacuum solution, such as a black hole with no matter outside it, the stress-energy tensor should be zero, so the field equations reduce to:

$$G_{\mu\nu} = 0. \tag{2}$$

For the solution of Einstein field equations, we assume the condition of spherical symmetry; which means the solution should be the same in all directions around the central mass. Further, we assume that the spacetime is not changing with time (i.e., the black hole is not rotating or evolving). Therefore, the most general form of a spherically symmetric, static metric is given by:

$$ds^2 = -\lambda_1(r)c^2dt^2 + \lambda_2(r)dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right). \tag{3}$$

By solving the vacuum Einstein equations $G_{\mu\nu} = 0$, we can determine the functions $\lambda_1(r)$ and $\lambda_2(r)$. The detailed solution process involves using the Ricci tensor and Ricci scalar to compute the Einstein tensor. For brevity, we focus on the result. The function $\lambda_1(r)$ takes the form:

$$\lambda_1(r) = 1 - \frac{2GM_B}{rc^2}. \tag{4}$$

Similarly, the function $\lambda_2(r)$ is the inverse of $\lambda_1(r)$ and defined as:

$$\lambda_2(r) = \left(1 - \frac{2GM_B}{rc^2}\right)^{-1}. \tag{5}$$

Substituting the functions $\lambda_1(r)$ and $\lambda_2(r)$ into the general spherically symmetric metric, see Eq. (3), we get the Schwarzschild metric as follows:

$$ds^2 = -\left(1 - \frac{2GM_B}{rc^2}\right)c^2dt^2 + \left(1 - \frac{2GM_B}{rc^2}\right)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right). \tag{6}$$

Here, the M_B represents the mass of black hole, r is the radial coordinate, t is the time coordinate, θ is the polar angle and ϕ represents the azimuthal angle. The Schwarzschild radius sR is the critical radius where the metric coefficient $\lambda_1(r) = 0$, leading to the event horizon of the black hole. The event horizon for a Schwarzschild black hole, is given by:

$$sR = \frac{2GM_B}{c^2}, \tag{7}$$

at $r = R_s$ the time component of the metric goes to zero, meaning time stops relative to an outside observer, and the radial component blows up, indicating that space becomes infinitely stretched.

3. Artificial neural networks

Neural networks models are particularly effective in predicting quantities [35–37] because of their capacity to generalize from training data, enabling accurate predictions even in dynamic or uncertain environments. Training functions play a critical role in constructing Artificial Neural Networks (ANNs) as they directly influence how the network learns and optimizes its performance. The primary purpose of a training function is to update the weights and biases of the network during the training process, enabling the model to minimize the error between predicted outputs and actual target values. These functions guide the optimization process, helping the network converge toward a solution. We have chosen two various training functions namely, the Levenberg-Marquardt training algorithm (LMTA) and the Scaled Conjugate Gradient training algorithm (SCGTA). Two different neural networks models are constructed to predict the *sR* of a black hole.

Both optimization techniques are used to train ANN to check their approach to optimization, speed, memory requirements, and suitability for the prediction of the event horizon of SBH. The mathematical framework for LMTA is as follows:

$$E(w) = \frac{1}{2}\sum_{i=1}^{n}(R_s - F(M_B, w))^2, \tag{8}$$

is the objective function and here, the real radius of a black hole is denoted by R_s, the objective function is given by $E(w)$, the number of samples is denoted by n and $F(M_B, w)$ is the predicted radius of black hole by neural network with weight w and mass of black hole M_B. It is important to note that the gradient of error function of LMTA is taken with respect to the weights and defined as:

$$\overleftrightarrow{G} = J(w)^T r(w), \tag{9}$$

here, $\overleftrightarrow{G}$ is the gradient vector, $J(w)$ is the Jacobian matrix and $r(w)$ is the residual vector. In LMTA, the weights are adjusted at each iteration:

$$\Delta w = -\left(J(w)^T J(w) + \lambda I\right)^{-1} J(w)^T r(w), \tag{10}$$

here, the weight update, damping factor, and identity matrix are denoted by $\Delta w, \lambda,$ and I respectively. For the SCGTA, the gradient of error function is computed by using the following relation:

$$\overleftrightarrow{G}_k = \nabla E(w_k), \tag{11}$$

here $\overleftrightarrow{G}_k$ denoted the gradient vector for k-iteration and $\nabla E(w_k)$ is the gradient of an error function subject to weights at the current stage. For SCGTA, the weights are updated towards direction p_k and the search direction is carried as follows:

$$p_k = -\overleftrightarrow{G}_k + \beta_k p_{k-1}, \tag{12}$$

here, p_k offers search direction for k-iteration and β_k denoted the scalar coefficient and it responsible for direction p_k is conjugate to the last iteration p_{k-1}. Further, Eq. (7) is used to evaluate the Schwarzschild black hole radius where G is the gravitational constant, c is the speed of light, sR is the Schwarzschild radius, and M_B is the black hole mass. This equation is the foundation of the ANN model where the black hole mass is chosen as the input and the Schwarzschild radius is taken as an output for the ANN model. The ANN is programmed to learn the mapping from mass to radius by modifying its weights and biases during training. The model is trained on data points generated using the Schwarzschild formula, with input-output pairs representing a wide range of black hole masses and radii. The neural network learns to mimic the theoretical relationship between mass and radius, thereby accurately modeling the equation. Thus, the ANN input-output architecture directly mirrors the physical law regulating black hole dynamics, with the network acting as an approximation tool for estimating the Schwarzschild radius based on the black hole mass. This connection between the physical equation and the ANN design allows the model to take advantage of the known relationship while being flexible when generalizing to new data points.

4. Results and discussion

The Schwarzschild black hole, which symbolizes an uncharged, non-rotating black hole, is the solution to Einstein's field equations in general relativity [38–41]. There is an event horizon on the Schwarzschild black hole, which is a boundary that nothing can pass through, not even light. The Schwarzschild radius is the name given to this event horizon's radius as given in Eq. (7). Our interest is to develop a neural networks model to predict *sR* by using two different training functions, namely, the LMTA and the SCGTA.

Table 1, Tables 2, and Table. 3 represents the Event-I, Event-II, and Event-III sample values of mass and *sR*. It is important to note that the data samples provided in Tables 1–3 were generated by utilizing the relation of Schwarzschild black hole mass with radius. Such mathematical relation is given by Eq. (7). The black hole masses are provided both in kilograms and solar masses (for reference). The wide range of masses is considered to reach various magnitudes of BH, and the corresponding Schwarzschild radii are calculated. To be more specific, for each value, firstly, the mass of the black hole is converted to kilograms and then used to get the Schwarzschild radius. Here, we chose the mass of the black hole as an input of the neural networks model while *sR* is the selected output for the neural model. The standard unit of mass in astrophysics is solar masses and it is commonly used in the study of black holes. Eq. (7) is a well-established equation that holds the direct relation of Schwarzschild radius with the mass of a black hole. Therefore, use of solar masses we obtained the calculations without losing generality because the formula is flexible from solar masses to other units like those used as kilograms. This enables a straightforward input-output relationship in the neural network.

For both networks, by using LMTA and SCGTA, we selected 100 sample values of the mass of the black hole as an input while 100 values of radius were chosen as output. Min-Max normalization technique was used to scale the data between 0 and 1. The neural network is trained using 70 samples, which make up the majority of the data. From this data, the model discovers links and patterns. This entails applying optimization techniques like backpropagation to update the model's weights based on the error between anticipated and actual values of *sR*. 15 samples each are selected for validation and testing. 10 number of neurons are chosen in the hidden layer. Both the schematic of the ANN architecture and the flow chart of training algorithms are given in Fig. 1 (a) and (b). For neural network N_1, we adopted LMTA for training of model and for neural network N_2, SCGTA is used for the training. The graphical outcomes in this regard are given by Figs. 2–5. In detail, Fig. 2 (a) and (b) offer the performance for constructed neural networks models namely N_1 and N_2. To be more specific, Fig. 2(a) offers the

Table 1
Event-I numerical sample data for Schwarzschild black hole in terms of mass and radius.

Sample	Black hole mass (Solar masses)	Black hole mass (Kg)	Schwarzschild Radius (m)
1.	10101011.09	2.00909E+37	29798392819
2.	20202021.18	4.01818E+37	59596782688
3.	30303031.27	6.02727E+37	89395172558
4.	40404041.36	8.03636E+37	1.19194E+11
5.	50505051.45	1.00455E+38	1.48992E+11
6.	60606061.55	1.20545E+38	1.7879E+11
7.	70707071.64	1.40636E+38	2.08589E+11
8.	80808081.73	1.60727E+38	2.38387E+11
9.	90909091.82	1.80818E+38	2.68186E+11
10.	101010101.9	2.00909E+38	2.97984E+11

Table 2
Event-II numerical sample data for Schwarzschild black hole in terms of mass and radius.

Sample	Black hole mass (Solar masses	Black hole mass (Kg	Schwarzschild Radius (m)
1.	151515152.4	3.01364E+38	4.46976E+11
2.	161616162.5	3.21455E+38	4.76774E+11
3.	171717172.5	3.41545E+38	5.06573E+11
4.	181818182.6	3.61636E+38	5.36371E+11
5.	191919192.7	3.81727E+38	5.66169E+11
6.	202020202.8	4.01818E+38	5.95968E+11
7.	212121212.9	4.21909E+38	6.25766E+11
8.	2222222230	4.42000E+38	6.55565E+11
9.	232323233.1	4.62091E+38	6.85363E+11
10.	242424243.2	4.82182E+38	7.15161E+11

Table 3
Event-III numerical sample data for Schwarzschild black hole in terms of mass and radius.

Sample	Black hole mass (Solar masses)	Black hole mass (Kg	Schwarzschild Radius (m)
1.	252525253.3	5.02273E+38	7.44960E+11
2.	262626263.4	5.22364E+38	7.74758E+11
3.	272727273.5	5.42455E+38	8.04557E+11
4.	282828283.5	5.62545E+38	8.34355E+11
5.	292929293.6	5.82636E+38	8.64153E+11
6.	303030303.7	6.02727E+38	8.93952E+11
7.	313131313.8	6.22818E+38	9.2375E+11
8.	323232323.9	6.42909E+38	9.53548E+11
9.	3333333340	6.63000E+38	9.83347E+11
10.	343434344.1	6.83091E+38	1.01315E+12

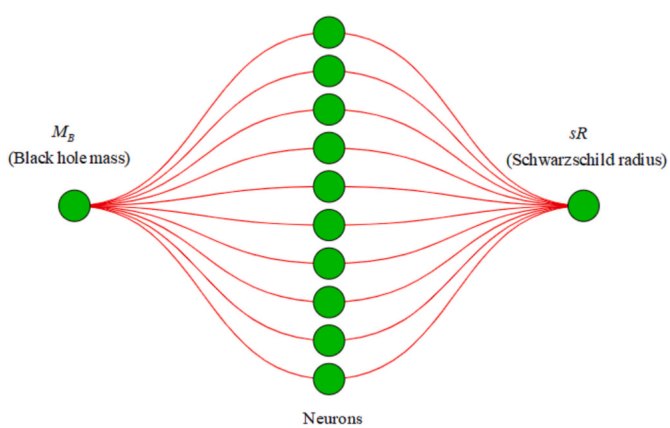

Fig. 1(a). Schematic of the ANN architecture.

performance of a constructed neural network model by using Levenberg-Marquardt as a training algorithm. During testing, it is observed that the error reduces as the neural network gains knowledge from the black hole mass training dataset. The model improves and the training error lowers over time when the weights are adjusted using the Levenberg-Marquardt optimization. Initially, the error may seem significant. Additionally, the validation error declines in a similar way as the training error, demonstrating the model's good generalization. After training, the model is tested on the black hole to determine its ultimate performance on unknown data. We noticed that the error for test data is ideally zero and consequently, the constructed model is successfully trained to forecast the radius of a black hole. Fig. 2(b) offers the performance of the neural networks model when it is trained by using the Scaled Conjugate Gradient algorithm. It appears that the model is not generalizing because it performs well on training data but poorly on validation data.

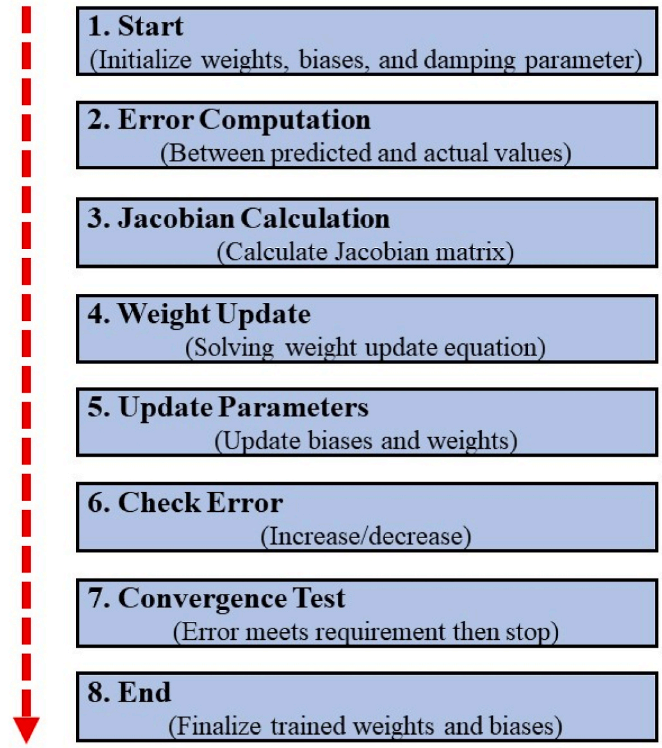

Fig. 1(b). Flow chart of training algorithms.

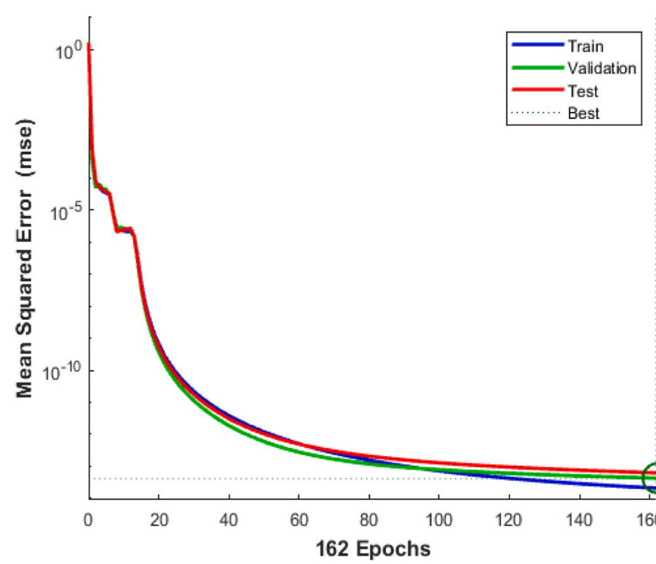

Fig. 2(a). Performance for neural network N_1.

Fig. 3(a) and (b) depicts the error histogram for neural networks models. In detail, Fig. 3(a) offers the pattern for networks being trained by the Levenberg-Marquardt algorithm. The majority of errors are relatively low, as indicated by the histogram's bars being centered around zero. This is encouraging since it shows that for the majority of the training data, the neural network model can predict the outputs with an accurate degree of accuracy. Consistent prediction accuracy is indicated by a low standard deviation. The model is balanced and does not show a discernible bias in any direction when both positive and negative errors are present.

Fig. 3(b) offers the error histogram of the neural networks model when the model is trained by using the Scaled Conjugate Gradient

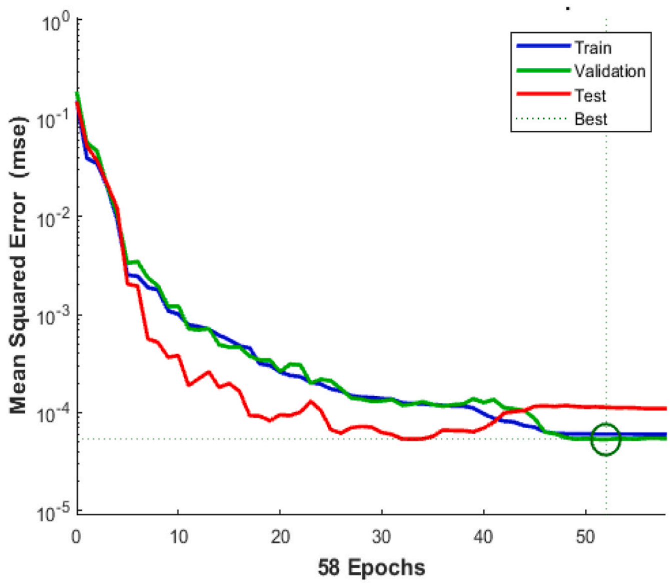

Fig. 2(b). Performance for neural network N_2.

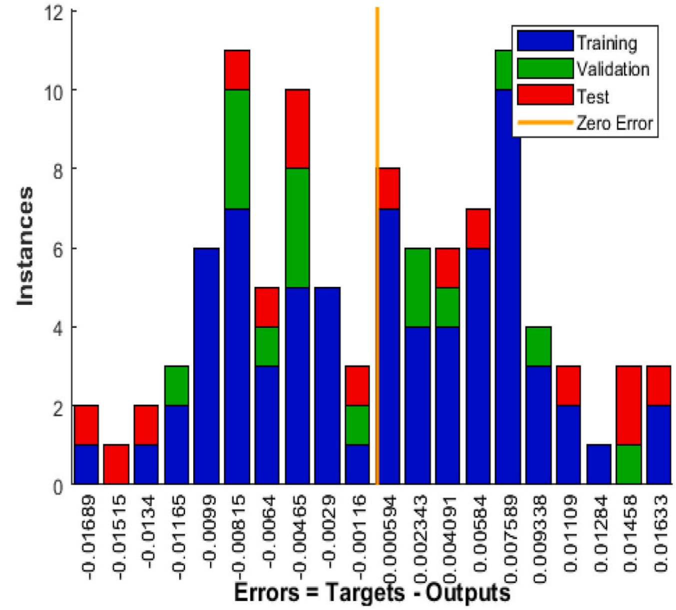

Fig. 3(b). Error histogram for neural network N_2.

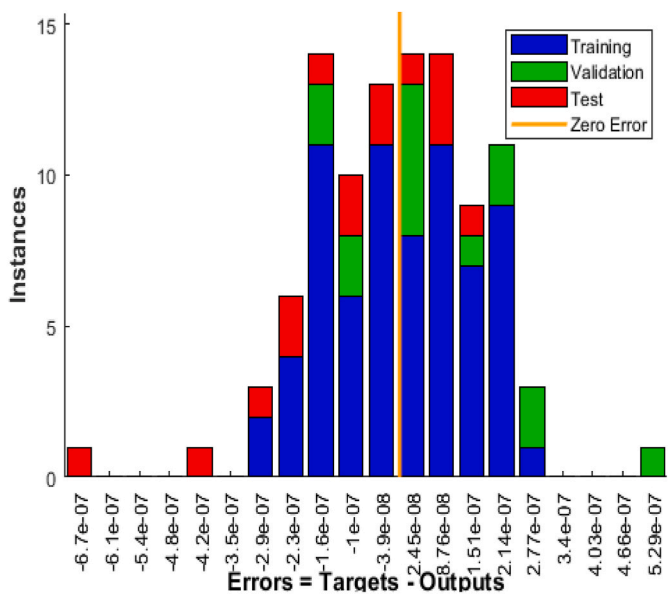

Fig. 3(a). Error histogram for neural network N_1.

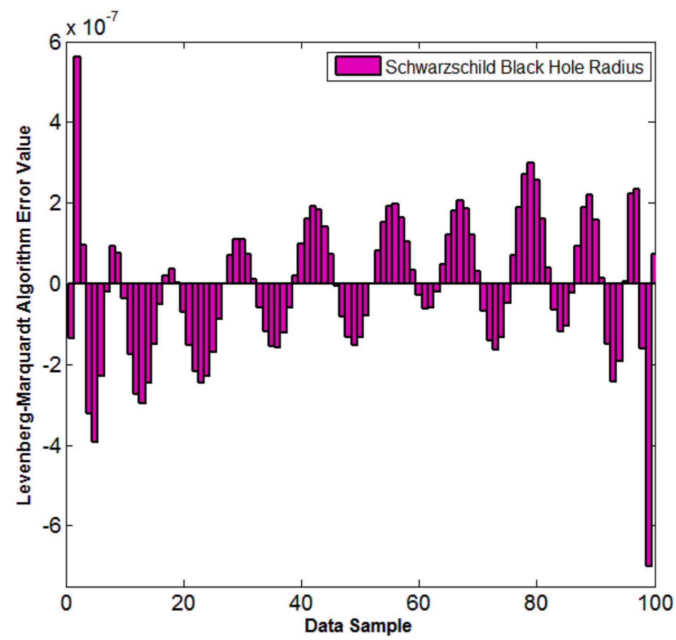

Fig. 4(a). Error value (EV) for neural network N_1.

algorithm. It is evident that the Levenberg-Marquardt used to train the neural networks model had a center peak that was closer to zero than the SCG histograms. In the SCG histogram, the distinction between validation and training error **is** more noticeable. Fig. 4(a) and (b) represent the error value (EV) between the original and ANN predicted data set of black hole radius. Particularly, Fig. 4(a) offers the EV for the data set being trained by the Levenberg-Mardquardt algorithm. One can see that the EV values are very low and hence ANN model is trained well by the use of LMTA. The error values produced by a neural network model employing the Conjugate Gradient algorithm are displayed in Fig. 4(b). The error data is precisely tied to sR over a range of data samples. The error range is approximately -0.02 to $+0.02$, indicating larger deviations than the prior EV of the model that was trained using LMTA, where the errors were probably more concentrated around zero. Some bars are significantly longer than others, which suggests that the error size occasionally rises. These spikes show that the model performs badly on specific data samples, most often as a result of noise, outliers, or inadequate conjugate gradient method convergence. In contrast to the earlier EV by a model trained by LMTA smoother error distributions, these spikes are more noticeable here.

Fig. 5(a) and (b) offer the comparison of ANN-predicted sR with the original values. In detail, a comparison between the real data set of Schwarzschild Black hole radius and the predictions of the ANN model trained with the LMTA method is shown in Fig. 5(a). The original data is closely followed by the predictions produced by the ANN model. It is clear from the close alignment that the ANN correctly identified the underlying relationship. An enlarged inset offers a better look, showing how the predictions of black hole radius (magenta) closely match the real data (blue circles), supporting the ANN model's accuracy. The predictions of the ANN, which was trained using the SCG approach, are compared with the original Schwarzschild black hole radius data, see Fig. 5(b). When compared to the earlier results obtained using the LMTA

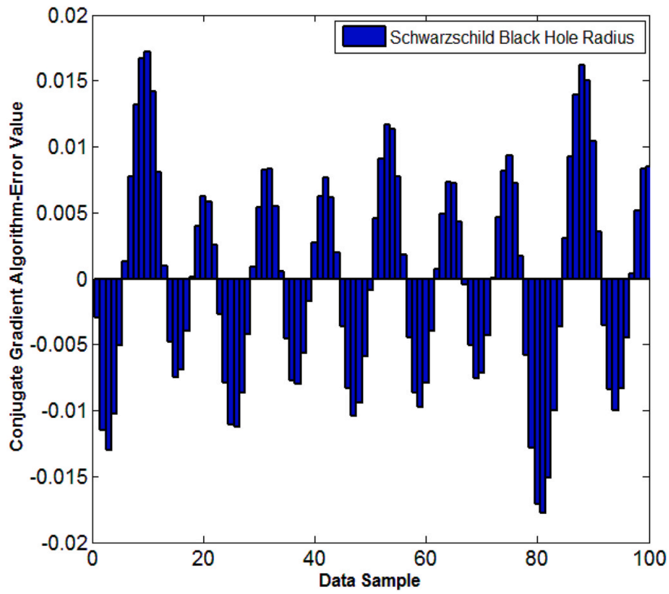

Fig. 4(b). Error value (EV) for neural network N_2.

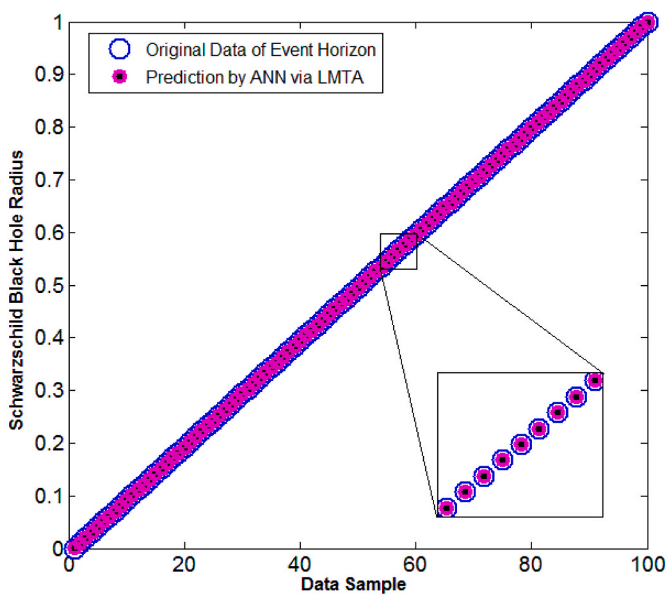

Fig. 5(a). Prediction of event horizon of black hole by neural network N_1.

method, the predictions seem to closely track the original data of the radius of the black hole, but with more noticeable variances also. The little to no divergence showed that the LMTA-based ANN was more accurate in forecasting the *sR*. Despite its continued relative accuracy, the SCG-trained ANN exhibits notable mismatches. For both neural networks, the comparison is constructed by using various performance parameters.

Table 4 is added in this direction. It compares two neural network models, LMTA and SCGTA, which were developed using different strategies. Coefficient of determination (R) and mean square error (MSE) are two examples of the metrics that are compared. The models were assessed for 70, 15, and 15 samples in the training, validation, and testing phases, respectively. When compared to the SCGTA model, the LMTA model obtains noticeably lower MSE values at every level, suggesting that it provides better precision and prediction accuracy at lower error rates. At every level, the LMTA model displays practically perfect correlation (very close to 1), meaning that the expected and actual

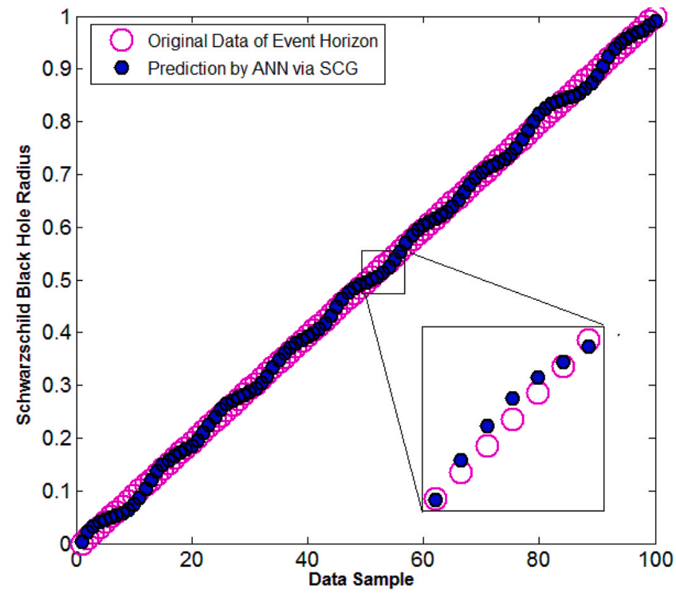

Fig. 5(b). Prediction of event horizon of black hole by neural network N_2.

Table 4
Comparative statistics for neural networks models to predict radius of black hole.

Neural networks model	Training function	Stages	Sample	MSE	R
N_1	LMTA	Training	70	2.09995E-14	9.99999E-1
		Validation	15	4.28200E-14	9.99999E-1
		Testing	15	6.28945E-14	9.99999E-1
N_2	SCGTA	Training	70	6.04138E-05	9.99642E-1
		Validation	15	5.32904E-05	9.99696E-1
		Testing	15	1.13613E-04	9.99366E-1

values of the black hole radius are virtually exactly in line with one another. The SCGTA model, on the other hand, has somewhat lower correlation values, which indicate a lower level of prediction accuracy. It is important to note that the synthetic nature of the dataset which is based on a well-defined physical formula (Eq. (7)) is the main cause of the low MSE reported for the LMTA model (10^{-14}). Such low error rates are achieved by the model in part because of the deterministic relationship between the black hole mass and radius and the lack of noise. Standard procedures for regression tasks were used to determine the convergence criteria and other hyperparameters for the LMTA and SCGTA. Given the problem's simplicity and the data's good behavior, the low MSE does not suggest overfitting and the training process for both algorithms were steady. Moreover, double precision was used to carefully regulate numerical precision guaranteeing the model's accuracy throughout training. It is important to note that real-world data with its inherent noise and complexities may produce different results even though the reported MSE represents the optimal performance on the dataset through Eq. (7). For the model evaluation, we found that the prediction error is minimal for black hole masses in the training data range which is consistent with the clearly defined mass-to-Schwarzschild radius relationship. Beyond this range, the models accuracy in extrapolation is constrained.

5. Limitations and implications

For the present study, the limitations and implications are itemized as follows.

5.1. Choice of data sample size

Each data point contains a black hole's mass and corresponding radius. A sample size of 100 data points was chosen as a reasonable compromise between attaining meaningful model training and maintaining computation manageability. When evaluating the model's ability to predict the *sR* based on mass, the selection of 100 data points is a reasonable place to start. Even though there may not seem to be many data points we think there are enough to demonstrate the fundamental connection between mass and Schwarzschild radius which is determined by a well-known physical equation (Eq. (7). Further, in machine learning the split; 70 (training):15 (validation):15 (testing) is a common practice [42–45] to make sure the model has enough data to learn from while also keeping a sufficient test set for assessment and a validation set for hyperparameter tuning. This split helps balance the models capacity to generalize across unseen data while preventing overfitting which is important given the limitations of data through Eq. (7).

5.2. Dataset representativeness

The model can learn the underlying scaling relationship between mass and *sR* because despite its small size the dataset is representative in that it covers a broad range of black hole masses from relatively smaller to larger black holes. However, the data points do not hold the intricacies and uncertainties found in actual observational data because they are produced using a theoretical formula, Eq. (7). As a result, although the dataset is helpful in showing how well the neural network model can learn the mass-radius relationship it might not accurately represent the noise and variations found in actual astrophysical datasets.

5.3. Simulated data vs observed data

There are various benefits to using simulated data as this work does including control over the input parameters (for example mass) as well as the capacity to produce a wide range of samples without requiring a large amount of observational data. However, using simulated data has drawbacks. The oversimplified formula used here does not account for the additional complexities that come with real astrophysical data such as measurement biases observational errors and the impact of outside variables. The use of simulated data has the consequence that although the model may be effective for the particular task of predicting the Schwarzschild radius given a black hole mass it may not generalize well to noisy real-world data. To improve the model account for measurement uncertainty and evaluate the models performance on actual black hole masses and radii, future research should ideally include observational data from telescopes or other astrophysical instruments.

5.4. Additional physical parameters

Incorporating other physical characteristics, like as the spin of a black hole, charge, or the existence of nearby matter like accretion disks and companion stars, may increase the model's forecasting capability in more complex scenarios. However, in the present work, we concentrated on the mass-radius relationship, which is a well-known feature of black hole physics.

5.5. Algorithm performance differences

Although the LMTA was more successful than the SCGTA for this well-defined dataset in the present study due to its faster convergence and lower error rates, its superior performance might indicate a possible overfitting risk, particularly when applied to real-world data that is inherently complex and noisy. SCGTA, however, offers more consistent convergence and might be more appropriate for handling datasets that are noisier or larger.

5.6. ANN vs traditional methods

In comparison to traditional methods [46–48] the artificial neural network approach offers significant advantages. The ANN can model complex, non-linear relationships between black hole mass and Schwarzschild radius without relying on fixed, deterministic formulas, making it more flexible in handling variations in observational data. Additionally, the ANN's ability to generalize well from small datasets, as demonstrated in our study, provides a notable advantage over methods that require larger datasets. While the ANN approach may struggle with extrapolation beyond the training range, it provides a robust alternative that can adapt to diverse and noisy data, offering improved performance and flexibility compared to traditional methods. However, it is worth noting that the model's physical interpretability may not match that of the analytical formula, and extrapolation beyond the trained data remains a challenge.

5.7. Future research

The comparative advantages and disadvantages of both training algorithms across a variety of increasingly complex datasets should be taken into account in future research even though LMTA performs well for the task at hand.

6. Conclusion

The artificial intelligence-based analysis is conducted to forecast the Schwarzschild black hole radius that corresponds to the event horizon of **a** black hole. Two various neural networks models are constructed by considering masses of black holes as an input. The first ANN model is developed by carrying the Levenberg-Marquardt algorithm as a training function while the second ANN model owns scaled conjugate gradient algorithm as a training function. The developed neural network models show notable variations in *sR* prediction when trained with two different techniques. For every evaluation parameter, such as mean squared error and correlation coefficient, the LMTA performed better. Both the graphical and tabular data showed that the LMTA-based model consistently generated more accurate predictions, with smaller errors and higher correlation values. However, despite its competence, the SCGTA model showed greater deviations, especially during the validation and testing phases, indicating a lesser capacity for generalization. Therefore, it is highly recommended to prefer LMTA for modeling astrophysical phenomena like black hole radius prediction.

CRediT authorship contribution statement

Khalil Ur Rehman: Writing – original draft, Validation, Software, Methodology, Investigation, Formal analysis, Data curation. **Wasfi Shatanawi:** Writing – original draft, Validation, Supervision, Software. **Weam G. Alharbi:** Investigation, Methodology, Validation, Writing – review & editing.

Declaration of competing interest

Authors have no conflict of Interest at this stage.

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Data availability

Data will be made available on request.

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