CS 3000: Algorithms & Data — Fall 2024
Homework 2

Due Friday Sep 27 at 11:59pm via Gradescope

Name:
Collaborators:

- Make sure to put your name on the first page. If you are using the LaTeX template we provided, then you can make sure it appears by filling in the `yourname` command.
- This homework is due Friday Sep 27 at 11:59pm via Gradescope. No late assignments will be accepted. Make sure to submit something before the deadline.
- Solutions must be typeset, preferrably in LaTeX. If you need to draw any diagrams, you may draw them by hand as long as they are embedded in the PDF. We recommend that you use the source file for this assignment to get started, which can be accessed from

this link:
https://www.overleaf.com/read/fpqrfpcbwkqp#711733

- We encourage you to work with your classmates on the homework problems, but also urge you to attempt all of the problems by yourself first. If you do collaborate, you must write all solutions by yourself, in your own words. Do not submit anything you cannot explain. Please list all your collaborators in your solution for each problem by filling in the `yourcollaborators` command.
- Finding solutions to homework problems on the web, or by asking students not enrolled in the class is strictly forbidden.

Problem 1. *(2 + 2 + 7 + 4 = 15 points) Lucky coincidence*

You are given an array of **distinct integers** $A[1,\ldots,n]$ **that is sorted in the increasing order**. You want to find out whether there is an index i such that $A[i] = i + 10$. For example, in the array $A = \{-3, 50, 60\}$, there is no i such that $A[i] = i + 10$ and the algorithm should return "no index found". On the other hand, for the array $A = \{0, 12, 17\}$ we have $A[2] = 2 + 10 = 12$ so the algorithm should return 2.

(a) What is the answer if $A[1] > 10$?

(b) What is the answer if $A[n] < n + 10$?

(c) Give an efficient divide-and-conquer algorithm for this problem. Hint: check if $A[\lfloor n/2 \rfloor] = \lfloor n/2 \rfloor + 10$. If not, try to recurse on a smaller problem.

(d) Write the recurrence of the running time and solve it.

Problem 2. *(15 points) Karatsuba Example*

Carry out Karatsuba's Algorithm to compute $73 \cdot 57$. What are the inputs for each recursive call, what does that recursive call return, and how do we compute the final product?

Problem 3. *($\frac{15}{4}+\frac{15}{4}+\frac{15}{4}+\frac{15}{4}$=15 Points) Asymptotics*

Decide whether each of the following statements are correct and give a proof for each part. (You may assume that all functions f, g, and h are non-negative for all inputs.)

1. If $f(n) = \Omega(g(n))$ and $g(n) = \Omega(f(n))$, then $f(n) = \Theta(g(n))$
2. $\max\{f(n), g(n)\} = \Theta(f(n) + g(n))$
3. If $f(n) = \Theta(g(n))$, then $2^{f(n)} = \Theta(2^{g(n)})$
4. $f(n) = \Theta(f(n/2))$

Problem 4. *(5+5+5 = 15 Points) Recurrences*

Solve the following recurrences and obtain tight asymptotic bounds on $T(n)$. You may ignore the floor operation while solving these recurrences.

(a) $T(n) = 9T(\lfloor n/3 \rfloor) + 9n$, $T(1) = 1$.

(b) $T(n) = 9T(\lfloor n/3 \rfloor) + 17n^2$, $T(1) = 1$.

(c) $T(n) = T(\lfloor n/3 \rfloor) + T(\lfloor 2n/3 \rfloor) + 6n$, $T(1) = 1$.