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On Bayesian approaches for real-time crack detection

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ABSTRACT: Fatigue is the most dangerous failure mode for mechanical components subject to alternating loads. Due to repeated loading and unloading, one or several cracks can be initiated and propagated through the cross section of the structure. Once a critical crack length is exceeded, the structure will catastrophically fail even for stress level much lower than the design stress limit. Non-destructive inspections may be performed at predetermined time intervals in order to detect the cracks. Alternatively, a continuous monitoring of the dynamic response of the structure can allow real-time cracks detection and corrective maintenance procedures might be taken in case the monitoring procedure identifies a crack.

In this paper, Bayesian model updating procedures is adopted for the detection of crack location and length on a suspension arm, normally used by automotive industry. Experimental data of the damaged structure (frequency response function) are simulated using a high-fidelity numerical model of the arm. A second, coarse-model represents the model to be updated where cracks of random locations and dimensions are introduced. The idea underlining the approach is to identify the most probable model consistent with the observations.

The likelihood is the key mathematical formulation to include the experimental knowledge in the updating of the probabilistic model. Different likelihood functions can be used based on different mathematical assumptions. In this work, the effect of different likelihood functions will be compared to verify the capability of Bayesian procedure for system health monitoring. The different likelihoods will be categorized according to the accuracy of the results and the efficiency of the numerical procedure.

1 INTRODUCTION

Failure for mechanical components subject to alternating loads may occur in several different ways, and failures due to fatigue are one of the most dangerous types. It is well known that cyclic loads can initiate cracks which propagate through the cross section of the structures. Once a critical crack length is exceeded, the structure will catastrophically and suddenly fail, even for stress level much lower than the design stress (Paris and Erdogan 1963). In particular, interactions may occur between the structural responses and cracks in components subject to high frequency dynamic excitations, leading to vibration-induced fatigue. Consequences may be a premature failure of the component or even worst the loss of the structures which rely on the component integrity.

Several strategies are accountable to prevent sudden failures; for instance, non-destructive inspections may be performed at predetermined time intervals, in order to detect the cracks (Faber et al. 1996); however failure can occur between inspections (Beaurepaire et al. 2012). Alternatively, a continuous monitoring of the dynamic response of the structure can allow for real-time crack detection and for a timely intervention with maintenance procedures (Chang et al. 2003). Repair actions are taken in case the monitoring procedure successfully identifies a crack which jeopardizes the structure.

In both cases, the procedure may fail in identifying a crack, leading to fatigue failure. Thus, an efficient crack detection procedure is required in order to avoid the loss of the structure. New emerging techniques are now available in the field of computational mechanics, which can be employed to assist in the monitoring of the health of the structures. These techniques modify some specific parameters in a numerical model to ensure a good agreement with the data, a so-called inverse problem. A computational framework well fitted for the solution of such inverse problems is the model updating (Fritzen et al. 1988).

In previous work (Beaurepaire et al. 2013), some of the authors implemented an updating framework for the detection of cracks in a suspension arm, as normally used by automotive industry. The mechanical behaviour of a device was characterized by collecting Frequency Response Functions (FRF) data at a specific location.

In this paper, the updating procedure for crack detection has been further investigated and extended, aiming at better understand its limitations and strengths. Different empirical likelihood expressions have been proposed in order to fit the procedure with different features of experimental evidence. The empirical formulation are compared and discussed in two representative cases; first, the detection of a single crack of known position and not-known length, secondly, the detection of a single crack of not-known position and not-known length.

Computational time is a an issue for real-time application of the procedure and it has been addressed. As a matter of fact, many model evaluations are required for the approach, thus a strategy for the parallelization of the simulations is provided. Moreover, adopting surrogate models such as Artificial Neural Networks (ANN) and Poly-Harmonic Spline (PHS) the computational time has been further reduced. The general purpose software Open Cossan (Patelli et al. 2014) has been employed in all the stages of the procedure.

The paper is structured as follows: Section 2 deals with the modelling of fracture in a Finite Element framework. Section 3 outlines the main concept of Bayesian model updating and the efficient simulation algorithm employed in the particular case of structure with cracks under dynamic excitation. A numerical example and results are presented and discussed in Section 4, and likelihood expressions are compared for the two considered cases. Finally, conclusions and remarks are drawn in Section 5.

2 MODELLING AND RELATED BACKGROUND

Finite Element (FE) analysis has become established as a powerful family of methods for the spatial approximation of systems of partial differential equations and variational problems. It has been used in a multitude of areas in the engineering field, e.g. in the analysis of mechanical components or structures. Nevertheless, the mechanical behaviour of structures may be altered if the elements are crossed by cracks. The cross section of the component is reduced, which causes a reduction of the stiffness. Moreover, the stress field is also modified in the vicinity of a crack.

Specific FE methods have been conceived to modelling efficiently the mechanical behaviour of structures containing cracks. The extended finite elements method (XFEM), first introduced by (Moës et al. 1999), has received considerable attention over the past few years. It consists of enriching the elements affected by a crack by introducing additional shape functions, which increases the number of degrees of freedom associated with the nodes. The stress field in these elements is then expressed using a combination of the standard and of the enrichment shape functions.

In case an element is crossed by a crack, a Heaviside function centred on the crack is introduced as an additional shape function. This step function accounts for the discontinuity of the displacements between the two lips of the crack. In case an element includes the crack tip, the corresponding nodes of the finite element model are enriched with specific shape functions. These functions correspond to the asymptotic displacement field at the vicinity of a crack tip, which can be determined analytically. For more details about the enrichment of the tip elements see (Moës et al. 1999). This allows capturing efficiently the displacement and strain fields near the crack tip, without excessive refinement of the mesh.

Details on the XFEM as implemented in the analysis here presented may be found in the works Zi and Belytschko 2003 and Abdelaziz and Hamouine 2008. However, mesh refinement in the vicinity of the crack tip may be necessary when the extended finite elements method is used, in spite of the enrichment of the nodes at the crack tip (Geniaut 2011). Nevertheless, the mesh does not have to be compatible with the crack, which considerably simplifies the re-meshing.

In case the behaviour of a cracked structure under dynamic excitation needs to be determined, the stiffness matrix may be computed using the XFEM, as stated above. The mass matrix is not modified by the presence of cracks, and no special action needs to be taken. The problem is subsequently solved using the standard procedure for linear dynamics: the modes and frequency of vibration are determined by solving the eigenvalue problem associated with the mass and stiffness matrices; and the FRF associated with any node of the finite element model are determined.

3 MODEL UPDATING PROCEDURE FOR CRACK DETECTION

3.1 *Bayesian updating of structural models*

A Bayesian model updating procedure is based on the very well-known Bayes theorem (Bayes 1763). The general formulation is the following:

$$P(\theta \mid D,I) = \frac{P(D \mid \theta,I)\,P(\theta \mid I)}{P(D \mid I)} \tag{1}$$

where θ represents any hypothesis to be tested, e.g., the value of the model parameters, D is the available data or observations, and I is the background information. Main terms can be identified in the Bayes theorem:

- $P(D \mid \theta, I)$ is the likelihood function of the data D;
- $P(\theta \mid I)$ is the prior Probability Density Function (PDF) of the parameters;
- $P(\theta \mid D, I)$ is the posterior PDF;
- $P(D \mid I)$ is a normalization factor ensuring that the posterior PDF integrates to 1.

The theorem introduces a way to update some a-priori knowledge on the parameters θ, by using data or observations D and conditional to some available information or hypothesis I.

Bayes law has been successfully applied in the updating of structural models see (Beck and Katafygiotis 1998) and (Katafygiotis and Beck 1998); in particular the Bayesian structural model updating has been successfully used to update large finite element models using experimental modal data (Goller et al. 2011). In a structural model updating framework, the initial knowledge about the unknown adjustable parameters, e.g. from prior expertise, is expressed through the prior PDF. A proper prior distribution can be a uniform distribution in the case when only a lower and upper bound of the parameter is known, or a Gaussian distribution when the mean and a relative error of the parameter is known.

The likelihood function gives a measure of the agreement between the available experimental data and the corresponding numerical model output. Particular care has to be taken in the definition of the likelihood, and the choice of likelihood depends on the type of data available, e.g. whether the data is a scalar or a vector quantity. Different likelihood leads to different accuracy and efficiency in the results of the updating procedure and should be selected with caution; as an example, the use of unsuitable likelihood function might cause that the model updating do not produce any relevant variation in the prior.

Finally, the posterior distribution expresses the updated knowledge about the parameters, providing information on which parameter ranges are more probable based on the initial knowledge and the experimental data.

3.2 *Transitional Markov-Chain Monte-Carlo*

The Bayesian updating expressed in equation 1 needs a normalizing factor, that can be very complex to obtain and computationally expensive. An effective stochastic simulation algorithm, called Transitional Markov Chain Monte-Carlo (Ching and Chen 2007), has been used in this analysis. This algorithm allows the generation of samples from the complex shaped unknown posterior distribution through an iterative approach. In this algorithm, m intermediate distributions P_i are introduced:

$$P_i \propto P(D \mid \theta, I)^{\beta_i} P(\theta \mid I) \tag{2}$$

where the contribution of the likelihood is scaled down by an exponent β_i, with $0 = \beta_0 < .. < \beta_i < .. < \beta_m = 1$, thus the first distribution is the prior PDF, and the last is the posterior. The value of these exponents β_i is automatically selected to ensure that the dispersion of the samples at each step meet a prescribed target. For additional information the reader is reminded to (Ching and Chen 2007). These intermediate distributions show a more gradual change in the shape from one step to the next when compared with the shape variation from the prior to the posterior.

In the first step, samples are generated from the prior PDF using direct Monte-Carlo. Then, sample from the P_{i+1} distribution are generated using Markov chains with the Metropolis-Hasting algorithm (Hastings 1970), starting from selected samples taken from the P_i distribution, and β_i is updated. This step is repeated until the distribution characterized by $\beta_i = 1$ is reached. By using the Metropolis-Hasting algorithm, samples are generated from the posterior PDF without the necessity of ever computing the normalization constant. By employing intermediate distributions, it is easier for the updating procedure to generate samples also from posterior showing very complex distribution, e.g., very peaked around a mean value or showing a multi-modal behaviour.

3.3 *Model updating for crack detection and likelihood expression*

In the FE model, the cracks are modelled using XFEM. Experimental data from the reference structure are taken into account in the form of Frequency Response Functions. Knowing that cracks will develop most probably in certain locations, characterized by high concentration of stresses, cracks are inserted in these specific positions assuming their lengths are random parameters.

Within the model updating framework, the cracks present in the damaged structure are seen as uncertain model properties. The prior will use uniform distribution for the crack parameters, allowing the possibility of crack in any stress concentration point and with any possible physically acceptable length, i.e. compatible with geometric constraints and material proprieties.

The likelihood is the key mathematical component of any Bayesian updating procedure. Within the proposed crack detection framework, synthetic experimental FRFs are compared with the numerical FRF of the numerical model. Within the case study, three empirical likelihood formulations are proposed and used to compare the experimental data with the numerical information. Expressions are discussed on the basis of the accuracy of the results, i.e. the ability in detecting true cracks positions and lengths.

The likelihoods have been be expressed as:

$$P(D\mid\theta,I)=\prod_{k=1}^{N_e}P\left(x_k^e;\theta\right) \tag{3}$$

or, equivalently, in the form of the log-likelihood:

$$P(D\mid\theta,I)=\sum_{k=1}^{N_e}\log\left(P\left(x_k^e;\theta\right)\right) \tag{4}$$

where x_k^e represent the kth experimental evidence, N_e is the number of available experimental data and θ is the vector of random crack lengths. The term $P\left(x_k^e;\theta\right)$ which include the experimental evidence have been assumed as exponentially distributed.

Three heuristic formulations have been defined as follows:

$$P\left(x_k^e;\theta\right)\propto\exp\left(\frac{\delta_k}{MSE_{pks}[h(\theta)-h_e^k]}\right) \tag{5}$$

$$P\left(x_k^e;\theta\right)\propto\exp\left(\frac{\delta_k}{MSE_{low}[h(\theta)-h_e^k]}\right) \tag{6}$$

$$P\left(x_k^e;\theta\right)\propto\exp\left(\frac{\delta_k}{MSE_{all}[h(\theta)-h_e^k]}\right) \tag{7}$$

where h_e^k is the experimental FRF k, $h(\theta)$ represents the numerical FRF, δ_k is the variance of the Means Square Errors (MSE) for the experiment k. The Means Square Errors have been obtained considering different frequency ranges, e.g. MSE_{all} is computed between experimental FRF and numerical FRF over the entire frequency domain, MSE_{low} is computed considering lower frequencies and MSE_{pks} is obtained around the main resonance peaks. The proposed expressions and the selection of the frequency ranges were defined on empirical basis therefore details will be discussed in the case study, Section 4.

After the updating procedure, the posterior distributions provide a qualitative indication of the crack length and positions, i.e. it will concentrate around the unknown length and position of the crack with most similar response if compared to the experimental observations.

It worth remark that even if the procedure in generally applicable to detect cracks, the component analysed have a specific FRF, which therefore limit the consideration on the validity of the defined likelihoods to the specific mechanical device in exam.

4 NUMERICAL EXAMPLES

This numerical example is similar to those used in the automotive industry (Mrzyglod and Zielinski 2006) and it has been recently applied (Beaurepaire et al. 2013) to detect cracks in a suspension arm, shown in Figure 1. It can freely rotate along the axis indicated by the dashed line; the suspension spring and the wheel structure are connected at the location indicated by "S". The stress concentration points, and candidate crack locations, are indicated in the figure by the numbers 1 to 6.

In this case study, "simulated" experimental data are generated using a Finite Element (FE) model. The software used to construct the model and in the analysis is Code Aster (Geniaut 2011). A crack with fixed length is inserted in one of the candidate position, and the reference FRF is computed at the position indicated by "O". Both the FRF in direction X and Y are considered, while no FRF is obtained in the direction Z since the structure is not constrained in that direction. Figure 2 display the FRFs in the directions X and Y when a 5 mm length crack is considered. Just three out of six possible position are shown for graphical reasons.

The crack lengths are considered as uncertain parameters, and are modelled using uniformly distributed random variables. Since the crack is physically constrained to not touch the flanges of the arm, a maximum crack length of 5 mm is assigned to the cracks in position 1 and 2, while the length is limited to 10 mm for the cracks in positions 3 to 6. The sampled values of the random variables are inserted into the FE model by using the ASCII file injection routine provided by OpenCossan (Patelli et al. 2014), providing a deterministic FRF. Additionally, the

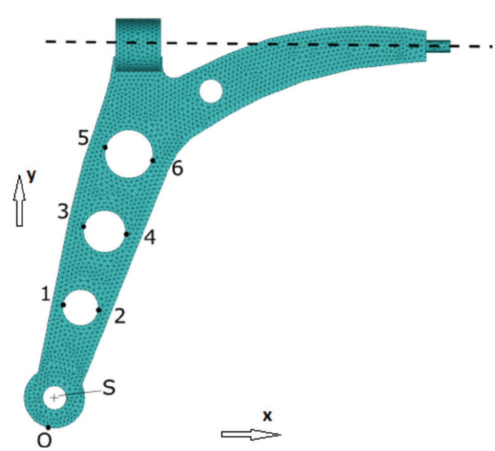


Figure 1. The suspension arm FE model with indicated the six possible crack positions and the FRF measurement point.

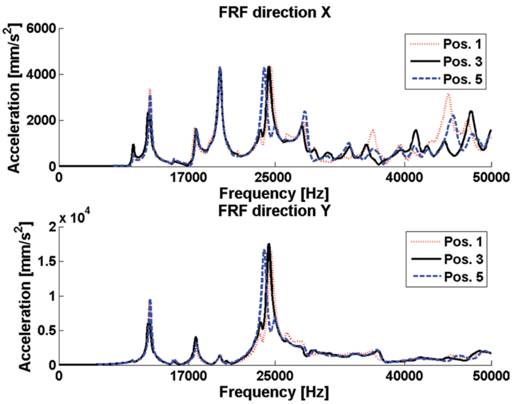


Figure 2. Frequency response functions of the high fidelity FE model. A single crack of 5 mm length is inserted in each of the stress concentration points.

simulations run in parallel on a computer cluster and surrogate model are used, allowing further reduction of the overall computational time.

The goodness of the procedure in detecting cracks has been tested for two updating cases:

- detection of single crack having known position and unknown length
- detection of a single crack of unknown position and length.

In both the cases analysis have been carried out by using empirical likelihood expressions. Results compared and discussed point out positive and negative features of the different formulations.

The likelihoods expressions and frequency ranges have been selected after considerations on the computational inaccuracy affecting numerical FRFs, especially in the high frequencies domain. It can be argued that likelihoods defined as in Equations 5–6 might detect cracks with higher precision if compared to Equation 7 which computes *MSE* including FRF values in the high frequency range. MSE_{low} in Equation 6 has been computed over the low frequency range from 0 Hz to 1.7×10^4 Hz while MSE_{all} is computed over the entire domain (from 0 Hz to 5×10^4 Hz). Furthermore, it has been noticed that the FRFs around the main resonance peaks (e.g. around 2.4×10^4 Hz in Fig. 2) appear to be particularly variable with respect to the crack positions and lengths. Hence, MSE_{pks} has been computed around the main resonance peaks (2.4×10^4 Hz) to verify if it is a good indicator to distinguish different crack lengths and locations.

4.1 *Surrogate model calibration and selection*

The Bayesian model updating procedure is computationally expensive, thus surrogate models have been adopted to reduce the computational time. Among the different surrogate models, Artificial Neural Networks (ANN) and Poly-Harmonic Spline (PHS) have been tested and the best model has been selected.

The classical architecture type for Artificial Neural Network (Chojaczyk et al. 2015) consists of one input layer, one or more hidden layers and one output layer. Each layer employs several neurons and each neuron in a layer is connected to the neurons in the adjacent layer with different weights. Poly-harmonic Spline is popular tool for model interpolation and has been considered as alternative surrogate model to be compared with the ANN due to their good proprieties, see e.g. in (Madych and Nelson 1990).

The coefficient of determination R^2 is often used to evaluate the quality of the regression model and have been adopted to choose the most suitable meta-model to use in the procedure. This coefficient can be expressed as follows:

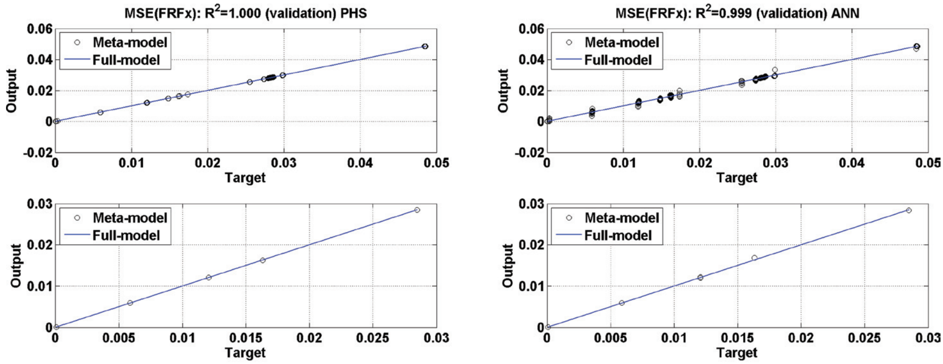
$$R^2 = 1 - \frac{\sum_i (y_i - \hat{y}_i)^2}{\sum_i (y_i - \bar{y})^2} \tag{8}$$

where y_i is the *ith* mean square error, $\hat{y}_i$ is the mean square error predicted by the surrogate model, $\bar{y}$ is the average of the mean square errors from the FE analysis.

It goes without saying that R^2 values have to be fairly compared between the two considered surrogates, therefore the coefficient have to be compared on the validation set. Otherwise the R^2 for the PHS, computed using the calibration set, will result by definition equal to 1.

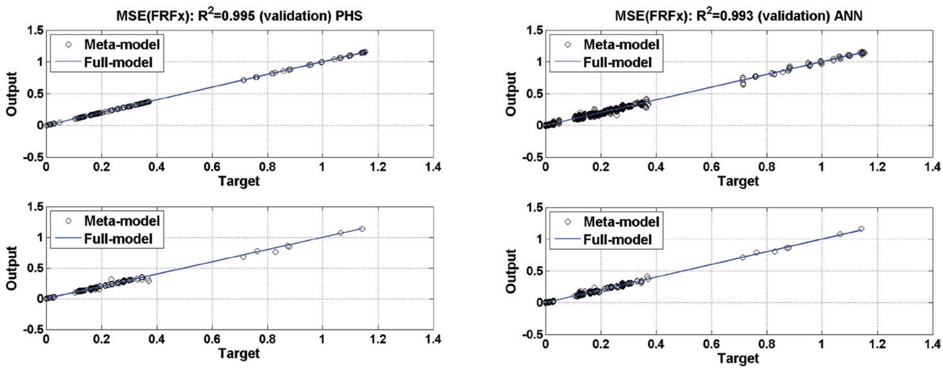
Input data for the surrogate models is the vector θ of simulated cracks, outputs are *MSE* between the simulated FRFs and a preselected synthetic experimental FRF. Both direction X and Y are considered, hence *MSE* in X and Y directions are the outputs of the surrogate models. In order to reduce complexity and limit the number of outputs, the entire FRF has not been considered as output for the surrogates model.

The proposed ANN layout consists of hidden layer with 11 nodes for the first two layers and 13 for the last layer. The described architecture have been selected based on precedent results and in order to capture the highly non-linear behaviours of the outputs. Cubic PHS has been considered for model the behaviour of the *MSE* in x and y directions. The models have both been calibrated using 90% of the data set and validated using the remaining 10%. Examples of results are proposed in Figures 3–4 where mean square errors between experimental and simulated FRFs in X direction are considered. The targets, in the X-axis, are MSE obtained using the



(a) PHS (b) ANN

Figure 3. Regression plots of meta-models for a single crack with fixed known position.



(a) PHS (b) ANN

Figure 4. Regression plots of meta-models for a single crack of unknown position and length.

FE model and the outputs, in the Y-axis, represent the output of the meta-model. The sub-plots in the top of the figures display results for the model calibration, while in the bottom are displayed results for the validation set. Experimental cracks is considered in position 6 of length 8.07 mm.

In the first case 700 single crack lengths with known position (crack in position 6 in Fig. 1) have been considered. Figures 3a–3b present linear regression results for the first analysed case. Similarly, Figures 4a–4b shows linear regression plots and comparisons for the second analysed case. In the second case, 2800 single cracks of random length with not-known position are the input for the surrogate model.

The R^2 values for the validation data show that PHS is the best candidate surrogate model for both the considered detection cases. Hence, PHS appear to be more suitable than ANN to mimic the *MSE* behaviour. This can be explained if considered the "step" behaviour of the outputs along the crack length. In order to reproduce this non-linear behaviour, an ANN need to be built with several nodes and several hidden layers; on the other hand, a PHS works well in capturing it because it is obliged to pass through the support points and better predict non-linearly in-between the supports.

4.1.1 *Single crack, known position*

Bayesian updating procedure has been used first to detect of single crack length which has a known position (position 6 in Fig. 1). The procedure starts by first selecting prior distribution for the crack length $P(\theta \mid I) \sim U[0,10]$ and sample number equal to 1500. Number of samples has been selected based on previous works, however future optimization of the initial setting might be considered to reduce the computational time. Three different synthetic experimental FRFs have been selected, corresponding to cracks of "short" length (2.4 mm) "medium" length (4.53 mm) and "long" length (8.07 mm). Updating procedure has been repeated to include the empirical likelihoods defined in Section 3.3. Results have been qualitatively ranked based on the accuracy of the posterior distributions; suitability of the likelihoods is discussed.

In Figure 5 shows posterior distributions for different likelihoods and for a crack lengths to be detected equal to 2.4 mm. The posterior distributions result peaked around the true crack length. The crack length is detected with high precision in this case; mean values for the distribution are almost equal to the true crack lengths 2.4 mm plotted in dotted line. Among the three analysed likelihoods, the one computed using Equation 5 result in a lower variance compared to the others, as can be qualitatively observed in the top sub-plot of Figure 5. Similar behaviour is observable for the other selected crack lengths.

Nevertheless, considered the final aim of the updating, the differences are of minor relevance. More specifically, in this stage of comparison which is intended to be mainly qualitative, detecting a crack with uncertainty ±0.06 mm or ±0.04 mm has been considered a minor difference. The crack length is detected with high precision in case of single crack with known position, and all the likelihood expressions seems well-suited to be used within the detection framework.

4.1.2 *Single crack, unknown position*

The procedure presented in 4.1.1 has been extended for detect both the length and position of a crack. The procedure, has been tested considering likelihoods described in Section 3.3, and assuming uniform prior distributions for cracks in position 3 to 6 ($P(\theta|I) \sim U[0,10]$) and in position 1 and 2 ($P(\theta|I) \sim U[0,5]$). The synthetic experimental FRF is the one of a crack in position six of length 8.07 mm.

Figures 6, 7 and 8 display the posterior distributions obtained by using likelihoods computed as in equations 5, 6 and 7 respectively. Results indicate correct detection of the crack within the true length range and the true position.

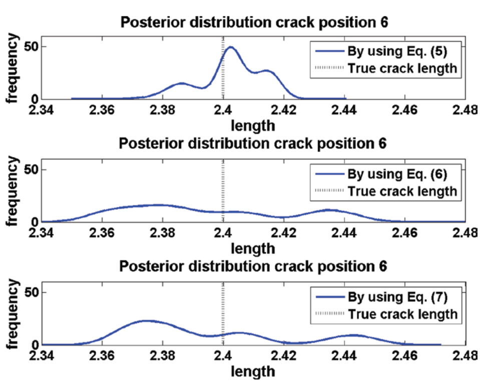


Figure 5. Posterior distribution of "short" crack length in position 6, different likelihoods expressions.

Computing likelihood considering main resonance frequency, as in Figure 6 crack is detected within the length interval [7–10 mm]. Similar results, displayed in Figures 7 and 8, are obtained by using different likelihood expressions. The results show higher uncertainty if compared to the results in Section 4.1.1.

The posterior distributions for cracks in positions from 1 to 5 result similar to the assumed prior uniform distributions. This mean that none of the simulated cracks length in positions form 1 to 5 can be fairly associated to the experimental evidence provided. However, results obtained by using Equation 6 wrongly detect a crack in position 4 of approximate length of 5 mm. Explanation might be found considering the

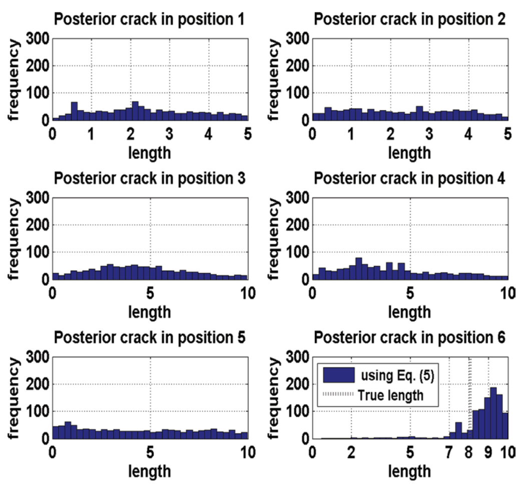


Figure 6. Likelihood computed with MSE_{pks}.

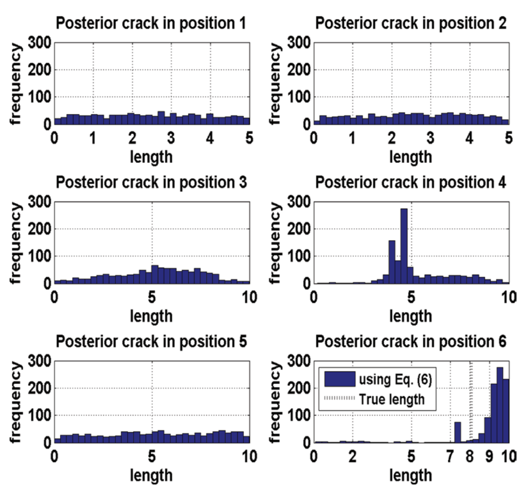


Figure 7. Likelihood computed with MSE_{low}.

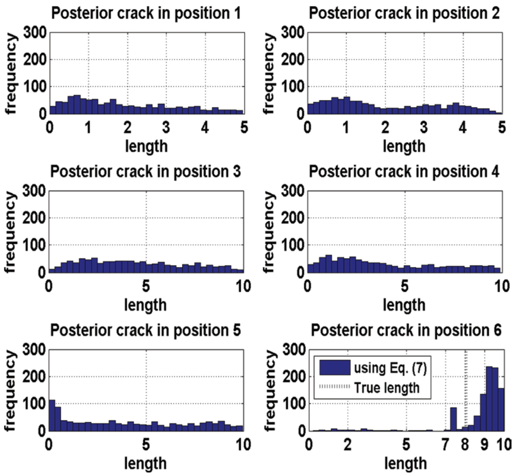


Figure 8. Likelihood computed with MSE_{all}.

FRF behaviour in the low frequencies range. The updating procedure may have revealed similarity between experimental data of "long" crack in position 6 and simulated FRF for cracks of mean length in position 4.

5 REMARKS AND CONCLUSIONS

A Bayesian model updating procedure for cracks detection has been presented and it has been applied to detect cracks in a suspension arm. Reference dynamic data from vibration analysis was used as target for the updating. The effects of different likelihood expressions and different experimental data on the crack detection strategy have been analysed. The procedure has been tested first to detect a single crack with unknown length but known position, the result comparison did not suggest major differences between the likelihood formulas. Nevertheless, the second case point out the limitations of some of them. It is possibly due to similarity in the FRF for different cracks or shortcoming in the computational accuracy. In both the analysed cases, the crack was detect correctly around the true length and position. Future development and additional research will be taken by using real experimental data to further validate and expand the proposed approach.

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