ENEL102, fall term 2017

Assignment 7

Matlab Symbolic Math

Due date: Dec 11

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This assignment will be based on questions taken from Chapter 11 of the Gilat textbook. Suggest you review this chapter before answering the questions. Fill in the following template with your answers using Matlab plots and screen shots as necessary. Then submit your Word document on D2L.

……

Q1 Consider the polynomial equation of

$$y=(x+1)^3(x+2)(x+3)$$

expand this equation symbolically. Then use factor() the result to ensure you get the same equation back.

(Matlab input)

```
syms x
symbolic = expand(((x+1)^3)*(x+2)*(x+3));
disp(symbolic)
f = factor(symbolic);
disp(f);
```

(Matlab Response)

Expanded: x^5 + 8*x^4 + 24*x^3 + 34*x^2 + 23*x + 6

Factored: [x + 3, x + 2, x + 1, x + 1, x + 1]

Q2 Find the coefficient of x^4 of the polynomial

$$y=(x+1)^3(x+2)(x+3)$$

hint use coeffs()

(Matlab input)

```
syms x
[coefficients,x] = coeffs(((x+1)^3)*(x+2)*(x+3), x);
disp(coefficients);
```

(Matlab Response)

[1, 8, 24, 34, 23, 6]

………

Q3 Next consider the polynomial equation of

$$y=(x+1)^2(x+2t)^3(x+3)+t^2$$

Collect the coefficients of the variable t. Give the coefficient of t^2.

(Matlab input)

```
syms t
syms x
[coefficients,t] = coeffs(((x+1)^2)*((x+2*t)^3)*(x+3)+t, t);
disp(coefficients);
disp(t);
a = coefficients(3);
disp(a);
```

(Matlab Response)

[8*(x + 1)^2*(x + 3), 12*x*(x + 1)^2*(x + 3) + 1, 6*x^2*(x + 1)^2*(x + 3), x^3*(x + 1)^2*(x + 3)]

[t^3, t^2, t, 1]

6*x^2*(x + 1)^2*(x + 3)

………

Q4 Find the inverse of $A=\begin{bmatrix} a & 1 \\ 1 & a \end{bmatrix}$ using the symbolic toolbox.

(Matlab input)

```
syms a
A = [a, 1; 1, a];
inverse = inv(A);
disp(inverse);
```

(Matlab Response)

[a/(a^2 - 1), -1/(a^2 - 1)]

[-1/(a^2 - 1), a/(a^2 - 1)]

……

Q5 The probability density function of a normal random variable, x, with a mean of a and a variance of v is given as

$$p(x) = \frac{1}{\sqrt{2\pi v}} \exp\left(-\frac{(x-a)^2}{2v} \right)$$

The probability that x is in the region between a lower bound of A and an upper bound of B is then

$$\Pr\left(A \le x \le B\right) = \int_A^B p(x)dx$$

Determine a symbolic expression of this probability $\Pr\left(A \le x \le B\right)$ and interpret the results.

(Matlab input)

```
syms x
syms a
syms v
syms A
syms B

expression = int((1/(sqrt(2*3.14*v)))*exp(-((x-a)^2)/(2*v)),x,B,A);
disp(expression);
```

(Matlab Response)

(2^(1/2)*pi^(1/2)*(erf((2^(1/2)*(A - a)*(1/v)^(1/2))/2) - erf((2^(1/2)*(B - a)*(1/v)^(1/2))/2)))/(2*((157*v)/25)^(1/2)*(1/v)^(1/2))

Q6 In certain problems we are interested in determining the change in $\Pr(x)$ as the mean of a is varied. As such we are interested in determining $\frac{d\Pr(A \le x \le B)}{da}$. Use Matlab to determine this derivative for the probability function, $\Pr(A \le x \le B)$, determined in **Q5**. Then solve this derivative directly by using calculus and show that Matlab's answer is correct.

(Matlab input)

```
syms x
syms a
syms v
syms A
syms B

Matlab_ans = int((1/(sqrt(3.14*v*2)))*exp(-((x-a)^2)/(v*2)),x,B,A);
disp(Matlab_ans);
Using_Calc = diff(Matlab_ans, a);
disp(Using_Calc);
```

(Matlab output)

(2^(1/2)*pi^(1/2)*(erf((2^(1/2)*(A - a)*(1/v)^(1/2))/2) - erf((2^(1/2)*(B - a)*(1/v)^(1/2))/2)))/(2*((157*v)/25)^(1/2)*(1/v)^(1/2))

Answer based on calculus

-(2^(1/2)*pi^(1/2)*((2^(1/2)*exp(-(A - a)^2/(2*v))*(1/v)^(1/2))/pi^(1/2) - (2^(1/2)*exp(-(B - a)^2/(2*v))*(1/v)^(1/2))/pi^(1/2)))/(2*((157*v)/25)^(1/2)*(1/v)^(1/2))

Q7 Solve the differential equation given as

$$\frac{dx}{dt} = x \quad x(t)\big|_{t=0} = 1$$

using the symbolic toolbox using dsolve() and interpret the output.

(Matlab input)

(Matlab output)

Q8 A transfer function of a system is given as

$$H(s) = \frac{1}{s+1}$$

Find the step response and unit slope ramp response based on the symbolic inverse Laplace transform. That is use ilaplace(). Plot the responses with ezplot().

(Matlab input)

(Matlab output)

……

Q9 The probability density function of a normal random variable, x, with a mean of 0 and a variance of 1 is given as

$$p(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2} \right)$$

Suppose we want to approximate p(x) as a polynomial around the neighborhood of x=1. Use a fourth order Taylor expansion to do this and then compare the polynomial expansion with p(x) in a plot.

hint: Note that Matlab regards the 'fourth order' expansion in terms of the error. Hence the error will go approximately as something times x^4 plus additional higher order terms and a bias offset. Hence for this question, use 'order' of 4 in taylor() and get a polynomial expression output that is a third order polynomial.

(Matlab input)

(Matlab Response)

Q10 Generalize the Taylor expansion problem of **Q9** to the more general expression of the Gaussian probability function as $p(x)=\frac{1}{\sqrt{2\pi v}}\exp\left(-\frac{(x-u)^2}{2v}\right)$

where v is the variance and u is the mean. This time do a 4^{th} order Taylor expansion around the expansion point of x=1. Specifically find the coefficient of x^3. (hint use coeffs())

(Matlab input)

(Matlab Response)

Q11 Take the coefficient of x^3 as calculated in Q10 and plot it for v=2 and u in the range of -2<u<3.

(Matlab input)

(Matlab Response)