贪心策略篇: 活动选择问题

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北京航空航天大学 计算机学院

中国大学MOOC北航《算法设计与分析》

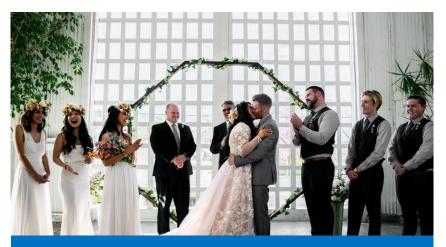


• 会场出租



公司年会: 10:00~19:00





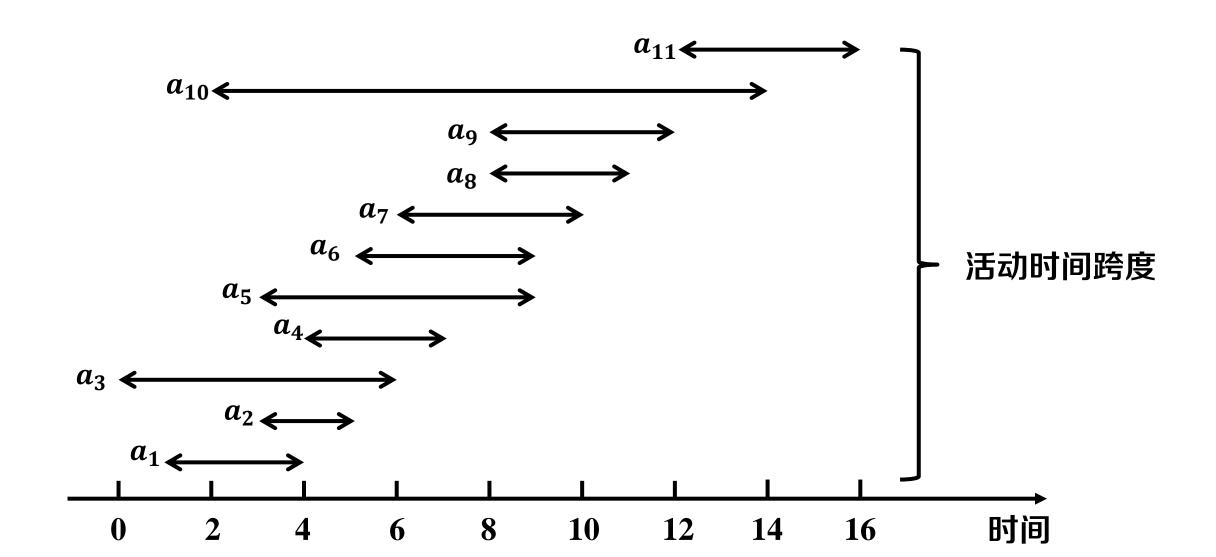
婚礼宴请: 11:00~14:00



学术研讨: 14:00~16:00

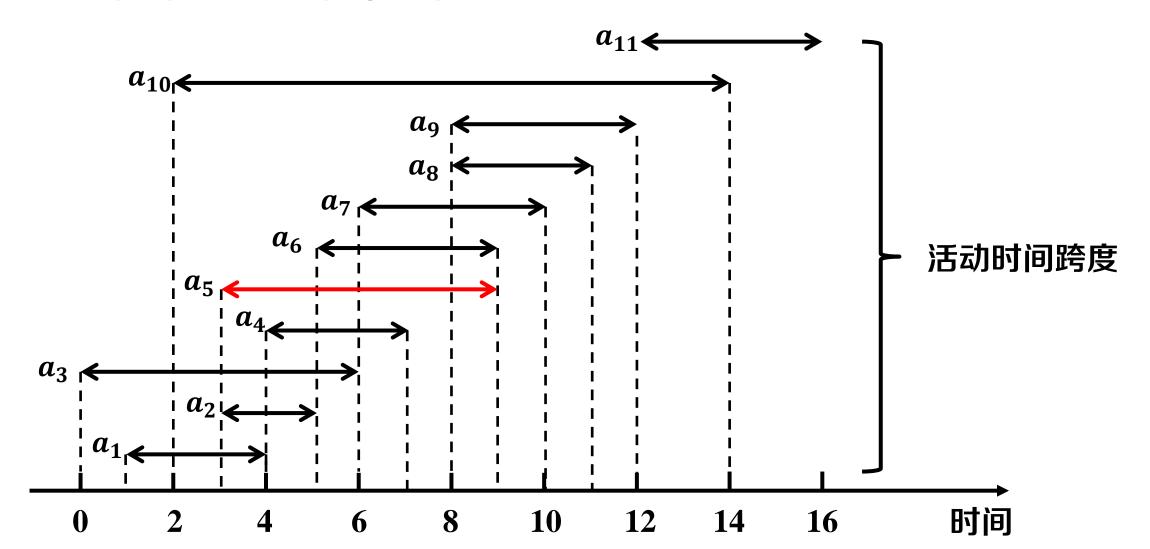


• 会场出租



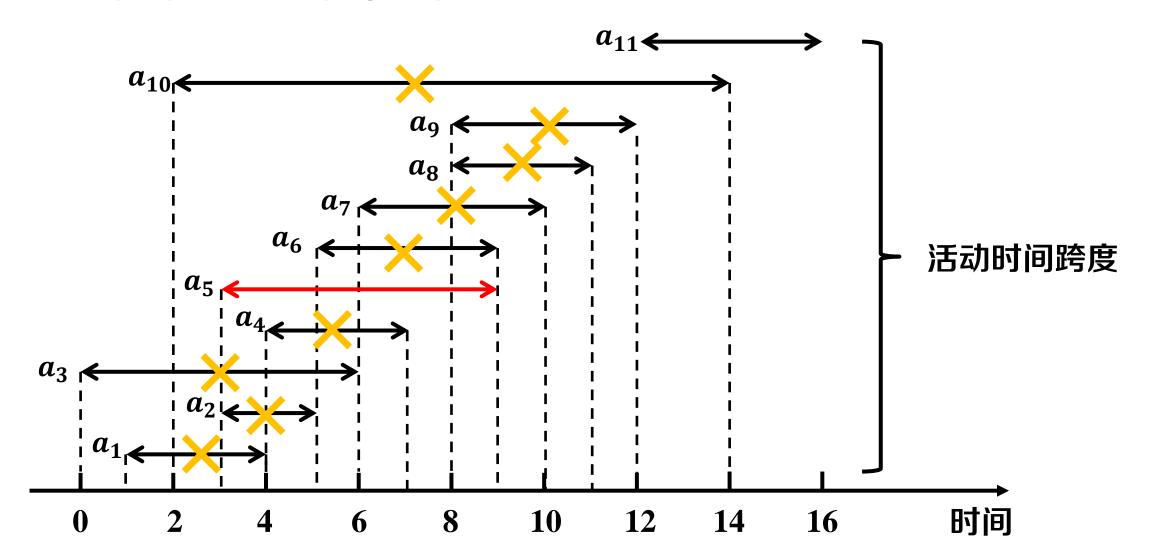


- 会场出租
 - 选择出租的活动时间不能冲突



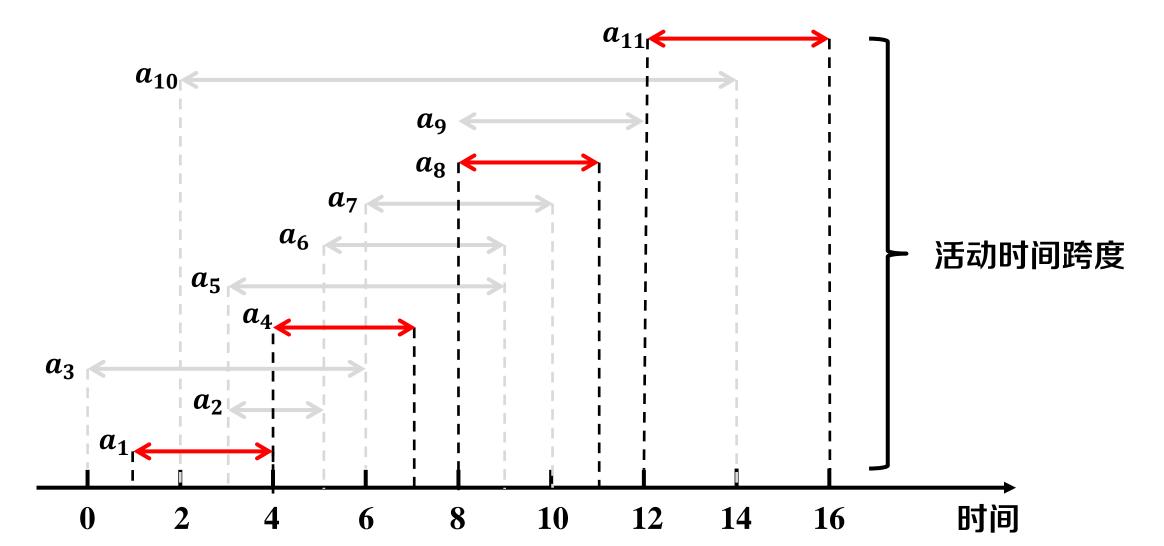


- 会场出租
 - 选择出租的活动时间不能冲突





- 会场出租
 - 选择出租的活动时间不能冲突,怎样选择才能选更多的活动?



问题定义



活动选择问题

Activity Selection Problem

输入

- n个活动组成的集合 $S = \{a_1, a_2, ..., a_n\}$
- 每个活动 a_i 的开始时间 s_i 和结束时间 f_i 输出

• 找出活动集合S的子集S',令

 $\max |S'|$

 $s.t. \forall a_i, a_j \in S', s_i \geq f_j$ 或 $s_j \geq f_i$

优化目标: 最大化选择活动个数



活动选择问题

Activity Selection Problem

输入

- n个活动组成的集合 $S = \{a_1, a_2, ..., a_n\}$
- 每个活动 a_i 的开始时间 s_i 和结束时间 f_i

输出

• 找出活动集合S的子集S',令

 $\max |S'|$

$$s.t. \forall a_i, a_j \in S', s_i \geq f_j$$
或 $s_j \geq f_i$

优化目标: 最大化选择活动个数



$$S_j$$
 a_j f_j a_i f_i S_i a_i f_j A_j A_j

贪心策略:一般步骤



提出贪心策略

观察问题特征,构造贪心选择



证明策略正确

假设最优方案,通过替换证明



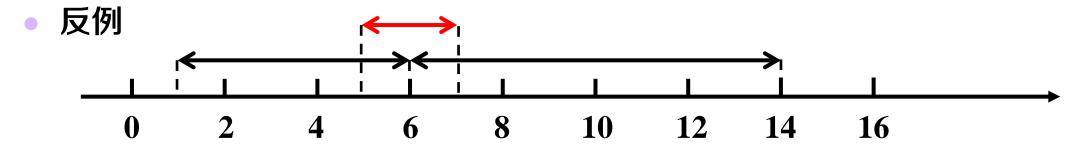
• 策略1: 最短活动优先

• 策略2: 最早开始活动优先

• 策略3: 最早结束活动优先



• 策略1: 最短活动优先

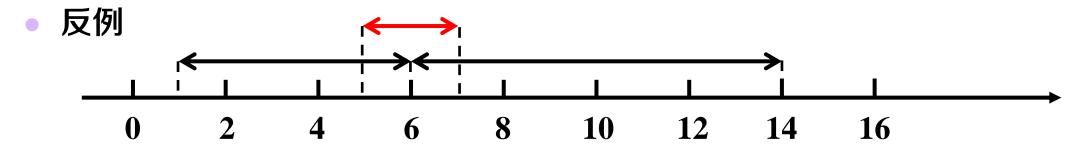


• 策略2: 最早开始活动优先

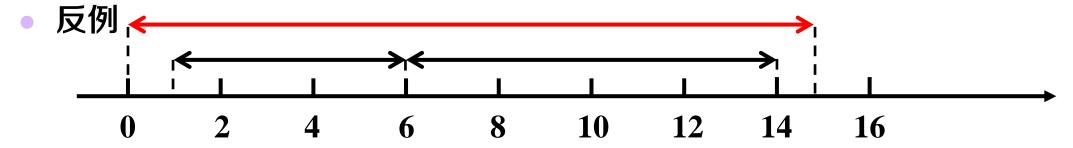
• 策略3: 最早结束活动优先



• 策略1: 最短活动优先



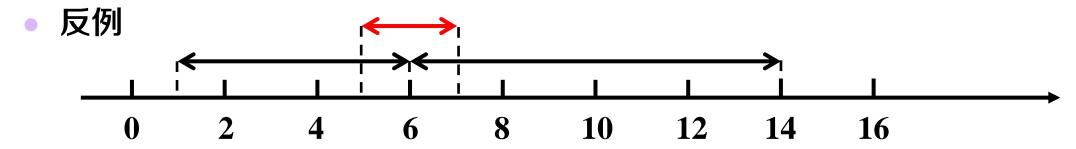
• 策略2: 最早开始活动优先



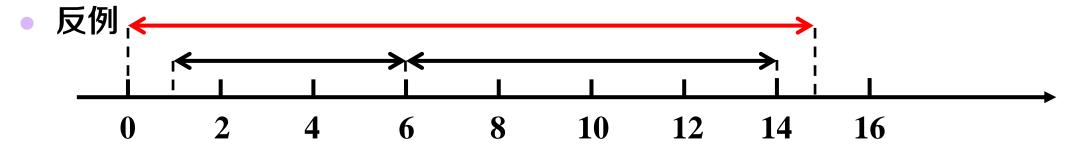
• 策略3: 最早结束活动优先



• 策略1: 最短活动优先



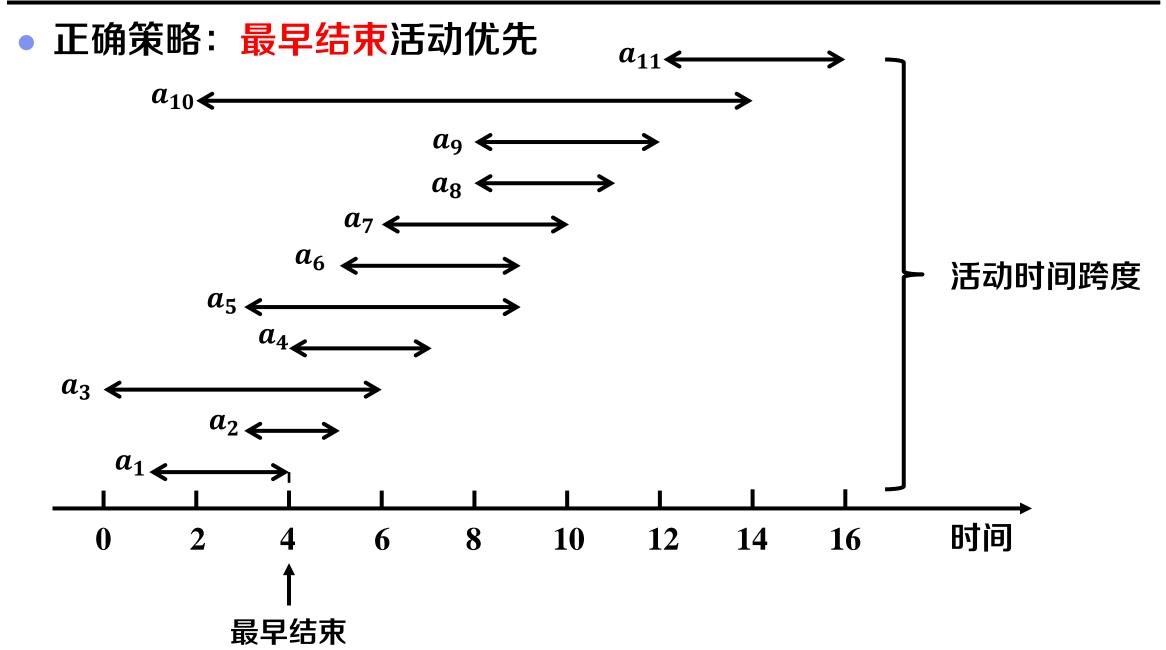
• 策略2: 最早开始活动优先



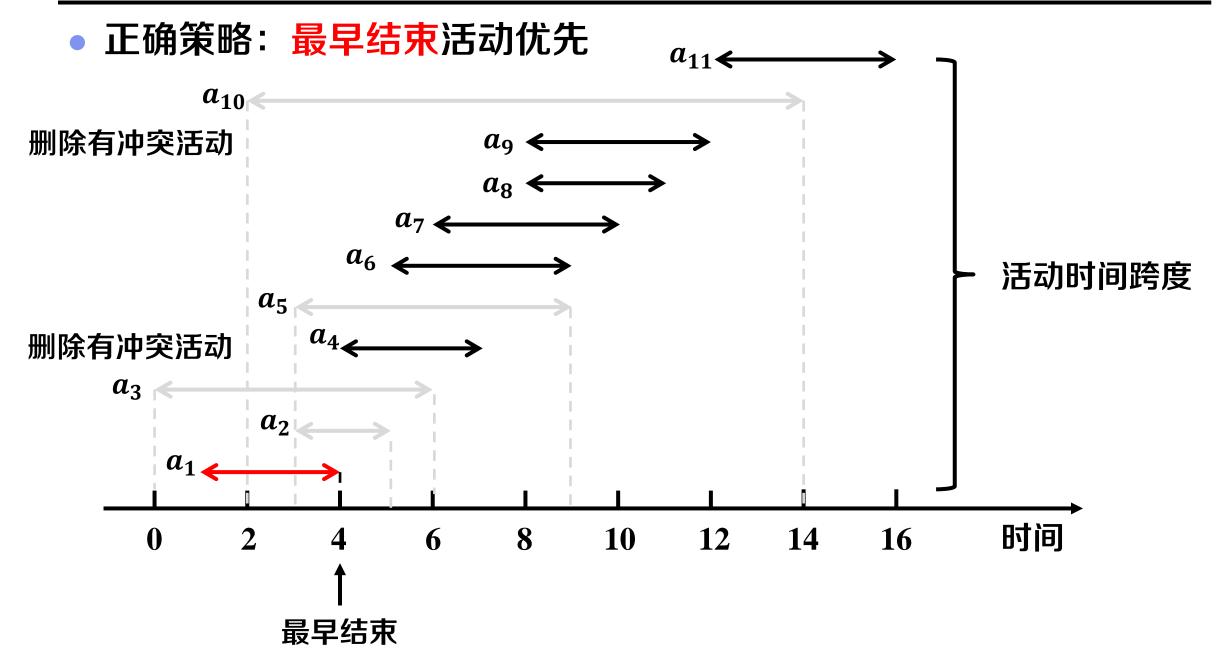
• 策略3: 最早结束活动优先

• 选择最早结束的活动,可以给后面的活动留更大的选择空间

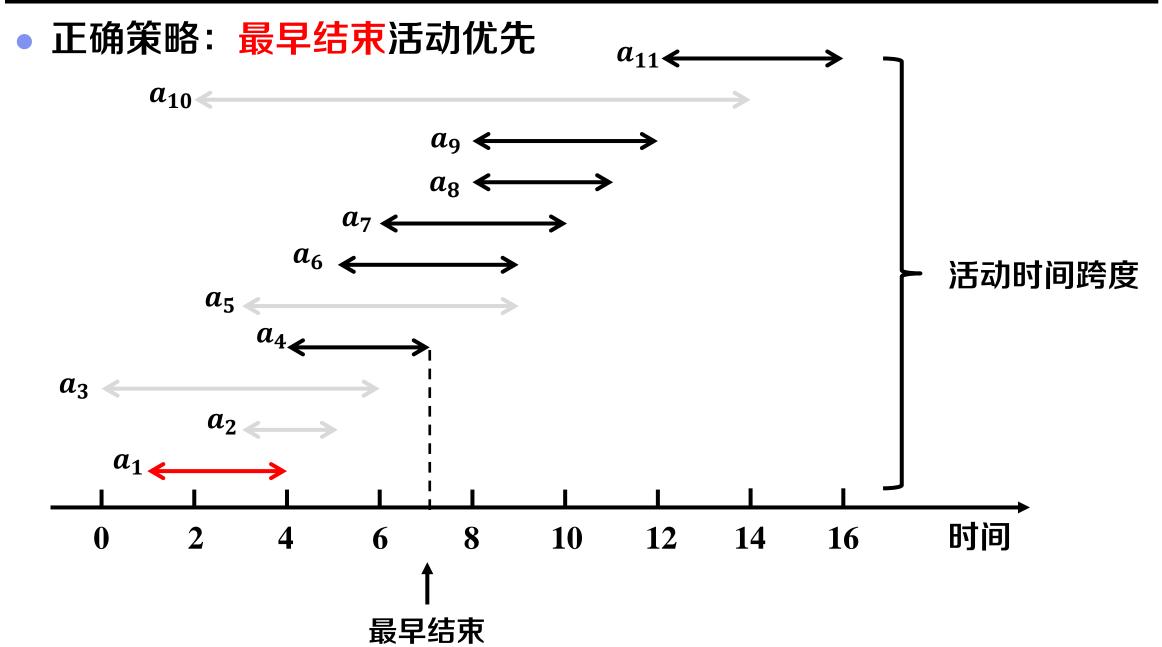




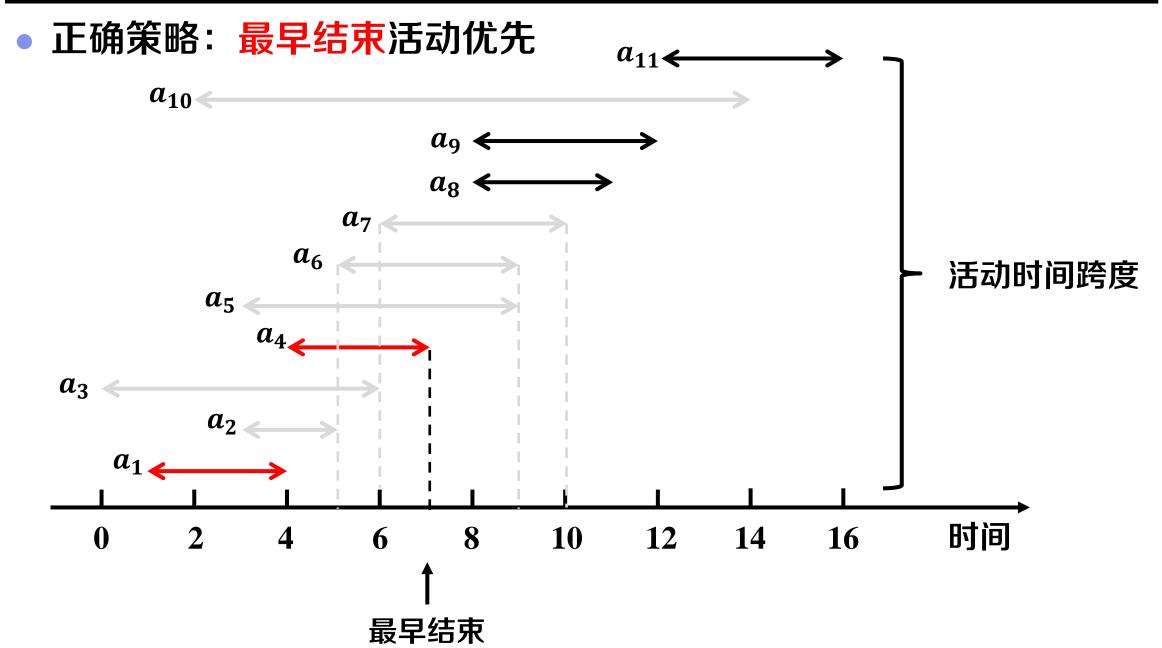




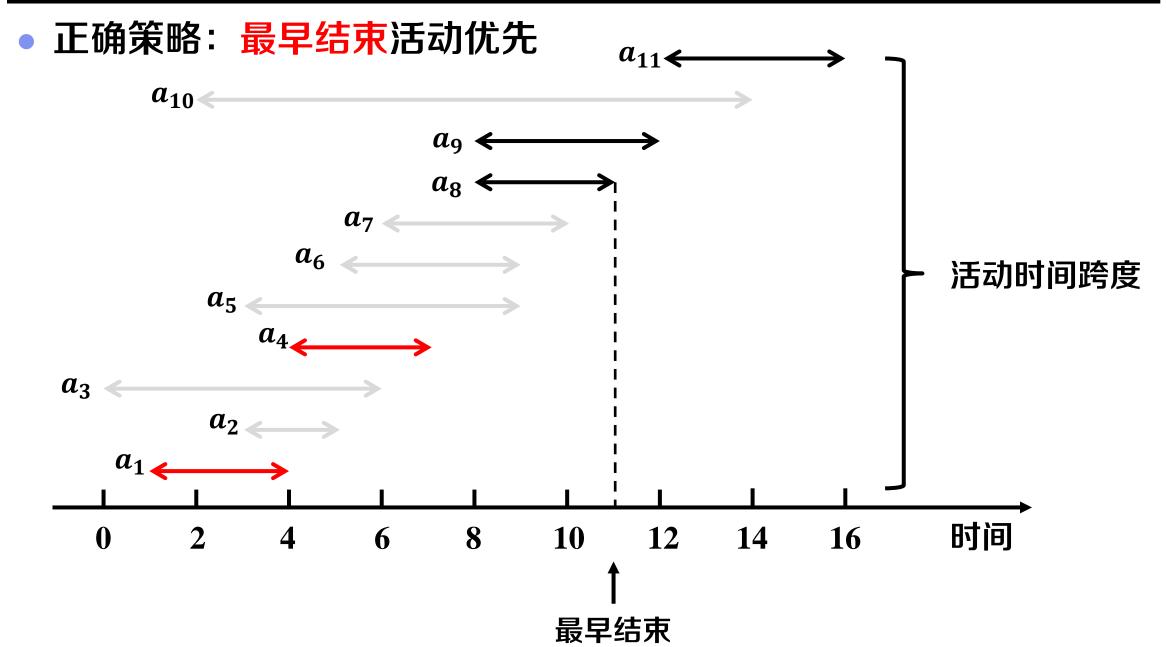




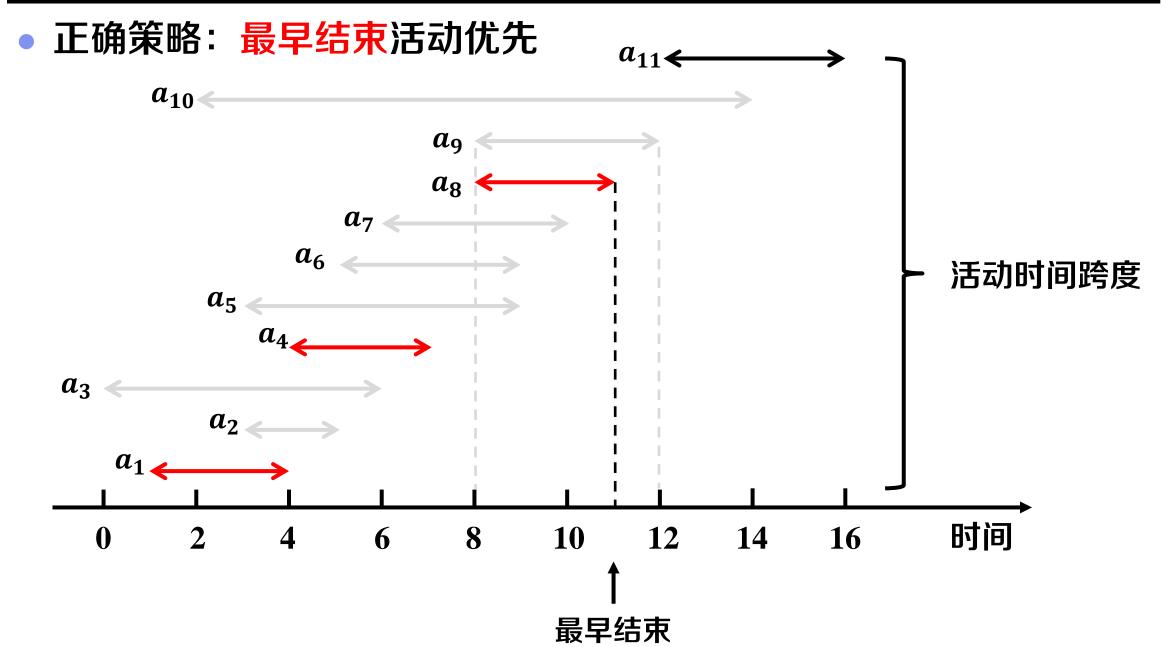




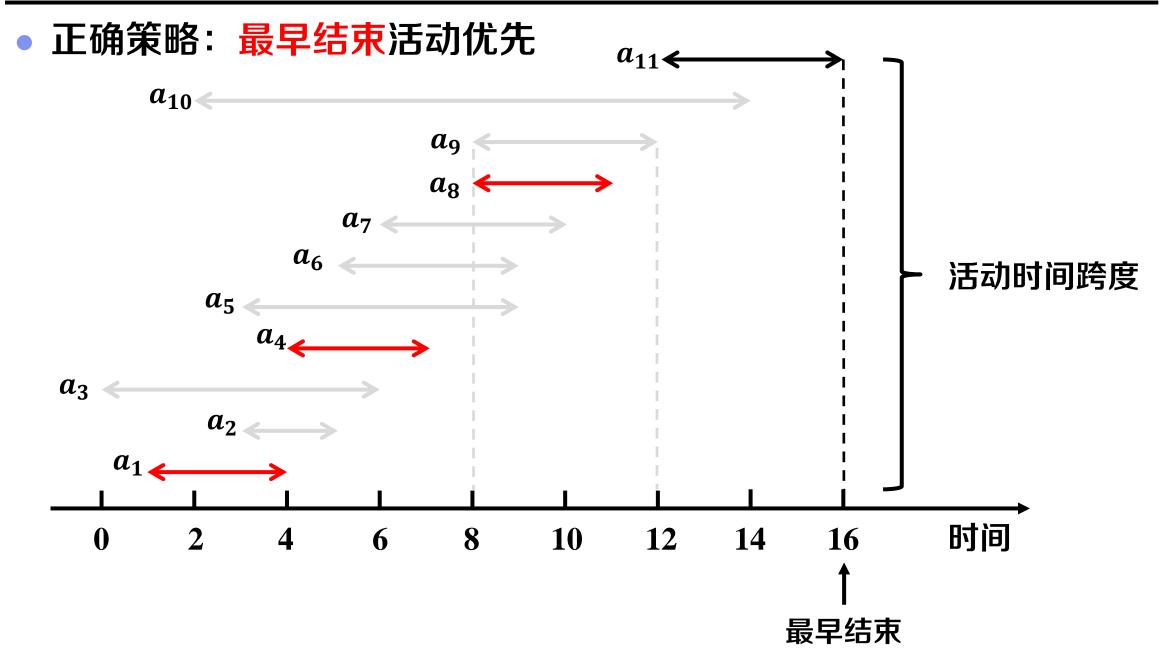




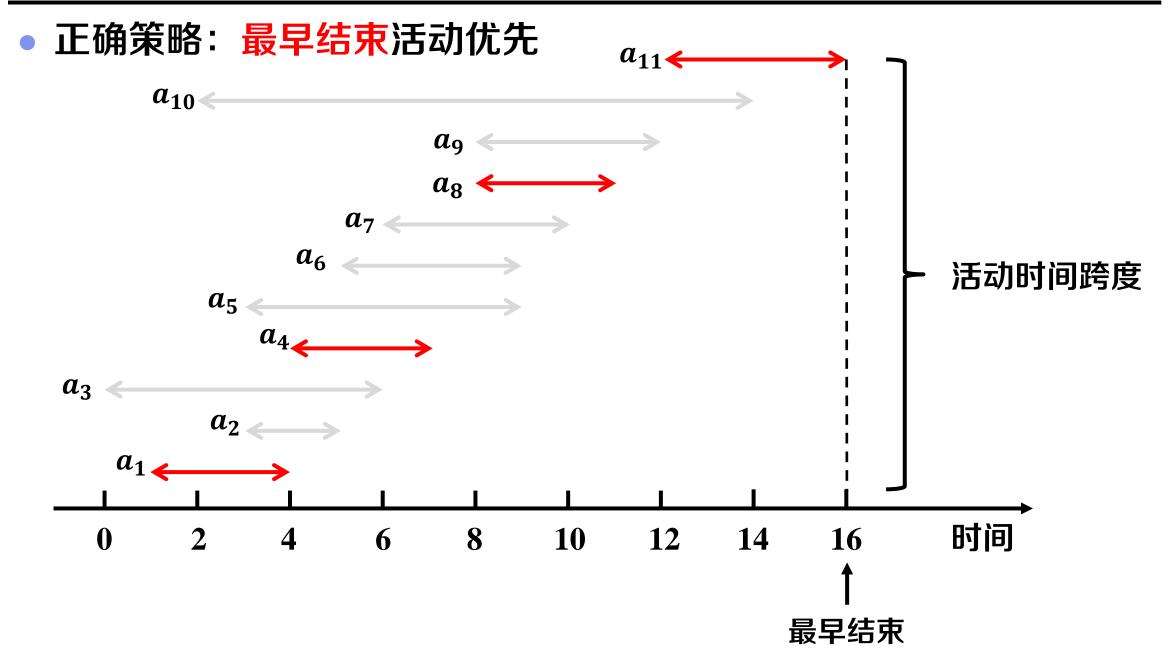






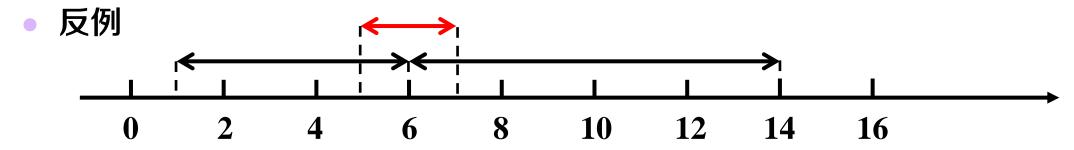




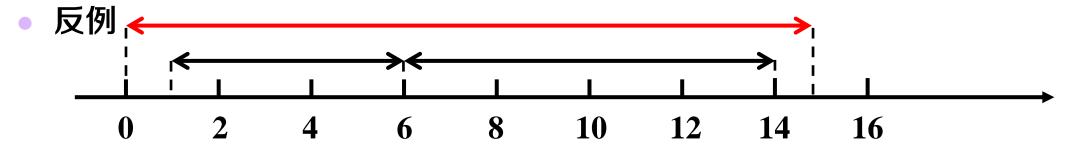




• 策略1: 最短活动优先



• 策略2: 最早开始活动优先



- 策略3: 最早结束活动优先
 - 选择最早结束的活动,可以给后面的活动留更大的选择空间

问题: 策略3是否可以保证最优解?

贪心策略:一般步骤



提出贪心策略

观察问题特征,构造贪心选择



证明策略正确

假设最优方案,通过替换证明

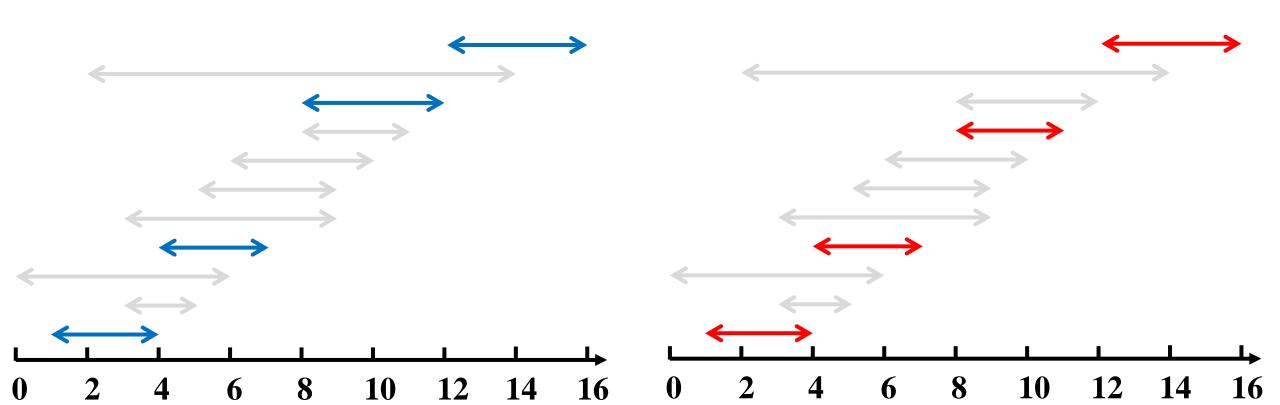


- 正确策略: 最早结束活动优先
- 证明: 贪心解不劣于最优解

任意最优活动集合

依次检查并替换

贪心所得活动集合





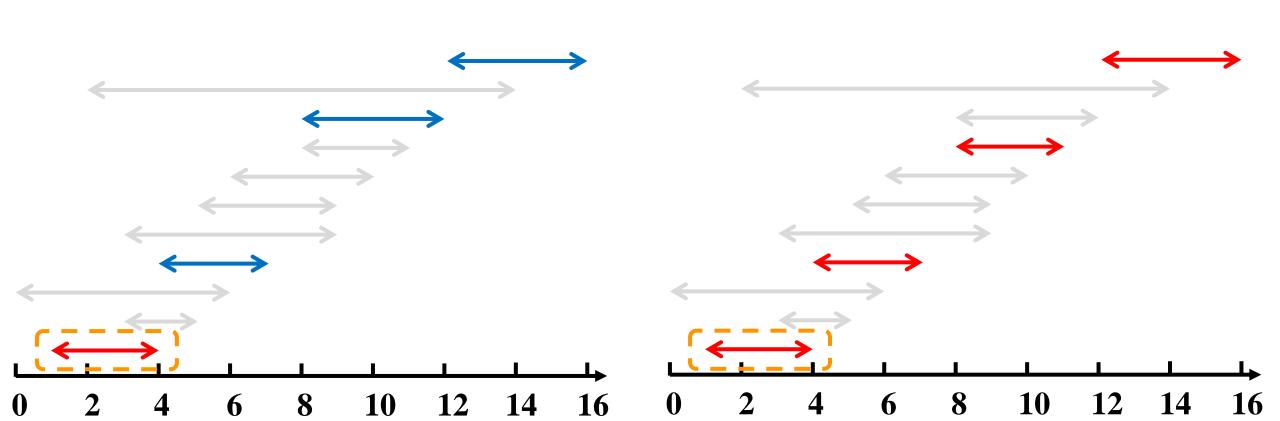
- 正确策略: 最早结束活动优先
- 证明: 贪心解不劣于最优解

任意最优活动集合

依次检查并替换

贪心所得活动集合

活动相同,无需替换





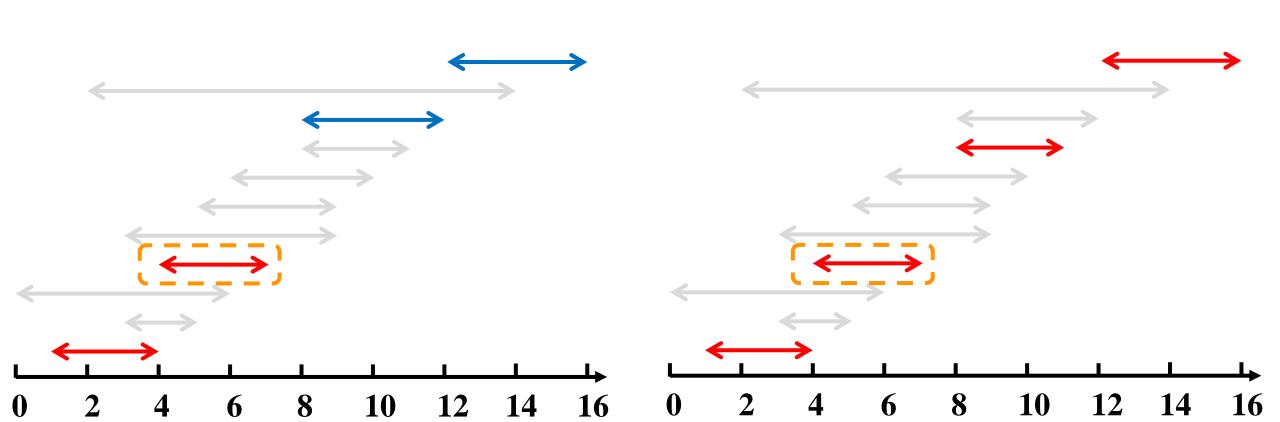
- 正确策略: 最早结束活动优先
- 证明: 贪心解不劣于最优解

任意最优活动集合

依次检查并替换

贪心所得活动集合

活动相同,无需替换





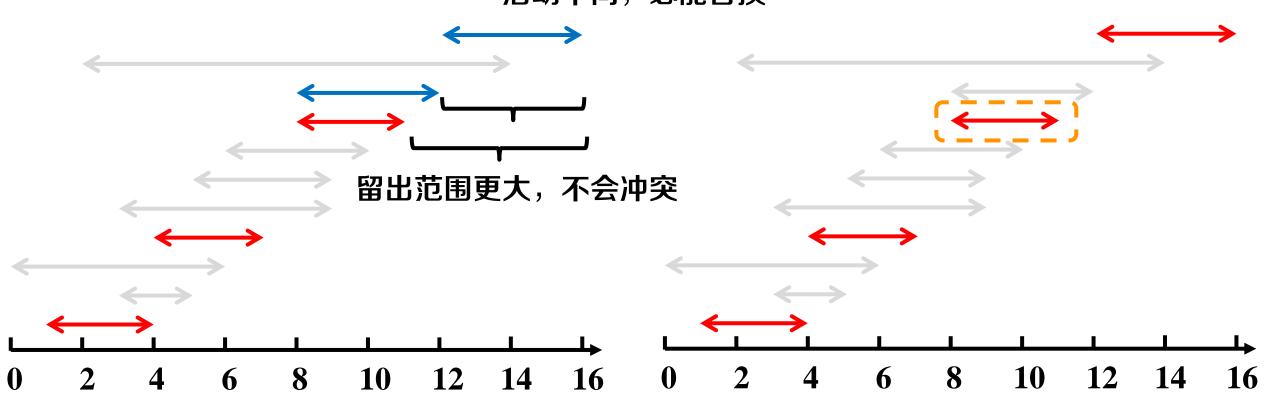
- 正确策略: 最早结束活动优先
- 证明: 贪心解不劣于最优解

任意最优活动集合

依次检查并替换

贪心所得活动集合

活动相同,无需替换活动不同,必能替换





- 正确策略: 最早结束活动优先
- 证明: 贪心解不劣于最优解

任意最优活动集合

依次检查并替换

活动相同,

无需替换

贪心所得活动集合

活动不同,必能替换 8 **10 10 12 14 16**

return S'



```
输入: 活动集合S = \{a_1, a_2, ..., a_n\}, 每个活动a_i的起止时间s_i, f_i
输出: 不冲突活动的最大子集S'
把活动按照结束时间升序排序 🛚
S' \leftarrow \{a_1\}
k \leftarrow 1
for i \leftarrow 2 to n do
   if s_i \geq f_k then
      S' \leftarrow S' \cup \{a_i\}
     k \leftarrow i
    end
end
```

为使用贪心策略作准备



```
输入: 活动集合S = \{a_1, a_2, ..., a_n\}, 每个活动a_i的起止时间s_i, f_i
```

输出: 不冲突活动的最大子集S'

<u>把活动按照结束时间升序排序</u>

$$S' \leftarrow \{a_1\}$$

$$k \leftarrow 1$$

把最早结束活动加入到集合

```
for i \leftarrow 2 to n do
\begin{vmatrix}
\mathbf{if} \ s_i \ge f_k \ \mathbf{then} \\
S' \leftarrow S' \cup \{a_i\} \\
k \leftarrow i \\
\mathbf{end}
\end{vmatrix}
```

return S'



```
输入: 活动集合S=\{a_1,a_2,...,a_n\}, 每个活动a_i的起止时间s_i,f_i
```

输出: 不冲突活动的最大子集S'

把活动按照结束时间升序排序

$$S' \leftarrow \{a_1\}$$

$$k \leftarrow 1$$

$$\text{for } i \leftarrow 2 \text{ to } n \text{ do}$$

记录当前选择的活动

```
\begin{array}{c|c} \mathbf{for} \ i \leftarrow 2 \ to \ n \ \mathbf{do} \\ & \mathbf{if} \ s_i \geq f_k \ \mathbf{then} \\ & S' \leftarrow S' \cup \{a_i\} \\ & k \leftarrow i \\ & \mathbf{end} \\ \end{array}
```

return S'



```
输入: 活动集合S = \{a_1, a_2, ..., a_n\}, 每个活动a_i的起止时间s_i, f_i
```

输出: 不冲突活动的最大子集S'

把活动按照结束时间升序排序

$$S' \leftarrow \{a_1\}$$
 $k \leftarrow 1$
 $\mathbf{for} \ i \leftarrow 2 \ to \ n \ \mathbf{do}$

检查每个活动

```
egin{array}{c|c} \mathbf{if} \ s_i \geq f_k \ \mathbf{then} \ S' \leftarrow S' \cup \{a_i\} \ k \leftarrow i \ \mathbf{end} \ \end{array}
```

return S'



```
输入: 活动集合S = \{a_1, a_2, ..., a_n\}, 每个活动a_i的起止时间s_i, f_i
输出: 不冲突活动的最大子集S'
把活动按照结束时间升序排序
S' \leftarrow \{a_1\}
k \leftarrow 1
for i \leftarrow 2 to n do
if s_i \geq f_k then
                                 没有冲突,则加入子集
k \leftarrow i
   \mathbf{end}
end
```

return S'



```
输入: 活动集合S = \{a_1, a_2, ..., a_n\}, 每个活动a_i的起止时间s_i, f_i
输出: 不冲突活动的最大子集S'
把活动按照结束时间升序排序
S' \leftarrow \{a_1\}
k \leftarrow 1
for i \leftarrow 2 to n do
   if s_i \geq f_k then
     S' \leftarrow S' \cup \{a_i\}
                                       更新当前选择的活动
end
```

贪心算法:复杂度分析



```
输入: 活动集合S = \{a_1, a_2, ..., a_n\}, 每个活动a_i的起止时间s_i, f_i
输出: 不冲突活动的最大子集S'
把活动按照结束时间升序排序
                                             ---- O(n \log n)
S' \leftarrow \{a_1\}
k \leftarrow 1
for i \leftarrow 2 to n do
   if s_i \geq f_k then
   \begin{vmatrix} S' \leftarrow S' \cup \{a_i\} \\ k \leftarrow i \end{vmatrix}
                                         O(n)
    end
end
                                                      时间复杂度: O(n \log n)
return S'
```

问题拓展



• 会场出租

收益很大



公司年会: 10:00~19:00

收益较多



收益良好



婚礼宴请: 11:00~14:00

收益较少

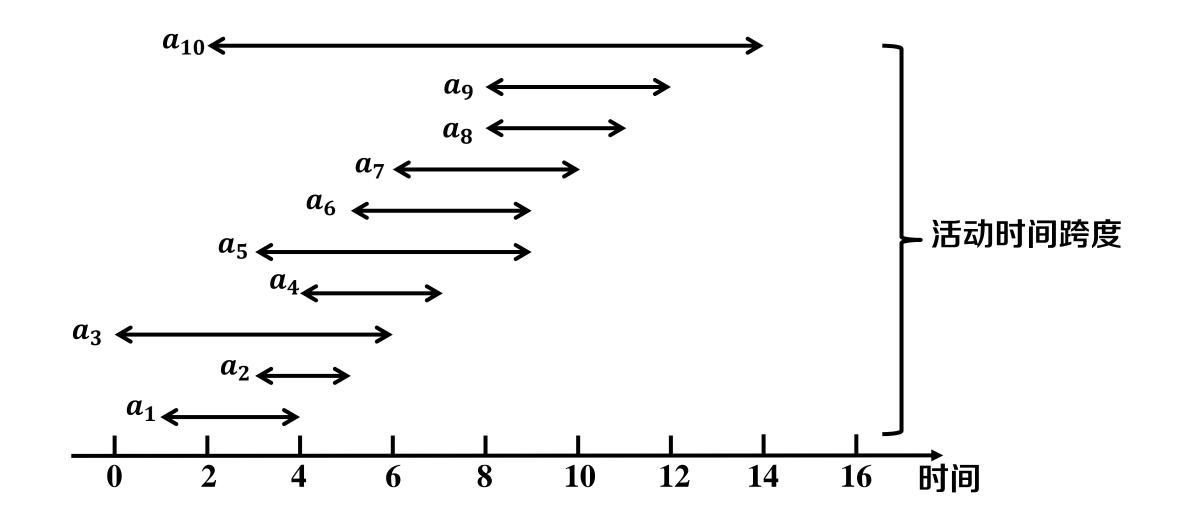


学术研讨: 14:00~16:00

问题拓展



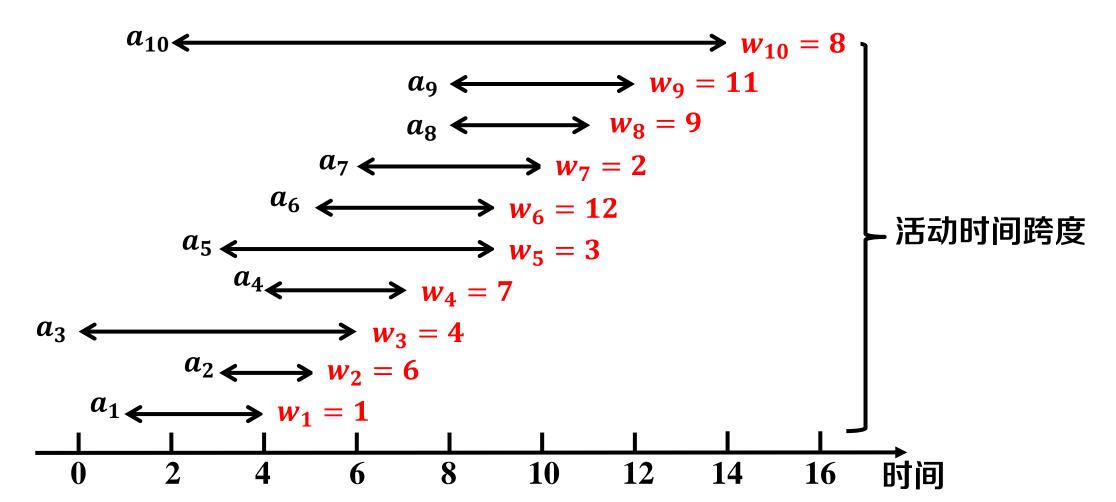
- 会场出租
 - 选择出租的活动时间不能冲突



问题拓展



- 会场出租
 - 选择出租的活动时间不能冲突,活动出租收益各不相同
 - 怎样选让收益总和最大?





带权活动选择问题

Weighted Activity Selection Problem

输入

- n个活动组成的集合 $S = \{a_1, a_2, ..., a_n\}$
- 每个活动 a_i 的开始时间 s_i ,结束时间 f_i 和权重 w_i

输出

• 找出活动集合S的子集S',令

优化目标: 最大化权重之和

$$\max \sum_{a_i \in S'} w_i$$

 $s.t. \forall a_i, a_j \in S', s_i \geq f_j$ 或 $s_j \geq f_i$

约束条件

问题比较



带权活动选择问题

$$\max \sum_{a_i \in S'} w_i$$

权重均为1

活动选择问题

 $\max |S'|$



带权活动选择问题

$$\max \sum_{a_i \in S'} w_i$$

权重均为1

活动选择问题

 $\max |S'|$

$$a_3 \longleftrightarrow w_3 = 1$$
 $a_2 \longleftrightarrow w_2 = 10000$ $a_2 \longleftrightarrow w_2 = 1$ $a_1 \longleftrightarrow w_1 = 1$ 时间



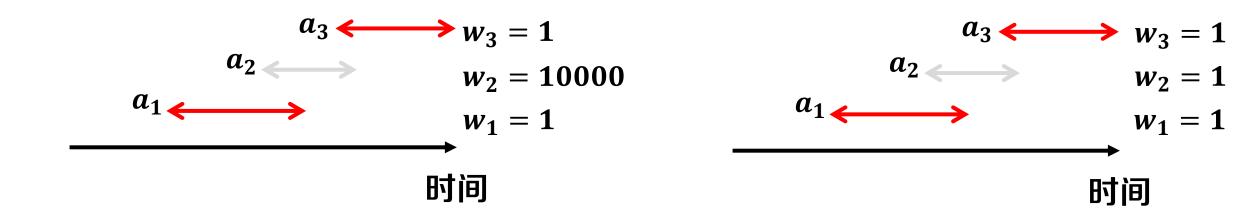


$$\max \sum_{a_i \in S'} w_i$$

权重均为1

活动选择问题

 $\max |S'|$



问题比较



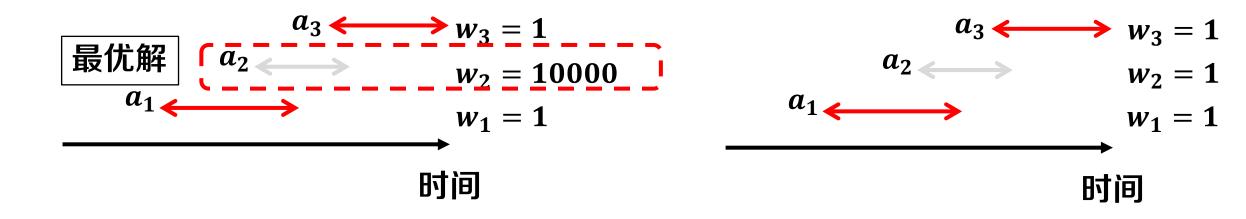


$$\max \sum_{a_i \in S'} w_i$$

权重均为1

活动选择问题

 $\max |S'|$

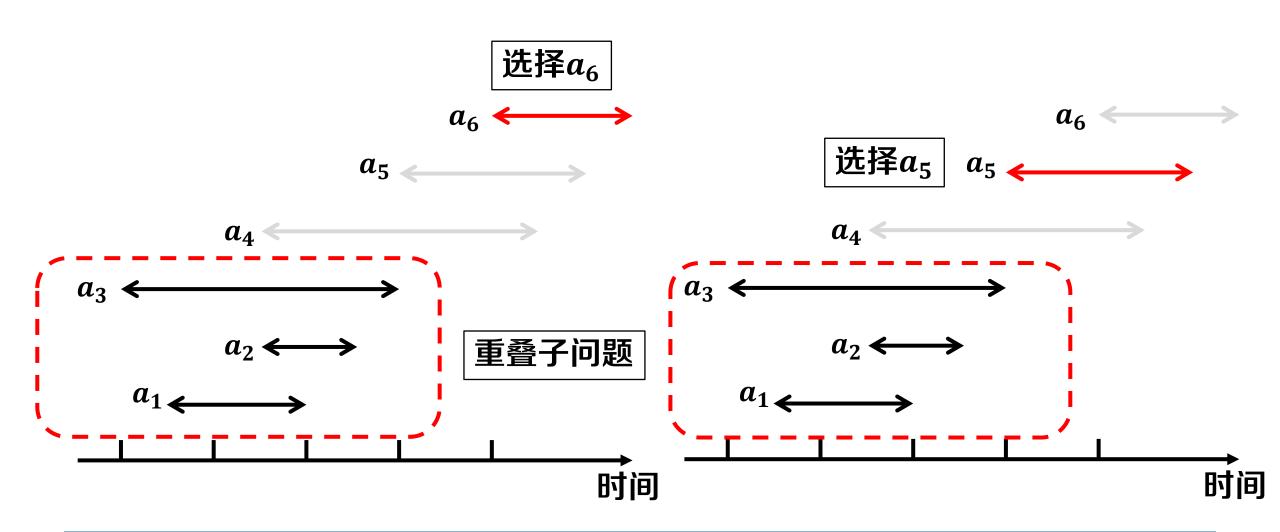


贪心策略不正确

贪心策略正确

从贪心策略到动态规划

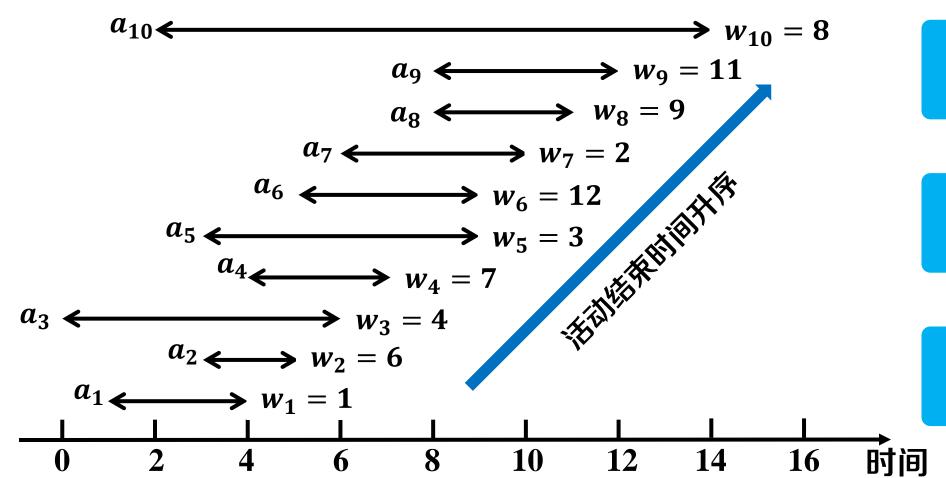




存在重叠子问题,使用动态规划求解



- 预处理
 - 排序: 按活动结束时间升序



问题结构分析



递推关系建立



自底向上计算

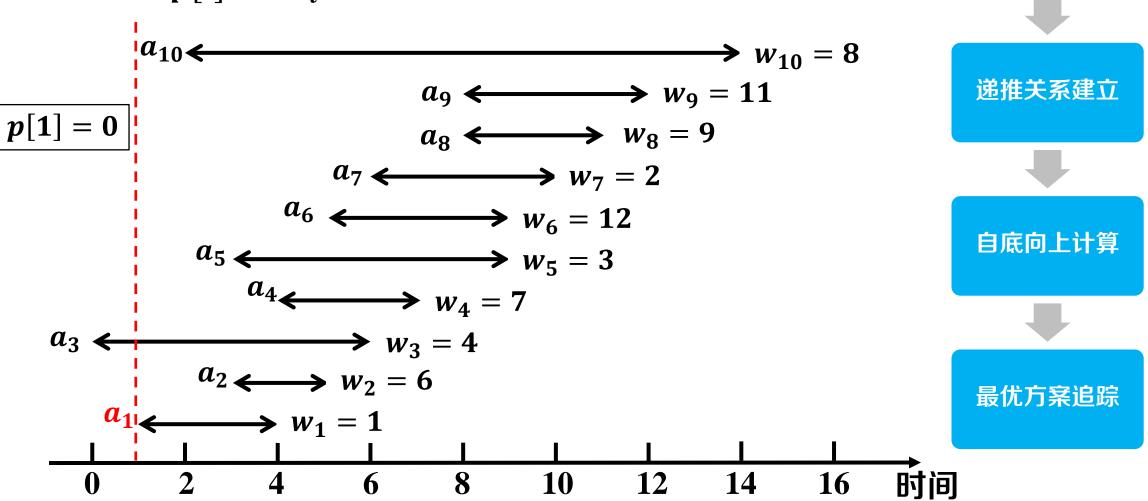




问题结构分析

预处理

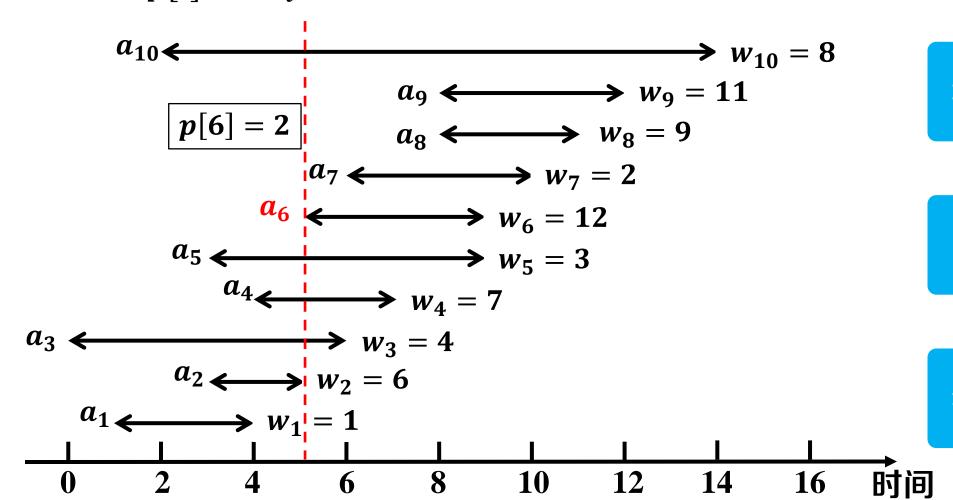
- 排序: 按活动结束时间升序
- 求p[i]: 在 a_i 开始前最后结束的活动





预处理

- 排序: 按活动结束时间升序
- 求p[i]: 在 a_i 开始前最后结束的活动



问题结构分析



递推关系建立



自底向上计算





预处理

- 排序: 按活动结束时间升序
- 求p[i]: 在 a_i 开始前最后结束的活动
 - 。 如何求解p[i]?
 - 。 排序后使用二分查找

问题结构分析



递推关系建立



自底向上计算





- 预处理
 - 排序: 按活动结束时间升序
 - 求p[i]: 在 a_i 开始前最后结束的活动
- 给出问题表示
 - D[i]: 集合 $\{a_1, a_2, a_3, ..., a_i\}$ 中不冲突活动最大权重和

- 明确原始问题
 - D[n]: 集合 $\{a_1, a_2, a_3, ..., a_n\}$ 中不冲突活动最大权重和

问题结构分析



递推关系建立



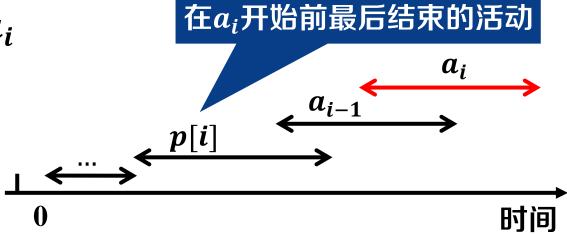
自底向上计算





• 考察活动 a_i

• 选择 a_i



问题结构分析



递推关系建立



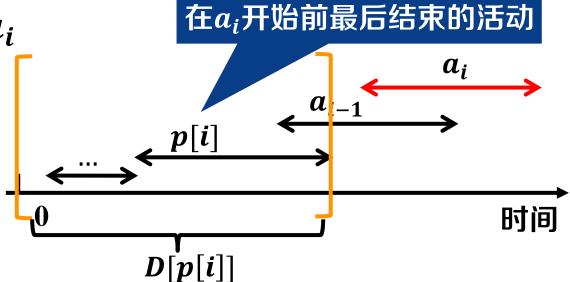
自底向上计算





• 考察活动 a_i

• 选择 a_i



问题结构分析



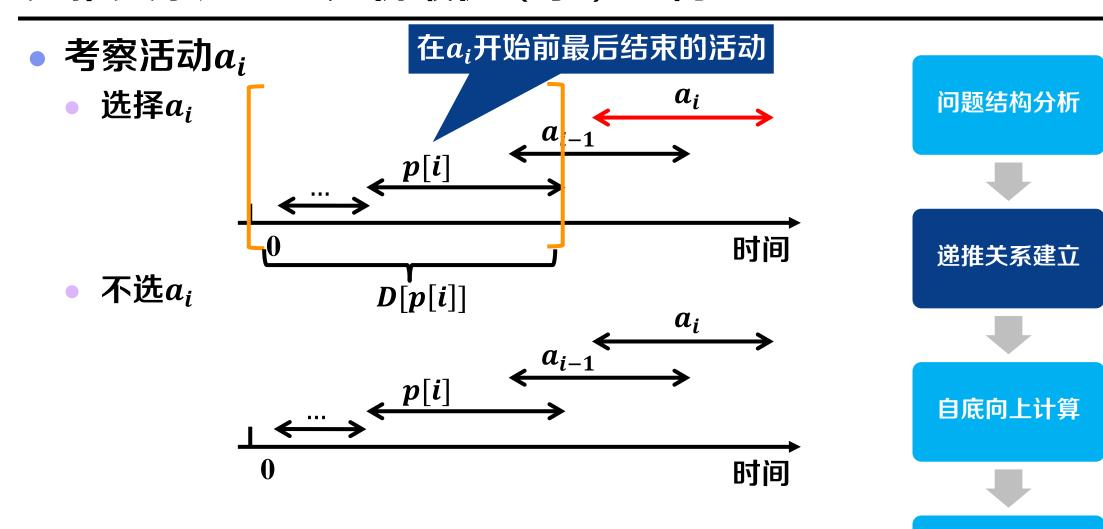
递推关系建立



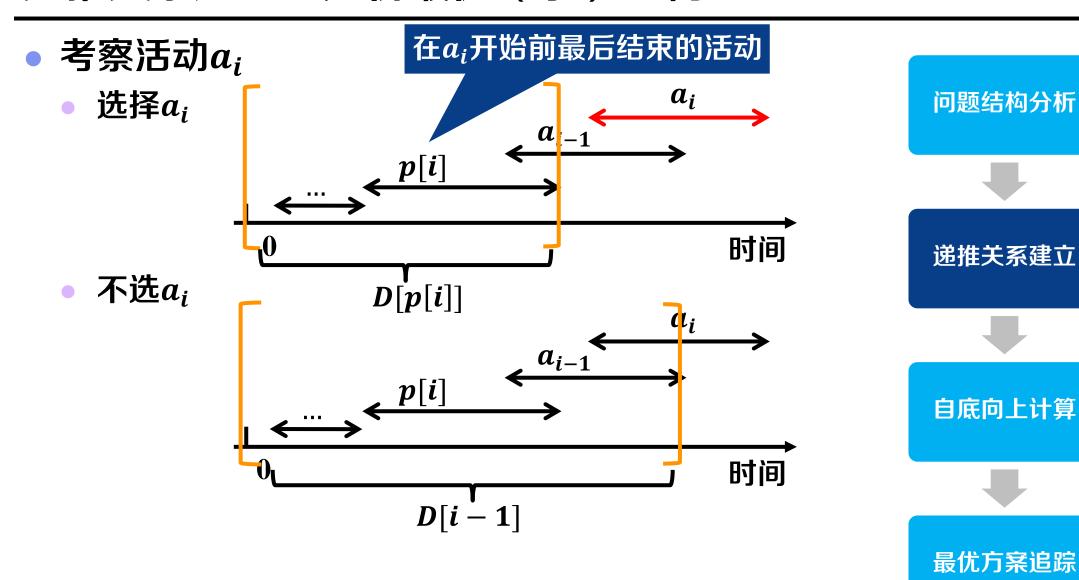
自底向上计算











递推关系建立: 构造递推公式

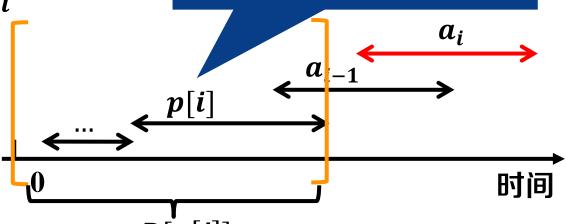


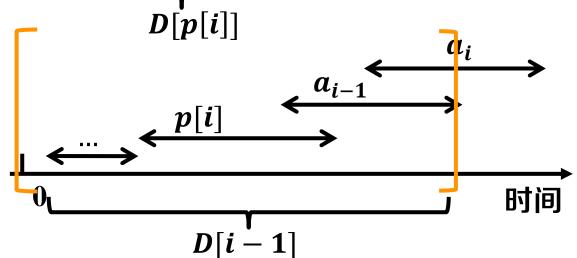


选择 a_i

• 不选 a_i

在 a_i 开始前最后结束的活动





- 递推公式
 - $D[i] = \max\{D[p[i]] + w_i, D[i-1]\}$

问题结构分析



递推关系建立



自底向上计算



递推关系建立: 构造递推公式

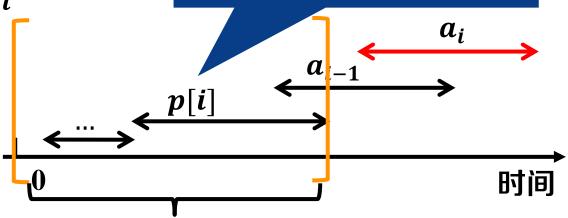


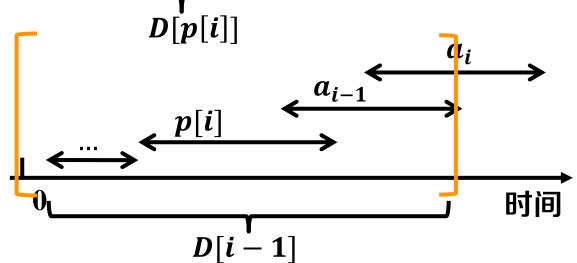


• 选择 a_i

• 不选 a_i

在 a_i 开始前最后结束的活动





递推公式

 $D[i] = \max\{D[p[i]] + w_i, D[i-1]\}$

问题结构分析



递推关系建立



自底向上计算



最优方案追踪

最优子结构

自底向上计算:确定计算顺序



- 初始化
 - D[0] = 0: 空活动集最大权重和为0

问题结构分析



递推关系建立



自底向上计算

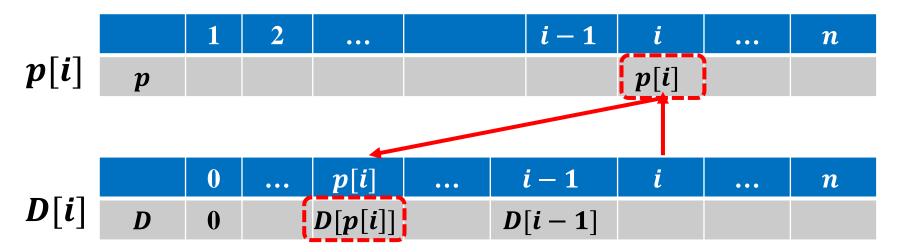


自底向上计算:确定计算顺序



- 初始化
 - D[0] = 0: 空活动集最大权重和为0
- 递推公式
 - $D[i] = \max\{D[p[i]] + w_i, D[i-1]\}$

| | | 1 | 2 | ••• | i-1 | i | ••• | n | |
|----|---|---|---|-----|-----|-------|-----|---|--|
| 已知 | W | | | | | w_i | | | |



问题结构分析



递推关系建立



自底向上计算



自底向上计算:确定计算顺序



• 初始化

已知

- D[0] = 0: 空活动集最大权重和为0
- 递推公式
 - $D[i] = \max\{D[p[i]] + w_i, D[i-1]\}$

| | 1 | 2 | ••• | i-1 | i | ••• | n |
|---|---|---|-----|-----|-------|-----|---|
| W | | | | | w_i | | |

| | | 1 | 2 | ••• | i-1 | i | ••• | n |
|------|---|---|---|-----|-----|------|-----|---|
| p[i] | p | | | | | p[i] | | |



问题结构分析



递推关系建立



自底向上计算



自底向上计算: 依次求解问题



- 初始化
 - D[0] = 0: 空活动集最大权重和为0
- 递推公式
 - $D[i] = \max\{D[p[i]] + w_i, D[i-1]\}$

| | | 1 | 2 | ••• | i-1 | i | ••• | n |
|----|---|---|---|-----|-----|-------|-----|---|
| 已知 | W | | | | | w_i | | |



问题结构分析



递推关系建立



自底向上计算



最优方案追踪



• 记录决策过程

•
$$Rec[i] = \begin{cases} 1, &$$
 选择活动 $a_i \\ 0, &$ 不选活动 $a_i \end{cases}$

问题结构分析



递推关系建立



自底向上计算



最优方案追踪



• 记录决策过程

•
$$Rec[i] = \begin{cases} 1, & \text{选择活动}a_i \\ 0, & \text{不选活动}a_i \end{cases}$$

• 输出最优方案

- Rec[i] = 1时,选择活动 a_i ,考察子问题D[p[i]]
- Rec[i] = 0时,不选活动 a_i ,考察子问题D[i-1]

| - | 烫结 | | | |
|---|----|------|-------------|------------|
| | | | ~~ <i>1</i> | A = = |
| | | / /_ | 7 1 /1 | N . |



递推关系建立

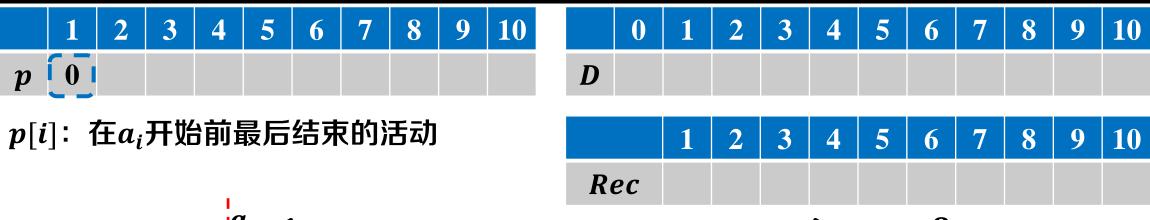


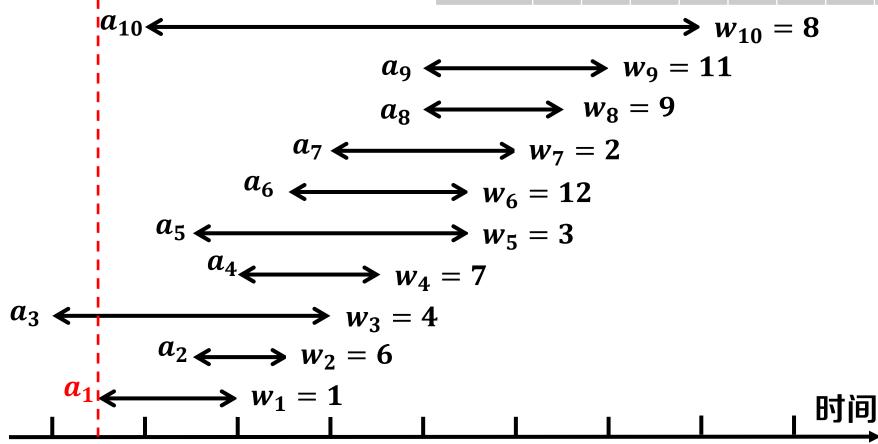
自底向上计算



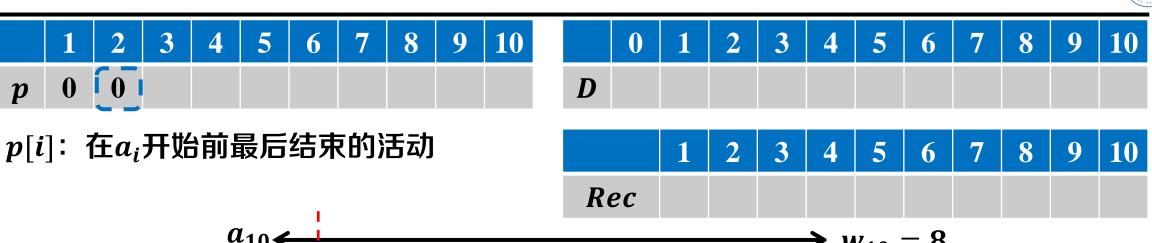
| | | 1 | 2 | ••• | i-1 | i | ••• | n-1 | \boldsymbol{n} |
|----|-----|---|-----|------|-----|------|-----|-----|------------------|
| 已求 | p | | | | | p[i] | | i | |
| | | | | | | | | | |
| | | 1 | ••• | p[i] | ••• | i | ••• | n-1 | n |
| | Rec | 1 | 0 | 0 | 0 | 1 | 0 | 1 + | 0 |

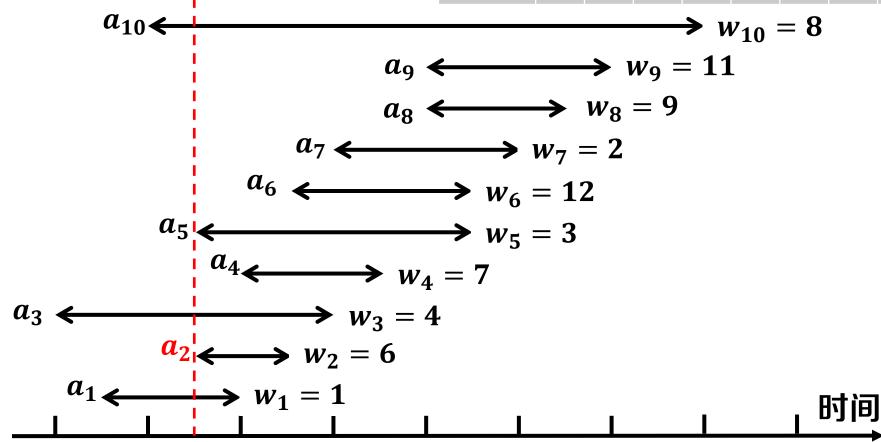




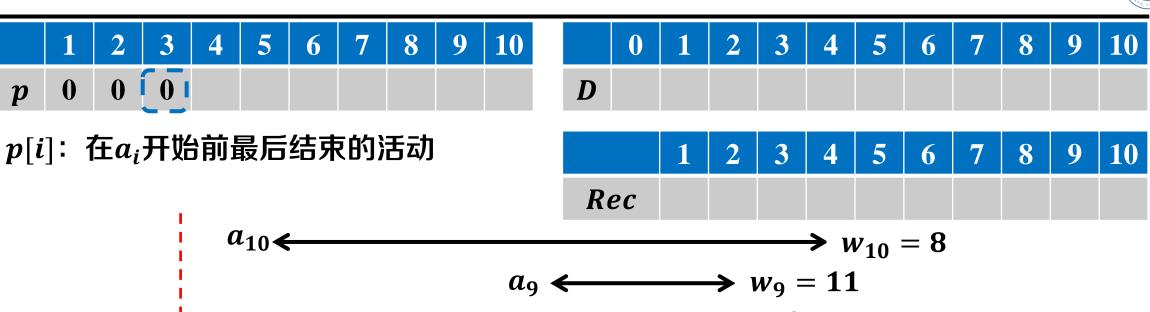


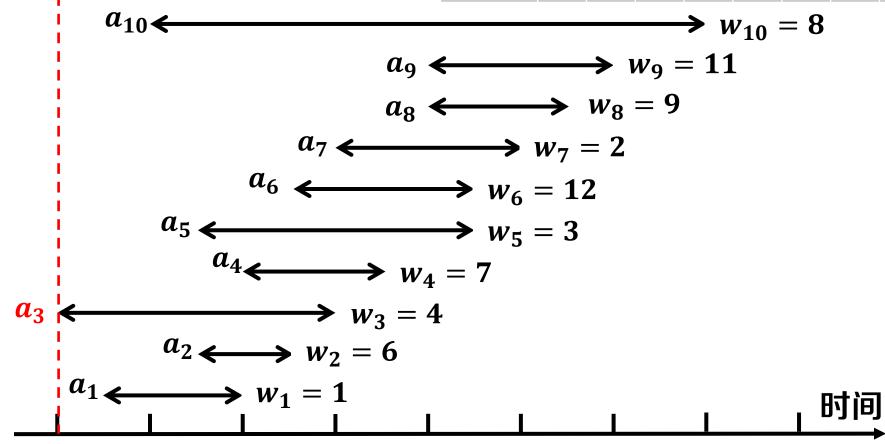




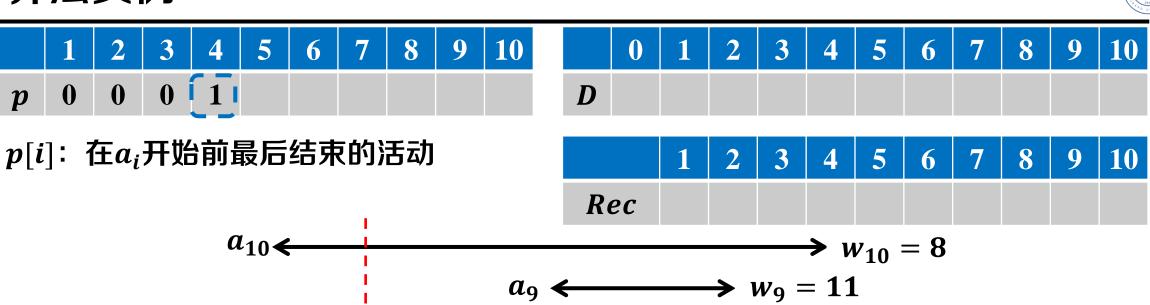


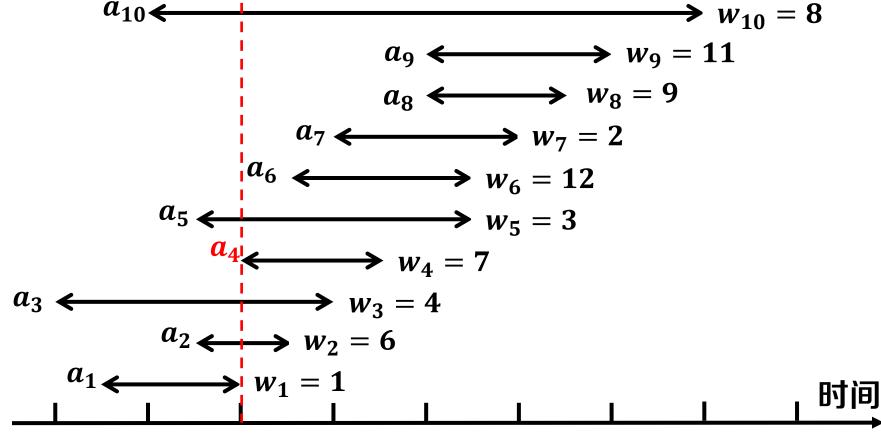




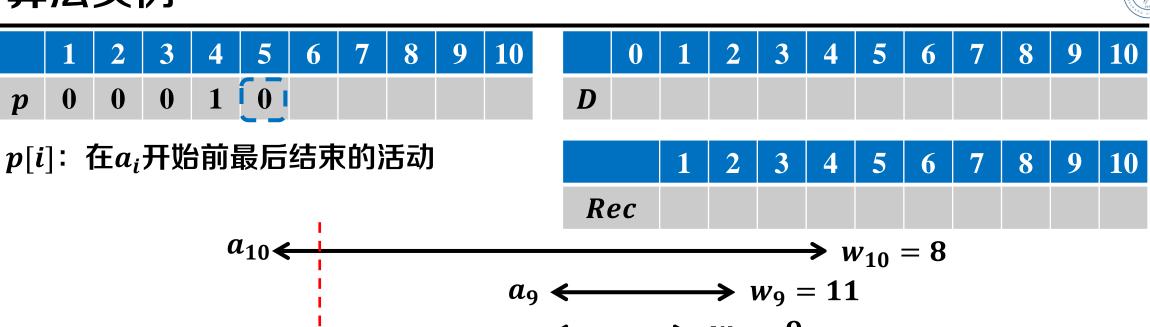


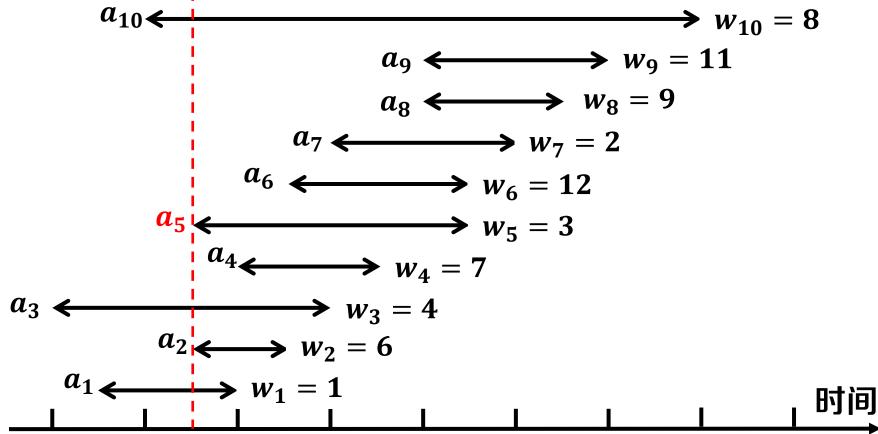






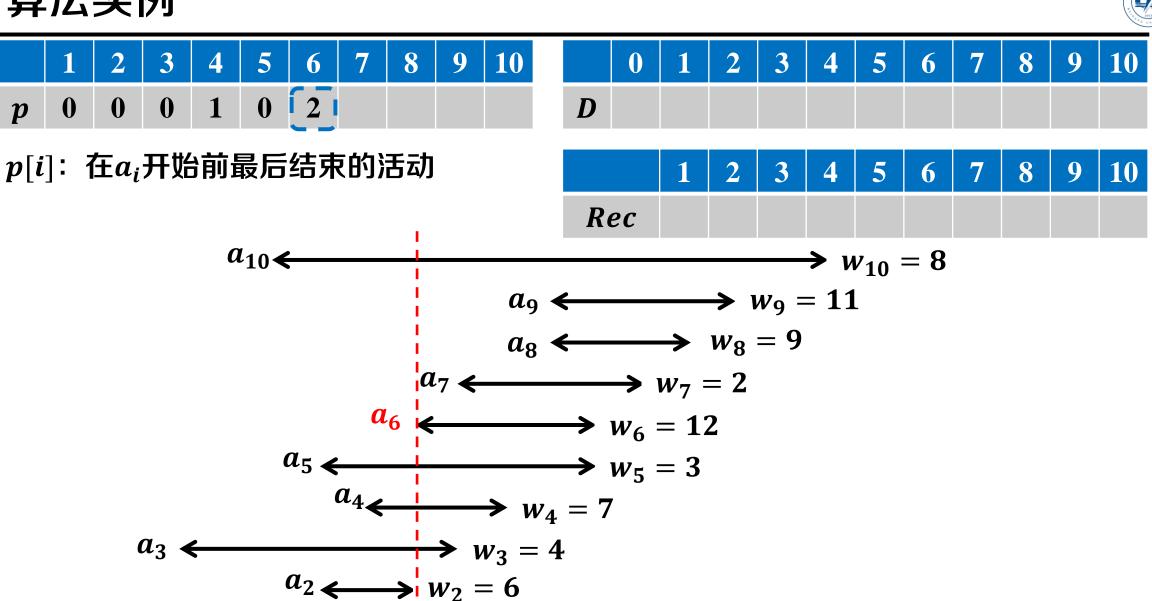




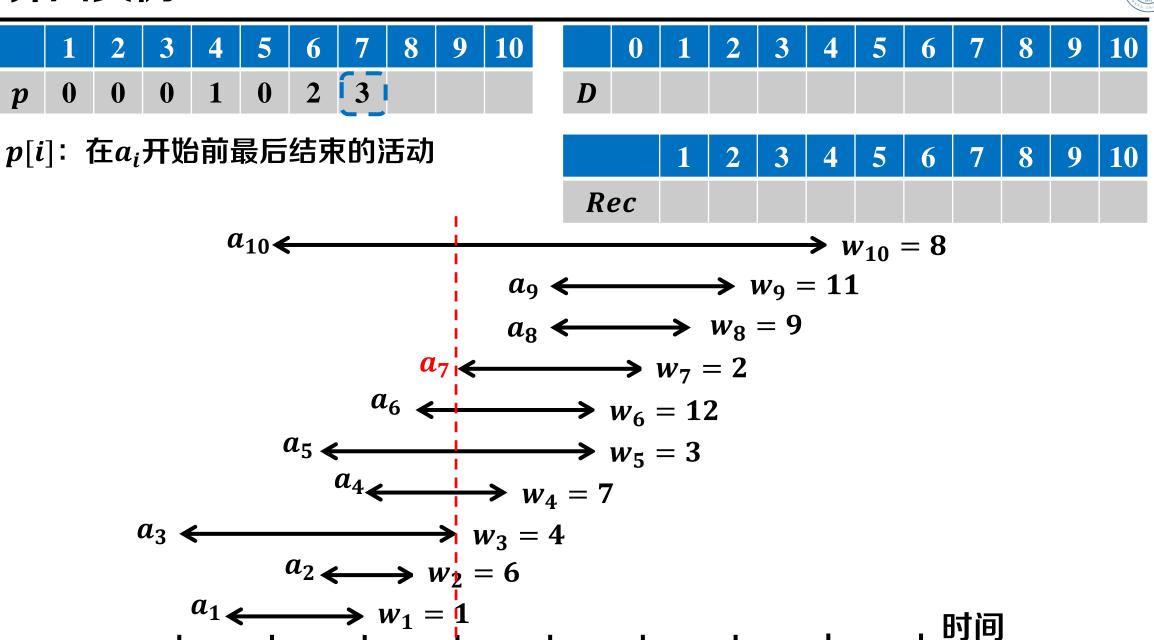




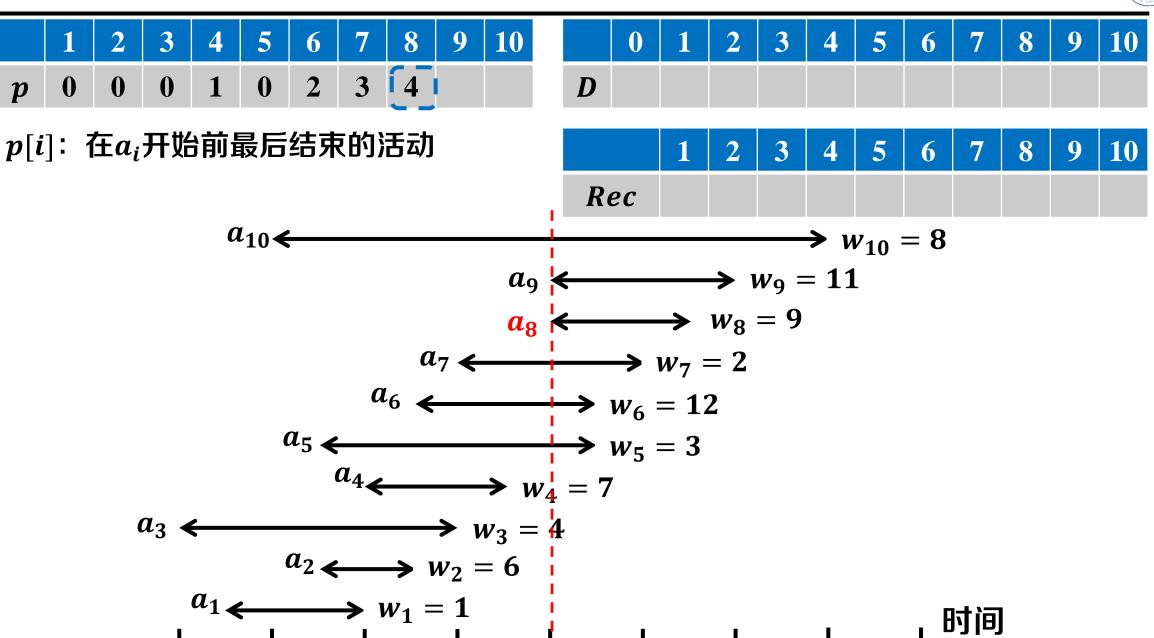
时间



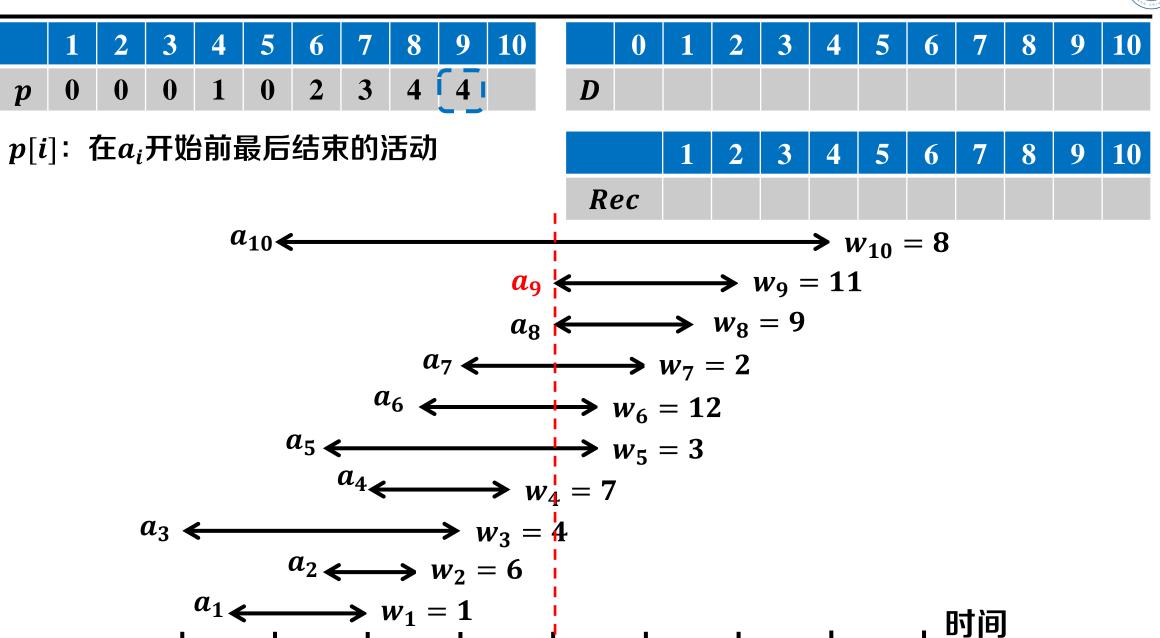










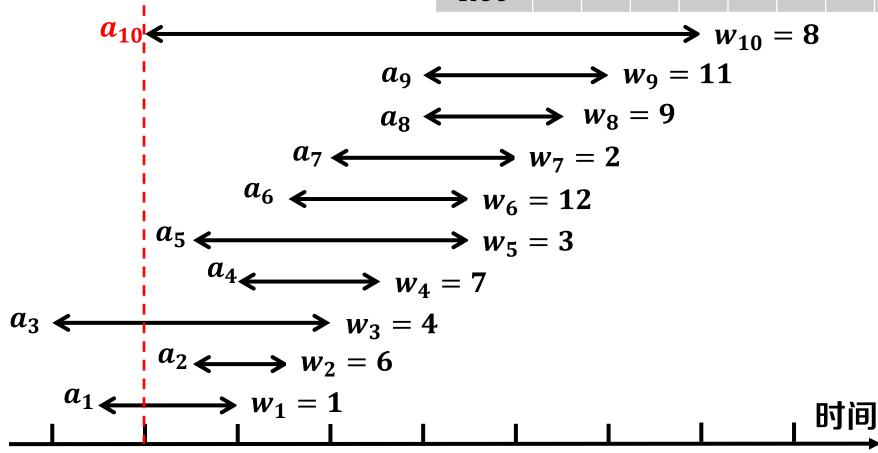




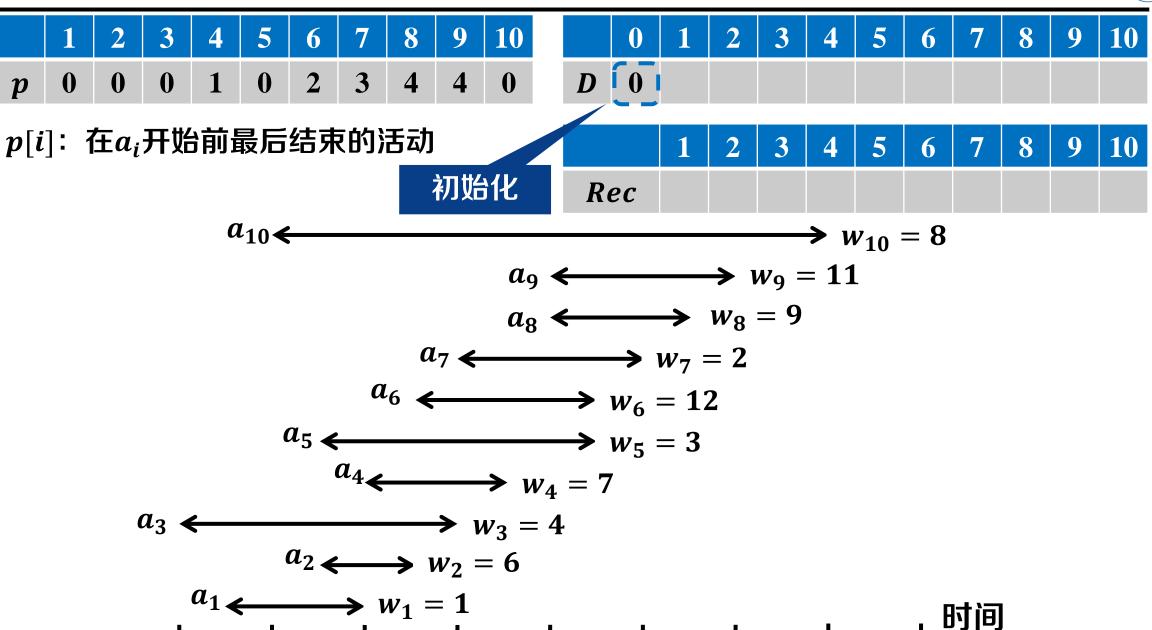
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|----|---|---|---|---|---|---|---|---|---|---|---|----|
| p | 0 | 0 | 0 | 1 | 0 | 2 | 3 | 4 | 4 | 0 | D | | | | | | | | | | | |

p[i]: 在 a_i 开始前最后结束的活动

 Rec
 1
 2
 3
 4
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 7
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 9
 10









| 开 | <i>1</i> ム: | 大 | ניל | | | | | | | | | | | | | | | | | | | 9 9 1952 d H G UNI | |
|---|-------------|----|------------------------------|------|--------------------------|---------|------|------------------|------------|-------|--------------|---------------|----------|------------|-------------------------|--------------|-----------------|-----|---|---|---|-----------------------|--|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | |
| p 0 0 1 0 2 3 4 4 0 D 0 | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | 10 | | | | | | | | |
| D[i] |] = | ma | $\mathbf{x}\{\boldsymbol{D}$ | p[i] |]] + | w_i , | D[i] | -1 |]} | | R | ec | | | | | | | | | | | |
| | | | | a | ¹ 10 ← | • | | | | | | | | | | → v | v ₁₀ | = 8 | | | | | |
| | | | | | | | | | | a_9 | | | | → 1 | <i>w</i> ₉ = | = 1 1 | l | | | | | | |
| | | | | | | | | | | a_8 | (| | → | w_8 | = 9 | | | | | | | | |
| | | | | | | | | \boldsymbol{a} | 7 ← | | | \rightarrow | W_7 = | = 2 | | | | | | | | | |



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|---|------|----|----------------------------|------|--------------------------|---------|------|------------|----|---------|-----|---|----|---------------|------------|------------|--------------|----------|-----|---|---|---|-------------------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| p | 0 | 0 | 0 | 1 | 0 | 2 | 3 | 4 | 4 | 0 | | D | 0 | | | | | | | | | | |
| $p[i]$: 在 a_i 开始前最后结束的活动 $1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9$ | | | | | | | | | | | | | | | 10 | | | | | | | | |
| D[i] | i] = | ma | x { <i>D</i> | p[i] |]]+ | w_i , | D[i] | - 1 |]} | | | R | ec | | | | | | | | | | |
| | | | | a | ¹ 10 ≺ | • | | | | | | | | | | | → v | v_{10} | = 8 | | | | |
| | | | | | | | | | | a_{0} | 9 < | | | | → 1 | W9 = | = 1 1 | 1 | | | | | |
| | | | | | | | | | | a_{8} | 3 ◀ | | | \rightarrow | w_8 | = 9 | | | | | | | |

$$a_{9} \longleftrightarrow w_{9} = 11$$

$$a_{8} \longleftrightarrow w_{8} = 9$$

$$a_{7} \longleftrightarrow w_{7} = 2$$

$$a_{6} \longleftrightarrow w_{6} = 12$$

$$a_{5} \longleftrightarrow w_{5} = 3$$

$$a_{4} \longleftrightarrow w_{4} = 7$$

$$a_{3} \longleftrightarrow w_{3} = 4$$

$$a_{2} \longleftrightarrow w_{2} = 6$$

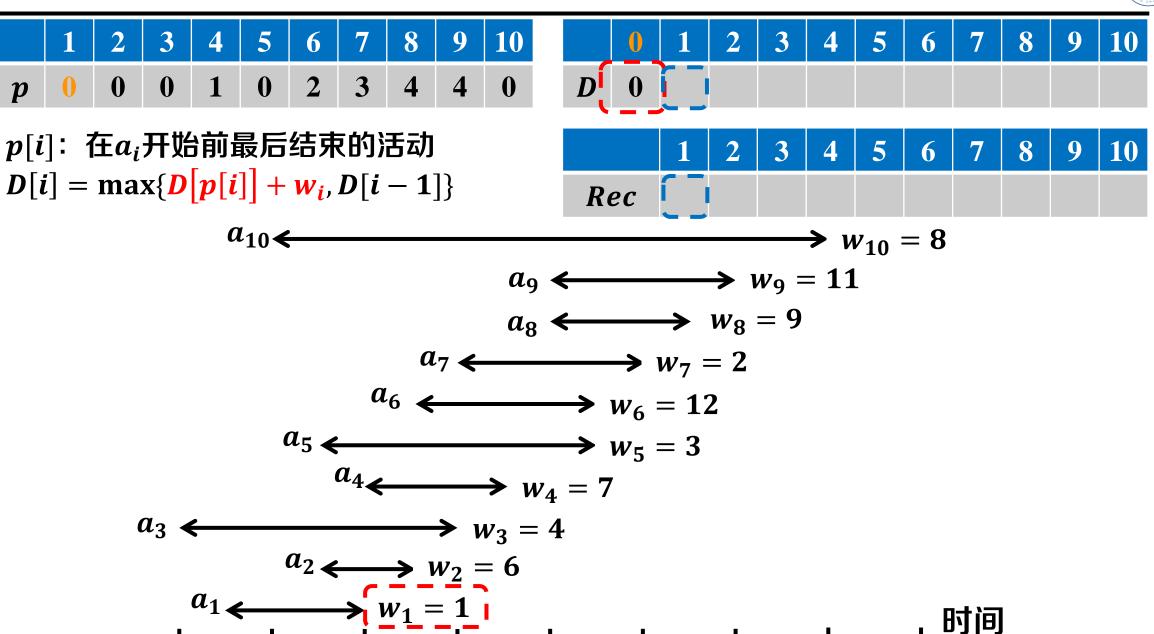
$$a_{1} \longleftrightarrow w_{1} = 1$$

$$||f|||$$

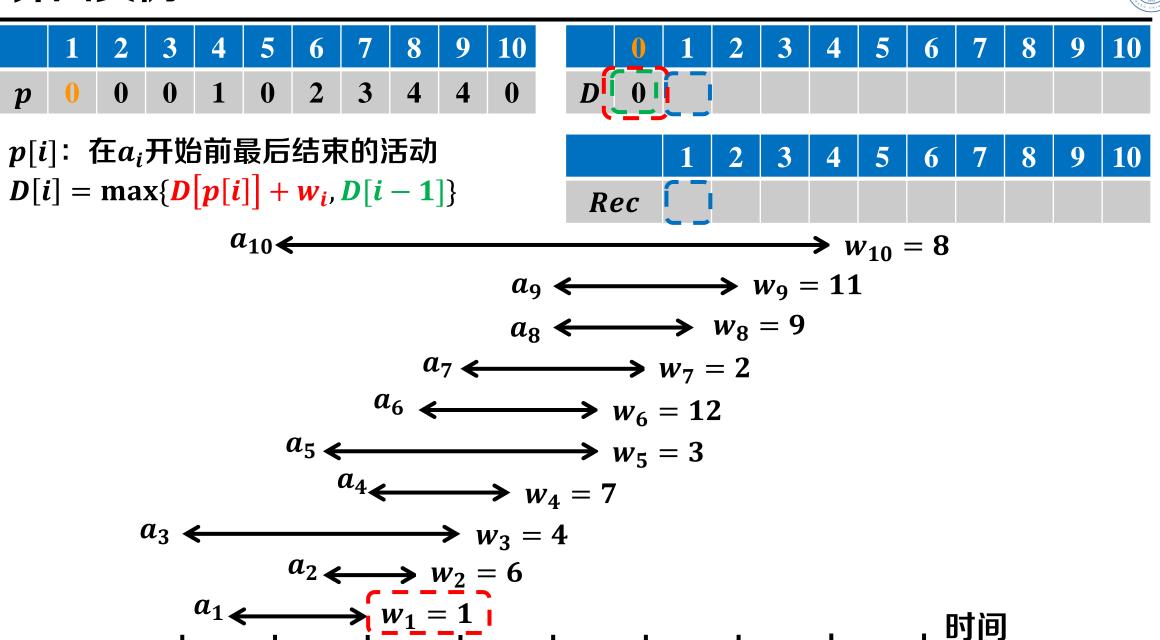


| | | | | | | | | | | | | | | | | | | | | | | | | 9 1952 4 NO UNIN |
|----|---------------|----|----------------------------|-------------|-------------------|------------------|------------------|-------------------------|-------------------------|-----------------|------------|-------------|-------|-------|---|---|-----|--------------|---|-----|----|---|---|---------------------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | | 0 | 1 | | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| p | 0 | 0 | 0 | 1 | 0 | 2 | 3 | 4 | 4 | 0 | | D | 0 | | | | | | | | | | | |
| | [] : 1 | - | | | | | | | | | | | | 1 | | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| D[| <i>i</i>] = | ma | x { <i>D</i> | p [i |]] + | w_i , | D[i] | -1 |]} | | | R | ec | | | | | | | | | | | |
| | | | | a | l ₁₀ ∢ | • | | | | | | | | | | | | | | = 8 | | | | |
| | | | | | | | | | | a_{ς} | | | | | | | - | = 1 : | 1 | | | | | |
| | | | | | | | | a | l- 0 | a_8 | • | | | | | | = 9 | | | | | | | |
| | | | | | | | a | 6 ← | ¹ 7 ← | | | | | w_7 | | | | | | | | | | |
| | | | | | | a ₅ ∢ | | | | | | → | w_6 | | | | | | | | | | | |
| | | | | | | | a ₄ ← | | | → v | 7 4 | | J | _ | • | | | | | | | | | |
| | | • | $a_3 \blacktriangleleft$ | | | | | | | $w_3 =$ | 4 | Ļ | | | | | | | | | | | | |
| | | | | | | | • | | | = 6 | | | | | | | | | | | | | | |
| | | | | a_1 | - | | → 1 | <i>w</i> ₁ = | = 1 | | | | | | | | | | | , | 时间 |] | | |











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|---------------|----|----|--------------------------|--------------|-------------------|--------------------------------|-------------------------|------------------|-------------------------|----------------|-------|-------------------------------------|----------|----------|------------------|--------------|----------|-----|----|---|---|--|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| p | 0 | 0 | 0 | 1 | 0 | 2 | 3 | 4 | 4 | 0 | | $D \begin{bmatrix} 0 \end{bmatrix}$ | 1 | | | | | | | | | |
| | _ | • | | | 最后 | | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| D[i] | | ma | $\mathbf{X}\{D$ | [p[i |]]+ | w_i , | D[i | -1 |]} | | | Rec | 1 | | | | | | | | | |
| | | | | a | l ₁₀ € | • | | | | | | | | | | → v | v_{10} | = 8 | | | | |
| | | | | | | | | | | a_{9} | , • | | | → | w ₉ = | = 1 1 | l | | | | | |
| | | | | | | | | | | a_{ϵ} | 3 | | → | w_8 | = 9 | | | | | | | |
| | | | | | | | | a | ¹ 7 ← | | | \longrightarrow | w_7 | = 2 | | | | | | | | |
| | | | | | | | a | 6 € | 1 | | | $\rightarrow w_6$ | = 1 | 2 | | | | | | | | |
| | | | | | (| <i>a</i> ₅ ∢ | | | | | | $\rightarrow w_5$ | = 3 | | | | | | | | | |
| | | | | | | (| <i>a</i> ₄ ← | | | → v | v_4 | _. = 7 | | | | | | | | | | |
| | | (| $a_3 \blacktriangleleft$ | \leftarrow | | | | | > 1 | $w_3 =$ | = 4 | 1 | | | | | | | | | | |
| | | | | | (| a_2 | | → _! | w ₂ = | = 6 | | | | | | | | | | | | |
| | | | | a_1 | - | | → [! | w ₁ = | = 1 | ì | | | | | | | | . 6 | 时间 |] | | |

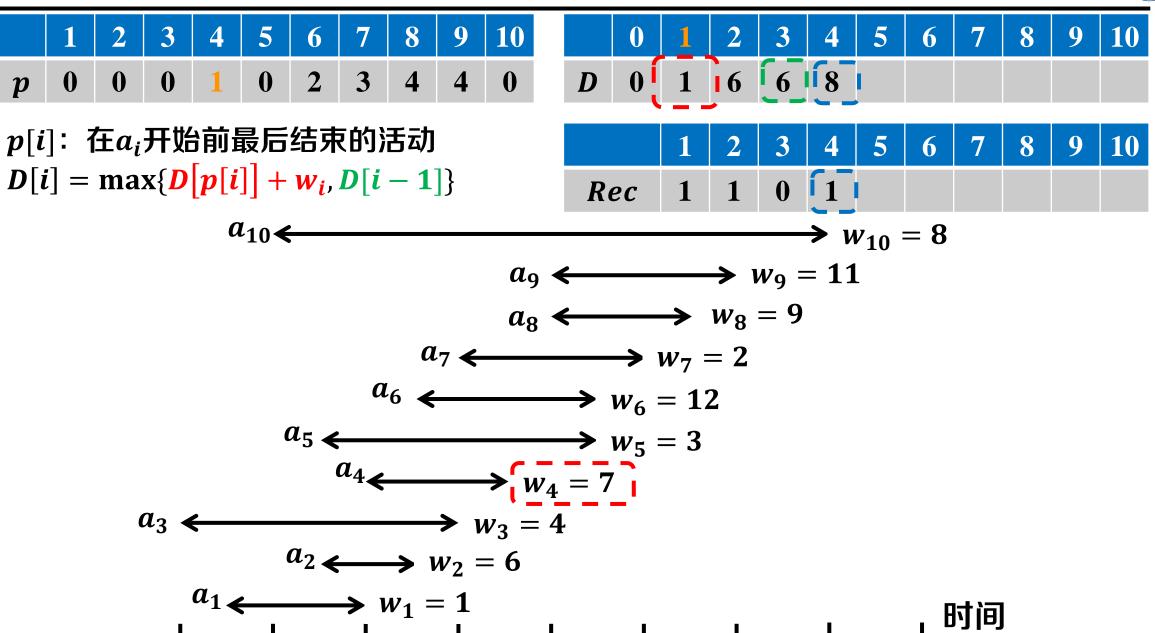


| | | | | | | | | | | | | | | | | | | | | | | | | 9 1952 1/0 UKINE |
|------------------|-----------------------------|---|--------------------------|-------------|--------------|--------------------------------|------------------|------------|-------------------------|------------------|----------------|---|-------------|----------|----|---|---|----|---|-----|----|---|---|---------------------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | | 0 | 1 | 2 | 3 | 2 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| \boldsymbol{p} | 0 | 0 | 0 | 1 | 0 | 2 | 3 | 4 | 4 | 0 | | D | 0 | 1 | 16 | | | | | | | | | |
| | _ | - | | | | | | | | | | | | 1 | 2 | 3 | 2 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| D[i] | $p[i]$: 在 a_i 开始前最后结束的活动 | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | , | | | | | | _ | | 11 | | | | | | |
| | | | | | | | | a | ر م _ا | a_8 | * | | | → | | | 9 | | | | | | | |
| | | | | | | | a | 6 ← | 7 ← | | | | | w_7 | | | | | | | | | | |
| | | | | | | a ₅ ∢ | | | | | | | w_6 w_5 | | | | | | | | | | | |
| | | | | | | | a ₄ ← | | | → и | ⁷ 4 | | J | — J | , | | | | | | | | | |
| | | (| $a_3 \blacktriangleleft$ | | | | | | | v ₃ = | | ı | | | | | | | | | | | | |
| | | | | a | | <i>a</i> ₂ ∢ | • | • | <i>W</i> ₂ = | = 6 | | | | | | | | | | | | | | |
| | | | | a_1 | - | | → 1 | v_1 = | = 1 | | | | | | | | | • | | . [| 时间 |] | | |



| | <i></i> | <u> </u> | | | | | | | | | | | | | | | | | | | | | # 1952 V V V V V V V V V V V V V V V V V V V |
|-------------|--|----------|--------------------------|-------|-------------|-------------------------|------------------|-------------------------|-------------------------|------------|--------------|-------------|----------|----------|-------|-----|---|---|-----|----|---|---|--|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| p | 0 | 0 | 0 | 1 | 0 | 2 | 3 | 4 | 4 | 0 | | D_{ζ} | 0 | 1 | 6 | 6 | | | | | | | |
| | | • | | | 最后:]] + | | | | | | ı | R | e.c | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | $D[i] = \max\{D[p[i]] + w_i, D[i-1]\}$ $a_{10} \longleftrightarrow w_{10} = 8$ | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | a_8 | « | | | → | w_8 | = 9 |) | | | | | | |
| | | | | | | | | a | ¹ 7 ← | | | | → | w_7 | = 2 | | | | | | | | |
| | | | | | | | a | 6 € | | | | → | w_6 | = 1 | 2 | | | | | | | | |
| | | | | | (| ¹ 5 ← | | | | | | → | w_5 | = 3 | | | | | | | | | |
| | | | | | | (| a ₄ ← | | | → v | 7 4 : | = 7 | 7 | | | | | | | | | | |
| | | (| $a_3 \blacktriangleleft$ | | | | | | → [] | $v_3 =$ | 4 | ì | | | | | | | | | | | |
| | | | | | (| $a_2 \leftarrow$ | | → 1 | w_2 = | = 6 | | | | | | | | | | | | | |
| | | | _ | a_1 | <u>.</u> | | → 1 | <i>w</i> ₁ = | = 1 | | _ | | _ | | _ | | _ | | . F | 可怕 | 1 | | |







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|---|--|---|--------------------------|-------------|--------------------------|--------------------------------|------------------|------------|------------------|------------|-----------------------|----------|---------------|------------|-----|---|------------|----------|-----|----|---|---|----------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| p | 0 | 0 | 0 | 1 | 0 | 2 | 3 | 4 | 4 | 0 | | D | 0 | 1 | 6 | 6 | 8 | 8 | | | | | |
| |]: 1 | • | | | | | | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $D[i] = \max\{D[p[i]] + w_i, D[i-1]\}$ $a_{10} \leftarrow \longrightarrow w_{10} = 8$ | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | a | ^l 10 ≺ | • | | | | | | | | | | | → 1 | w_{10} | = 8 | | | | |
| | | | | | | | | | | a_9 | ← | | | | | - | | 1 | | | | | |
| | $a_9 \longleftrightarrow w_9 = 11$ $a_8 \longleftrightarrow w_8 = 9$ | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | ¹ 7 ← | | | | \rightarrow | w_7 | = 2 | | | | | | | | |
| | | | | | | | a | 6 € | • | | | → | w_6 | <u>= 1</u> | 2 | | | | | | | | |
| | | | | | (| <i>a</i> ₅ € | <u>α</u> . | | | | | • | | = 3 | 1 | | | | | | | | |
| | | | | | | | a ₄ ← | | | → и | V ₄ | = 7 | 7 | | | | | | | | | | |
| | | (| $a_3 \blacktriangleleft$ | | | ~ | | | | $w_3 =$ | · 4 | ı | | | | | | | | | | | |
| | | | | ~ | | <i>a</i> ₂ ← | | → 1 | $w_2 =$ | = 6 | | | | | | | | | | | | | |
| | | | ı | a_1 | | | → \ | v_1 = | = 1 | | | | | | | | | | , 6 | 时间 |] | | |



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|-----------------------------|----|---|--------------------------|-------------|--------------------------|--------------------------------|-------------------------|------------|------------------|---------|----------------|-----|----------|----------|----------|------|------------|------------------------|-----|----------------|---|---|--------------------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| p | 0 | 0 | 0 | 1 | 0 | 2 | 3 | 4 | 4 | 0 | | D | 0 | 1 | 6 | 6 | 8 | 8 | 18 | | | | |
| | | - | | | | | | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $p[i]$: 在 a_i 开始前最后结束的活动 | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | a | ^l 10 ≺ |) | | | | | | | | | | | → 1 | <i>w</i> ₁₀ | = 8 | | | | |
| | | | | | | | | | | a_9 | ← | | | | → | W9 = | = 1 | 1 | | | | | |
| | | | | | | | | | | a_8 | • | | | → | w_8 | = 9 | 1 | | | | | | |
| | | | | | | | | | ¹ 7 ← | | | | → | w_7 | = 2 | | | | | | | | |
| | | | | | | | a | 6 € | • | | | →; | w_6 | = 1 | 2 | | | | | | | | |
| | | | | | (| <i>a</i> ₅ ← | , | | | | | • | w_5 | | | | | | | | | | |
| | | | | | | (| <i>a</i> ₄ ← | | | → v | V ₄ | = 7 | 7 | | | | | | | | | | |
| | | (| $a_3 \blacktriangleleft$ | | | | | | → 1 | $v_3 =$ | 4 | • | | | | | | | | | | | |
| | | | | | | $a_2 \leftarrow$ |) | → 1 | $w_2 =$ | = 6 | | | | | | | | | | | | | |
| | | | | a_1 | | | → 1 | w_1 = | = 1 | | | | | | | | | | , 1 | 对间 |] | | |
| | | | | | | | | | | | | | | | | | | | Ι. | <u>, הוע כ</u> | • | | |



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|---|---|---|--------------------------|-------------------------|--|--------------------------------|------------------|---------|------------|---------------|------------|----------|-------|-----|----------|------|--------------|---|----|----|---|---|----------------------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| p | 0 | 0 | 0 | 1 | 0 | 2 | 3 | 4 | 4 | 0 | | D | 0 | 1 | 6 | 6 | 18 | 8 | 18 | 18 | | | |
| |]: 1 | • | | | | | | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $D[i] = \max\{D[p[i]] + w_i, D[i-1]\}$ $a_{10} \longleftrightarrow w_{10} = 8$ $a_{10} \longleftrightarrow w_{10} = 11$ | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | $a_{\dot{q}}$ | 9 ◀ | | | | → | W9 : | = 1 1 | L | | | | | |
| | | | | | | | | | | a_8 | 3 ◀ | | | | | |) | | | | | | |
| | $a_{8} \longleftrightarrow w_{8} = 9$ $a_{7} \longleftrightarrow w_{7} = 2$ | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | 6 |) | | | → | w_6 | = 1 | 2 | | | | | | | | |
| | | | | | (| <i>a</i> ₅ € | , | | | | | → | w_5 | = 3 | | | | | | | | | |
| | | | | | | | α ₄ ← | | | → v | v_4 | = 7 | 7 | | | | | | | | | | |
| | | (| $a_3 \blacktriangleleft$ | | | ~ | | | | $w_3 =$ | = 4 | • | | | | | | | | | | | |
| | | | | ~ | | <i>a</i> ₂ ← | | | | = 6 | | | | | | | | | | | | | |
| | | | ı | <i>a</i> ₁ ∢ | : | | → ۱ | w_1 = | = 1 | | ı | | ı | | I | | ı | | , | 时间 | J | | |



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|---|--|---|--------------------------|--------------|----|--------------------------------|-------------------------|-------------------------|-------------------------|---------|-----------------------|----------|----------|---------|-----|---|---|---|-----|----|----|---|--------------------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| p | 0 | 0 | 0 | 1 | 0 | 2 | 3 | 4 | 4 | 0 | | D | 0 | 1 | 6 | 6 | 8 | 8 | 18 | 18 | 18 | | |
| | _ | - | | | 最后 | | | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $D[i] = \max\{D[p[i]] + w_i, D[i-1]\}$ $a_{10} \leftarrow \longrightarrow w_{10} = 8$ | | | | | | | | | | | | | | | | | | | | | | | |
| | $a_{10} \leftarrow \longrightarrow w_{10} = 8$ | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | a_{9} | ◆ | | | | | | | 1 | | | | | |
| | $a_{9} \longleftrightarrow w_{9} = 11$ $a_{8} \longleftrightarrow w_{8} = 9$ | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | ¹ 7 ← | | | | → | w_7 : | = 2 | | | | | | | | |
| | | | | | | | a | ¹ 6 ← |) | | | → | w_6 | = 1 | 2 | | | | | | | | |
| | | | | | (| <i>a</i> ₅ ← | , | | | | | → | w_5 | = 3 | | | | | | | | | |
| | | | | | | (| <i>a</i> ₄ € | | | → v | <i>V</i> ₄ | = 7 | 7 | | | | | | | | | | |
| | | (| $a_3 \blacktriangleleft$ | \leftarrow | | | | | → 1 | $w_3 =$ | 4 | • | | | | | | | | | | | |
| | | | | | | $a_2 \leftarrow$ | • | → 1 | w_2 = | = 6 | | | | | | | | | | | | | |
| | | | | a_1 | | | → 1 | w_1 = | = 1 | | | | | | | | | | , 6 | 时间 |] | | |
| | | | | | | | | | | | | | | | | | | | | | | | |



时间

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|---|------------------------------|-----------------------------|---|---|---|--|---|--|---|---|--|--|--|--|--|--|--|--|---|--|---|
| 1 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| p 0 0 0 1 0 2 3 4 4 0 D 0 1 6 6 8 18 18 18 18 19 p[i]: 在a _i 开始前最后结束的活动 1 2 3 4 5 6 7 8 9 | | | | | | | | | | | | | | | | | | | | | |
| $D[i] = \max(D[n[i]] \perp w D[i = 1])$ | | | | | | | | | | | | | | | 10 | | | | | | |
| $D[i] = \max\{D[p[i]] + w_i, D[i-1]\}$ $Rec 1 1 0 1 0 0 1$ | | | | | | | | | | | | | | | | | | | | | |
| | | a | ^l 10 ≺ | | | | | | | | | | | | | | = 8 | | | | |
| | | | | | | | | - | | | | | • | | | L _I | | | | | |
| | | | | | | a | | a_8 | • | | | | | = 9 | | | | | | | |
| | | | | | a | | | | | <u> </u> | | • | | | | | | | | | |
| | | | (| a ₅ | | | | | | | | | 2 | | | | | | | | |
| | | | | (| <i>a</i> ₄ ← | | | → и | ⁷ 4 | | | | | | | | | | | | |
| | $a_3 \blacktriangleleft$ | | | | | | > v | | | | | | | | | | | | | | |
|) | 1 2 0 0 在a _i = ma |) 0 0 在a _i 开始 | 1 2 3 4 0 0 0 1 在a _i 开始前的是 = max{D[p[i | 【 2 3 4 5) 0 0 1 0 在a _i 开始前最后 = max{D[p[i]] + a ₁₀ ◀ | 【 2 3 4 5 6) 0 0 1 0 2 在 a_i 开始前最后结束 = max{ $D[p[i]] + w_i$, a_{10} ← | $egin{array}{c c c c c c c c c c c c c c c c c c c $ | 1 2 3 4 5 6 7 8 0 0 0 1 0 2 3 4 在a _i 开始前最后结束的活动 = max{D[p[i]] + w _i , D[i − 1] a ₁₀ a ₆ a ₇ a ₈ a ₈ a ₈ a ₈ | $\begin{bmatrix} 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 0 & 0 & 0 & 1 & 0 & 2 & 3 & 4 & 4 \end{bmatrix}$ | $egin{array}{c ccccccccccccccccccccccccccccccccccc$ | $egin{array}{c ccccccccccccccccccccccccccccccccccc$ | $egin{aligned} egin{aligned} 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 0 & 0 & 0 & 1 & 0 & 2 & 3 & 4 & 4 & 0 & D \\ \hline & & & & & & & & & & & & \\ & & & & &$ | a_{1} a_{2} a_{3} a_{4} a_{5} a_{6} a_{7} a_{8} a_{9} a_{10} | 2 3 4 5 6 7 8 9 10 0 1 0 0 0 1 0 2 3 4 4 0 0 0 1 0 | a_{1} a_{2} a_{3} a_{4} a_{5} a_{6} a_{7} a_{8} a_{9} a_{10} | a_{1} a_{2} a_{3} a_{4} a_{5} a_{6} a_{7} a_{8} a_{9} a_{10} | は 2 3 4 5 6 7 8 9 10 0 1 2 3 4 0 0 0 1 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 | 在 a_i 开始前最后结束的活动 a_1 a_2 a_3 a_4 a_5 a_6 a_7 a_8 a_9 a_9 a_8 a_9 | a_{1} a_{2} a_{3} a_{4} a_{5} a_{6} a_{7} a_{8} a_{9} a_{10} | a_{1} a_{2} a_{3} a_{4} a_{5} a_{7} a_{1} a_{1} a_{2} a_{3} a_{4} a_{5} a_{7} a_{1} a_{1} a_{2} a_{3} a_{4} a_{5} a_{7} a_{1} a_{1} a_{1} a_{2} a_{3} a_{4} a_{5} a_{7} a_{1} a_{1} a_{1} a_{1} a_{2} a_{3} a_{4} a_{5} a_{7} a_{1} a_{1} a_{1} a_{1} a_{2} a_{3} a_{4} a_{5} a_{7} a_{1} a_{1} a_{1} a_{1} a_{2} a_{3} a_{4} a_{5} a_{7} a_{1} a_{1} a_{1} a_{1} a_{2} a_{3} a_{4} a_{5} a_{7} $a_{$ | 2 3 4 5 6 7 8 9 10 0 1 2 3 4 5 6 7 8 0 0 0 1 0 2 3 4 4 0 D 0 1 6 6 8 8 18 18 18 18 18 18 18 18 18 18 18 18 | $egin{array}{c ccccccccccccccccccccccccccccccccccc$ |

 $a_2 \longleftrightarrow w_2 = 6$

 $a_1 \longleftrightarrow w_1 = 1$



| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----|--------------|----|--------------------------|-------------------------|--------------------------|--------------------------------|------------------|-------------------------|-------------------------|---------|-------------|-----------------------|-----------------------|-----------------------|-----|-------------|---|-----|----|----|----|----|
| p | 0 | 0 | 0 | 1 | 0 | 2 | 3 | 4 | 4 | 0 | D | 0 | 1 | 6 | 6 | 8 | 8 | 18 | 18 | 18 | 19 | 19 |
| |]: 1 | • | | | | | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| D[| <i>i</i>] = | ma | $\mathbf{X}\{\mathbf{D}$ | p[i] |]]+ | w_i , | D[i | -1 |]} | | R | ec | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | |
| | | | | a | ^l 10 ≺ | • | | | | | | | | | | → [и | | = 8 | ì | | | |
| | | | | | | | | | | a_9 | | | | | - | = 11 | L | | | | | |
| | | | | | | | | 0 | | a_8 | | | | <i>w</i> ₈ | = 9 | | | | | | | |
| | | | | | | | a | | ¹ 7 ← | | | | <i>W</i> ₇ | | | | | | | | | |
| | | | | | | a ₅ | | 0 | | | | _ | = 1 | Z | | | | | | | | |
| | | | | | | _ | a ₄ ← | | | → w. | | <i>w</i> ₅ | = 3 | | | | | | | | | |
| | | (| $a_3 \blacktriangleleft$ | | | | | | → 1 | $v_3 =$ | 4 | | | | | | | | | | | |
| | | | | | | <i>a</i> ₂ ← | | → 1 | $w_2 =$ | = 6 | | | | | | | | | | | | |
| | | | ı | <i>a</i> ₁ ∢ | <u> </u> | | → \ | <i>w</i> ₁ = | = 1 I | | I | ı | | ı | | ı | | , E | 时间 | | | |



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|---|------------------------------------|-------|--------------------------|-------------|--------------------------|--------------------------------|-------------------------|------------|------------------|------------|----------|----------|----------|------------------|------------|---|------|-----|-----|----|----|----|------------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| p | 0 | 0 | 0 | 1 | 0 | 2 | 3 | 4 | 4 | 0 | | D | 0 | 1 | 6 | 6 | 8 | 8 | 18 | 18 | 18 | 19 | 19 |
| | | | | | | | | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | | | | | | | | | | | | R | ec | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 7 | 0 |
| | | | | a | ^l 10 ≺ |) | | | | | | | | | | | → 1/ | V40 | = 8 | | | | |
| | | | | | 10 | • | | | | a | | • | | | | | | | _ 0 | 重 | 號 | 解 | |
| | $u_9 \longleftrightarrow w_9 = 11$ | | | | | | | | | | | | | | | | | | | | | | |
| | $a_8 \longleftrightarrow w_8 = 9$ | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | a | ¹ 7 ← | | | | ~ | w ₇ : | = 2 | | | | | | | | |
| | | | | | | | a | 6 € |) | | | → | w_6 | = 1 | 2 | | | | | | | | |
| | | | | | (| <i>a</i> ₅ ← |) | | | | | → | W_5 | = 3 | | | | | | | | | |
| | | | | | | | <i>a</i> ₄ ← | | | → v | | | • | | | | | | | | | | |
| | | (| $a_3 \blacktriangleleft$ | | | | | | → 1 | $w_3 =$ | 4 | • | | | | | | | | | | | |
| | | | | | | $a_2 \leftarrow$ | • | → 1 | w_2 = | = 6 | | | | | | | | | | | | | |
| | | | | a_1 | _ | | _ | | = 1 | | ı | | | | | | ı | | , E | 时间 | | | |



| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|------------------|---|---|---|---|---|---|---|---|---|----|----|----|---|---|---|---|---|----|----|----|----|-----------------------------------|
| \boldsymbol{p} | 0 | 0 | 0 | 1 | 0 | 2 | 3 | 4 | 4 | 0 | D | 0 | 1 | 6 | 6 | 8 | 8 | 18 | 18 | 18 | 19 | 19 |
| · 活动集合S' = {} | | | | | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | |
| | | | | | | | | | | | Re | ec | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | $\begin{bmatrix} 0 \end{bmatrix}$ |

$$a_{10} \longleftrightarrow w_{10} = 8$$

$$a_{9} \longleftrightarrow w_{9} = 11$$

$$a_{8} \longleftrightarrow w_{8} = 9$$

$$a_{7} \longleftrightarrow w_{7} = 2$$

$$a_{6} \longleftrightarrow w_{6} = 12$$

$$a_{5} \longleftrightarrow w_{5} = 3$$

$$a_{4} \longleftrightarrow w_{4} = 7$$

$$a_{3} \longleftrightarrow w_{2} = 6$$

$$a_{1} \longleftrightarrow w_{1} = 1$$

$$| \textbf{bij} |$$



| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|----|----|-------|-----------|---|---|---|---|---|----|---|----|---|---|---|---|---|----|----|----|----|----|
| p | 0 | 0 | 0 | 1 | 0 | 2 | 3 | 4 | 4 | 0 | D | 0 | 1 | 6 | 6 | 8 | 8 | 18 | 18 | 18 | 19 | 19 |
| 活 | 动集 | 合S | ' = · | $\{a_9\}$ | - | | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | | | | | | | | | | | R | ec | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 |

$$a_{10}$$
 $w_{10} = 8$

$$a_{9} \longleftrightarrow w_{9} = 11$$

$$a_{8} \longleftrightarrow w_{8} = 9$$

$$a_{7} \longleftrightarrow w_{7} = 2$$

$$a_{6} \longleftrightarrow w_{6} = 12$$

$$a_{5} \longleftrightarrow w_{5} = 3$$

$$a_{4} \longleftrightarrow w_{4} = 7$$

$$a_{3} \longleftrightarrow w_{2} = 6$$

$$a_{1} \longleftrightarrow w_{1} = 1$$

时间



| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|----|------------|-----|-----------|---|---|---|---|---|----|---|----|---|---|---|---|---|----|----|----|----|----|
| p | 0 | 0 | 0 | 1 | 0 | 2 | 3 | 4 | 4 | 0 | D | 0 | 1 | 6 | 6 | 8 | 8 | 18 | 18 | 18 | 19 | 19 |
| 活 | 动集 | 合 <i>S</i> | ′ = | $\{a_9\}$ | } | | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | | | | | | | | | | | R | ec | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 |

$$a_{10}$$
 $w_{10} = 8$

$$a_{9} \longleftrightarrow w_{9} = 11$$

$$a_{8} \longleftrightarrow w_{8} = 9$$

$$a_{7} \longleftrightarrow w_{7} = 2$$

$$a_{6} \longleftrightarrow w_{6} = 12$$

$$a_{5} \longleftrightarrow w_{5} = 3$$

$$a_{4} \longleftrightarrow w_{4} = 7$$

$$a_{3} \longleftrightarrow w_{2} = 6$$

$$a_{1} \longleftrightarrow w_{1} = 1$$

时间



| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|-----------------------|---|---|---|---|---|---|---|---|----|--|---|----|---|---|---|-----------------------------------|---|----|----|----|----|----|
| p | 0 | 0 | 0 | 1 | 0 | 2 | 3 | 4 | 4 | 0 | | D | 0 | 1 | 6 | 6 | 8 | 8 | 18 | 18 | 18 | 19 | 19 |
| 活 | 活动集合 $S'=\{a_4,a_9\}$ | | | | | | | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | | | | | | | | | | | | R | ec | 1 | 1 | 0 | $\begin{bmatrix} 1 \end{bmatrix}$ | 0 | 1 | 0 | 0 | 1 | 0 |



| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|-----------------------|---|---|---|---|---|---|---|---|---|----|---|----|---|---|---|-----------------------------------|---|----|----|----|----|----|
| p | 0 | 0 | 0 | 1 | 0 | 2 | 3 | 4 | 4 | 0 | D | 0 | 1 | 6 | 6 | 8 | 8 | 18 | 18 | 18 | 19 | 19 |
| 活动集合 $S'=\{a_4,a_9\}$ | | | | | | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | |
| | | | | | | | | | | | R | ec | 1 | 1 | 0 | $\begin{bmatrix} 1 \end{bmatrix}$ | 0 | 1 | 0 | 0 | 1 | 0 |

$$a_{10}$$
 $w_{10} = 8$

$$a_{9} \longleftrightarrow w_{9} = 11$$

$$a_{8} \longleftrightarrow w_{8} = 9$$

$$a_{7} \longleftrightarrow w_{7} = 2$$

$$a_{6} \longleftrightarrow w_{6} = 12$$

$$a_{5} \longleftrightarrow w_{5} = 3$$

$$a_{4} \longleftrightarrow w_{4} = 7$$

$$a_{3} \longleftrightarrow w_{2} = 6$$

$$a_{1} \longleftrightarrow w_{1} = 1$$

时间



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|---------------------|---|---|------|---|---|-------------------------|---|-----|---|-------|----------|---------------|---------------|----------|------------|------|--------------|---|----|----|----|----|--------------------------|--|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | |
| p | 0 | 0 | 0 | 1 | 0 | 2 | 3 | 4 | 4 | 0 | | D | 0 | 1 | 6 | 6 | 8 | 8 | 18 | 18 | 18 | 19 | 19 | |
| | | | | | | | | | | | | | | | 10 | | | | | | | | | |
| Rec 1 1 0 1 0 0 1 0 | | | | | | | | | | | | | | | 0 | | | | | | | | | |
| | $a_{10} \leftarrow \qquad \qquad \qquad \Rightarrow w_{10} = 8$ | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | a_9 | ◆ | | | | → 1 | W9 = | = 1 1 | 1 | | | | | | |
| | | | | | | | | | | _ | | | | → | w_8 | = 9 | | | | | | | | |
| | | | | | | | _ | | | | | | \rightarrow | w_7 | = 2 | | | | | | | | | |
| | | | | | | | | 6 € |) | | | → | w_6 | = 12 | 2 | | | | | | | | | |
| | | | | | | a ₅ ∢ | • | | | | | \rightarrow | w_5 | = 3 | | | | | | | | | | |

$$a_{4} \longleftrightarrow w_{4} = 7$$

$$a_{3} \longleftrightarrow w_{3} = 4$$

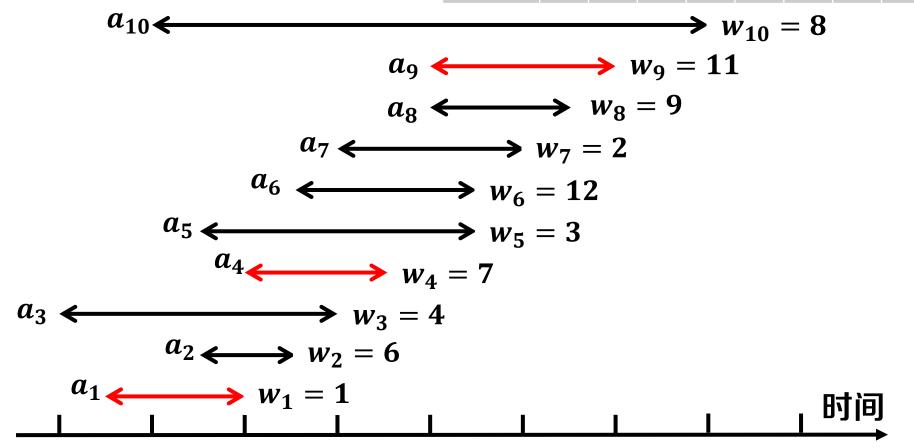
$$a_{2} \longleftrightarrow w_{2} = 6$$

$$a_{1} \longleftrightarrow w_{1} = 1$$

时间



| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---------------------------|---|---|---|---|---|---|---|---|---|----|---|----|---|---|---|---|---|----|----|----|----|----|
| p | 0 | 0 | 0 | 1 | 0 | 2 | 3 | 4 | 4 | 0 | D | 0 | 1 | 6 | 6 | 8 | 8 | 18 | 18 | 18 | 19 | 19 |
| 活动集合 $S'=\{a_1,a_4,a_9\}$ | | | | | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | |
| | | | | | | | | | | | R | ec | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 |





```
输入: 活动集合S = \{a_1, a_2, ..., a_n\},
    每个活动a_i的起止时间s_i, f_i和权重w_i
输出: 不冲突活动的最大子集S'
//预处理
把活动按照结束时间升序排序
                             预处理
for i \leftarrow 1 to n do
  二分查找求解p[i]
end
//初始化
新建数组D[0..n], Rec[1..n]
D[0] \leftarrow 0
```



```
输入: 活动集合S = \{a_1, a_2, ..., a_n\},
    每个活动a_i的起止时间s_i, f_i和权重w_i
输出: 不冲突活动的最大子集S'
//预处理
把活动按照结束时间升序排序
for i \leftarrow 1 to n do
  二分查找求解p[i]
end
//初始化
新建数组D[0..n], Rec[1..n]
                            初始化
D[0] \leftarrow 0
```



```
//动态规划
for j \leftarrow 1 to n do
                                               对每个子问题
  if D[p[j]] + w_j > D[j-1] then
       D[j] \leftarrow D[p[j]] + w_j
        Rec[j] \leftarrow 1
    end
    else
       D[j] \leftarrow D[j-1]
       Rec[j] \leftarrow 0
    end
end
```



//动态规划

```
for j \leftarrow 1 to n do
   (if D[p[j]] + w_j > D[j-1] then
      D[j] \leftarrow D[p[j]] + w_j
   Rec[j] \leftarrow 1
   end
    else
       D[j] \leftarrow D[j-1]
       Rec[j] \leftarrow 0
    end
end
```

选择活动 a_i



```
//动态规划
for j \leftarrow 1 to n do
   if D[p[j]] + w_j > D[j-1] then
       D[j] \leftarrow D[p[j]] + w_j
       Rec[j] \leftarrow 1
    end
   else
                                                       不选活动a_i
   D[j] \leftarrow D[j-1]
   Rec[j] \leftarrow 0
    \mathbf{end}
end
```



```
//输出方案
k \leftarrow n
while k > \theta do
  if Rec[k] = 1 then
                                                                选择活动a_k
    __print_选择a[k]___
     \overline{\phantom{a}} k \leftarrow \overline{p}[\overline{k}]
    end
    else
     k \leftarrow k-1
    \mathbf{end}
end
return D[n]
```



```
//输出方案
k \leftarrow n
while k > \theta do
    if Rec[k] = 1 then
      | print 选择 a[k]   k \leftarrow p[k] 
                                                          回溯子问题
    end
    else
     k \leftarrow k-1
    \mathbf{end}
end
return D[n]
```



```
//输出方案
k \leftarrow n
while k > \theta do
    if Rec[k] = 1 then
         \operatorname{print} 选择a[k]
        k \leftarrow p[k]
    \mathbf{end}
    else
                                                              不选活动a_k
     | k \leftarrow k-1
    \mathbf{end}
end
return D[n]
```

动态规划:复杂度分析



```
输入: 活动集合S = \{a_1, a_2, ..., a_n\},
     每个活动a_i的起止时间s_i, f_i和权重w_i
输出: 不冲突活动的最大子集S'
//预处理和初始化
把活动按照结束时间升序排序 ---O(n\log n)
for i \leftarrow 1 to n do
                                      O(n \log n)
   二分查找求解p[i]
end
新建数组D[0..n], Rec[1..n]
D[0] \leftarrow 0
//动态规划
for j \leftarrow 1 to n do
   if D[p[j]] + w_j > D[j-1] then
      D[j] \leftarrow D[p[j]] + w_j
     Rec[j] \leftarrow 1
   end
                                       O(n)
   else
     D[j] \leftarrow D[j-1]
     Rec[j] \leftarrow 0
   \mathbf{end}
end
//输出方案
k \leftarrow n
while k > \theta do
   if Rec[k] = 1 then
      print 选择a[k]
      k \leftarrow p[k]
                                       O(n)
   end
   else
   k \leftarrow k-1
   \mathbf{end}
                                                                        时间复杂度: O(n \log n)
end
return D[n]
```

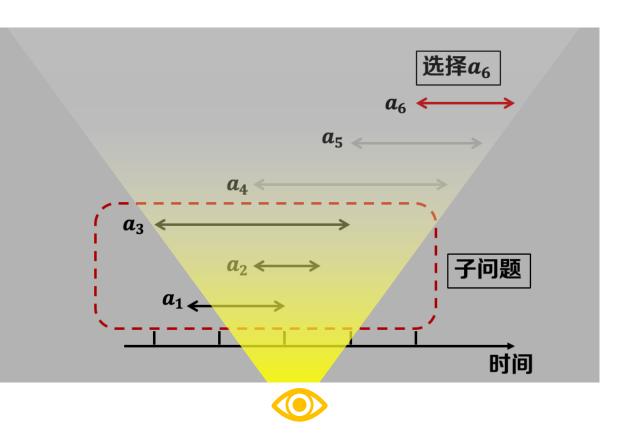
活动选择问题: 动态规划 vs. 贪心策略



带权活动选择问题

权重均为1 性质更好

活动选择问题



 $a_3 \leftarrow$ 时间

求解子问题,组合最优解

动态规划:考察全局

直接做决策,构造最优解

贪心策略:考察局部



算法设计与分析

分而治之篇

最大子数组问题 逆序计数问题

0-1背包问题

快速排序

动态规划篇

最长公共子序列问题Ⅱ最长公共子郎问题Ⅲ

最小编辑距离问题

钢条切割问题矩阵链乘法问题

贪心策略篇

部分背包问题霍夫曼编码

活动选择问题

课程总结



分而治之

动态规划

贪心策略

分解原问题

问题结构分析

提出贪心策略



₹



解决子问题

递推关系建立

证明策略正确



合并问题解

自底向上计算



最优方案追踪



