图算法篇：有向图的深度优先搜索
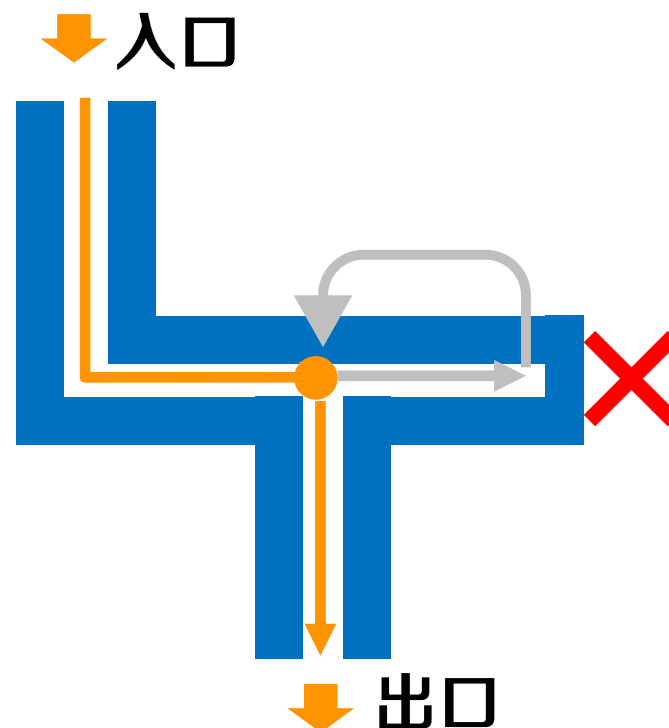
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- **算法步骤**
  - 分叉时，任选一条边深入
  - 无边时，后退一步找新边
  - 找到边，从新边继续深入

- **辅助数组**
  - *color*：表示顶点状态
    - *White*：白色顶点尚未被发现
    - *Black*：黑色顶点已被处理
    - *Gray*：正在处理，尚未完成
  - *pred*：顶点u由$pred[u]$发现
  - *d*：顶点发现时刻（变成灰色的时刻）
  - *f*：顶点完成时刻（变成黑色的时刻）

入口

出口

伪代码

● DFS(G)

输入: 图G

输出: 祖先数组$pred$, 发现时刻d, 结束时刻f

新建数组 $color[1..V]$, $pred[1..V]$, $d[1..V]$, $f[1..V]$

//初始化

for $v \in V$ **do**

    $pred[v] \leftarrow NULL$

    $color[v] \leftarrow WHITE$

end

$time \leftarrow 0$

for $v \in V$ **do**

    if $color[v] = WHITE$ **then**

        DFS-Visit(G, v)

    end

end

return $pred, d, f$

新建数组

$d[i], f[i]$分别记录顶点i的发现时刻与结束时刻

伪代码

- ## DFS(*G*)

```
输入: 图 G
输出: 祖先数组 pred，发现时刻 d，结束时刻 f
新建数组 color[1..V], pred[1..V], d[1..V], f[1..V]
//初始化
for v ∈ V do
    pred[v] ← NULL
    color[v] ← WHITE
end
time ← 0
for v ∈ V do
    if color[v] = WHITE then
        DFS-Visit(G, v)
    end
end
return pred, d, f
```

初始化

伪代码

- ## DFS(G)

输入: 图G
输出: 祖先数组$pred$，发现时刻d，结束时刻f
新建数组 $color[1..V], pred[1..V], d[1..V], f[1..V]$
//初始化
for $v \in V$ **do**
  $pred[v] \leftarrow NULL$
  $color[v] \leftarrow WHITE$
end
$time \leftarrow 0$
for $v \in V$ **do**
  if $color[v] = WHITE$ **then**
    DFS-Visit(G, v)
  end
end
return $pred, d, f$

保证搜索完全

伪代码

- ## DFS-Visit(G, v)

```
输入: 图G, 顶点v
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ G.Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(G, w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

修改当前顶点颜色、发现时刻

DFS-Visit(G, v)

```
输入: 图G, 顶点v
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ G.Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(G, w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

搜索相邻顶点

- **DFS-Visit(G, v)**

输入: 图G, 顶点v
$color[v] \leftarrow GRAY$
$time \leftarrow time + 1$
$d[v] \leftarrow time$
for $w \in G.Adj[v]$ **do**
    if $color[w] = WHITE$ **then**
        $pred[w] \leftarrow v$
        DFS-Visit(G, w)
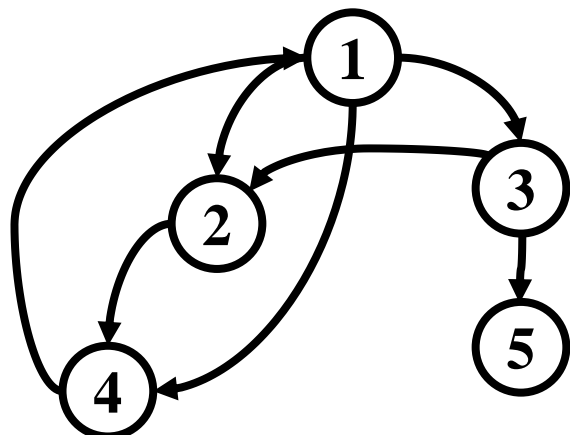   **end**
end
$color[v] \leftarrow BLACK$
$time \leftarrow time + 1$
$f[v] \leftarrow time$

结束搜索

$$time = 0$$

v	1	2	3	4	5
$pred$	N	N	N	N	N

v	1	2	3	4	5
$color$	W	W	W	W	W

v	1	2	3	4	5
d					

v	1	2	3	4	5
f					

$color[v] \leftarrow GRAY$
$time \leftarrow time + 1$
$d[v] \leftarrow time$
for $w \in Adj[v]$ **do**
$\quad$ **if** $color[w] = WHITE$ **then**
$\quad\quad pred[w] \leftarrow v$
$\quad\quad$ DFS-Visit(w)
$\quad$ **end**
end
$color[v] \leftarrow BLACK$
$time \leftarrow time + 1$
$f[v] \leftarrow time$

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
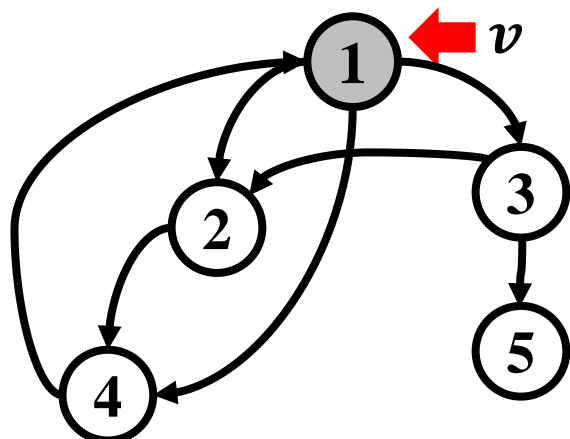
$time = 0$

v	1	2	3	4	5
$pred$	N	N	N	N	N

v	1	2	3	4	5
$color$	G	W	W	W	W

v	1	2	3	4	5
d					

v	1	2	3	4	5
f					

$time = 1$

v	1	2	3	4	5
$pred$	N	N	N	N	N

v	1	2	3	4	5
$color$	G	W	W	W	W

v	1	2	3	4	5
d					

v	1	2	3	4	5
f					

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
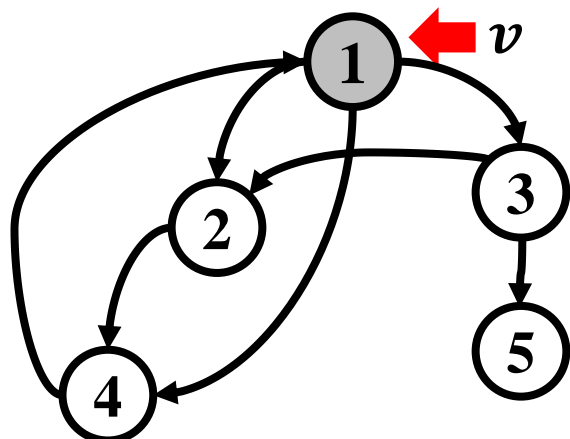
$time = 1$

v	1	2	3	4	5
$pred$	N	N	N	N	N

v	1	2	3	4	5
$color$	G	W	W	W	W

v	1	2	3	4	5
d	1				

v	1	2	3	4	5
f					

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
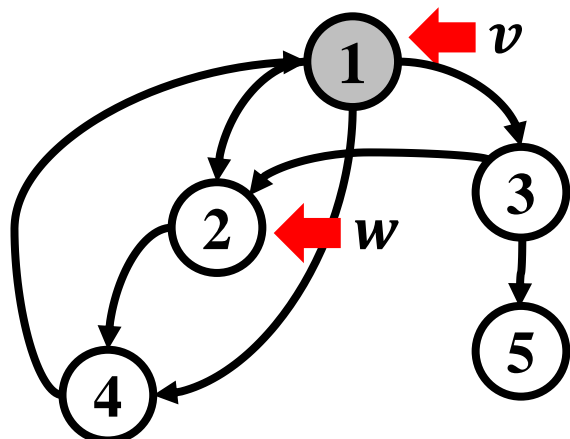
$$time = 1$$

v	1	2	3	4	5
$pred$	N	N	N	N	N

v	1	2	3	4	5
$color$	G	W	W	W	W

v	1	2	3	4	5
d	1				

v	1	2	3	4	5
f					

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
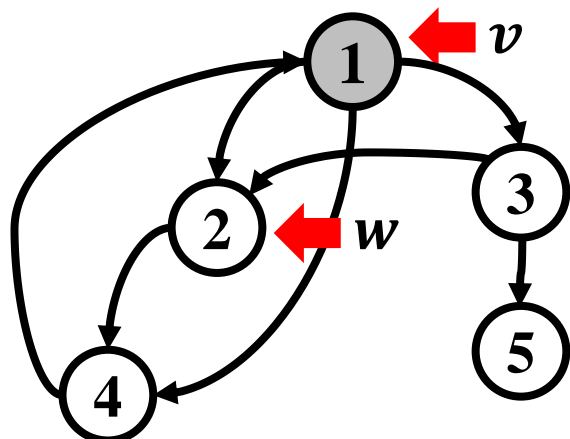
$time = 1$

v	1	2	3	4	5
$pred$	N	N	N	N	N

v	1	2	3	4	5
$color$	G	W	W	W	W

v	1	2	3	4	5
d	1				

v	1	2	3	4	5
f					

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
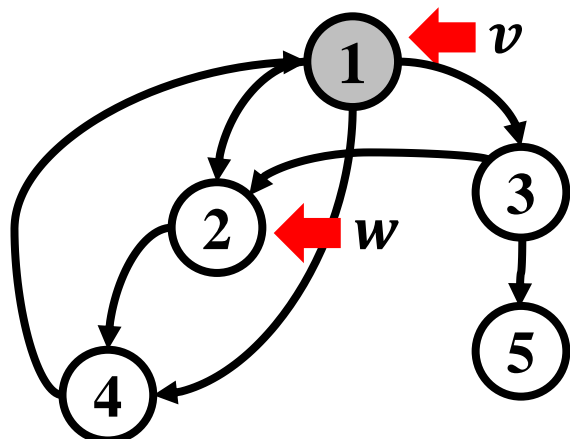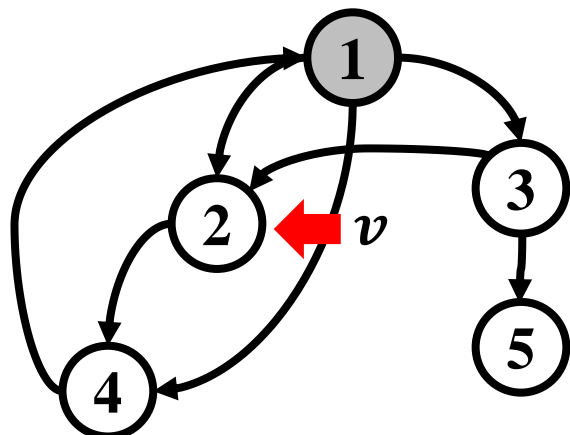
$$time = 1$$

v	1	2	3	4	5
$pred$	N	1	N	N	N

v	1	2	3	4	5
$color$	G	W	W	W	W

v	1	2	3	4	5
d	1				

v	1	2	3	4	5
f					

$time = 1$

v	1	2	3	4	5
$pred$	N	1	N	N	N

v	1	2	3	4	5
$color$	G	W	W	W	W

v	1	2	3	4	5
d	1				

v	1	2	3	4	5
f					

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$time = 1$

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
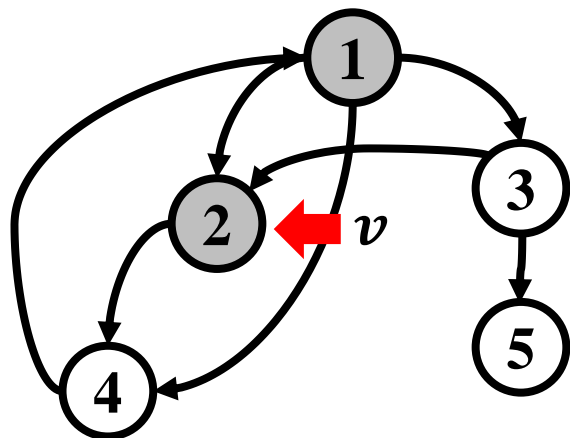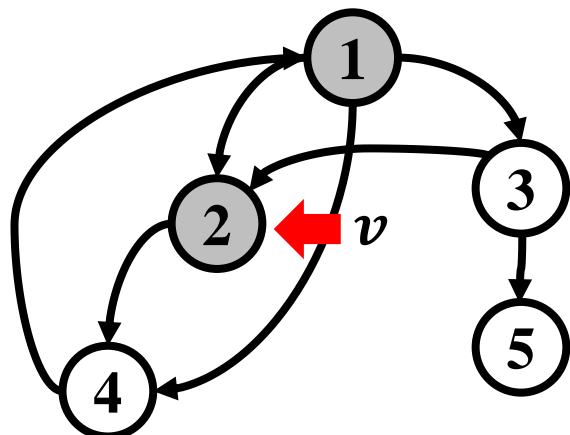
v	1	2	3	4	5
$pred$	N	1	N	N	N

v	1	2	3	4	5
$color$	G	G	W	W	W

v	1	2	3	4	5
d	1				

v	1	2	3	4	5
f					

$time = 2$

v	1	2	3	4	5
$pred$	N	1	N	N	N

v	1	2	3	4	5
$color$	G	G	W	W	W

v	1	2	3	4	5
d	1				

v	1	2	3	4	5
f					

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
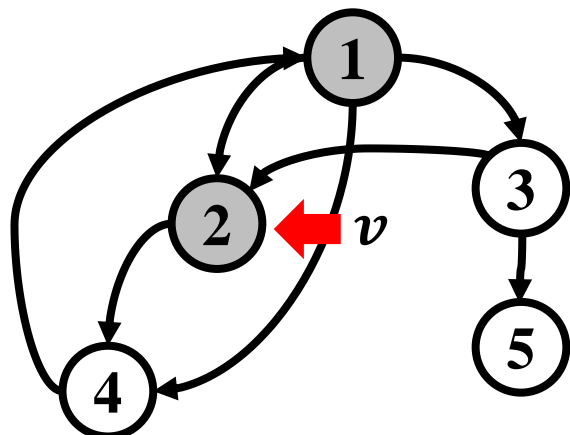
$$time = 2$$

v	1	2	3	4	5
$pred$	N	1	N	N	N

v	1	2	3	4	5
$color$	G	G	W	W	W

v	1	2	3	4	5
d	1	2			

v	1	2	3	4	5
f					

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
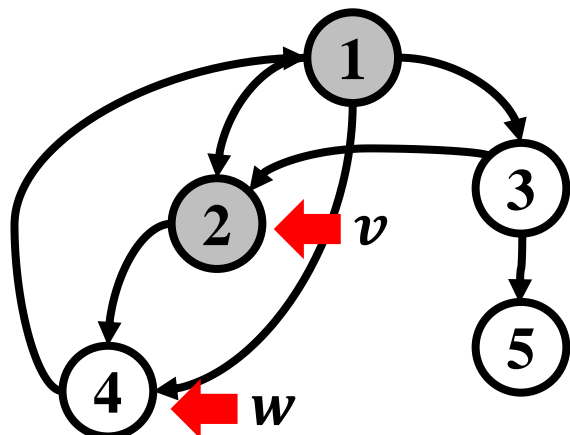
$$time = 2$$

v	1	2	3	4	5
$pred$	N	1	N	N	N

v	1	2	3	4	5
$color$	G	G	W	W	W

v	1	2	3	4	5
d	1	2			

v	1	2	3	4	5
f					

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
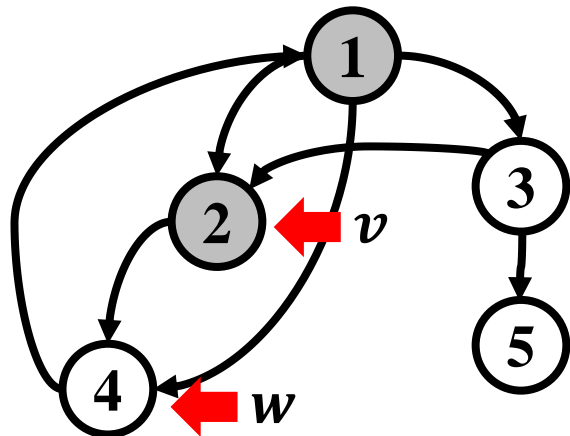
$time = 2$

v	1	2	3	4	5
$pred$	N	1	N	N	N

v	1	2	3	4	5
$color$	G	G	W	W	W

v	1	2	3	4	5
d	1	2			

v	1	2	3	4	5
f					

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
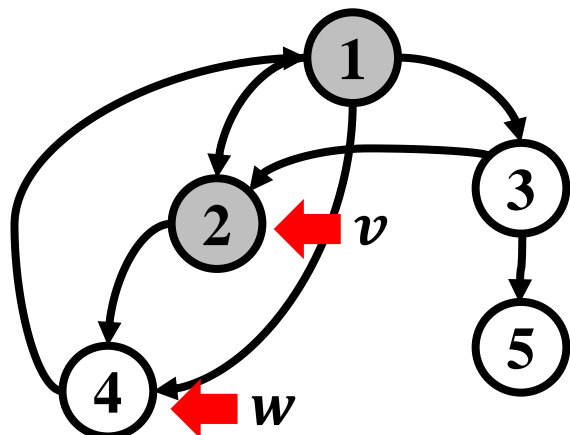
$$time = 2$$

v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	G	W	W	W

v	1	2	3	4	5
d	1	2			

v	1	2	3	4	5
f					

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
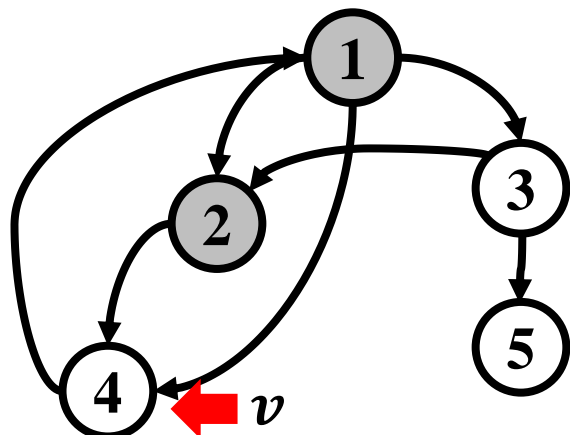
$time = 2$

v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	G	W	W	W

v	1	2	3	4	5
d	1	2			

v	1	2	3	4	5
f					

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
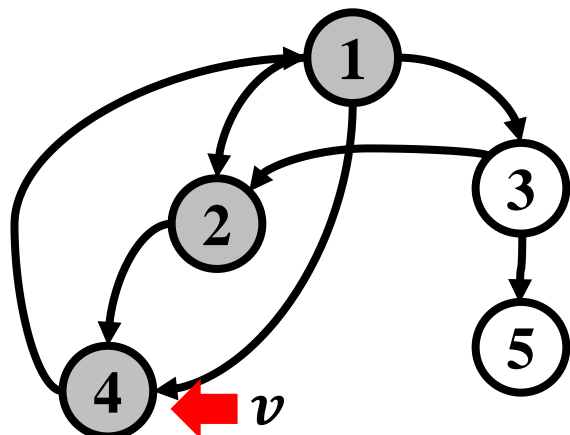
$$time = 2$$

v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	G	W	G	W

v	1	2	3	4	5
d	1	2			

v	1	2	3	4	5
f					

v

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
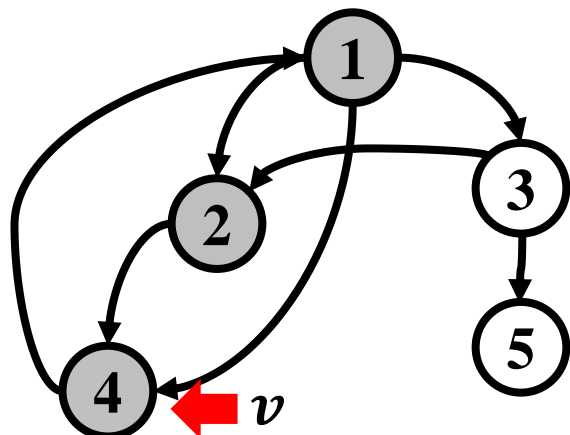
$time = 3$

v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	G	W	G	W

v	1	2	3	4	5
d	1	2			

v	1	2	3	4	5
f					

$color[v] \leftarrow GRAY$
$time \leftarrow time + 1$
$d[v] \leftarrow time$
for $w \in Adj[v]$ **do**
  if $color[w] = WHITE$ **then**
    $pred[w] \leftarrow v$
    DFS-Visit(w)
  end
end
$color[v] \leftarrow BLACK$
$time \leftarrow time + 1$
$f[v] \leftarrow time$

$$time = 3$$

v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	G	W	G	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f					

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
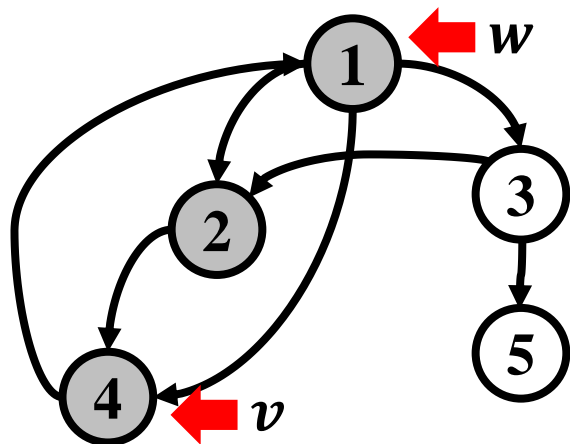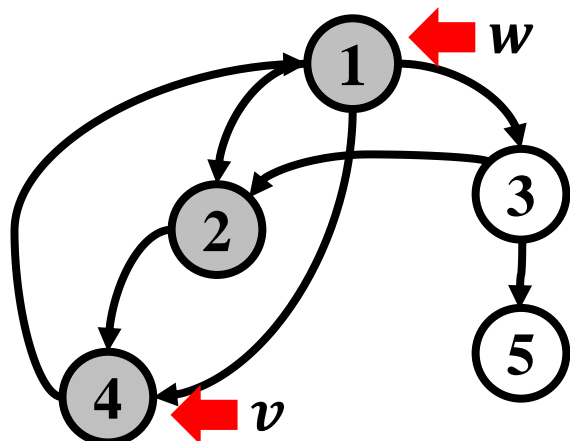
$time = 3$

v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	G	W	G	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f					

$time = 3$

v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	G	W	G	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f					

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
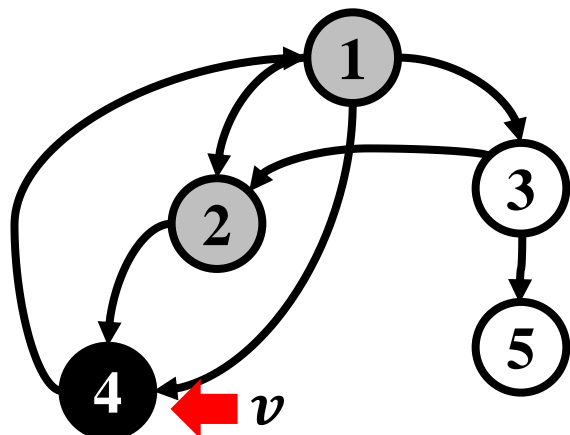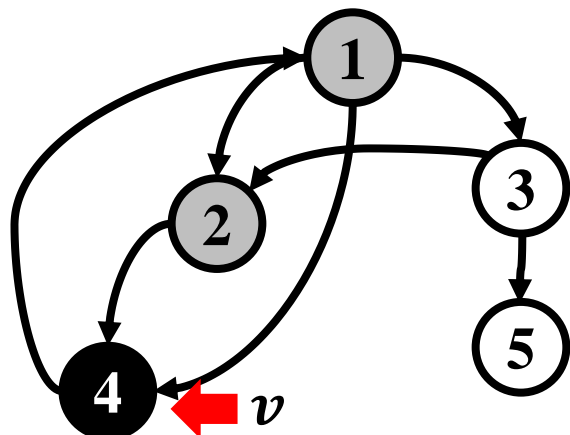
$time = 3$

v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	G	W	B	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f					

v

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
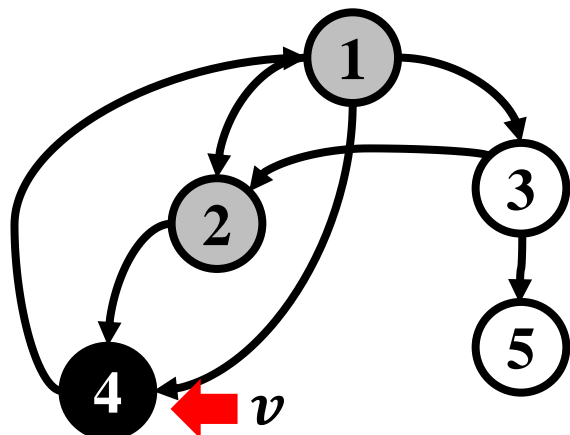
$time = 4$

v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	G	W	B	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f					

v

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
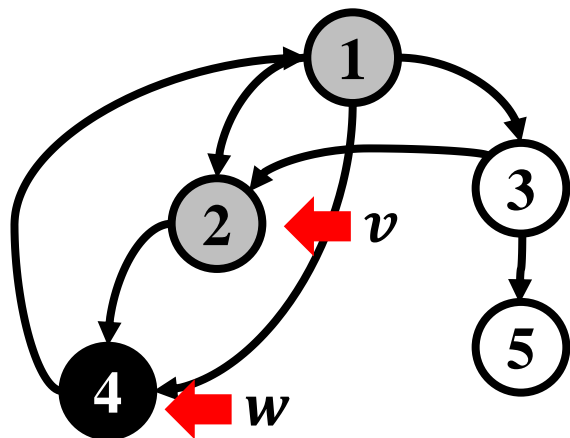
$$time = 4$$

v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	G	W	B	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f				4	

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$$time = 4$$
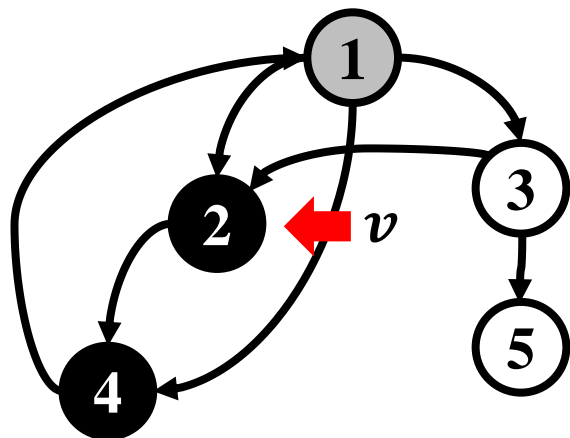
v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	G	W	B	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f				4	

v

w

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
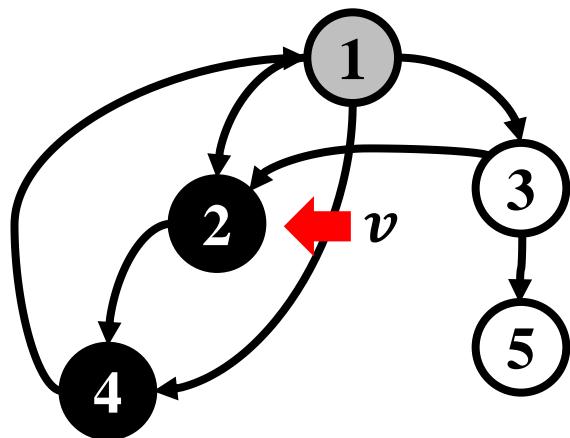
$time = 4$

v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	B	W	B	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f				4	

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
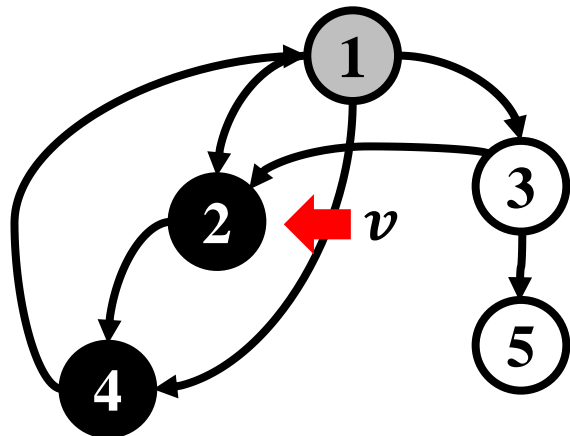
$time = 5$

v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	B	W	B	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f				4	

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
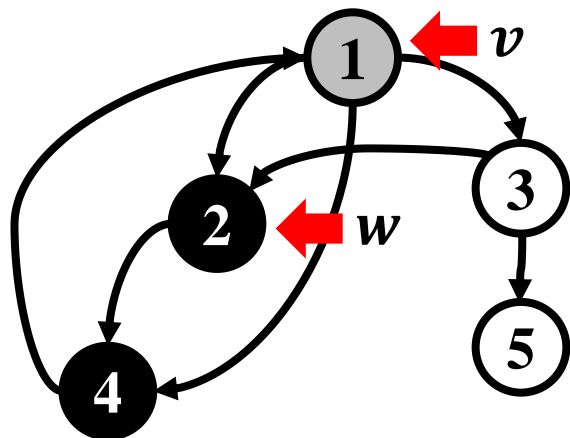
$$time = 5$$

v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	B	W	B	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f		5		4	

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
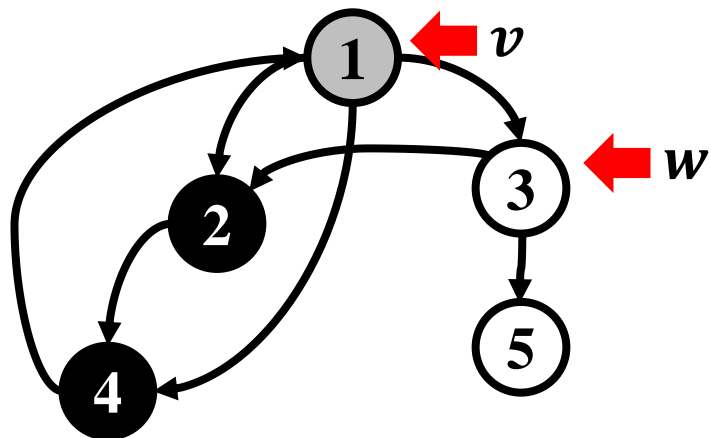
$$time = 5$$

v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	B	W	B	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f		5		4	

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
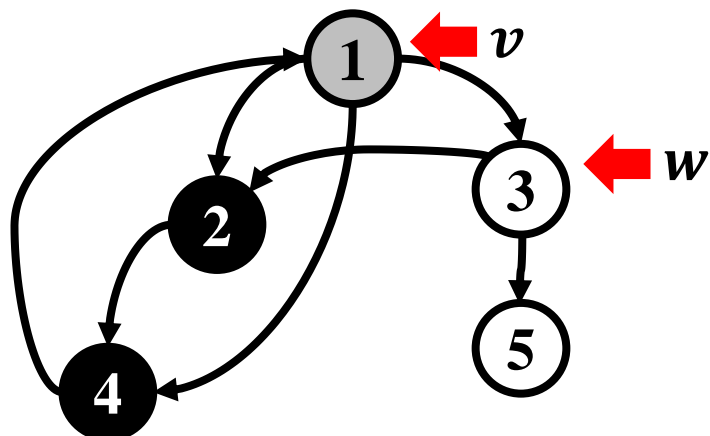
$time = 5$

v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	B	W	B	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f		5		4	

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
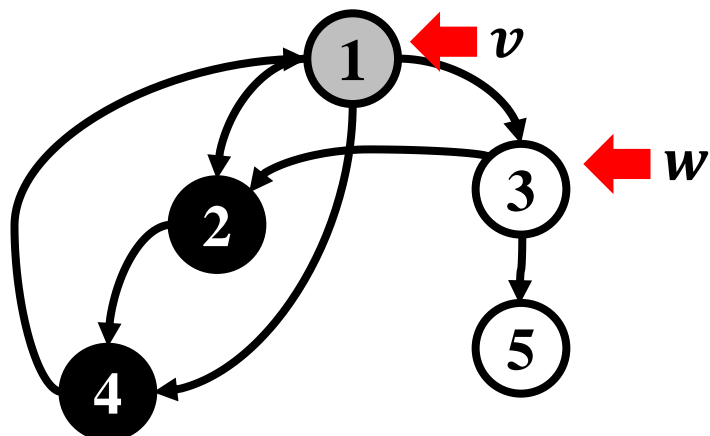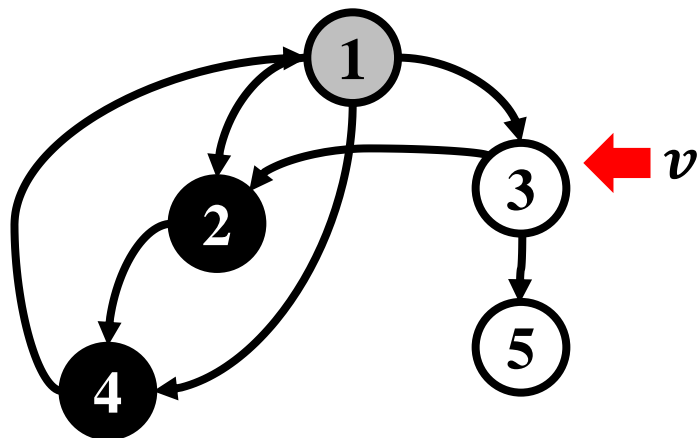
$time = 5$

v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	B	W	B	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f		5		4	

$color[v] \leftarrow GRAY$
$time \leftarrow time + 1$
$d[v] \leftarrow time$
for $w \in Adj[v]$ **do**
  if $color[w] = WHITE$ **then**
    $pred[w] \leftarrow v$
    DFS-Visit(w)
  end
end
$color[v] \leftarrow BLACK$
$time \leftarrow time + 1$
$f[v] \leftarrow time$

$$time = 5$$

v	1	2	3	4	5
$pred$	N	1	1	2	N

v	1	2	3	4	5
$color$	G	B	W	B	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f		5		4	

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$time = 5$

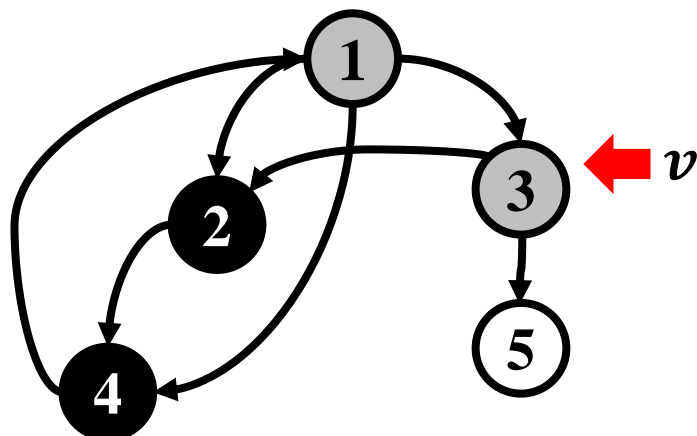| v	1	2	3	4	5
$pred$	N	1	1	2	N

v	1	2	3	4	5
$color$	G	B	W	B	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f		5		4	

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
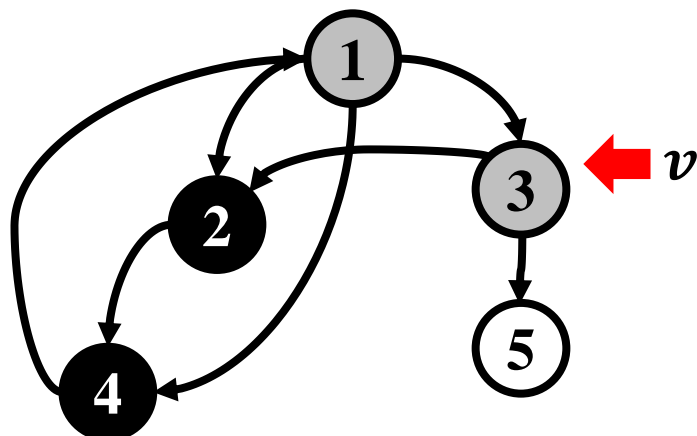
$time = 5$


```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
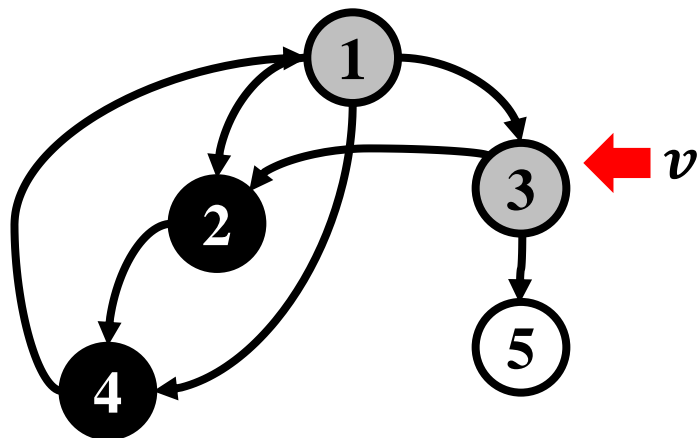
v	1	2	3	4	5
$pred$	N	1	1	2	N

v	1	2	3	4	5
$color$	G	B	G	B	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f		5		4	

$time = 6$

v	1	2	3	4	5
$pred$	N	1	1	2	N

v	1	2	3	4	5
$color$	G	B	G	B	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f		5		4	

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
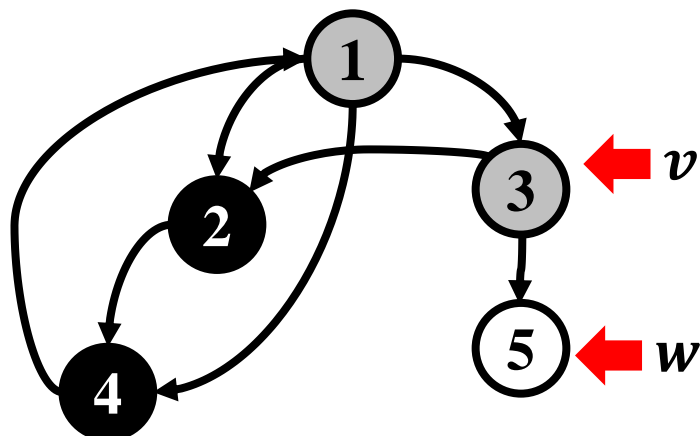
$time = 6$

v	1	2	3	4	5
$pred$	N	1	1	2	N

v	1	2	3	4	5
$color$	G	B	G	B	W

v	1	2	3	4	5
d	1	2	6	3	

v	1	2	3	4	5
f		5		4	

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
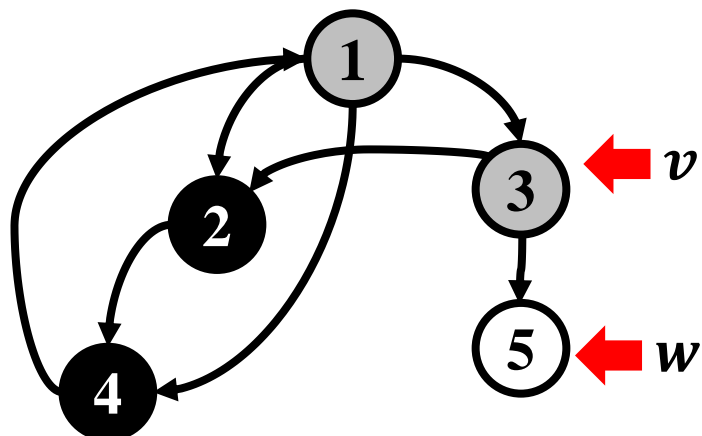
$$time = 6$$

v	1	2	3	4	5
$pred$	N	1	1	2	N

v	1	2	3	4	5
$color$	G	B	G	B	W

v	1	2	3	4	5
d	1	2	6	3	

v	1	2	3	4	5
f		5		4	

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
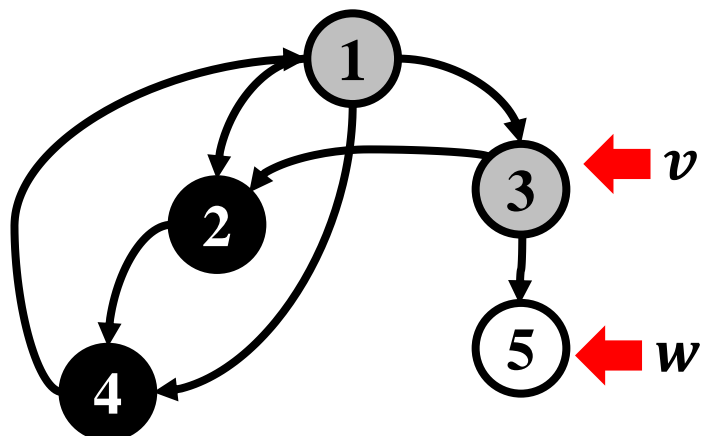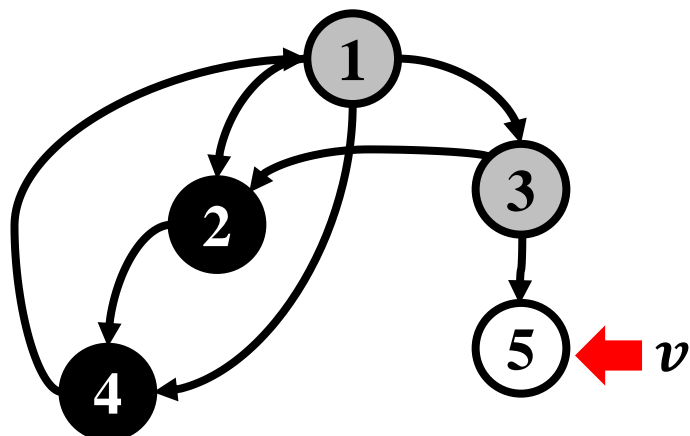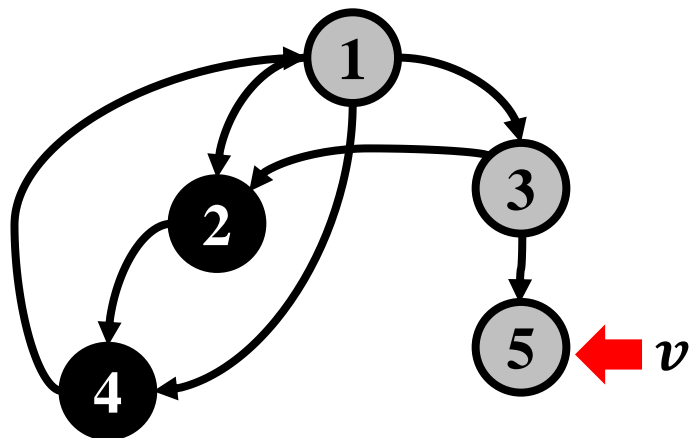
$$time = 6$$

v	1	2	3	4	5
$pred$	N	1	1	2	N

v	1	2	3	4	5
$color$	G	B	G	B	W

v	1	2	3	4	5
d	1	2	6	3	

v	1	2	3	4	5
f		5		4	

$color[v] \leftarrow GRAY$
$time \leftarrow time + 1$
$d[v] \leftarrow time$
for $w \in Adj[v]$ **do**
    if $color[w] = WHITE$ **then**
        $pred[w] \leftarrow v$
        DFS-Visit(w)
    end
end
$color[v] \leftarrow BLACK$
$time \leftarrow time + 1$
$f[v] \leftarrow time$

$time = 6$

v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	G	B	W

v	1	2	3	4	5
d	1	2	6	3	

v	1	2	3	4	5
f		5		4	

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$time = 6$

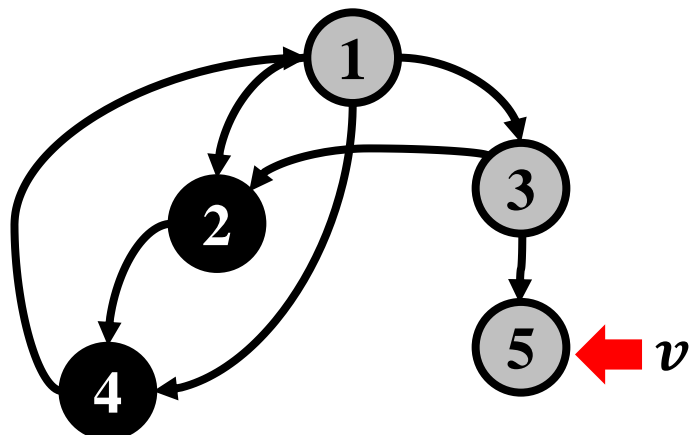| v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	G	B	W

v	1	2	3	4	5
d	1	2	6	3	

v	1	2	3	4	5
f		5		4	

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
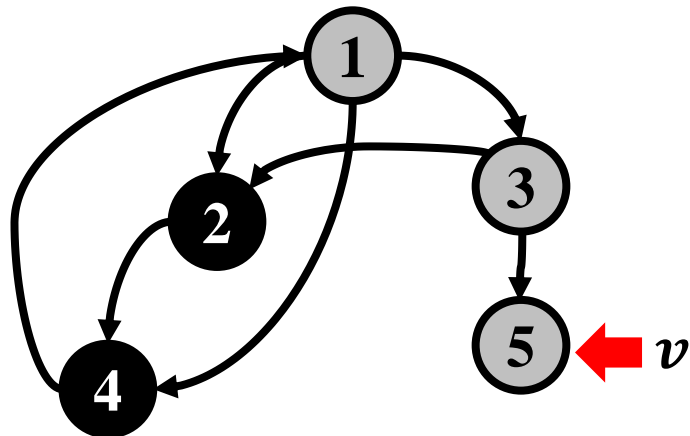
$$time = 6$$

v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	G	B	G

v	1	2	3	4	5
d	1	2	6	3	

v	1	2	3	4	5
f		5		4	

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
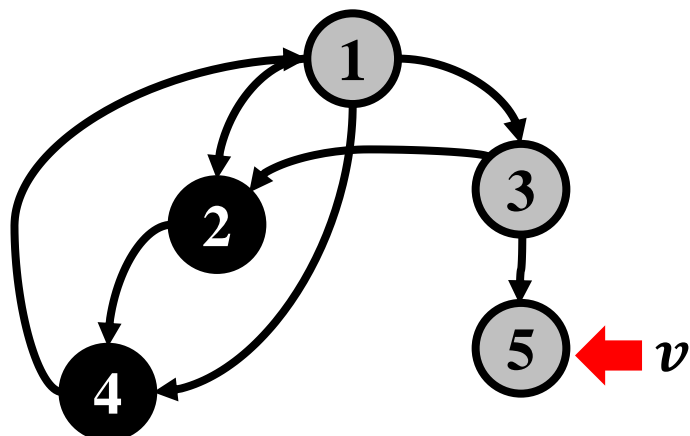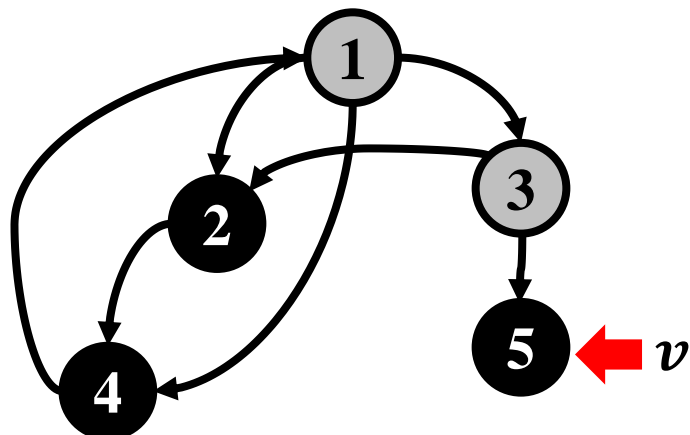
$time = 7$

v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	G	B	G

v	1	2	3	4	5
d	1	2	6	3	

v	1	2	3	4	5
f		5		4	

$$time = 7$$

v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	G	B	G

v	1	2	3	4	5
d	1	2	6	3	7

v	1	2	3	4	5
f		5		4	

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
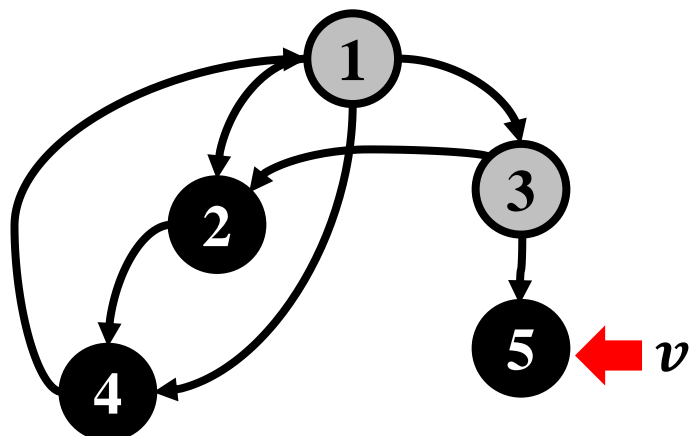
$$time = 7$$

v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	G	B	G

v	1	2	3	4	5
d	1	2	6	3	7

v	1	2	3	4	5
f		5		4	

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
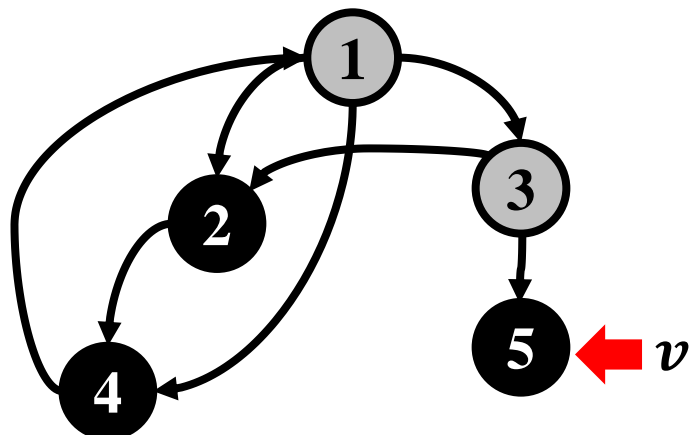
$$time = 7$$

v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	G	B	B

v	1	2	3	4	5
d	1	2	6	3	7

v	1	2	3	4	5
f		5		4	

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
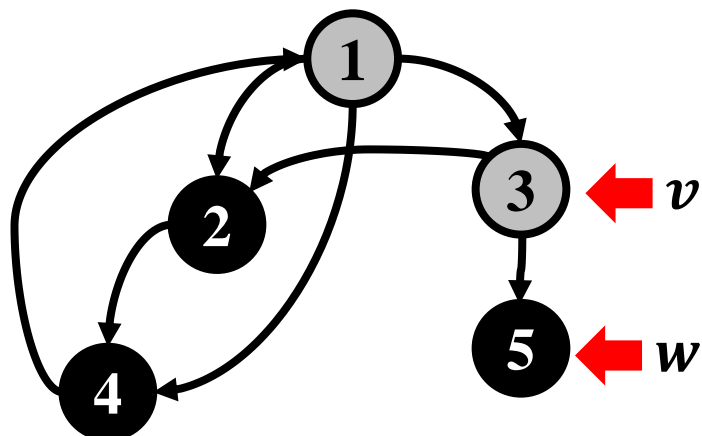
$time = \textcolor{red}{8}$

v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	G	B	B

v	1	2	3	4	5
d	1	2	6	3	7

v	1	2	3	4	5
f		5		4	

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
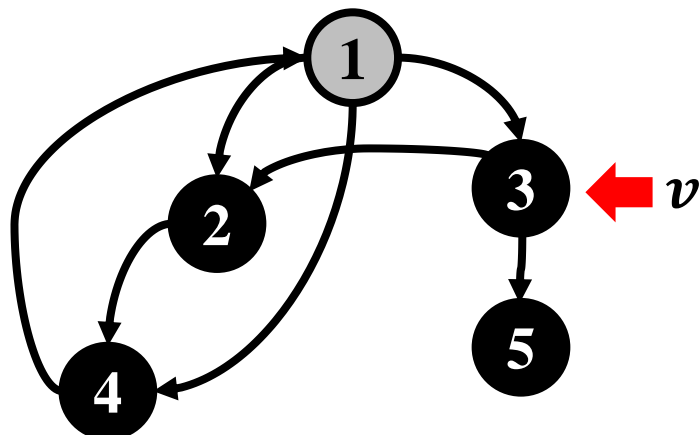
$time = 8$

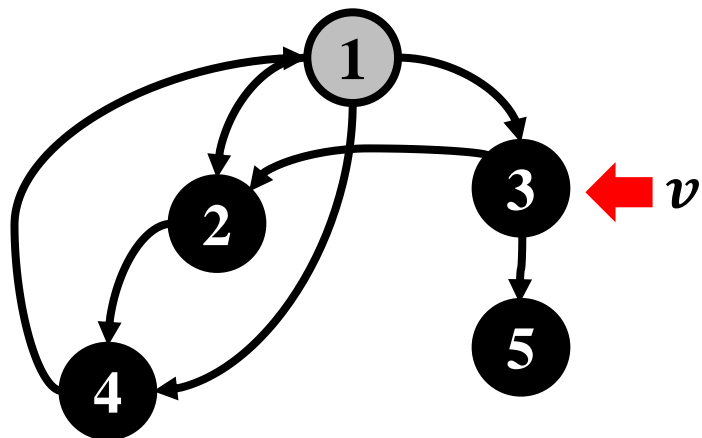| v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	G	B	B

v	1	2	3	4	5
d	1	2	6	3	7

v	1	2	3	4	5
f		5		4	8

$color[v] \leftarrow GRAY$
$time \leftarrow time + 1$
$d[v] \leftarrow time$
for $w \in Adj[v]$ **do**
    if $color[w] = WHITE$ **then**
        $pred[w] \leftarrow v$
        DFS-Visit(w)
    end
end
$color[v] \leftarrow BLACK$
$time \leftarrow time + 1$
$f[v] \leftarrow time$

$time = 8$


```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
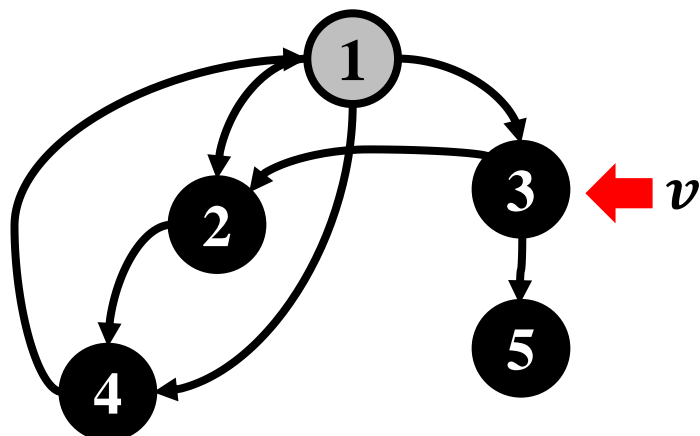
v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	G	B	B

v	1	2	3	4	5
d	1	2	6	3	7

v	1	2	3	4	5
f		5		4	8

$time = 8$

v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	B	B	B

v	1	2	3	4	5
d	1	2	6	3	7

v	1	2	3	4	5
f		5		4	8

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
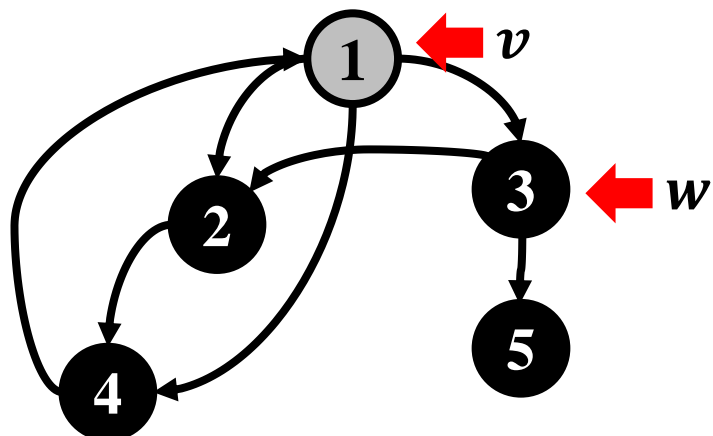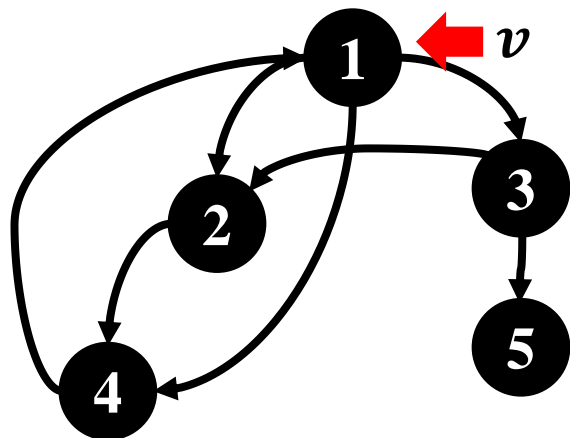
$time = 9$

v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	B	B	B

v	1	2	3	4	5
d	1	2	6	3	7

v	1	2	3	4	5
f		5		4	8

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
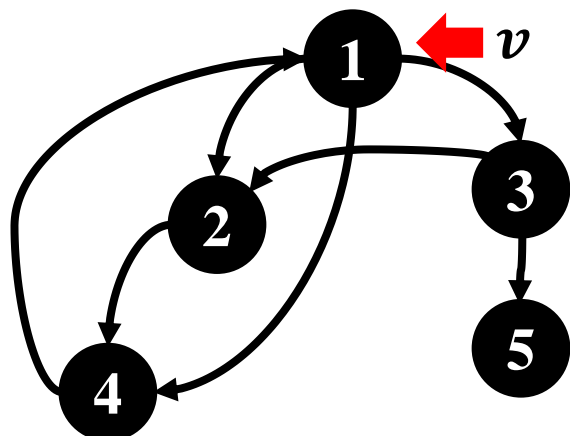
$time = 9$

v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	B	B	B

v	1	2	3	4	5
d	1	2	6	3	7

v	1	2	3	4	5
f		5	9	4	8

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```
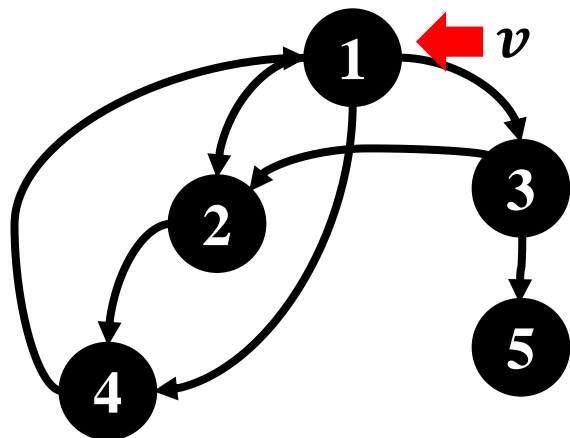
$time = 9$

v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	B	B	B

v	1	2	3	4	5
d	1	2	6	3	7

v	1	2	3	4	5
f		5	9	4	8

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$$time = 9$$

v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	B	B	B	B	B

v	1	2	3	4	5
d	1	2	6	3	7

v	1	2	3	4	5
f		5	9	4	8

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$time = 10$

v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	B	B	B	B	B

v	1	2	3	4	5
d	1	2	6	3	7

v	1	2	3	4	5
f		5	9	4	8

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$time = 10$

v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	B	B	B	B	B

v	1	2	3	4	5
d	1	2	6	3	7

v	1	2	3	4	5
f	10	5	9	4	8

连通无向图的优先树与连通有向图的优先森林

- ## 无向图
  - ### 树的形状：取决于搜索顺序
  - ### 树的数量：确定1棵优先树
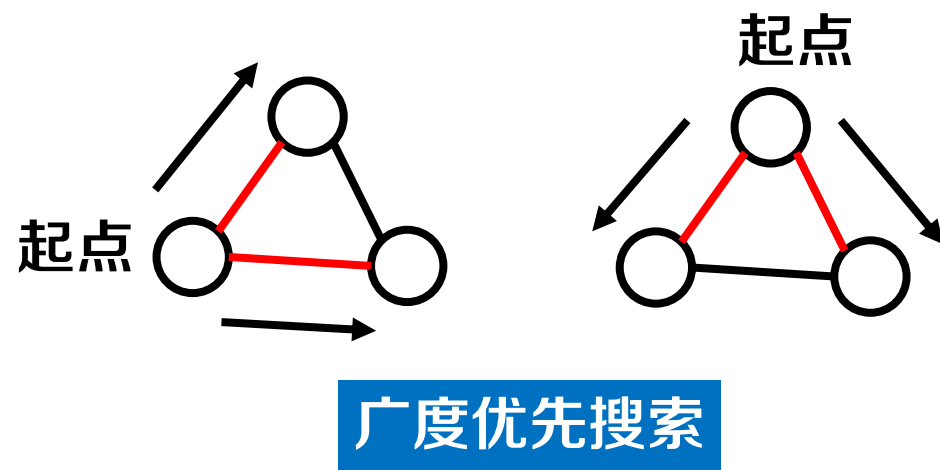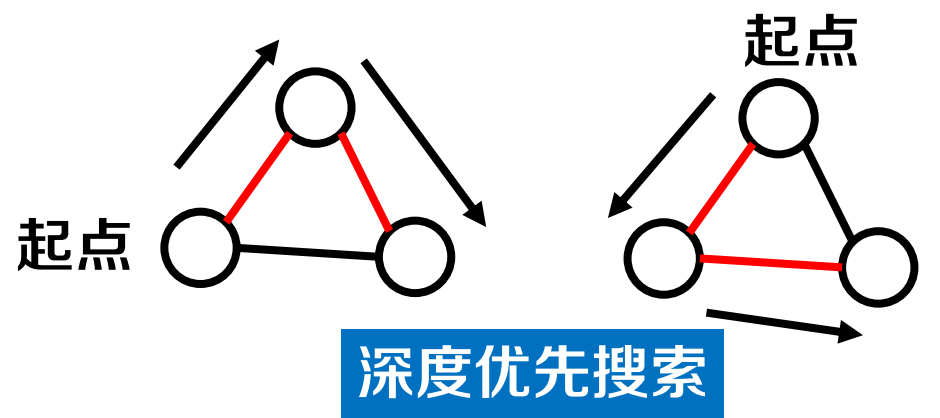
深度优先搜索

广度优先搜索

- ## 有向图
  - ### 树的形状：取决于搜索顺序
  - ### 树的数量：取决于搜索顺序

深度优先搜索
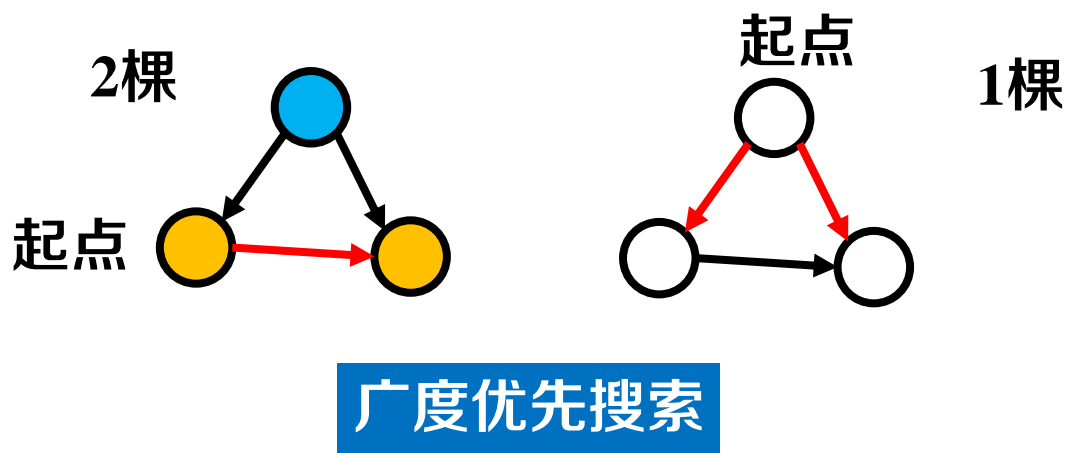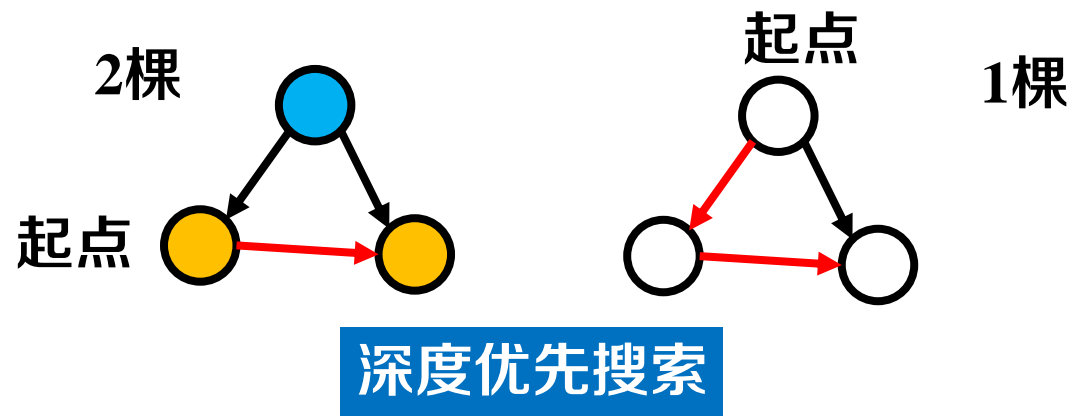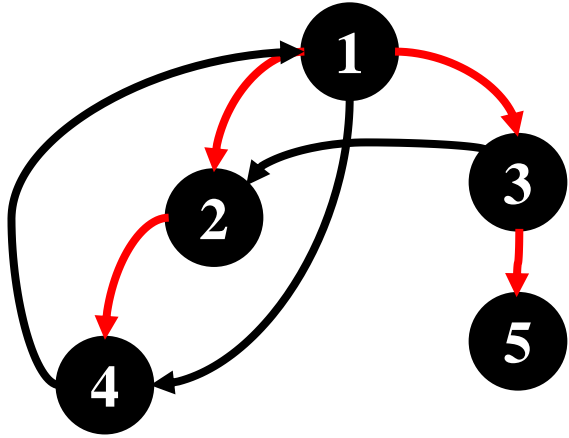
广度优先搜索

$$time = 10$$
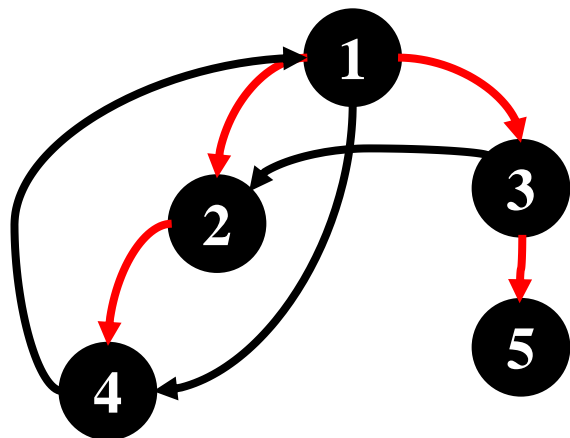
v	1	2	3	4	5
$pred$	N	1	1	2	3

$$time = 10$$

v	1	2	3	4	5
$pred$	N	1	1	2	3

- **回顾深度优先搜索边的性质**
  - 后向边：不是树边，但两顶点在深度优先树中是祖先后代关系
  - 对于**无向图**，非树边一定是后向边
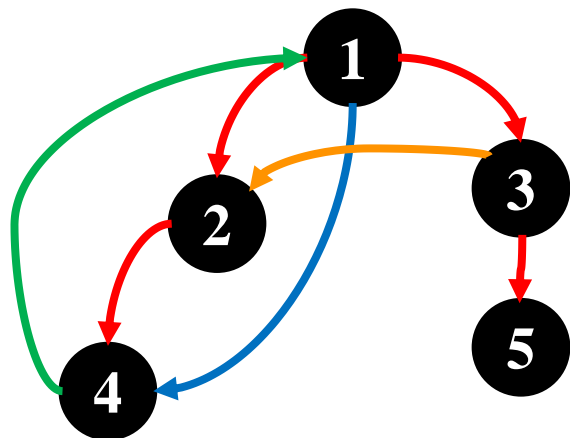
$$time = 10$$

v	1	2	3	4	5
$pred$	N	1	1	2	3

区别1：祖先指向后代？还是相反？

- **回顾深度优先搜索边的性质**
  - 后向边：不是树边，但两顶点在深度优先树中是**祖先后代关系**
  - 对于**无向图**，非树边一定是后向边

区别2：非树边出现在兄弟顶点之间

- **有向图，深度优先搜索有4类边**
  - **树边：在深度优先树中的边**
  - **前向边：不在深度优先树中，从祖先指向后代的边**
  - **后向边：从后代指向祖先的边**
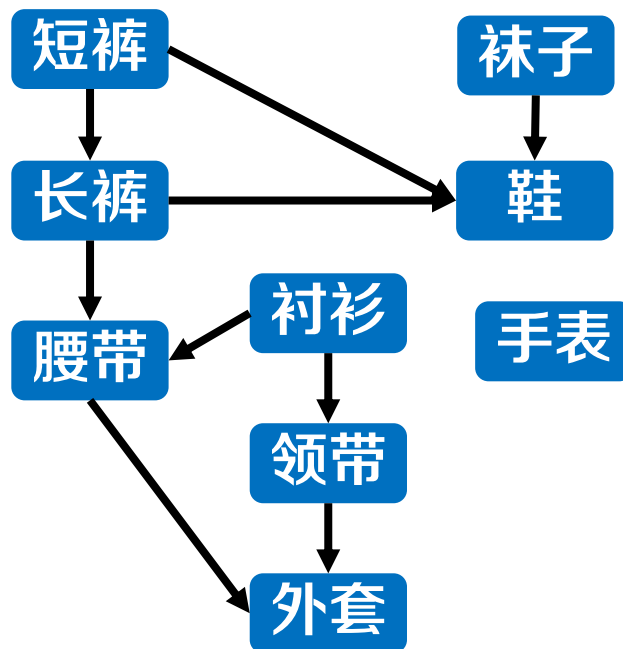 - **横向边：顶点不具有祖先后代关系的边**
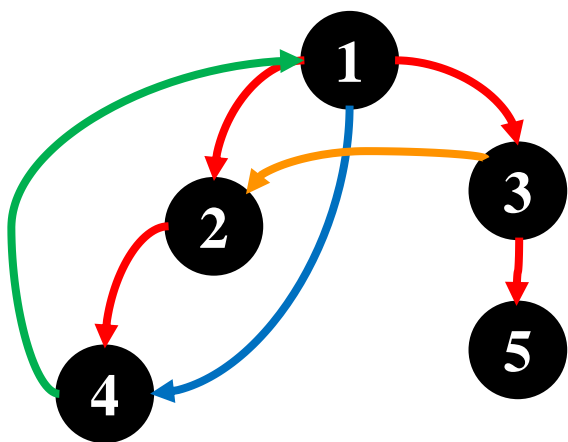
- **无向图，深度优先搜索有2类边**
  - **树边：在深度优先树中的边**
  - **后向边：两顶点有祖先后代关系的非树边**

环路的存在性判断

拓扑排序

强连通分量