

Successful Lax Pairs and Equations

Generated by Agentic Lax Pair Finder

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1.

$$L_1 = u + D(x) \quad (1)$$

$$P_1 = u D(x) + D(x) u + D(x, x) \quad (2)$$

$$(u)_t - 2(u)_x u \quad (3)$$

2.

$$L_2 = \begin{bmatrix} i(u + D(x)) & 0 \\ 0 & i(-u - D(x)) \end{bmatrix} \quad (4)$$

$$P_2 = \begin{bmatrix} i(2u D(x) + 2D(x)u + 2D(x, x)) & 0 \\ 0 & i(-2u D(x) - 2D(x)u - 2D(x, x)) \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} i(u)_t + 4(u)_x u & 0 \\ 0 & -i(u)_t + 4(u)_x u \end{bmatrix} \quad (6)$$

3.

$$L_3 = \begin{bmatrix} i D(x) & -iq \\ ip & -i D(x) \end{bmatrix} \quad (7)$$

$$P_3 = \begin{bmatrix} 2i D(x, x) & i(-q D(x) - D(x)q) \\ i(p D(x) + D(x)p) & -2i D(x, x) \end{bmatrix} \quad (8)$$

$$\begin{bmatrix} -(p)_x q - (q)_x p & -i(q)_t - (q)_{xx} \\ i(p)_t - (p)_{xx} & -(p)_x q - (q)_x p \end{bmatrix} \quad (9)$$

4.

$$L_4 = u + D(x) \quad (10)$$

$$P_4 = u D(x) + D(x) u + D(x, x) \quad (11)$$

$$(u)_t - 2(u)_x u \quad (12)$$

5.

$$L_5 = iu + D(x) \quad (13)$$

$$P_5 = i(u D(x) + D(x)u) + D(x, x) \quad (14)$$

$$i(u)_t + 2(u)_x u \quad (15)$$

6.

$$L_6 = \begin{bmatrix} i D(x) & -iq \\ ip & -i D(x) \end{bmatrix} \quad (16)$$

$$P_6 = \begin{bmatrix} i(pq + 2D(x, x)) & i(-qD(x) - D(x)q) \\ i(pD(x) + D(x)p) & i(-pq - 2D(x, x)) \end{bmatrix} \quad (17)$$

$$\begin{bmatrix} -2(p)_x q - 2(q)_x p & -i(q)_t - (q)_{xx} - 2pq^2 \\ i(p)_t - (p)_{xx} - 2p^2 q & -2(p)_x q - 2(q)_x p \end{bmatrix} \quad (18)$$

7.

$$L_7 = cu + D(x) \quad (19)$$

$$P_7 = cu D(x) + c D(x) u + D(x, x) \quad (20)$$

$$c((u)_t - 2(u)_x cu) \quad (21)$$

8.

$$L_8 = \begin{bmatrix} i D(x) & -iqu \\ ipu & -i D(x) \end{bmatrix} \quad (22)$$

$$P_8 = \begin{bmatrix} 2i D(x, x) & i(-qu D(x) - D(x)qu) \\ i(pu D(x) + D(x)pu) & -2i D(x, x) \end{bmatrix} \quad (23)$$

$$\begin{bmatrix} u(-(p)_x qu - (q)_x pu - 2(u)_x pq) & -i(q)_t u - (q)_{xx} u - 2(q)_x(u)_x - i(u)_t q - (u)_{xx} q \\ i(p)_t u - (p)_{xx} u - 2(p)_x(u)_x + i(u)_t p - (u)_{xx} p & u(-(p)_x qu - (q)_x pu - 2(u)_x pq) \end{bmatrix} \quad (24)$$

9.

$$L_9 = u + i D(x) \quad (25)$$

$$P_9 = u D(x) + D(x) u + i D(x, x) \quad (26)$$

$$(u)_t - 2(u)_x u \quad (27)$$

10.

$$L_{10} = \begin{bmatrix} i D(x) & -ip \\ iq & -i D(x) \end{bmatrix} \quad (28)$$

$$P_{10} = \begin{bmatrix} 2i D(x, x) & i(-pD(x) - D(x)p) \\ i(qD(x) + D(x)q) & -2i D(x, x) \end{bmatrix} \quad (29)$$

$$\begin{bmatrix} -(p)_x q - (q)_x p & -i(p)_t - (p)_{xx} \\ i(q)_t - (q)_{xx} & -(p)_x q - (q)_x p \end{bmatrix} \quad (30)$$

11.

$$L_{11} = \begin{bmatrix} i D(x) & -iuq \\ ipu & -i D(x) \end{bmatrix} \quad (31)$$

$$P_{11} = \begin{bmatrix} 2i D(x, x) & i(-uq D(x) - D(x)uq) \\ i(pu D(x) + D(x)pu) & -2i D(x, x) \end{bmatrix} \quad (32)$$

$$\begin{bmatrix} u(-(p)_x qu - (q)_x pu - 2(u)_x pq) & -i(q)_t u - (q)_{xx} u - 2(q)_x(u)_x - i(u)_t q - (u)_{xx} q \\ i(p)_t u - (p)_{xx} u - 2(p)_x(u)_x + i(u)_t p - (u)_{xx} p & u(-(p)_x qu - (q)_x pu - 2(u)_x pq) \end{bmatrix} \quad (33)$$

12.

$$L_{12} = \begin{bmatrix} i D(x) & -icq \\ icp & -i D(x) \end{bmatrix} \quad (34)$$

$$P_{12} = \begin{bmatrix} 2i D(x, x) & i(-cq D(x) - c D(x) q) \\ i(cp D(x) + c D(x) p) & -2i D(x, x) \end{bmatrix} \quad (35)$$

$$\begin{bmatrix} c^2(-(p)_x q - (q)_x p) & c(-i(q)_t - (q)_{xx}) \\ c(i(p)_t - (p)_{xx}) & c^2(-(p)_x q - (q)_x p) \end{bmatrix} \quad (36)$$