Seminar 12

1. Use Lagrange multipliers to find the extrema of the following functions subject to constraints:

   (a) $x^2 + y^2$ subject to $x - y + 1 = 0$.

   (b) $(x + y)^2$ subject to $x^2 + y^2 = 1$.

   (c) ★ $x^2 - y^2$ subject to $x^2 + y^2 = 1$.

   (d) $x + 2y + 3z$ subject to $x^2 + y^2 + z^2 = 1$.

   (e) $2x^2+y^2+3z^2$ subject to $x^2+y^2+z^2 = 1$.

   (f) ★ $x^3+y^3+z^3$ subject to $x^2+y^2+z^2 = 1$.

2. Find the minimum value of $\frac{1}{2}(x_1^2 + x_2^2 + x_3^2)$ subject to the following constraints:

   (a) $x_1 + x_2 + x_3 = 3$.

   (b) $x_1+x_2+x_3 = 3$ and $x_1+2x_2+3x_3 = 12$.

3. Compute the following integrals:

   (a) $\displaystyle\iint\limits_R \cos x \sin y \,\mathrm{d}x\,\mathrm{d}y$, where $R = [0, \pi/2] \times [0, \pi/2]$.

   (b) $\displaystyle\iint\limits_R \frac{1}{(x + y)^2} \,\mathrm{d}x\,\mathrm{d}y$ and $\displaystyle\iint\limits_R ye^{xy} \,\mathrm{d}x\,\mathrm{d}y$, where $R = [1, 2] \times [0, 1]$.

   (c) $\displaystyle\iint\limits_R \min\{x, y\} \,\mathrm{d}x\,\mathrm{d}y$, where $R = [0, 1] \times [0, 1]$.

4. Let $D \subseteq \mathbb{R}^2$ be the subset bounded by the parabola $y = x^2$ and the lines $x = 2$ and $y = 0$.

   (a) Express D as a simple set first w.r.t. the y-axis and then w.r.t. the x-axis.

   (b) Compute $\displaystyle\iint\limits_D xy\, dx\, dy$ in two ways.

Homework questions are marked with ★.
Solutions should be handed in at the beginning of next week's lecture.