

Seminar 3

1. Find the sum for each of the following series:

(a) $\sum_{n \geq 1} \frac{2}{3^n}$.

(c) $\sum_{n \geq 1} \frac{1}{4n^2 - 1}$.

(e) $\sum_{n \geq 1} \frac{n}{2^n}$.

(b) $\sum_{n \geq 1} \frac{2n+1}{n!}$.

(d) $\sum_{n \geq 1} \frac{1}{n(n+1)(n+2)}$.

(f) $\sum_{n \geq 1} \frac{1}{\sqrt[3]{n}}$.

2. ★ Find the sum for each of the following series:

(a) $\sum_{n \geq 2} \ln \left(1 - \frac{1}{n^2}\right)$.

(b) $\sum_{n \geq 1} \frac{n+1}{3^n}$.

(c) $\sum_{n \geq 1} \frac{n}{n^4 + n^2 + 1}$.

3. Study if the following series are convergent or divergent:

(a) $\sum_{n \geq 2} \frac{1}{\ln n}$.

(c) $\sum_{n \geq 1} \frac{\ln \left(1 + \frac{1}{n}\right)}{n}$.

(e) $\sum_{n \geq 1} \left(\frac{n}{n+1}\right)^{n^2}$.

(b) $\sum_{n \geq 1} \frac{1}{n\sqrt{n+1}}$.

(d) $\sum_{n \geq 1} \frac{n!}{n^n}$.

(f) $\sum_{n \geq 2} \frac{1}{n \ln(n)}$.

4. ★ Study if the following series are convergent or divergent:

(a) $\sum_{n \geq 1} \frac{x^n}{n^p}, x > 0, p \in \mathbb{N}$.

(b) $\sum_{n \geq 2} \frac{1}{(\ln n)^{\ln n}}$.

(c) $\sum_{n \geq 1} (\sqrt[n]{n} - 1)$.

5. ★ Start with an equilateral triangle of side 1. For each side, remove the middle third and add there another equilateral triangle. Repeat this process at each iteration (see the figure). How many sides are there at iteration n ? What is the limit of the perimeter and the area?



Homework questions are marked with ★.

Solutions should be handed in at the beginning of next week's lecture.