Extra homework 1

1. Prove that any union of open sets is open, and any finite intersection of closed sets is closed.

2. Let α be an irrational number and consider the set $S_\alpha := \big\{\{n\alpha\} \mid n \in \mathbb{N}\big\}$, where $\{\cdot\}$ denotes the fractional part. Show that the set S_α is dense in $[0,1]$, meaning that its closure is $[0,1]$. Then show that the set $\big\{\{n\alpha\} + m \mid n, m \in \mathbb{Z}\big\}$ is dense in $\mathbb{R}$.

These questions are extra. You will get bonus points for solving them.
Solutions should be uploaded on Teams before the next lecture.