

Extra homework 3

1. Start with the interval $[0, 1]$ and remove the middle third $(1/3, 2/3)$; then remove the middle third of each of the remaining intervals, and so on. Denote by \mathcal{C} set of points that are left – this is known as the Cantor set.

- What is the total length of the removed intervals?
- Prove that $x \in \mathcal{C}$ if and only if $x = \sum_{n \geq 1} a_n 3^{-n}$, with $a_n \in \{0, 2\}$.

In other words, $x \in \mathcal{C}$ iff it can be written in base 3 using only 0's and 2's.

- By constructing a surjective function from \mathcal{C} to $[0, 1]$, prove that \mathcal{C} is uncountable.

These questions are extra. You will get bonus points for solving them.
Solutions should be uploaded on Teams before the next lecture.