Project 5 (0.2 points)

- *Input*: natural numbers $m, n \geq 2$
- *Output*:
  1. the number of matrices $A \in M_{m,n}(\mathbb{Z}_2)$ in *reduced* (that is, each column containing a leading 1 has zeros everywhere else) echelon form
  2. the matrices $A \in M_{m,n}(\mathbb{Z}_2)$ in reduced echelon form (for $2 \leq m, n \leq 5$)

Example:

- *Input*: $m = 2$, $n = 3$
- *Output*:
  1. the number of matrices $A \in M_{2,3}(\mathbb{Z}_2)$ in reduced echelon form is 15
  2. the matrices $A \in M_{2,3}(\mathbb{Z}_2)$ in reduced echelon form are (the leading 1's are framed):

$$\begin{pmatrix} \boxed{1} & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} \boxed{1} & 1 & 0 \\ 0 & 0 & \boxed{1} \end{pmatrix} \quad \begin{pmatrix} \boxed{1} & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} \boxed{1} & 0 & 1 \\ 0 & \boxed{1} & 1 \end{pmatrix} \quad \begin{pmatrix} \boxed{1} & 0 & 1 \\ 0 & \boxed{1} & 0 \end{pmatrix} \quad \begin{pmatrix} \boxed{1} & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \boxed{1} & 0 & 0 \\ 0 & \boxed{1} & 1 \end{pmatrix} \quad \begin{pmatrix} \boxed{1} & 0 & 0 \\ 0 & \boxed{1} & 0 \end{pmatrix} \quad \begin{pmatrix} \boxed{1} & 0 & 0 \\ 0 & 0 & \boxed{1} \end{pmatrix} \quad \begin{pmatrix} \boxed{1} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & \boxed{1} & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & \boxed{1} & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & \boxed{1} \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & \boxed{1} & 0 \\ 0 & 0 & \boxed{1} \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Note:

- Any (reasonable) programming language may be used.
- The solutions will consist of the source code with comments (do not send executable files!) and at least 5 relevant input and output files, and will be sent to the e-mail address: septimiu.crivei@ubbcluj.ro.
- If necessary, you will be asked to explain your solution.
- The first 25 solutions will be rewarded.
- The final deadline is January 14, 2024.