$\dot{x} = x^3$. Study the stability of the equilibrium point $\eta^* = 0$.

$f(x) = x^3$

$\eta^* = 0$ is the only equilibrium

the orbits are $(-\infty, 0)$, $\{0\}$, $(0, \infty)$

From the phase portrait we deduce that $\eta^* = 0$ is a repeller.

Note that, since $f'(0) = 0$ the linearization method doesn't work.

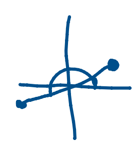
$r_1 = i \in \mathbb{C} \setminus \mathbb{R}$ $\quad$ $r - i = 0$ $\quad$ $x' - ix = 0$ $\quad$ we do not consider d.e. with coefficients that are not real !!!

$r_{1,2} = \pm i$ $\quad$ $r^2 + 1 = 0$ $\quad$ $x'' + x = 0$

$r^2 - \sqrt{3}\,r + 1 = 0$ $\qquad$ $\Delta = 3 - 4 = -1$ $\qquad$ $\frac{\sqrt{3}}{2} + i \cdot \frac{1}{2}$ $\qquad$ $\theta = \operatorname{arctg} \frac{1}{\sqrt{3}} = \frac{\pi}{6}$

$r_{1,2} = \frac{\sqrt{3} \pm i}{2}$ $\qquad$ $r_1 = \frac{\sqrt{3}}{2} + i \cdot \frac{1}{2}$ $\qquad$ $r_2 = \overline{r_1}$ $\qquad$ $-\frac{\sqrt{3}}{2} - i \cdot \frac{1}{2}$ $\qquad$ $\operatorname{arctg} \frac{1}{\sqrt{3}} = \frac{\pi}{6}$

$r_1^k = ?$ $\qquad$ $|r_1| = \sqrt{\frac{3}{4} + \frac{1}{4}} = 1$ $\qquad$ $\left.\begin{array}{l} \cos\theta = \frac{\sqrt{3}}{2} \\ \sin\theta = \frac{1}{2} \end{array}\right\} \Rightarrow \theta = \frac{\pi}{6}$



$r_1 = 1 \cdot \left(\cos\frac{\pi}{6} + i \sin\frac{\pi}{6}\right) \Rightarrow r_1^k = \cos\frac{k\pi}{6} + i \sin\frac{k\pi}{6} \Rightarrow \operatorname{Re}(r_1^k) = \cos\frac{k\pi}{6}$

$\operatorname{Im}(r_1^k) = \sin\frac{k\pi}{6}$

$x_k = c_1 \cos\frac{k\pi}{6} + c_2 \sin\frac{k\pi}{6}$, $\quad c_1, c_2 \in \mathbb{R}$.

IVP $\qquad x_{k+2} - \sqrt{3}\, x_{k+1} + x_k = 0, \quad x_0 = 1, \quad x_1 = 0$

$x_k = c_1 \cos\frac{k\pi}{6} + c_2 \sin\frac{k\pi}{6}$, $k \in \mathbb{N}$ $\qquad c_1, c_2 = ?$ s.t. $x_0 = 1$, $x_1 = 0$

$(x_k)_{k \in \mathbb{N}}$ $\qquad x_0 = c_1 \underbrace{\cos 0}_{=1} + c_2 \underbrace{\sin 0}_{=0} = c_1$ $\qquad x_1 = c_1 \cos\frac{\pi}{6} + c_2 \sin\frac{\pi}{6} = c_1 \frac{\sqrt{3}}{2} + c_2 \cdot \frac{1}{2}$

$(x_k)_{k\in\mathbb{N}}$ $\quad x_0 = c_1 \underbrace{\cos 0}_{=1} + c_2 \underbrace{\sin 0}_{=0} = c_1 \qquad x_1 = c_1 \cos\frac{\pi}{6} + c_2 \sin\frac{\pi}{6} = c_1\frac{\sqrt{3}}{2} + c_2\cdot\frac{1}{2}$

$$\begin{cases} c_1 = 1 \\ c_1\cdot\frac{\sqrt{3}}{2} + c_2\cdot\frac{1}{2} = 0 \end{cases} \Rightarrow \quad c_1 = 1, \quad c_2 = -\sqrt{3}$$

the unique sol of the IVP is $\quad \underline{x_k = \cos\frac{k\pi}{6} - \sqrt{3}\sin\frac{k\pi}{6}, \quad k\in\mathbb{N}}$

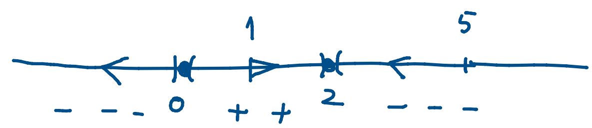
$\underbrace{x' - 2x = 7}_{x_p = a}$ $\qquad x_{k+1} - 2x_k = 7$, $\quad x_k^p = a, \forall k\in\mathbb{N}$ $\qquad \underbrace{x' - tx = 7}_{\text{Lagrange}}$.

$a - 2a = 7 \Rightarrow a = -7$

$x_k^p = -7, \quad \forall k\in\mathbb{N}$.

$\dot{x} = 2x(2-x)$

$f(x) = 2x(2-x) = 2(2x - x^2)$



1. $f(x) = 0 \iff \eta_1^* = 0, \quad \eta_2^* = 2$
2. the orbits are: $(-\infty, 0), \{0\}, (0,2), \{2\}, (2,+\infty)$
3. $f'(x) = 2(2-2x) = 4(1-x)$

   $f'(0) = 4 > 0 \Rightarrow 0$ is a repeller

   $f'(2) = -4 < 0 \Rightarrow 2$ is an attractor

$\varphi(t,1)$ is the unique solution of the IVP $\begin{cases} \dot{x} = 2x(1-x) \\ x(0) = 1 \end{cases}$.

$1 \in (0,2) \Rightarrow \gamma_1 = (0,2) \Rightarrow$ $\mathrm{Im}(\varphi(t,1)) = (0,2)$

$\varphi(t,1) \quad \varphi(t,2) \quad \varphi(t,5)$

$\varphi(t,1)$ is strictly increasing and bounded fc with values between 0 and 2.

$\lim_{t\to-\infty}\varphi(t,1) = 0, \quad \lim_{t\to+\infty}\varphi(t,1) = 2$.

2 is an equilibrium point $\Rightarrow \varphi(t,2) = 2, \quad \forall t\in\mathbb{R}$.