Sunday, June 16, 2024 5:14 PM

$$\begin{cases} x' = x(1-y) = f_1 \\ y' = -y(2-x) = f_2 \end{cases} \qquad \frac{dy}{dx} = \frac{-y(2-x)}{x(1-y)} \qquad \int \frac{1-y}{y}\,dy = \int \frac{x-2}{x}\,dx$$

$$\ln|y| - y = x - 2\ln|x| + c \qquad \ln|y| - y - x + 2\ln|x| = c, \quad c \in \mathbb{R}$$

$$H : (0,\infty)\times(0,\infty) \to \mathbb{R} \qquad H(x,y) = \ln y + 2\ln x - y - x$$

EDP: $x(1-y)\dfrac{\partial H}{\partial x} - y(2-x)\dfrac{\partial H}{\partial y} = 0, \quad \forall (x,y) \in (0,\infty)\times(0,\infty)$

$(\Leftrightarrow)$ $x(1-y)\left(\dfrac{2}{x} - 1\right) - y(2-x)\left(\dfrac{1}{y} - 1\right) = 0$ —"—

$(\Leftrightarrow)$ $(1-y)(2-x) - (2-x)(1-y) = 0$ —"— TRUE.

$$\boxed{f_1\frac{\partial H}{\partial x} + f_2\frac{\partial H}{\partial y} = 0} \qquad \frac{\partial H}{\partial x} f_1 + \frac{\partial H}{\partial y} f_2 = 0$$

$$\begin{cases} \dot{x} = -2x \\ \dot{y} = y \end{cases}$$

$$A = \begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$$

$\lambda_1 = -2 \qquad \lambda_2 = 1$

SADDLE

HAS A GLOBAL F.i.

$$\frac{dy}{dx} = \frac{y}{-2x}$$

Find a global f.i $H : \mathbb{R}^2 \to \mathbb{R}$

$$\int \frac{dy}{y} = -\frac{1}{2}\int \frac{dx}{x} \qquad \ln|y| = -\frac{1}{2}\ln|x| + c$$

$$\underbrace{\ln|y| + \frac{1}{2}\ln|x| = c} \qquad 2\ln|y| + \ln|x| = 2c$$

$$\ln|xy^2| = 2c \qquad xy^2 = k$$

$$H : \mathbb{R}^2 \to \mathbb{R}, \quad H(x,y) = xy^2$$

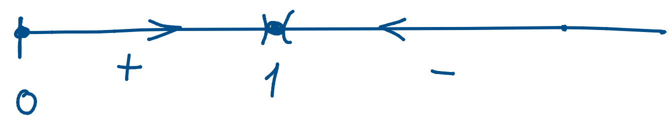
EDP: $-2x\dfrac{\partial H}{\partial x} + y\dfrac{\partial H}{\partial y} = 0, \quad \forall (x,y) \in \mathbb{R}^2$

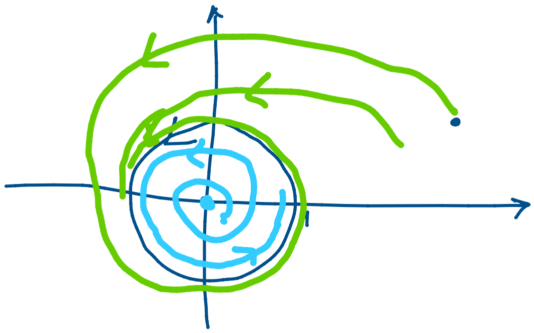
$(\Leftrightarrow)$ $-2x\,y^2 + y \cdot 2xy = 0$ —"— TRUE

$$\partial x \qquad \partial y$$

$$\Leftrightarrow \quad -2x\cdot y^2 + y\cdot 2xy = 0 \qquad -\text{''}- \qquad \text{TRUE}$$

$$\dot{\rho} = \rho(1-\rho^2)$$





$\underline{\rho = 1}$ is the unit circle

$\dot{\theta} = 1 \Rightarrow$ any orbit rotates around the origin in the trig. sense.

$$\begin{cases} x' = -x + xy \\ y' = -2y + 3y^2 \end{cases} \qquad \varphi\left(t, 0, \tfrac{2}{3}\right) = \left(0, \tfrac{2}{3}\right) \quad \forall t \in \mathbb{R}$$

$$\begin{cases} -x + xy = 0 \\ -2y + 3y^2 = 0 \end{cases} \Leftrightarrow \begin{cases} x(-1+y) = 0 \\ y(-2+3y) = 0 \end{cases} \qquad \begin{matrix} x_1 = 0,\ y_1 = 0 & (0,0) \\ x_2 = 0,\ y_2 = \frac{2}{3} & \left(0, \frac{2}{3}\right) \end{matrix}$$

$$\varphi(t, 4, 0) \qquad \varphi\left(t, 1, \tfrac{2}{3}\right) \qquad y' = -2y + 3y^2 \text{ has the equil. } 0 \text{ and } \tfrac{2}{3}$$

$$\varphi(t, 4, 0) \quad \left.\begin{cases} x' = -x + xy, & x(0) = 4 \\ \underline{y' = -2y + 3y^2, \ y(0) = 0}, & \Rightarrow y = 0 \end{cases}\right\} \Rightarrow \begin{cases} x' = -x \\ x(0) = 4 \end{cases} \Rightarrow x = 4e^{-t}$$

$$\varphi(t, 4, 0) = (4e^{-t}, 0), \quad \forall t \in \mathbb{R}$$

$$\varphi\left(t, 1, \tfrac{2}{3}\right) \quad \left.\begin{cases} x' = -x + xy, & x(0) = 1 \\ \underline{y' = -2y + 3y^2, \ y(0) = \frac{2}{3}} & \Rightarrow y = \frac{2}{3} \end{cases}\right\} \Rightarrow \begin{cases} x' = -x + \frac{2x}{3} \\ x(0) = 1 \end{cases} \Rightarrow$$

$$\left\{ x' = -\tfrac{1}{3}x \right. \Rightarrow x = e^{-\frac{t}{3}}$$

$$\begin{cases} x' = -\frac{1}{3}x \\ x(0) = 1 \end{cases} \Rightarrow \quad x = e^{-\frac{t}{3}}$$

$$\varphi\left(t, 1, \frac{2}{3}\right) = \left(e^{-\frac{t}{3}}, \frac{2}{3}\right), \quad \forall t \in \mathbb{R}.$$

Find the values of $h>0$ s.t. the attractor equil. of $\dot{x} = x^2 - x - 6$ is also an attractor fixed point of the discrete dyn. sys. associated to the Euler's numerical formula with stepsize $h>0$ for the given differential equation.

$\dot{x} = x^2 - x - 6$ Step 1. Find the attractor equil.

$x^2 - x - 6 = 0$ $\quad 3, -2$

$\boxed{f(x) = x^2 - x - 6}$ $\quad f'(x) = 2x - 1$ $\quad f'(3) = 5 > 0$ $\quad f'(-2) = -5 < 0.$

-2 is the attractor equil point.

Step 2. Write the Euler's numerical form

$$x_{k+1} = x_k + h f(x_k) \iff x_{k+1} = g(x_k)$$

$$g(x) = x + h f(x) \qquad g(-2) = -2 + h \underbrace{f(-2)}_{=0} = -2 \quad \forall h.$$

$? \; g(-2) = -2$ $\qquad g'(x) = 1 + h f'(x) \Rightarrow g'(-2) = 1 + h \cdot f'(-2) = 1 - 5h$

$$|g'(-2)| < 1 \iff |1 - 5h| < 1 \iff -1 < 1 - 5h < 1 \iff -2 < -5h < 0$$

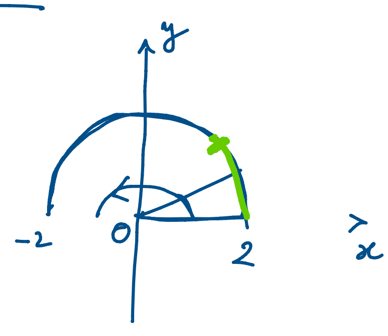
$$\iff 0 < h < \frac{2}{5} \iff h \in \left(0, \frac{2}{5}\right).$$

$$\{2e^{it} : t \in [0, \pi]\} \subset \mathbb{C}$$

$2e^{+it} = 2\cos t + i \cdot 2\sin t =$

$$\begin{cases} x = 2\cos t \\ y = 2\sin t, \quad t \in [0, \pi] \end{cases}$$

$x^2 + y^2 = 4\cos^2 t + 4\sin^2 t = 4$



$2e^{it} = 2(\cos t + i\sin t), \quad t \in [0, \pi]$

$$x^2+y^2 = 4\cos^2 t + 4\sin^2 t = 4$$

$$x^2+y^2=4$$

$$t \in \left[0, \tfrac{\pi}{4}\right]$$

$$2e^{it} = 2\left(\cos t + i \sin t\right), \quad t\in[0,\pi]$$

$$|2e^{it}| = 2$$

$$\boxed{z = \rho\left(\cos\theta + i\sin\theta\right)} \qquad \boxed{z = x+iy}$$

(ρ, θ) are the polar coordinates of the point in $\mathbb{R}^2$ of cartesian coord. (x,y)