Ain Shams University
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Specialized CHEP

Active Suspension System: Modern Control Techniques

BSc. Graduation Project (1)

MCT491s

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ABSTRACT

This thesis investigates the performance of various advanced control techniques applied to an active suspension system using a quarter-car model. The quarter-car model is widely used due to its simplicity and effectiveness in capturing the essential dynamics of a suspension system while allowing for efficient analysis and simulation. Simulation experiments are conducted to evaluate the effectiveness of these control algorithms.

The study focuses on the application of Linear Quadratic Regulator (LQR) for linear-time-invariant systems, Reinforcement Learning (RL), and Sliding Mode Control (SMC) to enhance suspension performance. LQR is particularly powerful for its ability to provide optimal control solutions by minimizing a defined cost function, balancing performance and energy efficiency. RL, on the other hand, offers a data-driven approach that adapts to system uncertainties and non-linearities, making it highly suitable for complex and dynamic environments. Additionally, the robustness of SMC makes it effective in handling system disturbances and parameter variations. Furthermore, the impact of state estimation using a Kalman filter is assessed, as it enables accurate estimation of unmeasured states, enhancing the overall control performance.

MATLAB/Simulink is employed to perform the simulations, beginning with the mathematical modeling of the system in state-space form, followed by the implementation of these modern control techniques. This research aims to contribute to a deeper understanding of control theory and its practical application in improving vehicle ride comfort and handling. By comparing the performance metrics of each control strategy, the study seeks to highlight the strengths and limitations of these advanced techniques, providing valuable insights into their applicability for real-world suspension systems.

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Nomenclature

Symbol	**Description**	**Unit**
b_s	Damper average damping coefficient	N.s/m
b_{tr}	Tire damping coefficient	N.s/m
e	Signal error	-
f	Frequency	Hz
f_{n-s}	Sprung mass natural frequency	Hz
f_{n-us}	Unsprung mass natural frequency	Hz
k	Stiffness of the Suspension Spring	N/m
k_t	Equivalent Spring Stiffness of the Tire	N/m
L	Lenght of Conecting rod	mm
M	Sprung Mass	kg
m	Unsprung Mass	kg
R	Lenght of Crank	mm
S_D	Sprung displacment	mm
STD	Static Tire Deflection	mm
T	Periodic Time	s
X	Position of the slider	mm
z	Vertical Position of The Car Body	mm
z_r	Road Excitation Displacement	m
z_t	Vertical position of Unsprung Mass	m
θ	Angle of the crank	degrees

Abbreviations

Symbol	Description
ASS	Active Suspension System
DOF	Degree-of-Freedom
DTD	Dynamic Tire Deflection
EMI	Electromagnetic Interference
FA	Actuator Force
FLC	Fuzzy Logic Control
IMU	Inertial Measurement Unit
LQR	Linear Quadratic Regulator
LVDT	Linear Variable Displacement Transducer
MPC	Model Predictive Control
MR	Magnetorheological
PI	Performance Index
PSS	Passive Suspension System
RF	Radio Frequency
RL	Reinforcement learning
RMS	Root Mean Square
RPM	Revolution per minute
SD	Sprung Mass Displacement
SMC	Sliding Mode Control
ST	Suspension Travel
STD	Static Tire Deflection
TR	Transmissibility Ratio
UD	Unsprung Mass Displacement

Chapter 1

Introduction

This chapter provides an introduction to vehicle suspension systems, outlining their fundamental characteristics and components. It begins by exploring various methods of classifying suspension systems and offers an overview of the terminology commonly used in suspension system technology. Detailed descriptions are included for several prevalent suspension components. It is important to highlight that this project specifically focuses on a prepared vertical test rig simulating a quarter-car model, which serves as the foundation for our study and analysis of suspension systems.

1.1 Importance of Suspension System

Suspension components, particularly spring, as the system elastic member, and damper have a profound effect as it serve a dual purpose contributing to the vehicle's handling and increasing the vehicle safety and improve the level of comfort of the passengers and keeping vehicle occupants comfortable and reasonably well isolated from road noise, bumps, and vibrations (better overall driving experience). It is important for the suspension to keep the vehicle wheel in contact with the road surface as much as possible, because all the forces acting on the vehicle do so through the contact patches of the tires. The suspension also helps protect the vehicle itself and any cargo or luggage from damage and wear. [5]

1.2 Suspension system components

This section provides brief overview of the essential components of suspension system, specifically springs and dampers, both of which have a profound effect on ride and handling performance. [6]

1.2.1 SPRING (Elastic member)

The suspension system incorporates an elastic link between the tires and the vehicle's body, enabling it to simultaneously press the tires onto the road surface to follow dips and absorb shocks or overloads. This system minimizes the transmission of impacts to the vehicle frame, ensuring both stability and comfort. There are three main types of springs commonly used in suspension systems: torsion bars, coil springs, and leaf springs.

- **Coil springs** are essentially wound torsion bars, prized for their exceptional endurance, compact design, and ease of mounting.
- **Leaf springs** consist of long, thin members that are loaded in bending. They are typically assembled using multiple thin metal layers to achieve the desired spring rate. Beyond providing suspension, they also function as linkage and damping elements.
- **Torsion bars** rely on the twisting motion of a long bar to provide a spring rate that reduces shock loading on the vehicle. However, their placement across the lower portion of the car makes them more difficult to package compared to other types. [7]

Characteristics of spring is detailed in Appendix 1.

1.2.2 DAMPER (Energy dissipation member)

As a car passes over a bump, the springs deflect and then rebound. Without a mechanism to dissipate the energy stored in the springs, the car would continue to bounce up and down. Dampers, or shock absorbers, fulfill this critical role. A damper consists of a piston and a cylinder equipped with adjustable valves that control the flow of hydraulic fluid (oil). These valves regulate the damping force during both the retraction (bounce) and extension (rebound) phases. The damper allows oil to flow through one-way valves in the piston and small control passages from one chamber to another, but this flow is deliberately restricted, causing it to move very slowly.

This controlled fluid movement slows down the spring's oscillations, helping the car return to a stable, level ride. The damper converts the kinetic energy from the vehicle's bounce into thermal energy, effectively dissipating it. Dampers are designed to retract under a lower force than is required for extension. This asymmetry allows them to absorb road bump forces while effectively dampening spring oscillations, resulting in improved ride comfort, vehicle stability, and control. [5]

1.3 Suspension System Performance

The performance of an automotive suspension system refers to how well it functions in terms of providing balance between passenger comfort, vehicle stability during all driving conditions.

Suspension dynamic performance:

Vehicle Handling: Vehicle stability and road holding (i.e. providing grip for the driver of the vehicle to control direction).

Ride Quality: Passenger's comfort or discomfort during movement of the vehicle.

The preferred performance of a suspension system is defined by its ability to seamlessly blend vehicle handling and roadholding, evaluated through dynamic tire deflection (DTD), with ride quality assessed via the transmissibility ratio (TR). An ideal suspension system ensures that passengers enjoy a smooth and comfortable ride, free from excessive body vibrations, while also providing the driver with optimal control, stability, and responsiveness under various driving conditions.

A well-designed suspension system strikes the perfect balance, delivering a luxurious ride for everyday driving and dynamic handling for sporty driving. It excels in vibration isolation by effectively dampening road-induced disturbances and offers sufficient suspension travel and dynamic tire deflection to adapt to diverse terrains.

Performance characteristics are detailed in Appendix 2.

1.4 Effect of suspension Parameters

Achieving both a luxurious ride and dynamic handling for sport driving presents a significant challenge in passive suspension systems (PSS). This is due to the opposing suspension characteristics required to fulfill these two objectives simultaneously. The inherent trade-offs in automotive passive suspension systems result in compromises between comfort and performance. Explanation of these trade-offs is detailed in Appendix 3.

1.5 Motivation

The experience of oscillations during vehicle motion is a well-documented challenge that arises primarily due to irregularities and stimuli encountered on road surfaces. These oscillations have far-reaching implications, affecting passenger comfort, cargo stability, and vehicle durability. To mitigate these effects, suspension systems are engineered to regulate and attenuate oscillations, ensuring that they remain within acceptable limits.

Automotive suspension systems have evolved significantly, encompassing various designs such as passive (mechanical), semi-active, active, and pneumatic systems. Passive suspension systems, characterized by their simplicity and cost-effectiveness, dominate the market and are commonly integrated into mass-produced vehicles. They typically consist of components such as coil springs, leaf springs, torsion bars, dampers, lever arms, and stabilizer bars. While these systems perform adequately under standard conditions, their inherent limitations—such as fixed properties and lack of adaptability to dynamic external stimuli—can compromise ride comfort, particularly in challenging driving environments. To address these shortcomings, advancements in suspension technology have introduced systems with enhanced adaptability. Air suspension systems, for instance, employ air springs capable of modulating stiffness through internal pressure adjustments, offering improved responsiveness to varying road conditions. Semi-active systems, incorporating technologies like magnetorheological (MR) dampers, provide another layer of adaptability, enabling the system to better respond to road-induced oscillations.

Active suspension systems represent the pinnacle of suspension technology, capable of dynamically adjusting to changing road conditions in real-time. By actively controlling the forces applied to the suspension components, these systems offer unparalleled performance in mitigating oscillations, ensuring optimal ride comfort, and maintaining vehicle stability. This thesis aims to explore and further develop control strategies for active suspension systems, emphasizing their potential to redefine vehicle dynamics and enhance overall driving experiences. In active suspension system, a hydraulic actuator (or electromagnetic actuator) generates an impact force acting on both masses of the vehicle, gradually diminishing the vehicle's oscillation.[8]

1.6 Scope and Objective

The main focus of this study is to perform a comparative analysis for the most commonly used modern control techniques, including LQR, SMC, and RL. The evaluation will focus on their effectiveness in reducing vertical acceleration and displacement of the sprung mass, and the suspension travel. The pros and cons of each technique will be assessed based on these results.

- **Objectives:**
  - Conducting various simulations using MATLAB/SIMULINK on the quarter car model to understand its behavior.
  - Compare between the response of the PSS and ASS for various road profiles
  - Implement the selected control algorithms on MATLAB/SIMULINK environment for different conditions.
  - Conduct comparative analysis to determine the suitable control algorithm for which case.

1.7 Limitations

- Simplified Model: The use of a quarter-car model represents a simplification of the actual vehicle dynamics. A full-vehicle model would provide a more accurate representation of real-world behavior, considering interactions between different axles and body motions.

- Limited Scope: The research focuses on a limited set of control strategies (LQR, RL, and SMC). Exploring other advanced control techniques, such as adaptive control and predictive control, could provide further insights into optimal suspension performance.

- Simulation-Based: The study relies entirely on simulations. Experimental validation on a physical test rig would be necessary to further validate the findings and assess the practical feasibility of the proposed control strategies.

- Idealized Assumptions: The simulations may involve idealized assumptions, such as perfect actuator dynamics and the absence of noise and disturbances in the measurements. Real-world implementations may encounter challenges due to these factors.

- Computational Cost: Some control algorithms, such as Reinforcement Learning, can be computationally expensive, especially for complex models or real-time applications.

1.8 Thesis contribution

This work contributes to the field of vehicle dynamics by:

- Demonstrating the effectiveness of advanced control techniques: The study provides a comparative analysis of LQR, RL, and SMC controllers in enhancing the performance of an active suspension system. This analysis will shed light on the strengths and weaknesses of each approach under different operating conditions.

- Validating the use of simulation tools: The research leverages MATLAB/Simulink for the design, implementation, and evaluation of control strategies. This demonstrates the effectiveness of simulation tools in developing and testing control systems for complex dynamic systems like vehicle suspensions.

- Providing insights into control system design: The findings of this research will contribute to a better understanding of how to design and implement effective control systems for active suspensions, considering factors such as ride comfort and handling.

- Establishing a foundation for future research: The results and insights gained from this research can serve as a foundation for further investigations into more advanced control techniques, such as adaptive control and predictive control, for improving vehicle suspension systems.

1.9 Thesis Organization

The content of thesis is as follows:

- **Chapter 1: Introduction**
  - Provided background on vehicle suspension systems, motivation for the research, research objectives, scope and limitations, and an outline of the thesis structure.
- **Chapter 2: Literature Review**
  - Reviews existing research on vehicle suspension systems, control theory fundamentals, active suspension control, simulation and modeling techniques, and state estimation methods.
- **Chapter 3: Mathematical Modeling**
  - Derives the equations of motion for the quarter-car model, develops the state-space representation.
- **Chapter 4: Modern Control Techniques**
  - Focuses on the design and implementation of LQR, RL, and SMC controllers in MATLAB/Simulink, including simulation setup, and analysis.
- **Chapter 5: Results and Discussion**
  - Presents and discusses the simulation results, compares the performance of the control strategies, analyzes the results, and investigates the impact of control parameters.
- **Chapter 6: Conclusion and Future Work**
  - Summarizes the key findings and conclusions, discusses the research contributions, acknowledges limitations, and suggests potential areas for future research.

Chapter 2

Literature Review

This chapter provides a brief overview of suspension system history and development, followed by reviewing the most recent studies concerned with modern control techniques in Active suspension system.

2.1 Suspension system history

Suspension is the term given to the system that connects a vehicle body to its wheels and allows relative motion between them. This Relative motion between the wheel and the body is necessary to isolate the vehicle's body from the road irregularities that are fed into the tire at the road/wheel interface. In general, some kind of linkage system that combines damping and stiffness controls this motion. We refer to this process as a suspension. This Damping and stiffness are fed to the system through the dampers, springs (shock absorbers) and the linkages that transmit motion or forces between various parts of the suspension system. [9], [5]

In the 15^{th} century, people understood that suspensions were crucial for making passengers feel comfortable. Back then, coaches had bodies that hung from a set of leaf springs attached to a strong chassis. This chassis held the wheel hubs. The loose ends of the springs were linked to the coach's body using leader belts. Figure 2.1 shows a coach[10] from around 1650 with this kind of suspension setup, providing a fascinating example. [1] [11]

Figure 2.1: This coach, built around 1650, shows a suspension made by four leaf springs and leader belts (National Automobile Museum of Torino). [1]

2.1.1 Suspension system evolution

1. **Passive Suspension Systems (Before 1980s):**
   - Early automotive suspension systems were mostly passive, relying on mechanical components such as springs and dampers to absorb shocks and vibrations.
   - These systems were simple and cost-effective but offered limited adaptability to varying road conditions.
2. **Active Suspension Systems (1980s - 1990s):**
   - The concept of active suspension, which actively adjusts the suspension settings in response to changing road conditions, gained popularity in the 1980s.

- One of the pioneering examples was the development of the Lotus Active Suspension in Formula 1 in the late 1980s.
- Some high-end road cars began to incorporate active suspension systems in the late 1980s and early 1990s, including models from manufacturers like Citroën and Cadillac.
- Active suspension systems used sensors to monitor various factors, and electronic control systems adjusted the suspension settings in real-time to optimize ride comfort and handling.

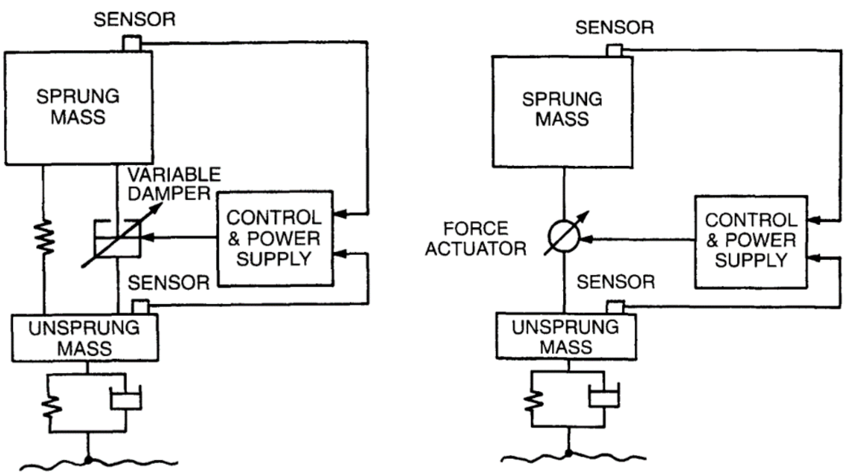
3. **Semi-Active Suspension Systems (1990s - Present):**

- Semi-active suspension systems represent a middle ground between passive and active systems.
- In a semi-active system, the suspension settings are adjusted in real-time, but they typically do not provide as much active control as fully active systems.
- Popular semi-active systems include electronically controlled shock absorbers (e.g., Delphi's MagneRide) that can adjust damping rates based on driving conditions.
- These systems offer improved ride quality and handling without the complexity and cost associated with fully active systems.

4. **Recent Developments (2000s - Present):**

- Advancements in sensor technology, computing power, and materials have allowed for more sophisticated suspension systems.
- Active and semi-active suspension systems continue to evolve, with a focus on enhancing performance, safety, and comfort.
- Some modern high-performance and luxury vehicles feature advanced adaptive suspension systems that can adjust multiple parameters, including ride height, stiffness, and damping rates.

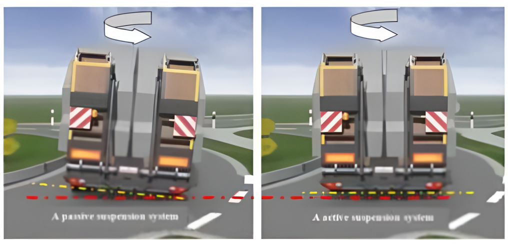
The semi-, figure 2.2 (a), and fully-, figure 2.2 (b), active suspension system are introduced. [9]



(a) (b)

Figure 2.2: Schematics of (a) Semi-active and (b) Fully-active suspension systems [2]

Active suspension system is even beneficial for the HVG (Heavy Goods Vehicle) in case of the vehicle negotiating a turn.



(a) (b)

Figure 2.3: Difference between the behavior of the truck body in a curve: with passive suspension (a) and with active suspension (b). [3]

2.2 Recent Research

2.2.1 PID controller

Many studies on control problems for active suspension systems have been introduced in the last few decades. [12] used PID (Proportional - Integral - Derivative) algorithm to control this system. This controller includes three stages corresponding to three coefficients, K_p, K_D and K_i. These coefficients will be self-tuning or selected based on Ziegler-Nichols's method [13].

However, the effectiveness of these methods is often low. [14] used the Fuzzy algorithm to tune the PID controller's variables based on excitation signals from the road. Based on the principle of genetics, the variables of the PID controller can be optimally chosen by the GA (Genetic Algorithm) solution. This was done by [15]. The size of the population and the number of generations will be determined based on the experience of the designer. Different from the GA algorithm, the PSO (Particle Swarm Optimization) method helps optimize the controller's coefficients based on the characteristics of the animals [16]. The animals used in this algorithm often have a herd lifestyle, such as whales, ants, bees, etc. Besides, many other intelligent algorithms have been used to search for the controller's parameters optimally [17].

If the system has many objects to be controlled, PID can not be used, buy multiple PID controllers is an option[18].

2.2.2 LQR controller

The Linear - Quadratic Regulator (LQR) controller can be used to replace the conventional PID controller for MIMO (Multi Input - Multi Output) systems[19]. This algorithm aims to minimize the cost function so the system can be more stable [20]. The mathematical model of this algorithm must be given in the form of a state matrix [21]. Once the Gaussian filter is combined with the LQR controller, it will be called an LQG (Linear - Quadratic Gaussian) controller [22].

Recent advancements in Linear Quadratic Regulator (LQR) control have led to significant developments in control theory and applications. For example, the LQR problem for singular systems has been revisited, focusing on enabling singular techniques for traditional control [23].

Additionally, the role of predictions in online LQR control under stochastic and adversarial disturbances is examined [24], while data-driven LQR frameworks with stability guarantees are proposed using recursive learning and policy gradient methods [25].

The above methods are only used for linear objects. If the object is nonlinear, more complex algorithms need to be used such as SMC MPC and RL.

2.2.3 SMC controller

The SMC algorithm is often used for complex nonlinear objects [26]. Kazemian et al. used the SMC algorithm to control the hydraulic actuator. To simplify the model, a linearization process of a hydraulic actuator should be done. This process is shown by Nguyen [27]. Then, a quarter-dynamic model that considers the influence of the actuator will have five state variables [28]. According to Zhao et al. [29], the sliding surface is an important component of the SMC controller. The controlled object can travel around the sliding surface in order to reach a steady state.

The "chattering" is a phenomenon that can occur when this algorithm is used. This problem can make the control signal vibrate continuously with high frequency and a small amplitude. To limit this problem, an SMC controller must be combined with a PID controller or a Fuzzy controller. For instance, Hsiao and Wang [30] introduced the design of an algorithm that combines SMC and Fuzzy, called STFSMC (Self-tuning Fuzzy Sliding Mode Controller). Another recommendation has been made by Suhail et al. [31] about using the ASMC (Adaptive Sliding Mode Control) algorithm. Overall, ride comfort can be greatly improved if this method is used [32]. Besides, many advanced control methods have been applied to the active suspension system, which also brings high efficiency.

2.2.4 RL controller

Recent advancements in the use of reinforcement learning for active suspension systems have shown promising results. For instance, a study on deep reinforcement learning for active suspension control highlights its potential for meeting ISO 2631-5 comfort requirements [33]. Another research work introduced a TD3-based control algorithm to address actuator delays in active suspension systems [34].

Optimization of full vehicle active suspension using advanced reinforcement learning controllers has also been explored, showcasing significant improvements in ride comfort [35]. Similarly, reinforcement learning-based vibration control for half-car active suspension systems demonstrated the effectiveness of adaptive dynamic programming algorithms [36].

The sim-to-real transfer of active suspension control has been studied to bridge the gap between simulated environments and real-world applications [37]. Additionally, a novel approach using a closed-chain five-bar active suspension system with deep reinforcement learning provided promising results in stability and obstacle traversal [38].

Research into magnetorheological-damped vehicle suspension systems using RL showed that the TD3 algorithm outperforms traditional control strategies [39]. Finally, iterative learning-based reinforcement methods for road profile estimation and active suspension control in connected vehicles highlight the potential for collaborative frameworks [40].

Chapter 3

Mathematical Model

3.1 Two-DOF Quarter Car Model

To analyze the parameters related to the suspension system, a simplified quarter-car model, as shown in 3.1(a), was utilized. The quarter-car model was chosen due to its simplicity and common use in analyzing the vertical vibrations caused by railway disturbances in vehicle dynamic models.

The vehicle's mass is divided into two: the sprung mass (representing the vehicle body) and the unsprung mass (tire). Suspension springs and dampers connect the sprung and unsprung masses and the road.

Both the transverse and longitudinal deflections are considered insignificant compared to the vertical deflections of the suspension system. For the passive suspension system, the actuator force is not considered as there is no control element, as shown in 3.1(b).

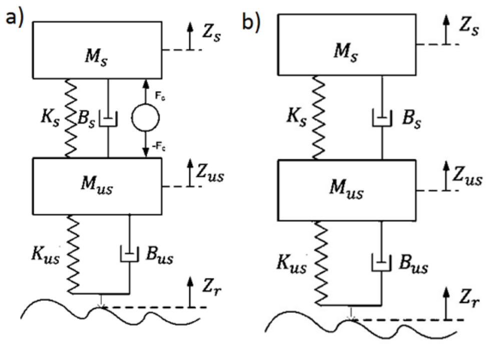


Figure 3.1: Active suspension system (a) and Passive suspension (b).
[41]

Table 3.1: Parameters of Quarter Car Model.

M_s	Vehicle body mass or sprung mass.
M_{us}	Unsprung mass (Tire, wheel, brake caliper, suspension links, etc.)
K_s	Spring constant for the sprung mass.
K_{us}	Spring constant for the unsprung mass.
B_s	Inherent damping coefficient for the suspension system.
B_{us}	Inherent damping coefficient for vehicle wheel assembly.
F_c	The active suspensions actuator control force.
Z_s	Vehicle (sprung mass) body displacement.
Z_{us}	Vehicles wheel displacement and the unsprung masses displacement
Z_r	Excitation due to the railway disturbance.

3.1.1 Passive Suspension Model

The parameters for the PSS are given in the following table:

Table 3.2: Passive Model Parameters

Parameter	Symbol	Value	Unit
Sprung mass	M	30	kg
Unsprung mass	m	13	kg
Spring stiffness	k	6921	N/m
Tire stiffness	k_t	81000	N/m
Damper average damping coefficient	b_s	900	N s/m
Tire damping coefficient	b_{tr}	0	N s/m

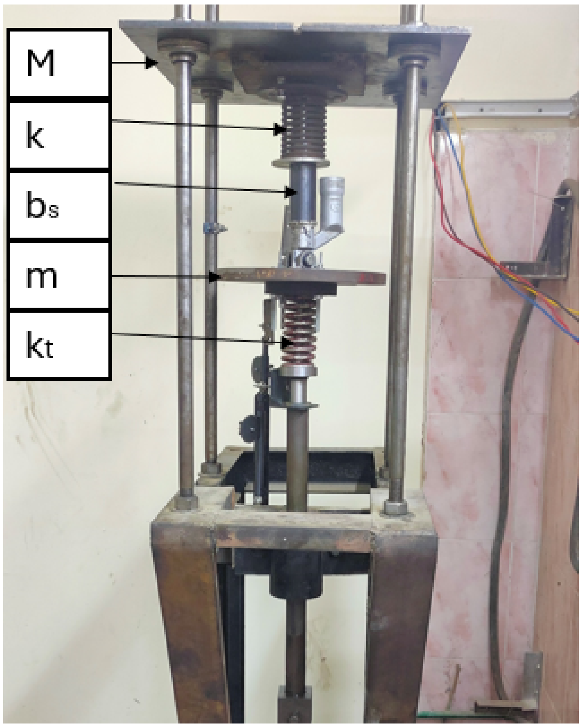


Figure 3.2: PSS Physical Model, by Ain Shams Univresity Students as Graduation Project, Fall 2024, Automotive Department.

3.1.2 Active Suspension Model

The parameters of the modified system is listed in the table below.

Table 3.3: Active Model Parameters

Parameter	Symbol	Value	Unit
Sprung mass	M	34	kg
Unsprung mass	m	11	kg
Spring stiffness	k	6921	N/m
Tire stiffness	k_t	81000	N/m

The setup of the modified Active suspension System is shown in the following figure:

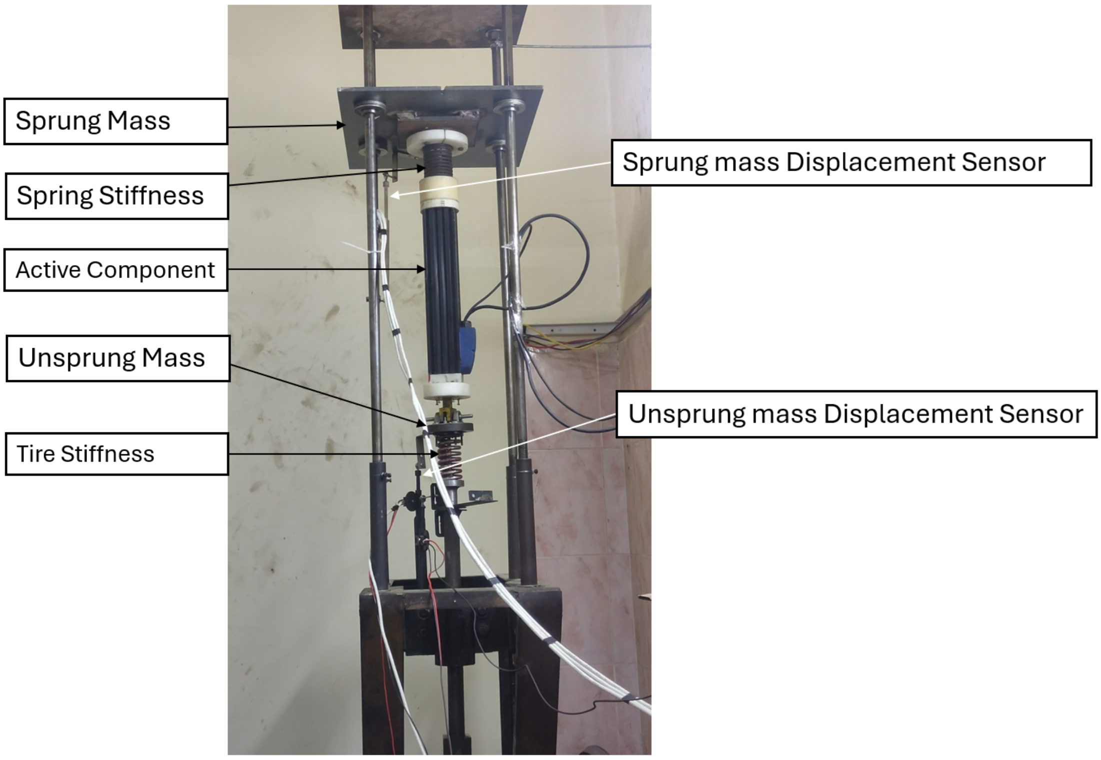


Figure 3.3: ASS Physical Model, by Ain Shams Univresity Students as Graduation Project, Spring 2024, Automotive Department.

3.1.3 Equation Of Motion

Sprung Mass

$$M_s\ddot{Z}_s = B_s\dot{Z}_{us} - B_s\dot{Z}_s - K_s(Z_s - Z_{us}) + F_c \tag{3.1}$$

$$\ddot{Z}_s = \frac{B_s\dot{Z}_{us}}{M_s} - \frac{B_s\dot{Z}_s}{M_s} - \frac{K_s(Z_s - Z_{us})}{M_s} + \frac{1}{M_s}F_c \tag{3.2}$$

Unsprung Mass

$$M_{us}\ddot{Z}_{us} = -B_s\dot{Z}_{us} - B_{us}\dot{Z}_{us} + B_s\dot{Z}_s + B_{us}\dot{Z}_r - K_s(Z_{us} - Z_s) - K_{us}(Z_{us} - Z_r) - F_c \tag{3.3}$$

$$\ddot{Z}_{us} = -\frac{B_s\dot{Z}_{us}}{M_{us}} - \frac{B_{us}\dot{Z}_{us}}{M_{us}} + \frac{B_s\dot{Z}_s}{M_{us}} + \frac{B_{us}\dot{Z}_r}{M_{us}} - \frac{K_s(Z_{us} - Z_s)}{M_{us}} - \frac{K_{us}(Z_{us} - Z_r)}{M_{us}} - \frac{1}{M_{us}}F_c \tag{3.4}$$

3.1.4 State Space Model

A state-space representation is a mathematical model used in modern control theory and system analysis to describe the behavior of a dynamic system. It offers a concise and systematic approach to representing the evolution of a system over time. In contrast to the frequency-domain representation (e.g., transfer functions), which characterizes a system's input-output relationship in terms of frequencies, the state-space representation provides a time-domain description.

The general state-space representation is given by the following:

$$\dot{x}_{(t)} = Ax_{(t)} + Bu_{(t)}$$

$$y_{(t)} = Cx_{(t)}$$

- x: State variables vector.
- $\dot{x}$: Represents the time derivative of the state variables vector.
- y: Output vector.
- u: Input vector.
- A: System matrix.
- B: Input matrix.
- C: Output matrix.
- D: Feedforward matrix.

State variables:

Variable	Description
$X_1 = Z_s - Z_{us}$	suspension travel
$X_2 = \dot{Z}_s$	sprung mass velocity
$X_3 = Z_{us} - Z_r$	wheel deflection
$X_4 = \dot{Z}_{us}$	wheel vertical velocity

Inputs: We will consider the input $\boldsymbol{u}$ into the system as the road disturbance velocity $\dot{Z}_r$ and the actuator input F_c.

Outputs: We will consider outputs $\boldsymbol{y}$ from the system as the suspension travel $Z_s - Z_{us}$ and the vehicle body (sprung mass) acceleration $\ddot{Z}_s$.

Using the above equations of motion, the state-space model of the active suspension system can easily be obtained and be written in the matrix form shown below:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{K_s}{M_s} & -\frac{B_s}{M_s} & 0 & \frac{B_s}{M_s} \\ 0 & 0 & 0 & 1 \\ \frac{K_s}{M_{us}} & \frac{B_s}{M_{us}} & -\frac{K_{us}}{M_{us}} & -\frac{B_s+B_{us}}{M_{us}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{M_s} \\ -1 & 0 \\ \frac{B_{us}}{M_{us}} & -\frac{1}{M_{us}} \end{bmatrix} \begin{bmatrix} \dot{Z}_r \\ F_c \end{bmatrix} \tag{3.5}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{K_s}{M_s} & -\frac{B_s}{M_s} & 0 & \frac{B_s}{M_s} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{M_s} \end{bmatrix} \begin{bmatrix} \dot{Z}_r \\ F_c \end{bmatrix} \tag{3.6}$$

Chapter 4

Modern Control Techniques

4.1 Controllability

Control systems face numerous challenges, including stabilizing unstable systems through feedback control. Controllability is a crucial factor in addressing these challenges. A system is considered controllable if a single control input, denoted as 'u', can effectively steer the system's state from any initial configuration to any desired final state. For a linear time-invariant state-space system to be controllable, the controllability matrix 'P' must have full rank, which implies that its rank must equal the number of states ('n') in the system.

$$\mathcal{C} = \begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix}$$
$$rank(\mathcal{C}) = n$$

```
% MATLAB script
ms  = 34;            % Sprung Mass (kg)
mus = 11;            % Unsprung Mass (kg)
ks  = 6921;          % Suspension Stiffness (N/m)
kus = 81000;         % Wheel stiffness (N/m)
bs  = 0;             % Suspension Inherent Damping coefficient (sec/m)
bus = 0;             % Wheel Inhenrent Damping coefficient (sec/m)

%% System Dynamics for the Active Suspension system.
A = [ 0 1 0 -1 ;
-ks/ms -bs/ms 0 bs/ms;
0 0 0 1;
ks/mus bs/mus -kus/mus -(bs+bus)/mus];

B = [0  0 ;
0 1/ms ;
-1  0 ;
bus/mus -1/mus ];

C = [ 1 0 0 0 ;
-ks/ms -bs/ms 0 bs/ms ];

D = [0 0;
0 0;
0 0;
0 0;
0 0;
0 1/ms];

%% Controllability
rank(ctrb(A,B))
```

The following figure shows that the system is controllable, because its controllability matrix is full rank which is equal to the number of states.

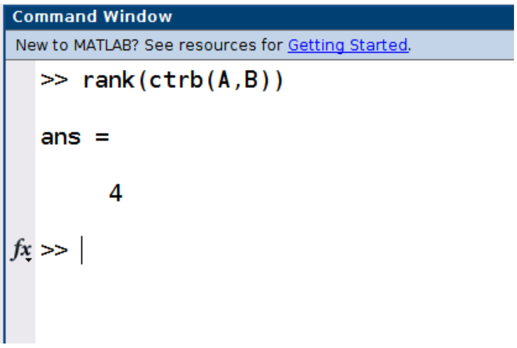


Figure 4.1: Conrollability matrix is full rank

4.2 Full State Feedback

A full state feedback controller, also referred to as a pole placement controller which is shown in figure 4.2, provides an optimal solution for achieving desired pole locations of a closed-loop system. This approach leverages the fact that all state variables are assumed to be known to the controller at all times and are available for feedback.

The state-space representation of the plant is utilized, where each state variable is fed back to the control input, u, through a gain matrix, K. This feedback gain matrix can be adjusted to achieve the desired closed-loop pole values.

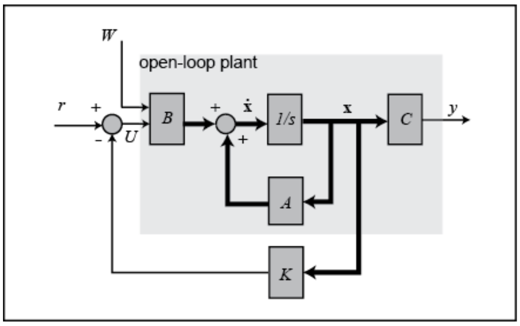


Figure 4.2: Full state feedback block diagram. [4]

Assuming no tracking ($r = 0$) and no external disturbance ($w = 0$), the system input is given by:
$u = -Kx$
$\dot{x} = Ax - BKx$
$\dot{x} = x(A - BK)$

4.3 Linear Quadratic regulator (LQR)

A widely used type of state feedback control that offers a systematic method for determining the control gain, K, is LQR controller. The LQR approach will be employed in the controller design for the active suspension system, as it is a classic and straightforward option for linear, time-invariant, multiple-input multiple-output (MIMO) systems. One of the key advantages of using an LQR controller is its ability to weight the factors affecting the performance index based on the desired outcome. For this project, the focus of the LQR approach will be on enhancing ride comfort and improving road-handling performance in the quarter-car model.

The function of an LQR controller is to minimize the cost function, J, which is shown in the following equation:

$$J = \frac{1}{2}\int_0^t (x^T Q x + u^T R u)\, dt$$

x^T = State vector.
u^T = Input vector.

4.3.1 LQR Implementation

The weighting matrices, Q and R, within the quadratic performance index significantly influence the LQR controller's behavior. These matrices allow for tuning the control system's priorities, such as emphasizing ride comfort or minimizing control effort. The optimal values for Q and R were determined through iterative simulations and tuning within the MATLAB environment. This process involved systematically adjusting the elements of Q and R and observing the resulting system response to determine the combination that best met the desired performance objectives.

Chapter 5

Results and Conclusion

Appendix A

Appendices

A.1 Appendix 1

The following are some characteristics of springs:

Suspension Rate or Stiffness (K) is one of the characteristics that mean that the force needed to compress or extend a suspension spring by a specific amount of distance is known as the spring rate, also known as stiffness. Newtons per millimeter (N/m) are the common units of measurement.

Increased spring rates give the suspension more stiffness, which enhances responsiveness and handling. But too stiff of springs can make for a rough ride. And this will be well discussed in the suspension system conflicts section. [7]

A.2 Appendix 2

- **Vibration Isolation:** This term means the response of the sprung mass (all components directly supported by the suspension system. Vehicle body and passenger passengers) to the various excitation types from the road. In most cases, **the transmissibility ratio** (transfer function) is used for evaluating a linear suspension system's vibration isolation ability.

$$TR = \frac{Z_{\mathrm{s}}}{Z_0} \tag{A.1}$$

  where:

  - TR is the transmissibility ratio of the suspension system.
  - Z_{s} is the sprung mass displacement.
  - Z_0 is the road excitation displacement.

- **Suspension Travel (ST):** This means the deflection of the suspension spring or the relative displacement between the sprung mass (Car Body) and unsprung mass (components that are not supported by the suspension system i.e. The wheels and tires).

$$ST = \frac{Z_{\mathrm{us}} - Z_{\mathrm{s}}}{Z_0} \tag{A.2}$$

  where:

  - ST is the suspension travel relative to road excitation displacement.
  - Z_{us} is the unsprung mass displacement.
  - Z_{s} is the sprung mass displacement.

- **Roadholding:** The usual force acting between the tire and the road varies as the vehicle system vibrates on the road. The roadholding capabilities, handling, and performance of the vehicle are all influenced by tire vibration, since the cornering force, tractive effort, and braking effort generated by the tire are all associated with the tire's normal load. The displacement of the unsprung mass with respect to the road surface can be used to depict the normal force between the tire and the road during vibration. **The dynamic tire deflection** is used as a measurable term for evaluating the suspension performance characteristic as this formula:

$$DTD = \frac{Z_0 - Z_{\mathrm{us}}}{Z_0} \tag{A.3}$$

where:

- DTD is the dynamic tire deflection.
- Z_0 is the road excitation displacement.
- Z_{us} is the unsprung mass displacement.

A.3 Appendix 3

As an illustration figures A.1 and A.2 shows the tradeoffs in suspension design, the response of a passive suspension system at different suspension parameters implemented by suspension mathematical modeling using MATLAB software, it resulted:

A stiffer suspension behavior is required to enhance tire contact with the road-dynamic tire deflection-and the vehicle's dynamic behavior during braking and turning, whereas a softer suspension behavior is required to increase vehicle vibration isolation and enhance passenger comfort. [42]

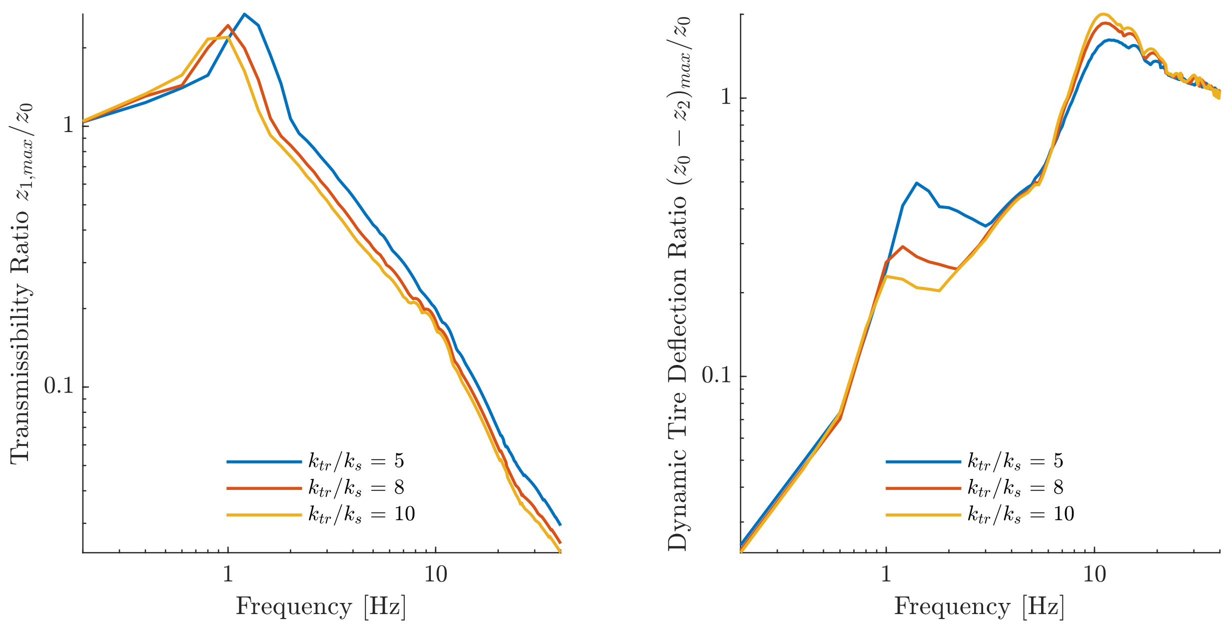


Figure A.1: Transmissibility ratio (left) and Dynamic Tire Deflection (right) as a function of frequency for the sprung mass of a suspension system with different ratios of tire stiffness to suspension spring stiffness

On the other hand, As shown in figures 1.3 and 1.4, talking about the damping ratio (ζ), the smaller damping ratio is required to high vibration isolation and good ride quality (lower transmissibility ratio), the natural frequency of the sprung mass or close to the natural frequency of the unsprung mass, to maintain good roadholding capability, higher damping is required. [2]

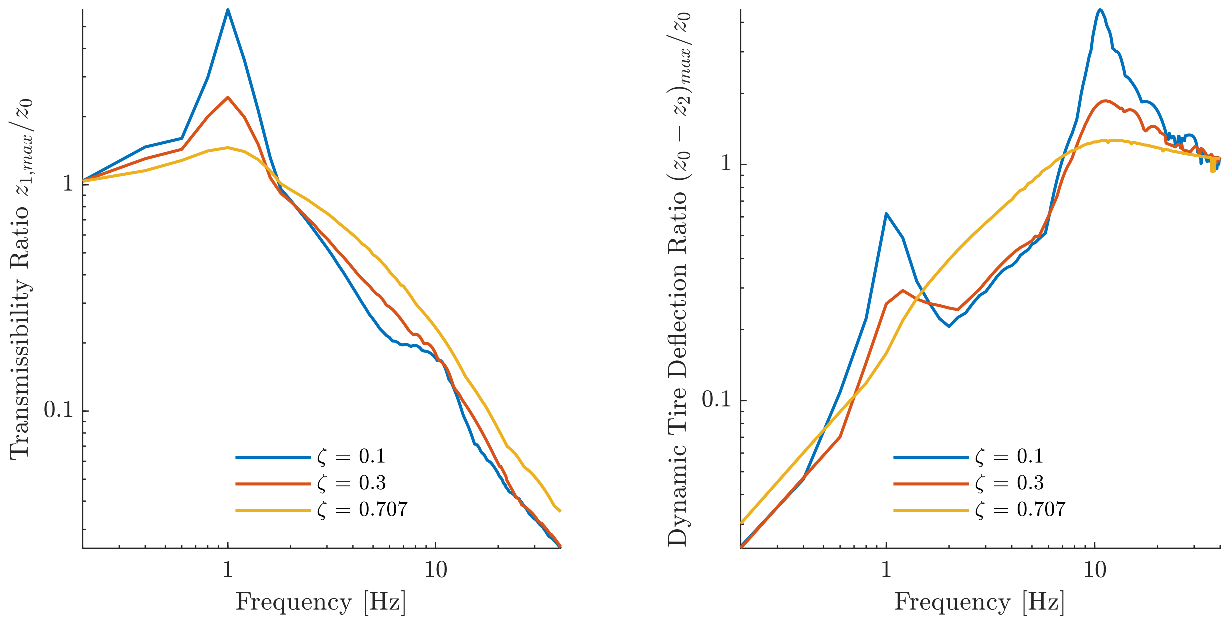


Figure A.2: Transmissibility ratio (left) and Dynamic Tire Deflection (right) as a function of frequency for the sprung mass of a suspension system with different damping ratios.

And this presents a very difficult challenge for the designers, who have to make some compromises to strike a balance barely that is appropriate for maintaining the luxurious and safe driving experience of the vehicle. Furthermore, these compromises affect overall driving experience or vehicle safety and handling. [42]

For talking about **compromises in better overall driving experience** or ride quality, the International Standard IS0 2631 provides an extensive basis for characterizing human tolerance to whole-body vibration. The guide provides three different limits for whole-body vibration: Exposure limit, fatigue, and Reduced comfort. And is suggested for use in industry and in the evaluation of vibratory environments in transport vehicles. And may be considered as guiding for designers in determining the trade-off limits between vehicle road holding and ride comfort reduction. [43]

Exposure limits: which, unless there is a specific reason, should not be exceeded in order to preserve safety (or health).

Fatigue or decreased proficiency boundaries: They pertain to maintaining operational efficiency and are applicable to jobs like operating a tractor or a road vehicle (driving for many hours).

Reduced comfort boundaries: which deal with maintaining comfort in transportation vehicles and are associated with activities like eating, writing, and reading in a car. [42], [43]

-Road Holding

On the other hand, talking about **compromises in road holding**, when a vehicle travels on the road, many factors may affect the vehicle holding cause instability. Among the problems related to car instability, reduced road holding in steering or braking or roll over condition, and this is the most dangerous phenomenon. All passengers and goods on board are threatened when the vehicle get instability in the road. Even the life of the car user may not be guaranteed once the instability accident occurs. Developed countries have better road conditions so that cars can travel at very high speeds. But this is not enough to avoid reduced road holding in all driving conditions. [44]

In general luxury cars typically have suspension systems that prioritize comfort over handling dynamics, which may cause the car to lose stability when braking and turning at specific high speeds. As a result, these vehicles offer a comfortable ride and are adept at swallowing bumps, but handling and control are compromised. However, the suspension of sports cars is usually designed with a focus on handling, meaning that while improving the road holding and stability, ride quality and comfort are compromised. Therefore, there is a trade-off between comfort and vehicle control when using passive suspension systems (PSS). [45]

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