MAT1856/APM466 Assignment 1

Tingyu Zhang, Student #: 1003371273

February, 2020

Fundamental Questions - 25 points

1. (a) **Why does a government issue bonds?**
   The aim of a government bond is to support government spending. The money raised from bonds is usually used for finance projects or day-to-day operations. It has also been used as a tool in controlling the money supply that flows into the market.

   (b) **From the government's perspective, why does the yield curve matter?**
   The yield curve is used as a benchmark for other debt in the market, such as mortgage rates or bank lending rates. The slope of the yield curve tells us how the bond market expects short-term interest rates (as a reflection of economic activity and future levels of inflation) to move in the future, which reflects the Policy Interest Rate from Bank of Canada. This means that the yield curve has a great impact on the money supply within the economy, government can use it to predict changes in economic output and growth and to regulate their central interest rate.

   (c) **How can a government reduce the money supply through bonds?**
   In open operations, the Treasury Board of Canada buys and sells government securities in the open market. If the Treasury Board of Canada wants to decrease the money supply, it sells bonds from its account, thus taking in cash and removing money from the economic system.

2. (CAN 1.5 Mar 1 2020); (CAN 0.75 Sep 1 2020); (CAN 0.75 Mar 1 2021); (CAN 0.75 Sep 1 2021) (CAN 0.5 Mar 1 2022); (CAN 2.75 Jun 1 2022); (CAN 1.75 Mar 1 2023); (CAN 1.5 Jun 1 2023) (CAN 2.25 Mar 1 2024); (CAN 1.5 Sep 1 2024);

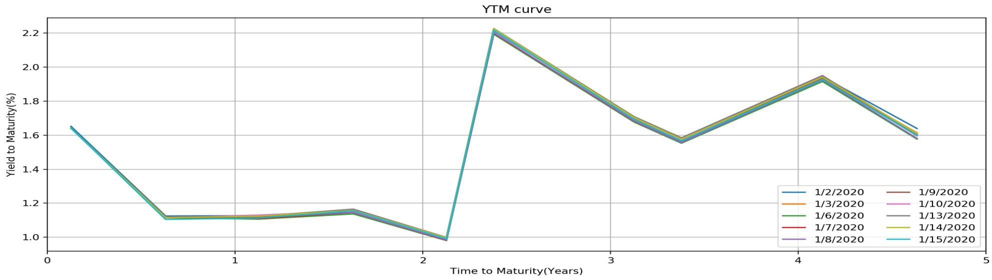
   First of all, since the government of Canada issues all of its bonds with a semi-annual coupon, it is required to find a semi-annual period. For the most of bonds from the given 32 bonds, they have a maturity date in June. This means that it is required to find a bond that has maturity date in December of the same year to be able to form the semi-annual period. However, given the requirement of bootstrapping 10 bonds that covers a selection range in 5 years, I have decided to choose bonds mature in March and September for each year to be able to suffice the semi-annual condition. (Exception in year 2022 and 2023, where September is missing. Data from June has been used.) Secondly, there are similar bond in terms of maturity date but with large difference on coupon. I would like to choose the bonds with small difference in terms of coupon for the stability of calculation. Finally, there are bonds with different issued date, some bonds with similar coupon and maturity date might not have similar characteristic due to its time length. By selecting bonds with more close and recent issue date, the problem can be improved.

3. The Principal Component Analysis tell us about how high dimensional data are distributed. It is usually being conveyed by the covariance matrix. Covariance matrix represents the direction and spread of your data. Diagonal spread along eigenvectors is expressed by the covariance, while x-and-y-axis-aligned spread is expressed by the variance. Eigenvectors trace the principal lines of force, and the axes of greatest variance and covariance illustrate where the data is most likely to change. When

covariance matrix performs a linear transformation, eigenvectors trace the lines of force it applies to input and reflect the forces that have been applied to the given, which is the direction of the principle component. Eigenvalues represent the magnitude of the variance of data corresponding to the eigenvectors. They are the measure of the data's covariance. By ranking eigenvectors in order of their eigenvalues from highest to lowest, the principal components in order of significance is acquired. The eigenvector with largest eigenvalue determines the first component, which is the direction of the largest variance.

Empirical Questions - 75 points

4.



(a) Since Canadian bonds are semi-annual bonds, the yield to maturity based on the equation:

$$price = \sum_{t=1}^{T} \frac{Coupon_t}{(1+\frac{ytm}{k})^{kt}} + \frac{Par\ Value}{(1+\frac{ytm}{k})^{kt}}$$

(b) If "Time to Maturity" $<$ 6 months:

the spot rate would be given by the equation:

$$r(T) = -\frac{log(\frac{Price}{Par\ Value})}{T}$$

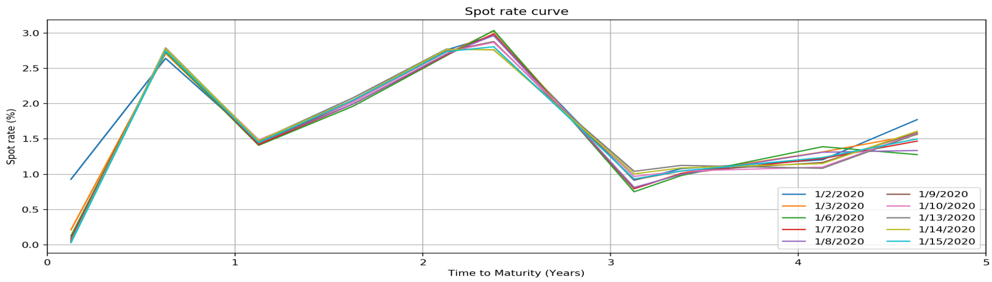
where T is the "Time to maturity".

If "Time to Maturity" $>$ 6 months:

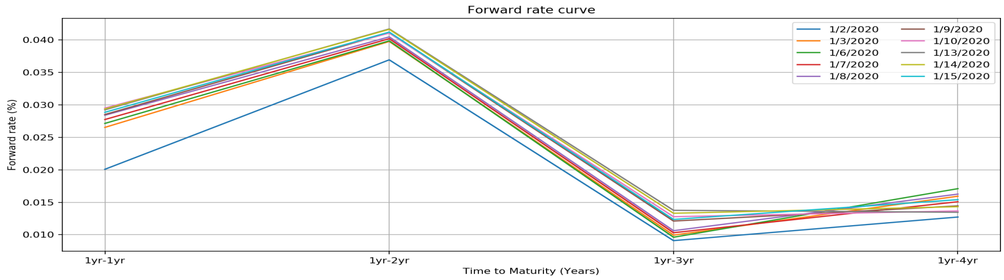
Use bootstrapping to generate the next spot rate by previous with following equation

$$P = C \cdot e^{-r(t_1)\cdot t_1} + (C + Par\ Value) \cdot e^{-r(t_2)\cdot t_2}$$

where, P is the dirty price = accrued interest + clean price; t_1 is current time to maturity and $t2$ is the next time to maturity; C is the Coupon, $r(t_1)$ and $r(t_2)$ are the spot rate in corresponding period. Then, by solving the equation above, we get the unknown spot rate $r(t_2)$ at next time.



(c) Step 1: Select bonds which mature in March, since only one-year forward curve is required.
Step 2: calculate the forward rate using following equation with corresponding time to maturity. t_1 ttm is longer in period and t_2 ttm is shorter in period, r_1 and r_2 are the respective spot rate.
$Forward\ Rate = \frac{(1+r_1)^{t_1}}{(1+r_2)^{t_2}}$



5. Covariance matrix of log-return of yield of maturity:

$$\begin{bmatrix} 4.20829608e-06 & 8.35094792e-06 & 8.96441258e-06 & 7.04529221e-06 & 5.55865878e-06 \\ 8.35094792e-06 & 6.08789517e-05 & 2.27782359e-05 & 2.97986333e-05 & 2.74060785e-05 \\ 8.96441258e-06 & 2.27782359e-05 & 4.86395639e-05 & 3.71676865e-05 & 3.35212957e-05 \\ 7.04529221e-06 & 2.97986333e-05 & 3.71676865e-05 & 3.41035118e-05 & 3.47306472e-05 \\ 5.55865878e-06 & 2.74060785e-05 & 3.35212957e-05 & 3.47306472e-05 & 5.13832779e-05 \end{bmatrix}$$

Covariance matrix of log-return of forward rate:

$$\begin{bmatrix} 0.00826815 & 0.00207085 & 0.0021004 & 0.00653509 \\ 0.00207085 & 0.00060387 & 0.00093998 & 0.00137782 \\ 0.0021004 & 0.00093998 & 0.00458169 & -0.00280122 \\ 0.00653509 & 0.00137782 & -0.00280122 & 0.01239235 \end{bmatrix}$$

6. The eigenvalue for log-return of yield to maturity are:
$[1.43175287e-04, 3.59807059e-05, 1.71332901e-05, 2.23233301e-06, 6.91985898e-07]$
The eigenvectors for log-return of yield to maturity are:
$[-0.10678883, -0.02307529, -0.17997387, -0.93274333, 0.29268236]^T$
$[-0.49401893, -0.8474293, -0.06496969, 0.03352302, -0.18017775]^T$
$[-0.49993539, 0.43597153, -0.58508853, 0.01300613, -0.46636432]^T$
$[-0.47683189, 0.13086032, -0.12287419, 0.32505989, 0.79670881]^T$
$[-0.51695008, 0.27227837, 0.77843541, -0.15176627, -0.17214065]^T$
The first eigenvalue $1.43175287e-04$ counts for the 71.8% of sum of eigenvalues.
The eigenvalue for log-return of forward rate are:
$[1.75665197e-02, 7.64217185e-03, 6.12887617e-04, 2.44796995e-05]$
The eigenvectors for log-return of forward rate are:
$[-0.57756754, -0.53465115, 0.60361487, 0.12733017]^T$
$[-0.13181261, -0.1882665, -0.08843116, -0.96920642]^T$
$[0.07017004, -0.74555784, -0.63397372, 0.19312429]^T$
$[-0.80256921, 0.35049591, -0.47529605, 0.08443308]^T$
The first eigenvalue $1.75665197e-02$ counts for the 67.9% of sum of eigenvalues.
Largest eigenvector of covariance matrix represents the largest variance of date, the eigenvalue represents its magnitude.

References and GitHub Link to Code

Intuition of PCA: `https://www.youtube.com/watch?v=FgakZw6K1QQ`
Github link to the code: `https://github.com/Robert54/APM466.git`