Danmarks Tekniske Universitet

Assignment 1

Demand-Side Flexibility in Active Distribution Grids

46750 - Optimization in modern power systems

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Individual Participation

The following table summarizes the contribution of each group member to the assignment. Each percentage represents the approximate share of total effort per category.

Table 1: Summary of individual contributions (%)

Contribution Category	**s214242**	**s215193**	**s214273**
Model formulation	20	40	40
Coding and implementation	0	50	50
Result analysis and discussion	33	33	33
Data visualization and plotting	25	50	25
Mathematical derivations (dual, KKT, etc.)	50	10	40
Report writing	30	30	40
Formatting and proofreading	33	33	33
Overall contribution	**27**	**35**	**37**

All group members contributed actively to discussions, validation of results, and review of the final report.

Contents

1 Question 1

1.1 (a)

1.1.1 Task i: Mathematical formulation

In formulating the problem we consider all the information given which results in the following collection of parameters, decision variables, constraints and objective function. There was stated to be a marginal cost of PV this was not found and therefore omitted, furthermore all ramping rates were omitted since these would require a MILP instead of an LP.
Input data:

$$
\begin{aligned}
&T : \text{Number of hours in day (24 h)}\\
&P_t^{PV} : \text{Hourly PV production for each hour } t \in T\\
&\lambda_t : \text{Hourly electricity price for hour } t \in T\\
&\tau^{imp} : \text{Import grid tariff fee}\\
&\tau^{exp} : \text{Export grid tariff fee}\\
&E_{min} : \text{Minimum daily energy consumption}\\
&D_{max} : \text{Maximum hourly demand}\\
&\overline{P}^{imp} : \text{Max import bound}\\
&\overline{P}^{exp} : \text{Max export bound}
\end{aligned}
$$

Decision Variables:

$$
\begin{aligned}
&D_t \geq 0 : \text{Cont. variable for the demand consumption for each hour } t \in T \; [kWh]\\
&C_t \geq 0 : \text{Cont. variable for the curtailment of PV for each hour } t \in T \; [kWh]\\
&P_t^{imp} \geq 0 : \text{Cont. variable for grid import each hour } t \in T \; [kWh]\\
&P_t^{exp} \geq 0 : \text{Cont. variable for grid export each hour } t \in T \; [kWh]
\end{aligned}
$$

Objective function:

$$\min_{D_t, C_t, P_t^{imp}, P_t^{exp}} \sum_{t=1}^{T} P_t^{imp} \cdot \left(\tau^{imp} + \lambda_t\right) + P_t^{exp} \cdot \left(\tau^{exp} - \lambda_t\right) \tag{1.1a}$$

Constraints:

$$
\begin{aligned}
&\text{Minimum consumption:} && \sum_{t=1}^{T} D_t \geq E_{min} && && (1.1b)\\
&\text{Power balance:} && P_t^{imp} - P_t^{exp} = D_t - P_t^{PV} + C_t && \forall t \in T && (1.1c)\\
&\text{Curtailment limit:} && C_t \leq P_t^{PV} && \forall t \in T && (1.1d)\\
&\text{Import limit:} && P_t^{imp} \leq \overline{P}^{imp} && \forall t \in T && (1.1e)\\
&\text{Export limit:} && P_t^{exp} \leq \overline{P}^{exp} && \forall t \in T && (1.1f)\\
&\text{Demand limit:} && D_t \leq D_{max} && \forall t \in T && (1.1g)\\
&\text{Limits:} && D_t, C_t, P_t^{imp}, P_t^{exp} \geq 0 && \forall t \in T && (1.1h)
\end{aligned}
$$

Here the main components of the problem are formulated. It's a clear LP seen from both objective function and decision variables. We see a full flexibility for the hourly curtailment and consumption with the caveat of there being a minimum daily consumption.

1.1.2 Task ii: Dual problem, Lagrangian and KKT

The Lagrangian
First, the Lagrangian is formulated; thus, the Lagrange multipliers for each constraint are formulated as

$$
\begin{aligned}
&\pi_t : \text{For the "power balance" constraint eq. 1.1c, } \pi_t \in \mathbb{R} \\
&\mu_t : \text{For the "curtailment limit" constraint eq. 1.1d, } \mu_t \geq 0 \\
&\gamma : \text{For the "minimum consumption" constraint eq. 1.1b, } \gamma \geq 0 \\
&\theta_t^{imp} : \text{For the "Import limit" constraint 1.1e, } \theta_t^{imp} \geq 0 \\
&\theta_t^{exp} : \text{For the "Export limit" constraint 1.1f, } \theta_t^{exp} \geq 0 \\
&\Delta_t : \text{For the "Demand limit" constraint 1.1g, } \Delta_t \geq 0
\end{aligned}
$$

Having the Lagrangian multipliers, the fact that all the decision variables should be greater than or equal to zero can be stated in the Lagrangian by introducing the dual multipliers

$$
\begin{aligned}
D_t &\geq 0 && \Rightarrow -D_t \leq 0 && \Rightarrow \sigma_t^D \geq 0 && (1.2a) \\
C_t &\geq 0 && \Rightarrow -C_t \leq 0 && \Rightarrow \sigma_t^C \geq 0 && (1.2b) \\
P_t^{imp} &\geq 0 && \Rightarrow -P_t^{imp} \leq 0 && \Rightarrow \sigma_t^{P^{imp}} \geq 0 && (1.2c) \\
P_t^{exp} &\geq 0 && \Rightarrow -P_t^{exp} \leq 0 && \Rightarrow \sigma_t^{P^{exp}} \geq 0 && (1.2d)
\end{aligned}
$$

Putting this together the Lagrangian becomes, an important notion is that we do not split the equality constraint but instead define it for $\pi \in \mathbb{R}$ which is why there's only one term.

$$
\begin{aligned}
\mathcal{L}(D, C, P^{imp}, P^{exp}, \pi, \mu, \gamma, \sigma, \theta^{imp}, \theta^{exp}, \Delta) = &\sum_{t=1}^{T} \Big(P_t^{imp}(\tau^{imp} + \lambda_t) + P_t^{exp}(\tau^{exp} - \lambda_t) \Big) \qquad (1.3) \\
&+ \sum_{t=1}^{T} \pi_t \big(P_t^{imp} - P_t^{exp} - D_t + P_t^{PV} - C_t \big) \\
&+ \sum_{t=1}^{T} \mu_t \left(C_t - P_t^{PV} \right) + \gamma \Big(E_{\min} - \sum_{t=1}^{T} D_t \Big) \\
&+ \sum_{t=1}^{T} \theta_t^{imp} (P_t^{imp} - \overline{P}^{imp}) + \sum_{t=1}^{T} \theta_t^{exp} (P_t^{exp} - \overline{P}^{exp}) \\
&+ \sum_{t=1}^{T} \Delta_t \left(D_t - D_{max} \right) \\
&- \sum_{t=1}^{T} \big(\sigma_t^D D_t + \sigma_t^C C_t + \sigma_t^{P^{imp}} P_t^{imp} + \sigma_t^{P^{exp}} P_t^{exp} \big).
\end{aligned}
$$

Dual formulation
Here the fact that the dual problem can be derived from the Lagrangian function using the "max-min" formulation.

$$
\max_{\substack{\pi \in \mathbb{R}^T,\, \mu \geq 0,\, \gamma \geq 0, \\ \sigma \geq 0, \theta^{imp} \geq 0, \theta^{exp} \geq 0, \\ \Delta \geq 0}} \left[\min_{D, C, P^{imp}, P^{exp} \geq 0} \mathcal{L}(\cdot) \right]
$$

Because $\mathcal{L}$ is linear in the primal variables and they are constrained to be nonnegative, the inner minimum is finite (and attained at the origin) if every coefficient of the primal variables is nonnegative. For each t the coefficients are

$$\begin{aligned}
\text{coef}(P_t^{imp}) &= \tau^{imp} + \lambda_t + \pi_t + \theta_t^{imp} - \sigma_t^{P^{imp}}, \\
\text{coef}(P_t^{exp}) &= \tau^{exp} - \lambda_t - \pi_t + \theta_t^{exp} - \sigma_t^{P^{exp}}, \\
\text{coef}(D_t) &= -\pi_t - \gamma + \Delta_t - \sigma_t^{D}, \\
\text{coef}(C_t) &= -\pi_t + \mu_t - \sigma_t^{C}
\end{aligned}$$

The existence of $\sigma_t^{\cdot} \geq 0$ making these coefficients nonnegative is equivalent to the following *dual feasibility* inequalities (after eliminating the σ-variables):

$$\begin{aligned}
&\tau^{imp} + \lambda_t + \pi_t + \theta_t^{imp} \geq 0, && \forall t \in T && (1.4a)\\
&\tau^{exp} - \lambda_t - \pi_t + \theta_t^{exp} \geq 0, && \forall t \in T && (1.4b)\\
&-\pi_t + \mu_t \geq 0, && \forall t \in T && (1.4c)\\
&-\pi_t - \gamma + \Delta_t \geq 0, && \forall t \in T && (1.4d)\\
&\mu_t, \gamma, \theta_t^{imp}, \theta_t^{exp}, \Delta_t \geq 0, \quad \pi_t \in \mathbb{R}. && && (1.4e)
\end{aligned}$$

When the inequalities above hold the inner minimum equals the constant terms of $\mathcal{L}$:

$$\min_{D,C,P^{imp},P^{exp}\geq 0} \mathcal{L} \;=\; \sum_{t=1}^{T} \Big[(\pi_t - \mu_t) P_t^{PV} \;+\; \gamma E_{\min} - (\theta_t^{imp} \overline{P}^{imp} + \theta_t^{exp} \overline{P}^{exp}) - \Delta_t D_{max} \Big] .$$

Hence the explicit dual maximization problem is

$$\max_{\pi,\mu,\gamma,\theta^{imp},\theta^{exp},\Delta_t} \quad \sum_{t=1}^{T} \Big[(\pi_t - \mu_t) P_t^{PV} \;+\; \gamma E_{\min} - (\theta_t^{imp} \overline{P}^{imp} + \theta_t^{exp} \overline{P}^{exp}) - \Delta_t D_{max} \Big] \quad (1.5a)$$

$$\begin{aligned}
\text{s.t.} \quad &\tau^{imp} + \lambda_t + \pi_t + \theta_t^{imp} \geq 0 && \forall t \in T && (1.5b)\\
&\tau^{exp} - \lambda_t - \pi_t + \theta_t^{exp} \geq 0 && \forall t \in T && (1.5c)\\
&-\pi_t + \mu_t \geq 0 && \forall t \in T && (1.5d)\\
&-\pi_t - \gamma + \Delta_t \geq 0 && \forall t \in T && (1.5e)\\
&\mu_t, \gamma, \theta_t^{imp}, \theta_t^{exp}, \Delta_t \geq 0, \quad \pi_t \in \mathbb{R} && \forall t \in T && (1.5f)
\end{aligned}$$

KKT conditions

1. *Primal feasibility:*
   The multipliers satisfy (1.1) for all t.

2. *Dual feasibility:*
   The multipliers satisfy (1.5) for all t.

3. *Stationarity:*
   Partial derivatives of $\mathcal{L}$ w.r.t. each primal variable set to zero (these produce the stationarity equalities that link primal variables, dual multipliers and the slack multipliers

σ):

$$\frac{\partial \mathcal{L}}{\partial P_t^{imp}}: \quad \tau^{imp} + \lambda_t + \pi_t + \theta_t^{imp} - \sigma_t^{P^{imp}} = 0 \tag{1.6}$$

$$\frac{\partial \mathcal{L}}{\partial P_t^{exp}}: \quad \tau^{exp} - \lambda_t - \pi_t + \theta_t^{exp} - \sigma_t^{P^{exp}} = 0 \tag{1.7}$$

$$\frac{\partial \mathcal{L}}{\partial D_t}: \quad -\pi_t - \gamma + \Delta_t - \sigma_t^{D} = 0 \tag{1.8}$$

$$\frac{\partial \mathcal{L}}{\partial C_t}: \quad -\pi_t + \mu_t - \sigma_t^{C} = 0 \tag{1.9}$$

Eliminating the σ-variables from these equalities together with $\sigma \geq 0$ yields the dual feasibility inequalities (1.5).

4. *Complementary slackness:*
Each nonnegative multiplier multiplies its primal slack to zero:

$$\begin{aligned}
&\mu_t\,(C_t - P_t^{PV}) = 0,\\
&\gamma\Big(E_{\min} - \sum_{t=1}^{T} D_t\Big) = 0,\\
&\theta_t^{imp}(P_t^{imp} - \overline{P}^{imp}) = 0,\\
&\theta_t^{exp}(P_t^{exp} - \overline{P}^{exp}) = 0,\\
&\Delta_t\,(D_t - D_{max}) = 0\\
&\sigma_t^D D_t = 0, \quad \sigma_t^C C_t = 0, \quad \sigma_t^{P^{imp}} P_t^{imp} = 0, \quad \sigma_t^{P^{exp}} P_t^{exp} = 0.
\end{aligned}$$

(No complementarity is associated with the equality multiplier λ_t; the power balance equality is enforced directly.)

The formulated dual problem inherits the linear nature of the primal problem which in turn ensures that the dual problem is also convex and that strong duality holds. This in turn means that the optimal values of both primal and dual problem will coincide and that the derived KKTs are both necessary and sufficient.

With an economic perspective one can look at the dual variables and use theses as marginal values or "shadow prices". A specific example is the variable π_t which actually describes the shadow price of the electricity for this consumer. The Lagrangian in turn describes mathematically the link between these marginal values and their primary decisions.

1.1.3 Qualitative analysis

The formulated optimization problem is a linear program (LP), meaning that both the objective function and all constraints are linear. As a result, the feasible region is convex, and the global optimum will always lie at one of its corner points. The model represents the hourly energy flow/schedule of a consumer over a 24 hour period and is therefore separable by hour, except for the daily minimum energy constraint. This constraint couples all hours together, ensuring that the total energy consumption over the day meets the required minimum $E_{\min}$, while still allowing the load to shift freely between hours to minimize total cost.

The key components of the LP that drives the solution is the electricity price λ_t, the grid tariffs τ and minimum load E_{min}. The electricity prices determines whether the consumer prefers to import or export. E.g. when the price is low the solver will tend to favor import, and when the price is high favor exporting the PV generation in order to make profit (as long as export tariffs are not too high). The grid tariffs impact the amount of import and export as well. Through high tariffs the solver will tend to promote self consumption or curtailment. For the minimum load constraint, it ensures at least some load must be satisfied and thus can often be the binding constraint when import from the grid is necessary.

The problem's dual variables (π_t, μ_t, γ) have clear economic interpretations as marginal values associated with the corresponding constraints. The variable π_t represents the marginal value of electricity at the consumer node in hour t, that is, the consumer's shadow price of consuming or supplying one additional kWh in that hour. The multiplier μ_t reflects the marginal cost of reaching the PV curtailment limit, corresponding to the opportunity cost of being forced to curtail production (or equivalently, the value of exporting an additional kWh). Finally, γ denotes the marginal cost of the daily minimum energy consumption requirement, representing the shadow price associated with increasing the required daily load by one additional kWh.

Import, export, or self-sufficiency behavior
Using the KKT conditions and complementary slackness stated in previous section, the interpretation of the the optimal solutions can be seen as:

- *Import:* Occurs when $P_t^{imp} > 0$. This means the consumer is buying electricity from the grid because the cost of importing (price plus tariff) is lower than the value of using their own generation.
- *Import limit reached:* When $P_t^{imp} = \overline{P}^{imp}$ the import maximum limit becomes active ($\theta^{imp} > 0$) meaning no more power can be imported at that time.
- *Export:* Happens when $P_t^{exp} > 0$. In this case, the consumer sells electricity to the grid because the export price (after the tariff) is higher than keeping the energy for self-use.
- *Export limit reached:* When $P_t^{exp} = \overline{P}^{exp}$ the export maximum limit becomes active ($\theta^{exp} > 0$) meaning no more power can be exported at that time.
- *Curtailment:* Occurs when $0 < C_t < P_t^{PV}$. This means not all available PV power is used, and $\mu_t = 0$. The reason could be that exporting is not profitable due to low prices or high export tariffs.
- *Curtailment limit reached:* When $C_t = P_t^{PV}$, the PV curtailment limit becomes active ($\mu_t > 0$), meaning it would not be allowed to curtail more PV at that time.
- *Load:* When $0 < D_t < D_{max}$ power is consumed at the load to satisfy demand, this could origin from all "generating" units.
- *Load limit reached:* When $D_t = D_{max}$ load consumption limit becomes active ($\Delta_t > 0$) and no more demand can be satisfied in this hour t
- *Minimum load not binding:* If the total daily load is higher than the minimum requirement ($\sum_{t=1}^{T} D_t > E_{\min}$), then $\gamma = 0$. The consumer is already using more energy than required, so this constraint does not affect the solution.
- *Minimum load binding:* When the total load equals the minimum required amount ($\sum_{t=1}^{T} D_t = E_{\min}$), then $\gamma > 0$. This means increasing the minimum requirement would make the total cost go up.

LP simplification
The LP simplifies under certain conditions in relation to the data:

- *If PV generation is zero:* The LP reduces to a simple load scheduling problem, where E_{min} has to be distributed in order to minimize the total cost of buying (importing) electricity
- *If $E_{min} = 0$:* The consumer only sells PV generated electricity when it is profitable - a pure arbitrage problem
- *If tariffs are zero:* The problem becomes a pure arbitrage problem between load, PV generation and the grid

In summary, the structure of the LP shows that the consumer's flexibility is mainly driven by the time variation in energy prices and tariffs. When these are constant, flexibility has no value and the problem collapses to a static energy balance. When they vary, the KKT and dual conditions reveal how the marginal values (π_t) drive optimal import/export, PV curtailment, and load allocation decisions across hours.

1.1.4 Model Implementation

The model has been implemented using gurobipy and the code can be found in the "Part 1.A_new.ipynb" file on either Github or attached to the hand-in in table 1.1 below is the optimal solution presented

Table 1.1: Optimal Energy Dispatch Results

Hour	Load (kW)	Curtailment (kW)	Grid Import (kW)	Grid Export (kW)	PV Production (kW)	Electricity Price (DKK/kWh)
0	0.00	0.00	0.00	0.00	0.00	1.10
1	0.00	0.00	0.00	0.00	0.00	1.05
2	0.00	0.00	0.00	0.00	0.00	1.00
3	0.00	0.00	0.00	0.00	0.00	0.90
4	0.00	0.00	0.00	0.00	0.00	0.85
5	0.15	0.00	0.00	0.00	0.15	1.01
6	0.11	0.00	0.00	0.31	0.42	1.05
7	0.00	0.00	0.00	0.63	0.63	1.20
8	0.00	0.00	0.00	0.45	0.45	1.40
9	0.00	0.00	0.00	0.36	0.36	1.60
10	0.00	0.00	0.00	0.63	0.63	1.50
11	0.00	0.00	0.00	0.75	0.75	1.10
12	2.55	0.00	0.00	0.00	2.55	1.05
13	2.25	0.00	0.00	0.00	2.25	1.00
14	1.65	0.00	0.00	0.00	1.65	0.95
15	1.29	0.00	0.00	0.00	1.29	1.00
16	0.00	0.00	0.00	0.69	0.69	1.20
17	0.00	0.00	0.00	0.15	0.15	1.50
18	0.00	0.00	0.00	0.75	0.75	2.10
19	0.00	0.00	0.00	0.75	0.75	2.50
20	0.00	0.00	0.00	0.00	0.00	2.20
21	0.00	0.00	0.00	0.00	0.00	1.80
22	0.00	0.00	0.00	0.00	0.00	1.40
23	0.00	0.00	0.00	0.00	0.00	1.20
						Total cost: -6.37 DKK

1.1.5 Numerical analysis

In this section, we will review plots of how the load, import and export are scheduled. Given the fully flexible load it is expected that the movement of load in time will be the models greatest attribute to save costs, i.e. scheduling load during PV production hours.

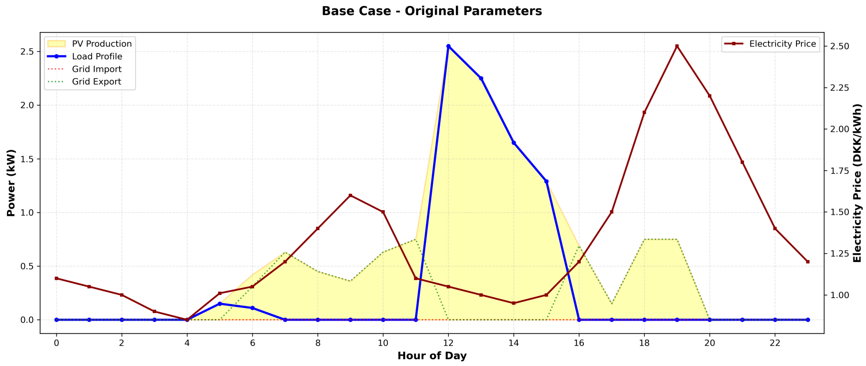

Figure 1.1: Scheduled Load in Base case scenario a)

In order to test our hypothesis that the model is most sensitive to "Free" pv energy we create a scenario with flat energy prices throughout the day, being the mean of all energy prices.

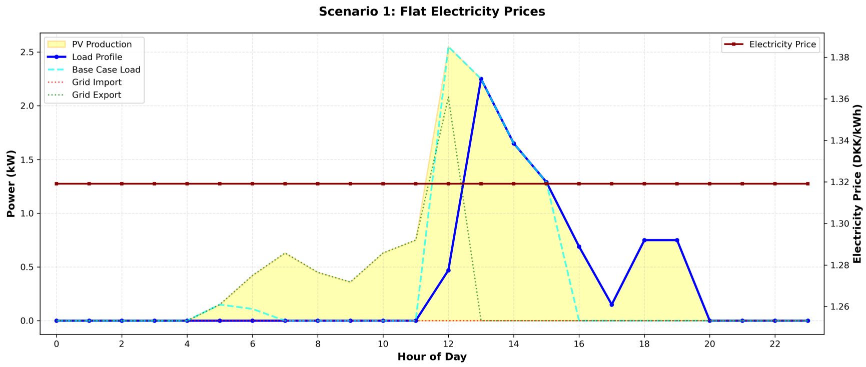

Figure 1.2: Scheduled Load in Flat Energy Prices scenario a)

When we compare Figure 1.1 and Figure 1.2 we see that the optimal load schedule is affected very little by the change in prices and still schedules the load around the PV production. We do see that since there is no variation in the prices the model simply satisfies the minimum daily load and sells the rest of the power. The difference between the two scenarios being pricing.

As the load can be fully satisfied by the PV production, we investigate a scenario in which this cannot be true by making the minimum daily load 2x PV production. We expect the model to still use the PV production mainly and supplement this with the cheapest energy prices, while limited by the max import.

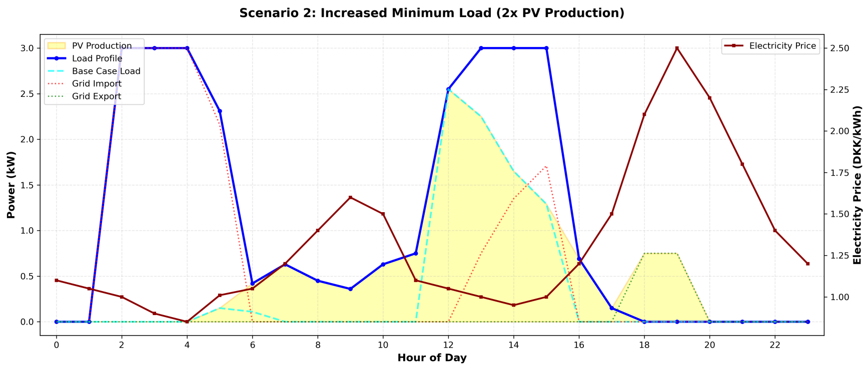

Figure 1.3: Scheduled Load in Increased Daily Load scenario a)

Figure 1.3 Shows that the expected behaviour is true. The model still prefers to ustilise the PV production supplementing with purchase of power in the cheapest period in the early morning. It is seen that the spread of prices is high enough that some arbitrage is made during the high price hours of the evening. However we still observe the PV production as a very important parameter. To further test this we construct a scenario in which the PV load is halfed, to simulate a cloudy day.

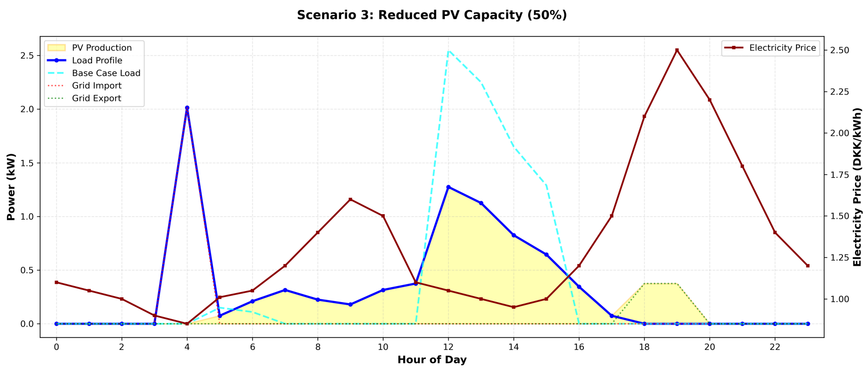

Figure 1.4: Scheduled Load in Reduced PV Production scenario a)

As expected Figure 1.4 shows that the model still prefers to maximise its use of the PV production, with the only exception being a large purchase of energy in the cheapest hour in order to export energy in the most expensive hours that still have PV production, as the spread of prices is high enough. Finally we would like to observe how the model will schedule load given full arbitrage powers. This is done by reducing the grid tariff to a very small number (setting to 0 causes the problem to become unbounded!)

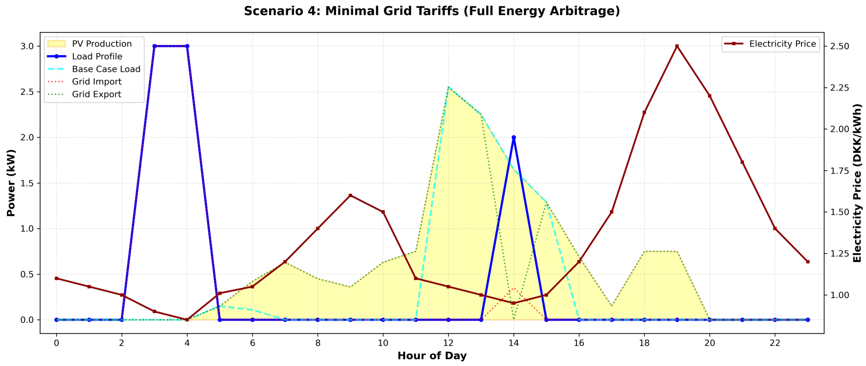

Figure 1.5: Scheduled Load in Minimal Grid Tariffs scenario a)

The result shown in Figure 1.5 passes the sanity test. The model, having a very small penalty for trading energy and full flexibility on timing, buys as much energy as possible in the cheapest hours of the day and sells all the PV production at a higher price. One expection is a 14hrs, being the 3rd cheapest price the model has already imported maximum energy at the 2 cheapest and decides to use the PV in that hour + a little import.

1.2 (b)

1.2.1 Task i: Mathematical formulation

With the problem now changing from a fixed daily energy requirement to a discomfort factor, the underlying mathematical formulation must be amended. Here it's assumed that the discomfort is symmetrical, meaning that the discomfort caused by having too little power is equal to that of having too much.

To implement this, two new parameters are introduced.
New Parameters:

$$\alpha : \text{Comfort factor } [DKK/kWh]$$
$$D_t^{ref} : \text{Reference load for each hour } t \in T \ [kWh]$$

Now the objective function is changed to include the weighting, by multiplying a cost/comfort factor to the absolute value of the deviation from the desire schedule. This ensure that the problem will try to minimize the deviations as they will modeled as a cost.

$$\min_{D_t, C_t, P_t^{imp}, P_t^{exp}} \sum_{t=1}^{T} \left[P_t^{imp} \cdot \left(\tau^{imp} + \lambda_t \right) + P_t^{exp} \cdot \left(\tau^{exp} - \lambda_t \right) \right] + \alpha \cdot \sum_{\text{t=1}}^{\mathcal{T}} |D_t - D_t^{ref}| \tag{1.10}$$

This inclusion gives the ability to change the importance of the discomfort caused by the load flexibility, which effectively is a way of changing the flexibility it self with $\alpha = 0$ being fully flexible as in section 1.1.1 and $\alpha = \infty$ being fully inflexible
Here α has the units of $\frac{DKK}{kWh}$ and represents the cost in DKK of each unit of energy scheduled in a different time block than the reference, therefore penalizing the objective cost function for deviations. The objective function in 1.10 is not linear as such, since an LP is the requirement we introduce an auxiliary variable L_t instead of $|D_t - D_t^{ref}|$ which then changes the objective function to

$$\min_{D_t, C_t, P_t^{imp}, P_t^{exp}, L_t} \sum_{t=1}^{T} \left[P_t^{imp} \left(\tau^{imp} + \lambda_t \right) + P_t^{exp} \left(\tau^{exp} - \lambda_t \right) + \alpha L_t \right] \tag{1.11}$$

This in turn also changes the constraints.
New constraints:

Positive deviation constraint:	$L_t \geq D_t - D_t^{ref}$	$\forall t \in T$	(1.12)
Negative deviation constraint:	$L_t \geq -\left(D_t - D_t^{ref} \right)$	$\forall t \in T$	(1.13)
Minimum load constraint:	$\sum_{t=1}^{T} [D_t] \geq \sum_{t=1}^{T} \left[D_t^{ref} \right]$		(1.14)
Limits:	$D_t, C_t, P_t^{imp}, P_t^{exp}, L_t \geq 0$	$\forall t \in T$	(1.15)

It's noticed that the minimum consumption constraint is essentially the same, E_min is just switched for $\sum_{t=1}^{T} L_t^{ref}$. The remaining constraints, parameters and decision variables are the same as in section 1.1.1 giving the full optimization problem in equations 1.1

$$\min_{D_t, C_t, P_t^{imp}, P_t^{exp}, L_t} \sum_{t=1}^{T} \left[P_t^{imp} \left(\tau^{imp} + \lambda_t \right) + P_t^{exp} \left(\tau^{exp} - \lambda_t \right) + \alpha \cdot L_t \right] \tag{1.16a}$$

$$
\begin{aligned}
\text{s.t.} \quad & P_t^{imp} - P_t^{exp} = D_t - PV_t + C_t && \forall t \in T && (1.16b)\\
& C_t \leq P_t^{PV} && \forall t \in T && (1.16c)\\
& L_t \geq D_t - D_t^{ref} && \forall t \in T && (1.16d)\\
& L_t \geq -\left(D_t - D_t^{ref}\right) && \forall t \in T && (1.16e)\\
& P_t^{imp} \leq \overline{P}^{imp} && \forall t \in T && (1.16f)\\
& P_t^{exp} \leq \overline{P}^{exp} && \forall t \in T && (1.16g)\\
& D_t \leq D_{max} && \forall t \in T && (1.16h)\\
& \sum_{t=1}^{T} [D_t] \geq \sum_{t=1}^{T} \left[D_t^{ref}\right] && && (1.16i)\\
& D_t, C_t, P_t^{imp}, P_t^{exp}, L_t \geq 0 && \forall t \in T && (1.16j)
\end{aligned}
$$

1.2.2 Task ii: Dual problem, Lagrangian and KKT

The Lagrangian:
Firstly it's realised that a lot of the items will be the same as described in section 1.1.2. Since no constraints are removed the lagrange multipliers in eq. 1.17 are simply added to the ones used in section 1.1.2, once again the equality constrain 1.16b is not split instead the langrange multiplier λ_t is defined for $\mathbb{R}$

$$\zeta_t^{pos} : \text{For the "Positive deviation" constraint eq. 1.16d, } \zeta_t^{pos} \geq 0 \tag{1.17a}$$
$$\zeta_t^{neg} : \text{For the "Negative deviation" constraint eq. 1.16e, } \zeta_t^{neg} \geq 0 \tag{1.17b}$$

Since there hasn't been removed any decision variables the dual multipliers stated in equations 1.2 are used with equation 1.18 added to them.

$$L_t \geq 0 \qquad \Rightarrow -L_t \leq 0 \qquad \Rightarrow \sigma_t^L \geq 0 \tag{1.18}$$

This results in the following Lagrangian function for the problem

$$
\begin{aligned}
\mathcal{L}(D, C, P^{imp}, P^{exp}, L, \pi, \mu, \theta^{imp}, \theta^{exp}, \Delta_t, \zeta^{pos}, \zeta^{neg}, \sigma) = & \qquad (1.19)\\
& \sum_{t=1}^{T} \left[P_t^{imp} \left(\tau^{imp} + \lambda_t \right) + P_t^{exp} \left(\tau^{exp} - \lambda_t \right) + \alpha L_t \right] \\
& + \sum_{t=1}^{T} \left[\pi_t \left(P_t^{imp} - P_t^{exp} - D_t + P_t^{PV} - C_t \right) \right] \\
& + \sum_{t=1}^{T} \left[\mu_t \left(C_t - P_t^{PV} \right) \right] + \gamma \left(\sum_{t=1}^{T} \left[D_t^{ref} \right] - \sum_{t=1}^{T} \left[D_t \right] \right) \\
& + \sum_{t=1}^{T} \left[\theta_t^{imp} (P_t^{imp} - \overline{P}^{imp}) \right] + \sum_{t=1}^{T} \left[\theta_t^{exp} (P_t^{exp} - \overline{P}^{exp}) \right] \\
& + \sum_{t=1}^{T} \Delta_t \left(D_t - D_{max} \right) \\
& + \sum_{t=1}^{T} \left[\zeta_t^{pos} \left(L_t - D_t + D_t^{ref} \right) \right] + \sum_{t=1}^{T} \left[\zeta_t^{neg} \left(L_t + D_t - D_t^{ref} \right) \right] \\
& - \sum_{t=1}^{T} \left[\sigma_t^D D_t + \sigma_t^C C_t + \sigma_t^{P^{imp}} P_t^{imp} + \sigma_t^{P^{exp}} P_t^{exp} + \sigma_t^L L_t \right]
\end{aligned}
$$

The dual problem:
Using the same convention and methods as described in section 1.1.2, the dual problem of the expanded primal is derived from the Lagrangian function, by separating the constant terms and isolating the primal decision variables in the Lagrangian. This yields the following dual problem.

$$
\begin{aligned}
\max_{\pi, \mu, \gamma, \theta^{imp}, \theta^{exp}, \Delta_t, \zeta^{pos}, \zeta^{neg}} \sum_{t=1}^{T} \Big[& P_t^{PV} \left(\pi_t - \mu_t \right) - \left(\theta_t^{imp} \overline{P}^{imp} + \theta_t^{exp} \overline{P}^{exp} \right) \\
& + D_t^{ref} \left(\zeta_t^{pos} - \zeta_t^{neg} \right) + \gamma D_t^{ref} \Big]
\end{aligned} \qquad (1.20a)
$$

$$
\begin{aligned}
\text{s.t.} \quad & \tau^{imp} + \lambda_t + \pi_t + \theta_t^{imp} \geq 0 && \forall t \in T && (1.20b)\\
& \tau^{exp} - \lambda_t - \pi_t + \theta_t^{exp} \geq 0 && \forall t \in T && (1.20c)\\
& -\pi_t - \zeta_t^{pos} + \zeta_t^{neg} - \gamma + \Delta_t \geq 0 && \forall t \in T && (1.20d)\\
& -\pi_t + \mu_t \geq 0 && \forall t \in T && (1.20e)\\
& \alpha + \zeta_t^{pos} + \zeta_t^{neg} \geq 0 && \forall t \in T && (1.20f)\\
& \mu_t, \gamma, \theta_t^{imp}, \theta_t^{exp}, \Delta_t, \zeta_t^{pos}, \zeta_t^{neg} \geq 0, \quad \pi_t \in \mathbb{R} && \forall t \in T && (1.20g)
\end{aligned}
$$

KKT conditions:

- *Primal feasibility* here the constraints from eq. 1.16 apply.
- *Dual feasibility* here the constraints from eq. 1.20 apply.

- *Stationarity* again all partial derivatives from the primal problem are set equal to zero

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial P_t^{imp}} &: \tau^{imp} + \lambda_t + \pi_t + \theta_t^{imp} - \sigma_t^{P^{imp}} = 0 \\
\frac{\partial \mathcal{L}}{\partial P_t^{exp}} &: \tau^{exp} - \lambda_t - \pi_t + \theta_t^{exp} - \sigma_t^{P^{exp}} = 0 \\
\frac{\partial \mathcal{L}}{\partial D_t} &: -\pi_t - \zeta_t^{pos} + \zeta_t^{neg} - \gamma + \Delta_t - \sigma_t^{D} = 0 \\
\frac{\partial \mathcal{L}}{\partial C_t} &: -\pi_t + \mu_t - \sigma_t^{C} = 0 \\
\frac{\partial \mathcal{L}}{\partial L_t} &: \alpha + \zeta_t^{pos} + \zeta_t^{neg} - \sigma_t^{L} = 0
\end{aligned}$$

- *Complementary slackness* we apply the same rule of each inequality constrain being binding or its multiplier being zero which for every $t \in T$ gives

$$\begin{aligned}
&\mu_t(C_t - P_t^{PV}) = 0 \\
&\theta_t^{imp}(P_t^{imp} - \overline{P}^{imp}) = 0 \\
&\theta_t^{exp}(P_t^{exp} - \overline{P}^{exp}) = 0 \\
&\Delta_t\,(D_t - D_{max}) = 0 \\
&\zeta_t^{pos}(D_t - D_t^{ref} - L_t) = 0 \\
&\zeta_t^{neg}(-D_t + D_t^{ref} - L_t) = 0 \\
&\gamma\left(\sum_{t=1}^{T}\left[D_t^{ref}\right] - \sum_{t=1}^{T}[D_t]\right) = 0 \\
&\sigma_t^{D} D_t = 0, \sigma_t^{C} C_t = 0, \sigma_t^{P^{imp}} P_t^{imp} = 0, \sigma_t^{P^{exp}} P_t^{exp} = 0, \sigma_t^{L} L_t = 0
\end{aligned}$$

Once again we see that the problem remains linear due to the style of implementation of the discomfort, meaning that the convexity holds and also the strong duality. From this it can then once again be said that the KKT's are necessary and sufficient. Economically the introduction of the two ζ values, shows us the marginal value of relaxing the deviation limits of the problem. Mathematical the dual constraints show how α creates the coupling between each hour and the load and effectively deciding how much load can actually be shifted.

1.2.3 Qualitative Analysis

In the new formulation the strictly energy requirement from before is changed to include a penalty term weighted by α which changes both the structure and behavior of the problem. However, the problem remains a linear program because of the auxiliary variable L_t, thus, convexity and strong duality still hold. On the other hand the objective now couples hours through the deviation penalty, rather than through the previous daily energy constraint. Therefore, flexibility is no longer enforced globally, but locally for each hour through the deviation from the reference load D_t^{ref}. In the case when $\alpha = 0$ the model becomes identical to in part *a*, and the consumer is fully flexible and the objective only cost optimizing. When $\alpha \to \infty$ the deviation from the reference load becomes too 'uncomfortable' and the optimal solution converges to $D_t = D_t^{ref}$ for all t, resulting in completely inflexibility.

Comparison to numerical results from question A

As presented in section 1.1.5 the key numerical takeaway from was that the consumer's behavior is dominated by economic arbitrage, thus, it freely shifts its load and trading decisions to exploit PV output and price variations, with no concern for temporal comfort. Introducing the discomfort penalty weighted by α fundamentally changes this focus. The key behavioral impacts are

- **Reduced load shifting:**
  The strong reshaping of load seen in figures 1.1-1.4 would now be dampened. For small values of α, we would still see some alignment of load with PV generation, but as α grows, load deviations from the reference would shrink until $D_t \approx D_t^{ref}$.

- **Flattened scheduling:**
  The large spikes in import and export (seen when tariffs were small or PV was limited) would be less pronounced. The model would avoid extreme hour-to-hour load changes because these now incur discomfort 'costs'.

- **Less arbitrage:**
  In scenarios with significant price variations, the fully flexible model would by low and sell high. With the inclusion of discomfort, such behavior would decline, as only the most profitable shifts would outweigh the discomfort value.

- **PV utilization trade-off:**
  Previously, PV production was always maximized (since it was effectively 'free'). Now, if aligning load perfectly with PV requires large deviations from the reference profile, the consumer might accept some curtailment rather than fully reshaping its load. PV self-consumption remains desirable but is no longer absolute.

- **Price sensitivity damping:**
  In contrast to the model in 1.1.1, which reacted strongly to hourly price variations, the introduction of α adds a damping effect, creating a form of *price inertia* where only significant price differentials motivate load adjustments.

Constraints Activity
Which and whether the constraints are binding highly rely on the value of α.

For a high value of α:

- The deviation constraints ($L_t \geq (D_t - D_t^{ref})$) will often be binding at hours where the optimal value of L_t deviates from D_t^{ref}.
- The power balance constraint is always binding, in order to ensure the problem being feasible
- The curtailment constraint ($C_t \leq P_t^{PV}$) may or may not be active depending on PV generation versus price levels.
- Import and export limits ($P_t^{\cdot} \leq \overline{P}^{\cdot}$) typically bind only if the grid limits flexibility.

For a low value of α:

- The deviation constraints ($L_t \geq (D_t - D_t^{ref})$) will likely be non binding, since deviations from the reference load are 'cheap'. Then, the import and export constraints will be binding depending on the electricity prices.

In conclusion the degree to which deviation constraints are binding grows with α. This transition explains how consumer flexibility shrinks continuously as comfort preference increases.

1.2.4 Model Implementation

The model has been implemented using gurobipy and the code can be found in the "Part 1.B_new.ipynb" file on either Github or attached to the hand-in in table 1.2 below is the optimal solution presented

Table 1.2: Optimal Energy Dispatch Results

Hour	Load (kW)	Curtailment (kW)	Grid Import (kW)	Grid Export (kW)	PV Production (kW)	Electricity Price (DKK/kWh)
0	0.17	0.00	0.17	0.00	0.00	1.10
1	0.12	0.00	0.12	0.00	0.00	1.05
2	0.12	0.00	0.12	0.00	0.00	1.00
3	0.12	0.00	0.12	0.00	0.00	0.90
4	1.17	0.00	1.17	0.00	0.00	0.85
5	1.44	0.00	1.29	0.00	0.15	1.01
6	2.28	0.00	1.86	0.00	0.42	1.05
7	2.40	0.00	1.77	0.00	0.63	1.20
8	1.89	0.00	1.44	0.00	0.45	1.40
9	0.66	0.00	0.30	0.00	0.36	1.60
10	0.75	0.00	0.12	0.00	0.63	1.50
11	1.05	0.00	0.30	0.00	0.75	1.10
12	2.55	0.00	0.00	0.00	2.55	1.05
13	2.25	0.00	0.00	0.00	2.25	1.00
14	1.65	0.00	0.00	0.00	1.65	0.95
15	1.95	0.00	0.66	0.00	1.29	1.00
16	2.34	0.00	1.65	0.00	0.69	1.20
17	2.70	0.00	2.55	0.00	0.15	1.50
18	0.75	0.00	0.00	0.00	0.75	2.10
19	0.75	0.00	0.00	0.00	0.75	2.50
20	0.00	0.00	0.00	0.00	0.00	2.20
21	0.45	0.00	0.45	0.00	0.00	1.80
22	0.22	0.00	0.22	0.00	0.00	1.40
23	0.17	0.00	0.17	0.00	0.00	1.20
						Total cost: 29.25 DKK

1.2.5 Numerical Analysis

With the added constraint of consumer behaviour on inflexible demand, we first would like to investigate how flexibility affects the optimal solution. To do so, we test a sensitivity on the α parameter we introduced.

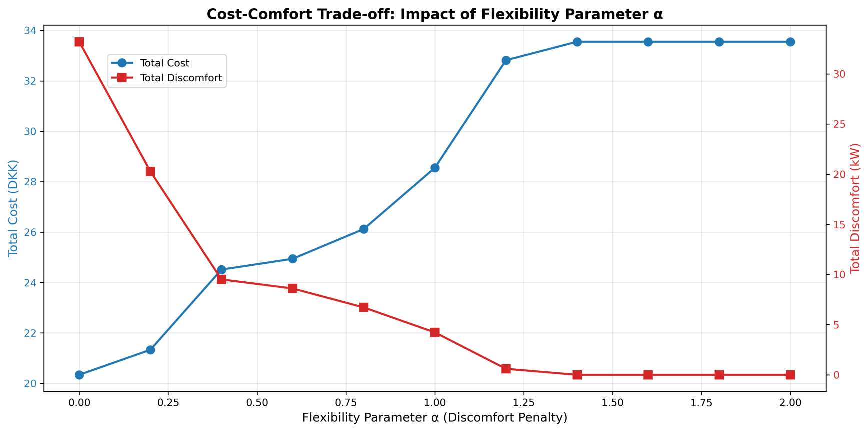

Figure 1.6: Cost-Comfort Trade-off: Impact of Flexibility Parameter α

Figure 1.6 shows, as expected, that discomfort and cost are inversely correlated, as the consumers' behaviour resembles that of other consumers; therefore, they demand more energy in more expensive hours of the day. As we increase the penalty for discomfort, we see that around $1.3\frac{DKK}{kWh}$ penalty for discomfort, the solutions stabilise and the model sees it most optimal to just stick to L_{ref}

While varying α could be up to the discretion of the user, perhaps the above solutions may not be completely feasible in the real world, as there is not much restriction on what energy is moved from what time to another. There would be a avenues of investigation here. fx only allow energy to move to the previous or next hour. However to keep this paper within reasonable scope we have decided to investigate scenarios with a fixed baseload. The reason a consumer has a usage profile is due to schedules, habits and other things that cant easily be moved to save on energy, even if they would like to. Some appliances run all day, like fridges and some may have electric heating therefore will there have a stable use amount.

The scenarios are set in the following way. A percentage of L_{ref} is seen as completely inflexible, the remaining percentage is then seen as flexible by the parameter α. Figure 1.7 shows the results of this investigation. While the we continue seeing expected behaviour we also find an interesting result. Costs rise with decreasing flexibility consistently, however, the same is not true for discomfort falling. Between 50% and 30% flexibility, while the cost is, as expected, higher for the 30% scenario, the discomfort is not lower.

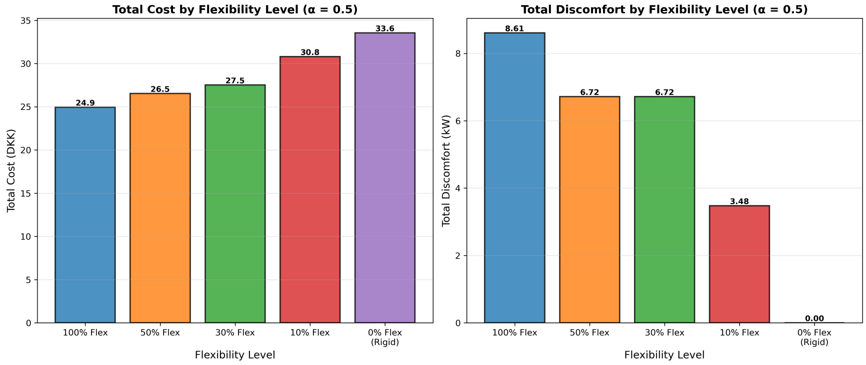

Figure 1.7: Total Cost and Discomfort by Flexibility Level

This result was quite surprising and required further investigation. Figure 1.8 shows a pareto plot of the same scenarios as in Figure 1.7 with varying α values. We see that this interesting behaviour around the $30\% - 50\%$ range is exacerbated with increased α. The discomfort is actually lower for the 50% case while being at a larger cost. This is likely due to there not being more costs to be saved, and making the 30-50% region a transition point where additional flexibility provides some economic benefit but little to no comfort benefit for this specific load and price profile.

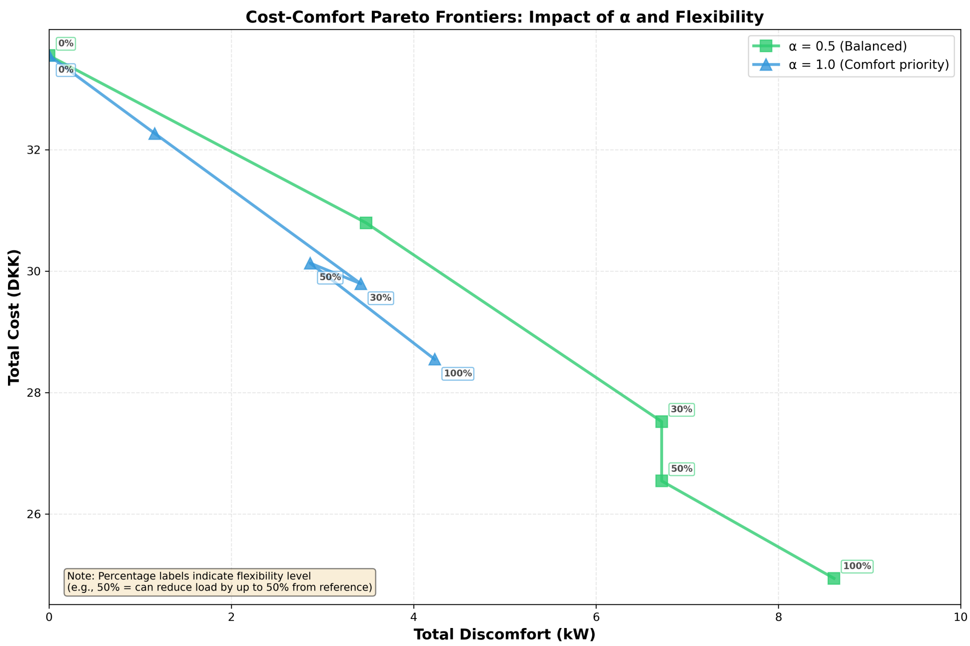

Figure 1.8: Cost-Comfort Pareto Frontiers: Impact of α and Flexibility

1.3 (c)

1.3.1 Task i: Mathematical formulation

When adding a battery to the system, this allows for the movement of energy through time, at the cost of losing some efficiency. The battery is considered to be in the scenario and is therefore assumed to have no cost or revenue, and the goal of minimising cost has not changed; no reformulation of the objective function in equation 1.11 is needed.
A battery does however have a lot of technical parameters that need to be taken into account, these are as follows
Added parameters

$$
\begin{aligned}
&SoC^{\max} : \text{Battery storage capacity } [kWh]\\
&r^{ch} : \text{Max charging rate } [kW]\\
&r^{dis} : \text{Max discharging rate } [kW]\\
&\eta^{ch} : \text{Charging efficiency}\\
&\eta^{dis} : \text{Disharging efficiency}\\
&SoC_0 : \text{Initial SoC for hour } t = 0 \ [kWh]\\
&SoC^{end} : \text{Min allowed SoC for hour } t = T \ [kWh]
\end{aligned}
$$

To actually incorporate the behaviour of the battery, 3 new decision variables are introduced, P_t^{dis}, $P_t^{ch}$and SoC_t. P_t^{dis} and P_t^{ch} are used for the decision of whether to charge or discharge the battery, per the convention described in the beginning of the assignment, SoC_t is an auxiliary variable used to track the actual state of charge for the battery after each timestep t, ensuring that no limits are exceeded and to track the amount that can be charges/discharged.
New decision variables

$$
\begin{aligned}
&P_t^{dis} \geq 0 : \text{Battery discharging power for hour } t \in T \ [kW]\\
&P_t^{ch} \geq 0 : \text{Battery charging power for hour } t \in T \ [kW]\\
&SoC_t \geq 0 : \text{Battery SoC at the end of hour } t \in T \ [kWh]
\end{aligned}
$$

These should now be implemented as constraints to model the actual behavior of the battery
Added and modified Constraints

$$
\begin{aligned}
&\text{Power balance:} && P_t^{imp} - P_t^{exp} + P_t^{dis} - P_t^{ch} = D_t - P_t^{PV} + C_t && \forall t \in T && (1.21a)\\
&\text{SoC dynamics:} && SoC_t = SoC_{t-1} + \eta^{ch} P_t^{ch} - \frac{P_t^{dis}}{\eta^{dis}} && \forall t \in T && (1.21b)\\
&\text{SoC limit:} && SoC_t \leq SoC^{max} && \forall t \in T && (1.21c)\\
&\text{Charging limit:} && P_t^{ch} \leq r^{ch} && \forall t \in T && (1.21d)\\
&\text{Discharging limit:} && P_t^{dis} \leq r^{dis} && \forall t \in T && (1.21e)\\
&\text{SoC-end limit:} && SoC_T \geq SoC^{end} && && (1.21f)\\
&\text{Limits:} && D_t, C_t, P_t^{imp}, P_t^{imp}, L_t, P_t^{dis}, P_t^{ch}, SoC_t \geq 0 && \forall t \in T && (1.21g)
\end{aligned}
$$

All this combined gives the following LP, with SoC_0 being a given parameter. It's worth noting that this appears as an extra constraint in the physical implementation of the code.

$$
\min_{D_t, C_t, P_t^{imp}, P_t^{exp}, L_t, P_t^{dis}, P_t^{ch}, SoC_t} \sum_{t=1}^{T} \Big[P_t^{imp} \left(\tau^{imp} + \lambda_t\right) + P_t^{exp} \left(\tau^{exp} - \lambda_t\right) + \alpha L_t \Big] \qquad (1.22a)
$$

$$
\begin{aligned}
\text{s.t.}\quad & P_t^{imp} - P_t^{exp} + P_t^{dis} - P_t^{ch} = D_t - P_t^{PV} + C_t && \forall t \in T && (1.22b)\\
& C_t \leq P_t^{PV} && \forall t \in T && (1.22c)\\
& L_t \geq D_t - D_t^{ref} && \forall t \in T && (1.22d)\\
& L_t \geq -\left(D_t - D_t^{ref}\right) && \forall t \in T && (1.22e)\\
& P_t^{imp} \leq \overline{P}^{imp} && \forall t \in T && (1.22f)\\
& P_t^{exp} \leq \overline{P}^{exp} && \forall t \in T && (1.22g)\\
& D_t \leq D_{max} && \forall t \in T && (1.22h)\\
& \sum_{t=1}^{T} [D_t] \geq \sum_{t=1}^{T} \left[D_t^{ref}\right] && && (1.22i)\\
& SoC_t = SoC_{t-1} + \eta^{ch} P_t^{ch} - \frac{P_t^{dis}}{\eta^{dis}} && \forall t \in T && (1.22j)\\
& SoC_t \leq SoC^{max} && \forall t \in T && (1.22k)\\
& P_t^{dis} \leq r^{dis} && \forall t \in T && (1.22l)\\
& P_t^{ch} \leq r^{ch} && \forall t \in T && (1.22m)\\
& SoC_T \geq SoC^{end} && && (1.22n)\\
& D_t, C_t, P_t^{imp}, P_t^{exp}, L_t, P_t^{dis}, P_t^{ch}, SoC_t \geq 0 && \forall t \in T && (1.22o)
\end{aligned}
$$

1.3.2 Task ii: Dual problem, Lagrangian and KKT

The Lagrangian:
Again a lot can be borrowed from the previous sections. There's not removed anything from section 1.2.2, once again the equality constrain 1.16b is not split instead the Lagrange multiplier λ_t is defined for $\mathbb{R}$. This yields the following Lagrange multipliers added to those of section 1.2.2:

β_t : For the "SoC dynamics" constraint eq. 1.22j, $\beta_t \in \mathbb{R}$

ξ_t : For the "SoC limit" constraint eq. 1.22k, $\xi_t \geq 0$

ρ_t^{ch} : For the "charging limit" constraint eq. 1.22m, $\rho_t^{ch} \geq 0$

ρ_t^{dis} : For the "discharging limit" constraint eq. 1.22l, $\rho_t^{dis} \geq 0$

κ : For the "SoC-end limit" constraint eq. 1.22n, $\kappa \geq 0$

Since there hasn't been removed any decision variables the dual multipliers stated in equations 1.18 are used with equation 1.23 added to them.

$$
\begin{aligned}
& P_t^{dis} \geq 0 && \Rightarrow -P_t^{dis} \leq 0 && \Rightarrow \sigma_t^{P^{dis}} \geq 0 && (1.23)\\
& P_t^{ch} \geq 0 && \Rightarrow -P_t^{ch} \leq 0 && \Rightarrow \sigma_t^{P^{ch}} \geq 0 && (1.24)\\
& SoC_t \geq 0 && \Rightarrow -SoC_t \leq 0 && \Rightarrow \sigma_t^{SoC} \geq 0 && (1.25)
\end{aligned}
$$

This results in the following Lagrangian function for the problem

$$\mathcal{L}(D, C, P^{imp}, P^{exp}, L, P^{ch}, P^{dis}, SoC_t, \pi, \mu, \gamma, \theta^{imp}, \theta^{exp}, \Delta_t, \zeta^{pos}, \zeta^{neg}, \beta, \xi, \rho^{ch}, \rho^{dis}, \sigma) = \tag{1.26}$$

$$\begin{aligned}
&\sum_{t=1}^{T}\left[P_t^{imp}\left(\tau^{imp}+\lambda_t\right)+P_t^{exp}\left(\tau^{exp}-\lambda_t\right)+\alpha L_t\right] \\
&+\sum_{t=1}^{T}\left[\pi_t\left(P_t^{imp}-P_t^{exp}+P_t^{dis}-P_t^{ch}-D_t+P_t^{PV}-C_t\right)\right]+\sum_{t=1}^{T}\left[\mu_t\left(C_t-P_t^{PV}\right)\right] \\
&+\sum_{t=1}^{T}\left[\theta_t^{imp}(P_t^{imp}-\overline{P}^{imp})\right]+\sum_{t=1}^{T}\left[\theta_t^{exp}(P_t^{exp}-\overline{P}^{exp})\right]+\gamma\left(\sum_{t=1}^{T}\left[D_t^{ref}\right]-\sum_{t=1}^{T}\left[D_t\right]\right) \\
&+\sum_{t=1}^{T}\left[\zeta_t^{pos}\left(L_t-D_t+D_t^{ref}\right)\right]+\sum_{t=1}^{T}\left[\zeta_t^{neg}\left(L_t+D_t-D_t^{ref}\right)\right]+\sum_{t=1}^{T}\Delta_t\left(D_t-D_{max}\right) \\
&+\sum_{t=1}^{T}\left[\beta_t\left(SoC_t-SoC_{t-1}-\eta^{ch}P_t^{ch}+\frac{P_t^{dis}}{\eta^{dis}}\right)\right]+\sum_{t=1}^{T}\left[\xi_t\left(SoC_t-SoC^{max}\right)\right] \\
&+\sum_{t=1}^{T}\left[\rho_t^{dis}\left(P_t^{dis}-r^{dis}\right)\right]+\sum_{t=1}^{T}\left[\rho_t^{ch}\left(P_t^{ch}-r^{ch}\right)\right]+\sum_{t=1}^{T}\left[\kappa\left(SoC^{end}-SoC_T\right)\right] \\
&-\sum_{t=1}^{T}\left[\sigma_t^{D}D_t+\sigma_t^{C}C_t+\sigma_t^{P^{imp}}P_t^{imp}+\sigma_t^{P^{exp}}P_t^{exp}+\sigma_t^{L}L_t\right. \\
&\qquad\left.+\sigma_t^{P^{dis}}P_t^{dis}+\sigma_t^{P^{ch}}P_t^{ch}+\sigma_t^{SoC}SoC_t\right]
\end{aligned}$$

The dual problem:
Using the same convention and methods as described in section 1.1.2, the dual problem of the expanded primal is derived from the Lagrangian function, by separating the constant terms and isolating the primal decision variables in the Lagrangian. This yields the following dual problem.

$$\begin{aligned}
\max_{\pi,\mu,\theta^{imp},\theta^{exp},\Delta_t,\zeta^{pos},\zeta^{neg},\beta,\xi,\rho^{dis},\rho^{ch},\kappa}\ &\sum_{t=1}^{T}\left[P_t^{PV}\left(\pi_t-\mu_t\right)-\left(\theta_t^{imp}\overline{P}^{imp}+\theta_t^{exp}\overline{P}^{exp}\right)\right. && (1.27a)\\
&\left.+D_t^{ref}\left(\zeta_t^{pos}-\zeta_t^{neg}\right)+\gamma D_t^{ref}-SoC^{max}\xi_t-r^{dis}\rho_t^{dis}-r^{ch}\rho_t^{ch}\right] \\
&+SoC^{end}\kappa-\beta_1 SoC_0
\end{aligned}$$

$$\begin{aligned}
\text{s.t.}\quad & \tau^{imp}+\lambda_t+\pi_t+\theta_t^{imp}\geq 0 && \forall t\in T && (1.27b)\\
& \tau^{exp}-\lambda_t-\pi_t+\theta_t^{exp}\geq 0 && \forall t\in T && (1.27c)\\
& -\pi_t-\zeta_t^{pos}+\zeta_t^{neg}-\gamma+\Delta_t\geq 0 && \forall t\in T && (1.27d)\\
& -\pi_t+\mu_t\geq 0 && \forall t\in T && (1.27e)\\
& \alpha+\zeta_t^{pos}+\zeta_t^{neg}\geq 0 && \forall t\in T && (1.27f)\\
& \pi_t+\frac{\beta_t}{\eta^{dis}}+\rho_t^{dis}\geq 0 && \forall t\in T && (1.27g)\\
& -\pi_t-\eta^{ch}\beta_t+\rho_t^{ch}\geq 0 && \forall t\in T && (1.27h)\\
& \beta_t-\beta_{t+1}+\xi_t\geq 0 && \forall t\in T && (1.27i)\\
& \beta_T+\xi_T-\kappa\geq 0 && && (1.27j)\\
& \mu_t,\theta_t^{imp},\theta_t^{exp},\gamma,\Delta_t,\zeta_t^{pos},\zeta_t^{neg},\xi_t,\rho_t^{dis},\rho_t^{ch},\kappa\ \geq 0,\quad \pi_t,\beta_t\in\mathbb{R} && \forall t\in T && (1.27k)
\end{aligned}$$

KKT conditions:

- *Primal feasibility* here the constraints from eq. 1.22 apply.
- *Dual feasibility* here the constraints from eq. 1.27
- *Stationarity* again all partial derivatives from the primal problem are set equal to zero

$$\frac{\partial \mathcal{L}}{\partial P_t^{imp}} : \tau^{imp} + \lambda_t + \pi_t + \theta_t^{imp} - \sigma_t^{P^{imp}} = 0$$
$$\frac{\partial \mathcal{L}}{\partial P_t^{exp}} : \tau^{exp} - \lambda_t - \pi_t + \theta_t^{exp} - \sigma_t^{P^{exp}} = 0$$
$$\frac{\partial \mathcal{L}}{\partial D_t} : -\pi_t - \zeta_t^{pos} + \zeta_t^{neg} - \gamma + \Delta_t - \sigma_t^{D} = 0$$
$$\frac{\partial \mathcal{L}}{\partial C_t} : -\pi_t + \mu_t - \sigma_t^{C} = 0$$
$$\frac{\partial \mathcal{L}}{\partial L_t} : \alpha + \zeta_t^{pos} + \zeta_t^{neg} - \sigma_t^{L} = 0$$
$$\frac{\partial \mathcal{L}}{\partial P_t^{dis}} : \pi_t + \frac{\beta_t}{\eta^{dis}} + \rho_t^{dis} - \sigma_t^{P^{dis}} = 0$$
$$\frac{\partial \mathcal{L}}{\partial P_t^{ch}} : -\pi_t - \eta^{ch}\beta_t + \rho_t^{ch} - \sigma_t^{P^{ch}} = 0$$

For t = 1,...,T-1

$$\frac{\partial \mathcal{L}}{\partial SoC_t} : \beta_t - \beta_{t+1} + \xi_t - \sigma_t^{SoC} = 0$$

For t = T

$$\frac{\partial \mathcal{L}}{\partial SoC_T} : \beta_T + \xi_T - \kappa - \sigma_T^{SoC} = 0$$

- *Complementary slackness* we apply the same rule of each inequality constrain being binding or its multiplier being zero which for every $t \in T$ gives

$$\mu_t(C_t - P_t^{PV}) = 0$$
$$\theta_t^{imp}(P_t^{imp} - \overline{P}^{imp}) = 0$$
$$\theta_t^{exp}(P_t^{exp} - \overline{P}^{exp}) = 0$$
$$\Delta_t\,(D_t - D_{max}) = 0$$
$$\zeta^{pos}(D_t - D_t^{ref} - L_t) = 0$$
$$\zeta^{neg}(-D_t + D_t^{ref} - L_t) = 0$$
$$\gamma\left(\sum_{t=1}^{T}\left[D_t^{ref}\right] - \sum_{t=1}^{T}[D_t]\right) = 0$$
$$\xi_t(SoC_t - SoC^{max}) = 0$$
$$\rho_t^{dis}(P_t^{dis} - r^{dis}) = 0$$
$$\rho_t^{ch}(P_t^{ch} - r^{ch}) = 0$$
$$\kappa(SoC^{end} - SoC_T) = 0$$
$$\sigma_t^{D}D_t = 0, \sigma_t^{C}C_t = 0, \sigma_t^{P^{imp}}P_t^{imp} = 0, \sigma_t^{P^{exp}}P_t^{exp} = 0, \sigma^{L_t}L_t = 0$$
$$\sigma_t^{P^{dis}}P_t^{dis} = 0, \sigma_t^{P^{ch}}P_t^{ch} = 0, \sigma_t^{SoC}SoC_t = 0$$

The implementation again ensures that the problem is kept linear and convex which in turn results in strong duality holding and that the KKT's are necessary and sufficient. If we once again look economically at the dual problem we see that e.g. β_t sctually describes the marginal value of stored energy, which is interesting because this would be expected to have a major impact on the system and decisions as a whole.

1.3.3 Qualitative Analysis

Economic Benefits
The availability of a battery gives the consumer economic benefits in case of arbitrage and increased PV self consumption. Arbitrage occurs because the battery enables the consumer to buy (or import) electricity during low price hours and discharge (or reduce imports / export) during high price hours. This works through the SoC dynamics constraint and the dual variable β_t, representing the marginal value of stored energy. For PV self consumption the battery allows storage of excess PV production during midday and use in the evening/night, reducing grid dependency. Thus, excess PV that would otherwise be curtailed (binding μ_t constraint) can be stored instead of exported at a lower price. Another economic benefit is reducing the discomfort weighted by α. By buffering energy shifts, the battery allows the consumer to stay closer to the reference load L_t^{ref} while still exploiting favorable prices, effectively reducing both discomfort penalties and energy costs simultaneously.

The economic value of the battery depends strongly on external conditions:

- With a large spread between high and low electricity prices, the arbitrage potential and thus the shadow value of stored energy (β_t) increases.
- High PV production promotes storage charging during mid-day and discharging in the evening, reducing curtailment.
- Elevated grid tariffs or export restrictions further enhance the incentive to store energy for self-consumption.
- When the discomfort factor α is large, the battery provides an indirect flexibility buffer, enabling cost savings without visibly shifting the consumer's load L_t away from its reference L_t^{ref}.

Binding constraints and marginal values Compared to the formulation in section 1.2.2, the set of binding constraints changes notably. The SoC dynamics constraint (1.22j) and capacity limits (1.22k) often become binding in high-utilization cases, while PV curtailment (1.22c) becomes less restrictive due to the storage buffer. The end-of-day constraint (1.22n) typically binds at optimality, ensuring energy neutrality across the horizon. The dual variables β_t associated with SoC capture the intertemporal marginal value of stored energy, which rises in high price periods and declines in low price periods.

In conclusion, the battery profitability is greatest under conditions of:

- Large hourly price variations or volatile PV production
- High import tariffs or export limitations
- Moderate discomfort penalties α limiting direct load shifting
- High battery capacity or efficiency (low losses)

Conversely, when electricity prices are flat, PV generation is low, or α is small, the marginal value of storage β_t tends to zero, and the battery provides limited additional benefit.

1.3.4 Implement model

The model has been implemented using gurobipy and the code can be found in the "Part 1.C_new.ipynb" file on either Github or attached to the hand-in in table 1.3 below is the optimal solution presented

Table 1.3: Optimal Energy Dispatch Results with Battery Storage

Hour	Load (kW)	Curtailment (kW)	Grid Import (kW)	Grid Export (kW)	PV Production (kW)	Batt Charge (kW)	Batt Discharge (kW)	SOC (kWh)	Electricity Price (DKK/kWh)
0	0.17	0.00	0.17	0.00	0.00	0.00	0.00	3.00	1.10
1	0.12	0.00	0.12	0.00	0.00	0.00	0.00	3.00	1.05
2	0.12	0.00	1.02	0.00	0.00	0.90	0.00	3.81	1.00
3	1.59	0.00	2.49	0.00	0.00	0.90	0.00	4.62	0.90
4	3.00	0.00	3.90	0.00	0.00	0.90	0.00	5.43	0.85
5	1.44	0.00	1.29	0.00	0.15	0.00	0.00	5.43	1.01
6	2.28	0.00	1.86	0.00	0.42	0.00	0.00	5.43	1.05
7	2.40	0.00	1.77	0.00	0.63	0.00	0.00	5.43	1.20
8	1.89	0.00	0.00	0.00	0.45	0.00	1.44	3.83	1.40
9	0.66	0.00	0.00	0.00	0.36	0.00	0.30	3.50	1.60
10	0.75	0.00	0.00	0.00	0.63	0.00	0.12	3.36	1.50
11	1.05	0.00	0.30	0.00	0.75	0.00	0.00	3.36	1.10
12	1.65	0.00	0.00	0.00	2.55	0.90	0.00	4.17	1.05
13	1.35	0.00	0.00	0.00	2.25	0.90	0.00	4.98	1.00
14	1.35	0.00	0.60	0.00	1.65	0.90	0.00	5.79	0.95
15	1.95	0.00	0.89	0.00	1.29	0.23	0.00	6.00	1.00
16	2.34	0.00	1.65	0.00	0.69	0.00	0.00	6.00	1.20
17	2.70	0.00	1.65	0.00	0.15	0.00	0.90	5.00	1.50
18	0.75	0.00	0.00	0.00	0.75	0.00	0.00	5.00	2.10
19	0.00	0.00	0.00	2.55	0.75	0.00	1.80	3.00	2.50
20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.00	2.20
21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.00	1.80
22	0.22	0.00	0.22	0.00	0.00	0.00	0.00	3.00	1.40
23	0.17	0.00	0.17	0.00	0.00	0.00	0.00	3.00	1.20
									Total cost: 26.02 DKK

1.3.5 Numerical Analysis

With the addition of a battery one expects this to be used as a tool for flexibility. This is to say an ability for the model to find a cheaper solution for the same level of comfort as in the previous scenario. To test this we look to a Discomfort-Cost Pareto chart, noting this has a flipped axis compared to the previous Pareto. This can be seen in Figure 1.9. As shown the model can now consistently achieve lower cost and discomfort at on the full range of α

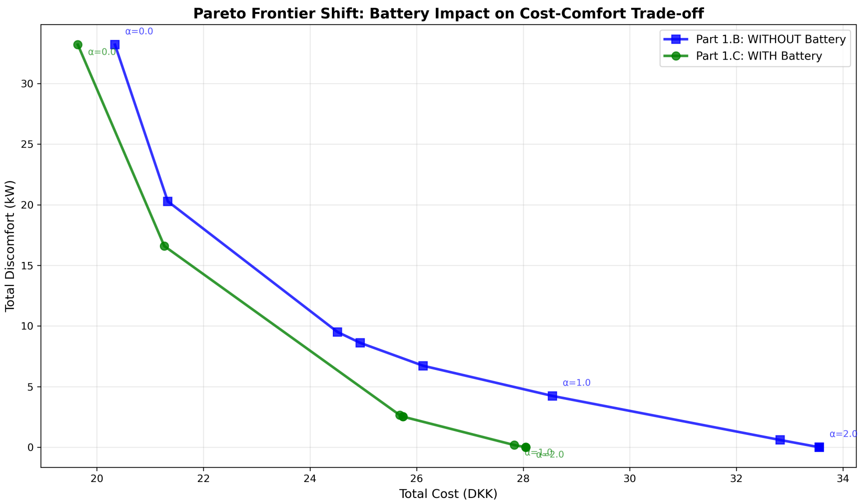

Figure 1.9: Cost-Comfort Pareto Frontiers: Impact of α and Flexibility

To give a direct comparison to the previous model, we look at the same type of plot as Figure 1.6. For the battery base case, the same sweep of α values was performed, with expected yet still interesting results.

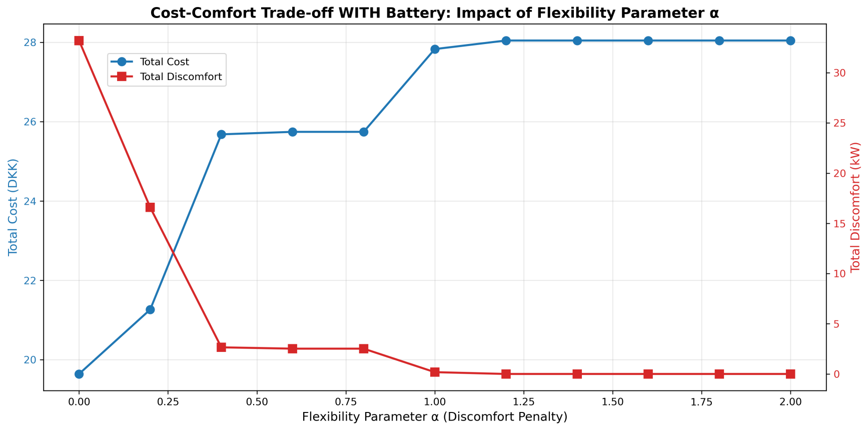

Figure 1.10: Cost-Comfort Pareto Frontiers: Impact of α and Flexibility

Figure 1.10 shows that the battery makes discomfort fall quite rapidly and stabilises around 0.4 before falling to zero at $\alpha = 1$.
To further investigate the impact of the battery we constructed 3 scenarios to compare with the base case. This will be done at 2 different alpha values.

- Base case: The objective and constraints presented above with one of the 2 given alpha values
- Reduced PV: As in section 1, reducing the PV production by 50%
- High Volatility: Increasing the volatility of the prices using $\lambda_{volatile} = \lambda_{mean} + 1.5 \cdot (\lambda_t - \lambda_{mean})$
- Zero Tariffs: Using same methodology as previously, using basically negligible tariffs, however, non-zero to stop import/export maxing

Table 1.4: Battery Scenario Comparison Summary ($\alpha = 0.5$)

Scenario	Objective	Cost (DKK)	Discomfort	Import (kWh)	Export (kWh)	Batt Cycle (kWh)
Base_Case	28.27	25.75	2.52	15.72	0.00	5.91
Reduced_PV	39.21	36.75	2.46	22.69	0.00	7.06
High_Volatility	29.50	18.49	11.01	19.89	4.18	5.91
Zero_Tariffs	21.50	9.89	11.61	26.40	10.10	8.72

Table 1.5: Scenario Comparison Summary ($\alpha = 0.3$)

Scenario	Objective	Cost (DKK)	Discomfort	Import (kWh)	Export (kWh)	Batt Cycle (kWh)
Base_Case	33.72	22.71	11.01	18.09	2.55	5.09
Reduced_PV	43.82	33.86	9.96	24.60	2.17	5.80
High_Volatility	33.61	15.55	18.06	20.64	4.93	5.91
Zero_Tariffs	22.91	9.08	13.83	27.06	10.76	8.72

Several conclusions can be drawn from the results shown in Table 1.5 and Table 1.4. First, a recurring for all scenarios, the PV availability is seemingly one of the most important factors. Halving PV raises the objective significantly in both α scenarios, and that cost is directly to the consumer's pocket, as discomfort is similar to the base for both αs. This makes sense as the consumer must replace the lost PV energy with imports. We also see that price volatility increases the value of the battery; that is to say, the spread of electricity prices is more important than the value of electricity prices. Meaning value of a battery would most accurately be assessed through a measure of standard deviation rather than the mean value. In both α scenarios, we see cost drop significantly, and when comfort is less prioritised, the cost savings are even greater. As was the point of the test, the near-zero grid tariff scenarios show that this is one of the greatest limiting factors to the model. Without this limit it can aggressively trade and has the lowest objective (sum of discomfort and cost) by quite some margin. This comparison does have one slight problem in that it is not entirely apples to apples as the management of the grid is paid through these tariffs and some other payment structure would have to be put in place for the cost to be represented.

2 Question 2

2.1 (b)

2.1.1 Task i: Mathematical formulation

Disclaimer: We assume that this assignment builds on task 1.c and not 1.b as stated in the assignment formulation

Firstly the model from assignment 1.c is used where it's realized that an implementation of the battery cost has to be made therefore the following parameters are introduced
New parameters:

$$c^{bat} : \text{Cost of the battery per kWh } [DKK/kWh]$$
$$N^{bat} : \text{Battery life time horizon } [days]$$

Furthermore a scalar for the battery size is needed which in this case is a decision variable
New decision variables:

$$\gamma^{bat} : \text{Scalar for battery size and cost}$$

This is then implemented in the objective function. It's notable that the cost in this case is chosen to be per kWh which is the common practice in the base case this is a arbitrarily chosen to be $c^{batt} = 1200$ DKK/kWh and can be used for the numerical analysis, furthermore $N^{bat} = 3650$ is used to normalize the capital cost to a single day, thereby keeping the optimization for a single day. This makes the LP the following

$$\min_{D_t, C_t, P_t^{imp}, P_t^{exp}, L_t, P_t^{dis}, P_t^{ch}, SoC_t, \gamma^{bat}} \sum_{t=1}^{\mathcal{T}} \left[P_t^{imp} \left(\tau^{imp} + \lambda_t \right) + P_t^{exp} \left(\tau^{exp} - \lambda_t \right) + \alpha L_t \right] + \frac{\gamma^{bat} c^{bat} SoC^{max}}{N^{bat}} \tag{2.1a}$$

$$
\begin{aligned}
\text{s.t.}\quad & P_t^{imp} - P_t^{exp} + P_t^{dis} - P_t^{ch} = D_t - P_t^{PV} + C_t && \forall t \in T && \text{(2.1b)}\\
& C_t \leq P_t^{PV} && \forall t \in T && \text{(2.1c)}\\
& L_t \geq D_t - D_t^{ref} && \forall t \in T && \text{(2.1d)}\\
& L_t \geq -\left(D_t - D_t^{ref}\right) && \forall t \in T && \text{(2.1e)}\\
& P_t^{imp} \leq \overline{P}^{imp} && \forall t \in T && \text{(2.1f)}\\
& P_t^{exp} \leq \overline{P}^{exp} && \forall t \in T && \text{(2.1g)}\\
& D_t \leq D_{max} && \forall t \in T && \text{(2.1h)}\\
& \sum_{t=1}^{T} [D_t] \geq \sum_{t=1}^{T} \left[D_t^{ref}\right] && && \text{(2.1i)}\\
& SoC_t = SoC_{t-1} + \eta^{ch} P_t^{ch} - \frac{P_t^{dis}}{\eta^{dis}} && \forall t \in T && \text{(2.1j)}\\
& SoC_t \leq SoC^{max}\gamma^{bat} && \forall t \in T && \text{(2.1k)}\\
& P_t^{dis} \leq r^{dis}\gamma^{bat} && \forall t \in T && \text{(2.1l)}\\
& P_t^{ch} \leq r^{ch}\gamma^{bat} && \forall t \in T && \text{(2.1m)}\\
& SoC_T \geq SoC^{end}\gamma^{bat} && && \text{(2.1n)}\\
& D_t, C_t, P_t^{imp}, P_t^{exp}, L_t, P_t^{dis}, P_t^{ch}, SoC_t, \gamma^{bat} \geq 0 && \forall t \in T && \text{(2.1o)}
\end{aligned}
$$

2.1.2 Task ii: Qualitative discussion

Overall 5 major assumptions are made in this part

- Linear scaling of battery parameters
- Constant battery efficiencies
- Linear Capital cost
- Fixed lifetime & no degradation
- Constant parameters & no discounting

Linear scaling of battery parameters
Overall this is probably the most important assumption, in regards to the real world it would not hold up perfectly, especially as you start approaching very high levels in the parameters.

The fact that this is assumed in this problem enables us to optimize the size of the battery without actually changing the structure of the problem and most importantly to keep the problem linear and also convex.
If there was a wish to represent this in a non-linear fashion, the major impact would be a nonlinear objective function which could be either convex or non-convex depending on the problem. computationally this would mean that either non-linear programming or maybe a MILP is needed which computationally is much harder for the solver.

Constant battery efficiencies
The constant battery efficiencies make a lot more sense in a real world context, since efficiencies are so high and would change very little with respect to the already existing level.
In this problem the fact that the efficiencies do not scale with the power and/or SoC, ensures that the SoC-dynamics constraint eq. 2.1j is kept linear and in the feasible region. Should this not be the case the LP structure is broken and the problem would be much harder to solve and potentially non-convex.

Linear capital cost
This follows the same principle, it does make somewhat sense for the cost to increase in this way with regards. and i fits perfectly with keeping the problem linear. One could imagine there being some variable parts, volume discounts or the like. in this case a MILP would be needed at the cost of computational power.

Fixed lifetime & no degradation
This overall allows us to condense the cost of the battery to a single day, which does make sense overall, however a battery does degrade over the lifetime. Implementing this would require the problem to be expanded to a multi day problem or implementing some form of dynamic optimization to include a degradation constraint. Again this would be computationally much heavier.

Constant parameters & no discounting
This just enables us to optimize for a single "typical" day. The more proper solution to actually catch the variability over a multi year period would be to implement either stochastic or robust optimization, this again deviates from the regular LP and the problem dimensionality would increase, along with an introduction of some uncertainty modeling. Again a high rise in complexity which would result in a computationally much heavier problem.
$\overline{P}^{imp}$

2.1.3 Implementaiton

The model has been implemented using gurobipy and the code can be found in the "Part 2.B_new.ipynb" file on either Github or attached to the hand-in in table 2.1 below is the optimal solution presented with the optimal scaling factor below

Table 2.1: Optimal Battery Sizing and Hourly Dispatch Results

Hour	Load (kW)	Curtailment (kW)	Grid Import (kW)	Grid Export (kW)	PV Production (kW)	Electricity Price (DKK/kWh)	SOC (kWh)	Batt Charge (kW)	Batt Discharge (kW)
0	0.17	0.00	0.17	0.00	0.00	1.10	3.12	0.00	0.00
1	0.12	0.00	0.12	0.00	0.00	1.05	3.12	0.00	0.00
2	0.12	0.00	0.27	0.00	0.00	1.00	3.25	0.15	0.00
3	0.12	0.00	1.06	0.00	0.00	0.90	4.10	0.94	0.00
4	0.22	0.00	1.16	0.00	0.00	0.85	4.94	0.94	0.00
5	1.44	0.00	1.29	0.00	0.15	1.01	4.94	0.00	0.00
6	2.28	0.00	1.86	0.00	0.42	1.05	4.94	0.00	0.00
7	2.40	0.00	1.77	0.00	0.63	1.20	4.94	0.00	0.00
8	1.89	0.00	0.00	0.00	0.45	1.40	3.34	0.00	1.44
9	0.66	0.00	0.00	0.00	0.36	1.60	3.01	0.00	0.30
10	0.75	0.00	0.00	0.00	0.63	1.50	2.87	0.00	0.12
11	1.05	0.00	0.30	0.00	0.75	1.10	2.87	0.00	0.00
12	1.61	0.00	0.00	0.00	2.55	1.05	3.71	0.94	0.00
13	1.31	0.00	0.00	0.00	2.25	1.00	4.56	0.94	0.00
14	1.35	0.00	0.64	0.00	1.65	0.95	5.40	0.94	0.00
15	1.95	0.00	1.60	0.00	1.29	1.00	6.24	0.94	0.00
16	2.34	0.00	1.65	0.00	0.69	1.20	6.24	0.00	0.00
17	2.70	0.00	2.55	0.00	0.15	1.50	6.24	0.00	0.00
18	1.77	0.00	0.00	0.00	0.75	2.10	5.11	0.00	1.02
19	2.62	0.00	0.00	0.00	0.75	2.50	3.03	0.00	1.87
20	0.22	0.00	0.00	0.00	0.00	2.20	2.78	0.00	0.22
21	0.45	0.00	0.00	0.00	0.00	1.80	2.28	0.00	0.45
22	0.22	0.00	0.22	0.00	0.00	1.40	2.28	0.00	0.00
23	0.17	0.00	1.10	0.00	0.00	1.20	3.12	0.94	0.00

Total cost: 28.98 DKK
Scaling factor (γ): 1.0405
Optimal battery capacity: 6.24 kWh (base: 6.00 kWh)
Optimal max charging power: 0.94 kW (base: 0.90 kW)
Optimal max discharging power: 1.87 kW (base: 1.80 kW)
Daily battery cost: 2.0525 DKK

2.1.4 Numerical Analysis

Our numerical experiment will be to understand the sizing of battery for different consumer types. For this we create 3 consumer types based on their flexibility preferences by varying α.

- Highly Flexible, cost-conscious consumer: $\alpha = 0.1$
- Moderately Flexible, balance consumer: $\alpha = 0.5$
- Inflexible, comfort consumer: $\alpha = 2.0$

A standard Capex of 1200 DKK/kWh was chosen as this is the mid-range for a battery and installation costs in a residential context. To justify this assumption, a sensitivity analysis is conducted. Research showed a range of 800-1500 DKK/kWh, so a sweep from 600-1800 DKK/kWh was taken.

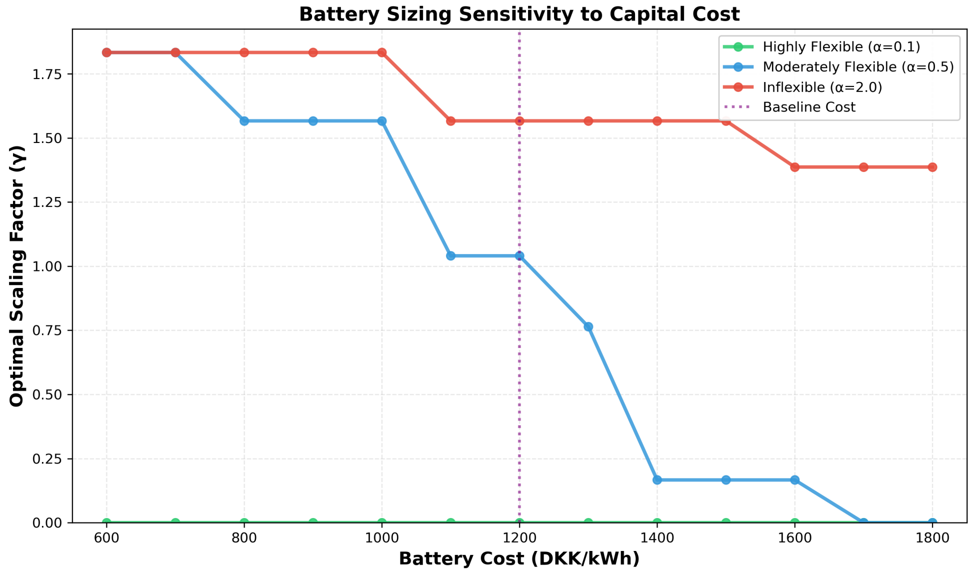

Figure 2.1: Battery cost sensitivity by consumer type

We see that the highly flexible consumer actually sees no benefit in installing a battery. This is understandable, as the factor the battery gives is flexibility, which the consumer already has, and the added ability of arbitrage is not enough to offset the cost of installing the battery. The moderately flexible consumer is very sensitive to CAPEX changes in the 1000-1400 DKK/kWh range and the inflexible consumer is sensitive in steps at 1000 DKK/kWh and 1500 DKK/kWh. With a fixed, baseline cost we now want to further investigate what happens in with each consumer by comparing the breakdown of costs. The results of this can be seen in Figure 2.2.

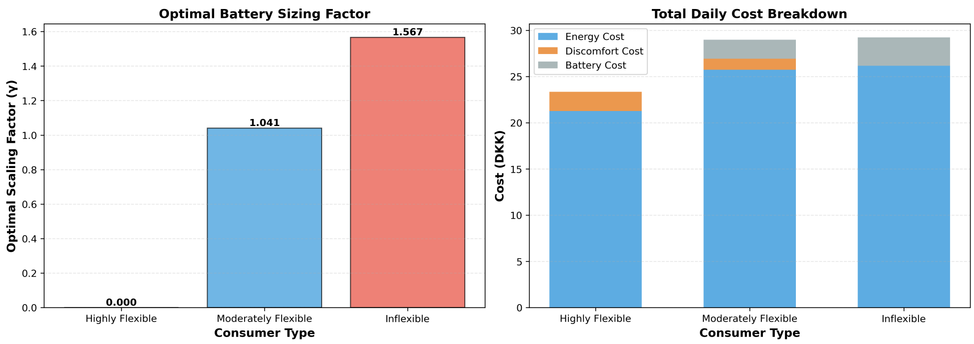

Figure 2.2: Optimal Battery Sizing and Daily Cost Breakdown by consumer type

Figure 2.2 Gives some interesting insights. When comparing the moderately flexible and inflexible consumer we see that the inflexible consumer prefers a larger battery to make sure they take no discomfort cost. We see that their respective energy costs are almost the same and the saving to the moderately flexible consumer is trading battery capex for discomfort. The highly flexible consumer still sees the lowest cost and the battery itself is not enought to bring the Energy Cost down to the level of the Highly flexible consumer. They can fit their consumption very well with PV while the maximum SOC constraint is likely forcing the less flexible consumer to pay a little extra.
This insight leads to the final investigation on what is the most important factor of the battery, it is its Maximum SOC or charge/discharge power. Perhaps it may be possible to have a battery at a certain capacity with larger flow capacity. To investigate this, we look at the Shadow Prices using the dual variable. We calculate the Average shadow price for SOC, charging and discharging power, the highly flexible consumer is omitted as they do not install a battery.

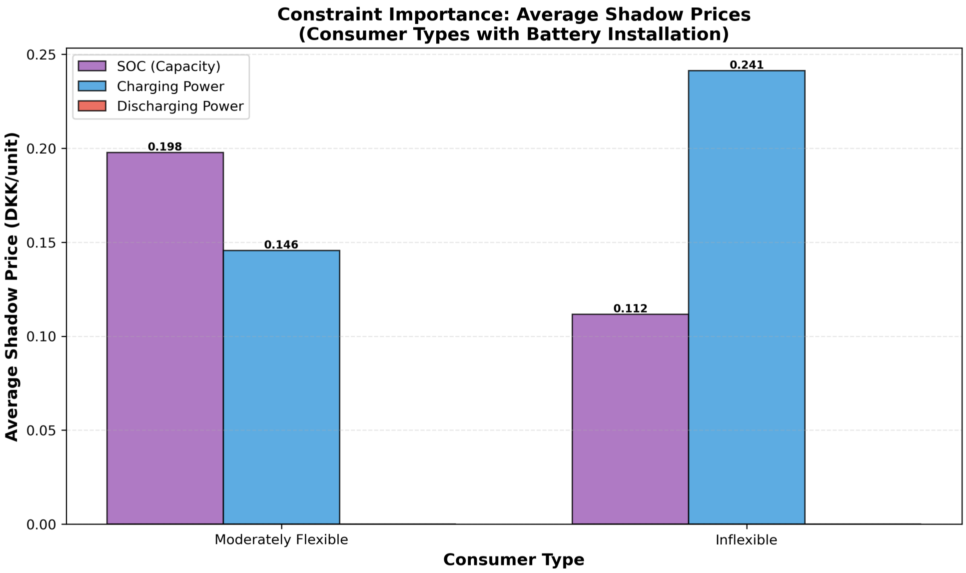

Figure 2.3: Optimal Battery Sizing and Daily Cost Breakdown by consumer type

Figure 2.3 shows a few interesting results. The discharge power is not a binding constraint, as the shadow price is 0. Now, for the less flexible consumers, we actually find opposite results. The moderately flexible is constrained by SOC, while the inflexible, who has installed a larger battery, is more constrained by charging power. This is a result of the linear scaling applied. While the moderately flexible consumer may want a larger battery, at this optimal the next step does not offset enough discomfort.

Discussion of Method

While this analysis does provide some valuable insights, there are major restrictions to be aware of. Firstly, this is solved as an optimisation problem with perfect foresight. This is not applicable to the real world and some changes would have to made to account for uncertainty. Furthermore, there is major assumption that each day will have the same profile to account for the battery lifetime. While one could argue that the profile represents an average that argument fails to account for battery's performance being best during periods of variability. To conclude the calculation is a complex napkin calculation. There are many broad assumptions and while this can give some insights much more indepth calculation would be needed for accurate decision making.