References

- *Collision Detection between Geometric Models: A Survey*, by M. Lin and S. Gottschalk, Proc. of IMA Conference on Mathematics of Surfaces 1998.
- *I-COLLIDE: Interactive and Exact Collision Detection for Large-Scale Environments*, by Cohen, Lin, Manocha & Ponamgi, Proc. of ACM Symposium on Interactive 3D Graphics, 1995. (More details in Chapter 3 of M. Lin's Thesis)
- *A Fast Procedure for Computing the Distance between Objects in Three-Dimensional Space*, by E. G. Gilbert, D. W. Johnson, and S. S. Keerthi, In IEEE Transaction of Robotics and Automation, Vol. RA-4:193--203, 1988.

Geometric Proximity Queries

- **Given two object, how would you check:**
  - *If they intersect with each other while moving?*
  - *If they do not interpenetrate each other, how far are they apart?*
  - *If they overlap, how much is the amount of penetration*

Collision Detection

- Update configurations w/ TXF matrices
- Check for edge-edge intersection in 2D
  (Check for edge-face intersection in 3D)
- Check every point of A inside of B &
  every point of B inside of A
- Check for pair-wise edge-edge intersections

Imagine larger input size: N = 1000+ ……

Classes of Objects & Problems

- 2D vs. 3D
- Convex vs. Non-Convex
- Polygonal vs. Non-Polygonal
- Open surfaces vs. Closed volumes
- Geometric vs. Volumetric
- Rigid vs. Non-rigid (deformable/flexible)
- Pairwise vs. Multiple (N-Body)
- CSG vs. B-Rep
- Static vs. Dynamic

And so on… This may include other geometric representation schemata, etc.

Some Possible Approaches

- Geometric methods
- Algebraic Techniques
- Hierarchical Bounding Volumes
- Spatial Partitioning
- Others (e.g. optimization)

Voronoi Diagrams

- Given a set S of n points in R^2, for each point p_i in S, there is the set of points (x, y) in the plane that are closer to p_i than any other point in S, called Voronoi polygons. The collection of n Voronoi polygons given the n points in the set S is the ***"Voronoi diagram"***, *Vor(S)*, of the point set S.

Intuition: To partition the plane into regions, each of these is the set of points that are closer to a point p_i in S than any other. The partition is based on the set of closest points, e.g. bisectors that have 2 or 3 closest points.

Generalized Voronoi Diagrams

- **The extension of the Voronoi diagram to higher dimensional features (such as edges and facets, instead of points); i.e. the set of points closest to a *feature*, e.g. that of a polyhedron.**
- **FACTS:**
  – In general, the generalized Voronoi diagram has quadratic surface boundaries in it.
  – If the polyhedron is convex, then its generalized Voronoi diagram has planar boundaries.

Voronoi Regions

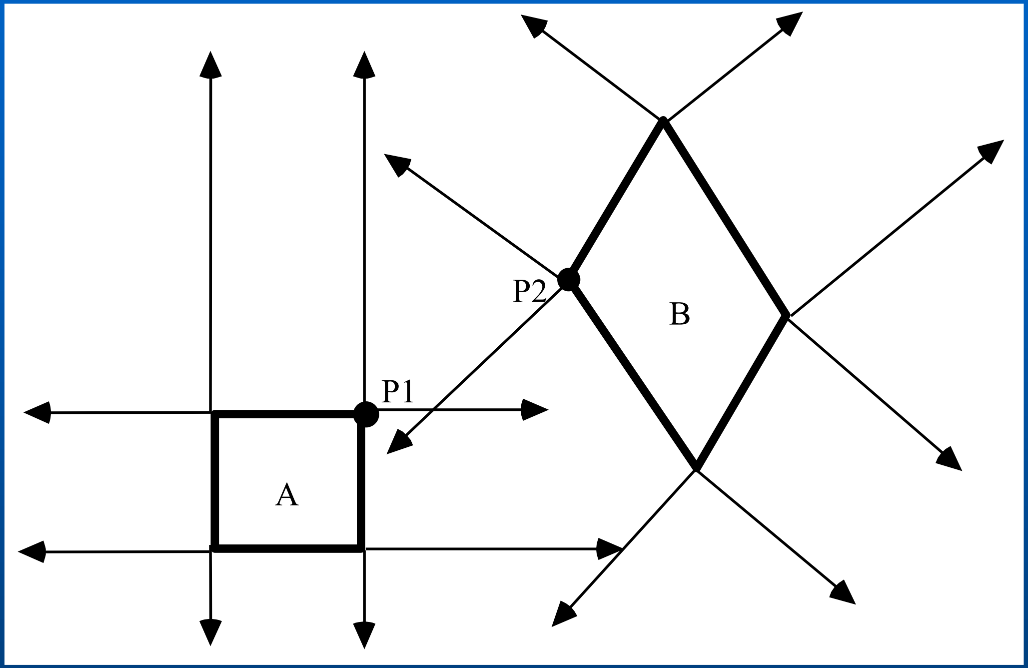
- A ***Voronoi region*** associated with a ***feature*** is a set of points that are closer to that feature than any other.
- **FACTS:**
  - The Voronoi regions form a partition of space outside of the polyhedron according to the closest feature.
  - The collection of Voronoi regions of each polyhedron is the generalized Voronoi diagram of the polyhedron.
  - The generalized Voronoi diagram of a convex polyhedron has linear size and consists of polyhedral regions. And, all Voronoi regions are convex.

Voronoi Marching

Basic Ideas:

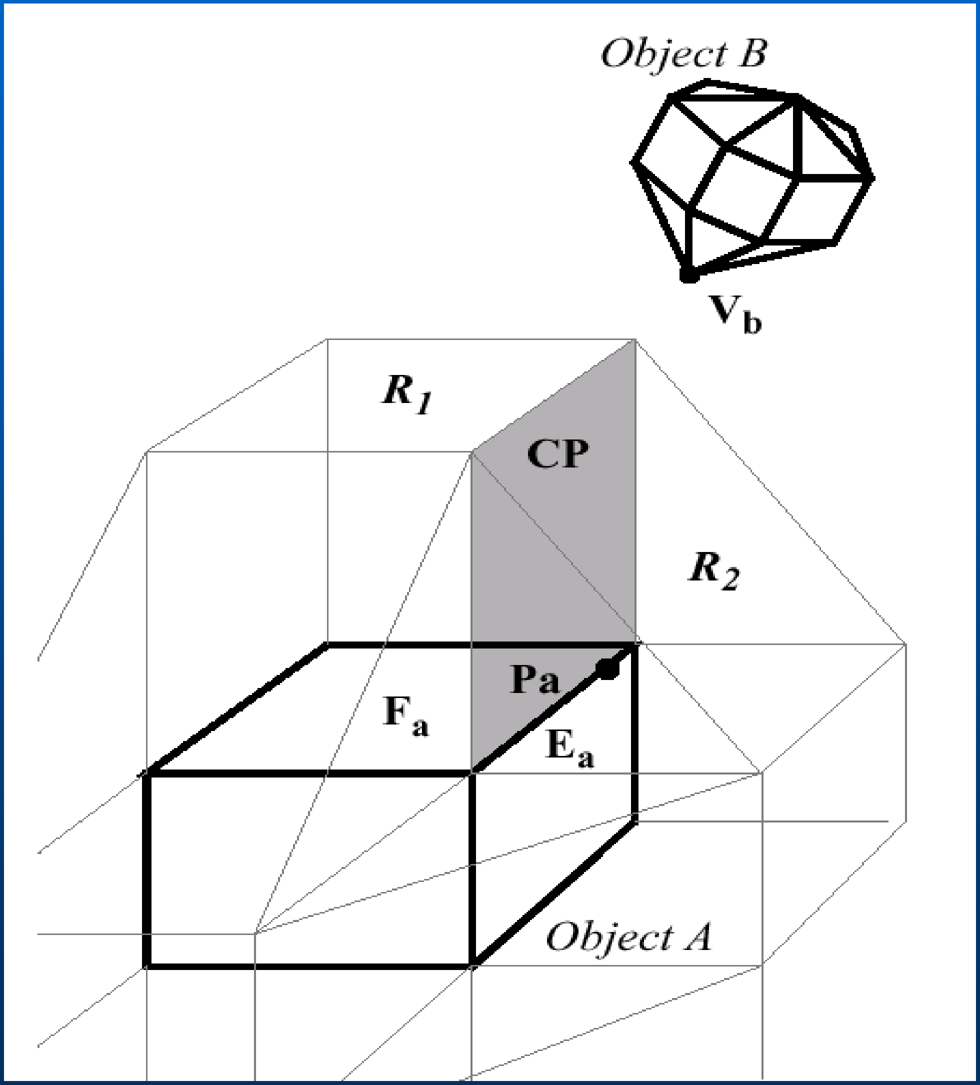
- **Coherence:** local geometry does not change much, when computations repetitively performed over successive small time intervals
- **Locality:** to "***track***" the pair of closest features between 2 moving convex polygons(polyhedra) w/ Voronoi regions
- **Performance:** expected **constant** running time, independent of the geometric complexity

Simple 2D Example



Objects A & B and their Voronoi regions: P1 and P2 are the pair of closest points between A and B. Note P1 and P2 lie within the Voronoi regions of each other.

Basic Idea for Voronoi Marching



Linear Programming

In general, a d-dimensional linear program-ming (or linear optimization) problem may be posed as follows:

- **Given a finite set A in R^d**
- **For each a in A, a constant K_a in R, c in R^d**
- **Find x in R^d which minimize $<x, c>$**
- **Subject to $<a, x> \geq K_a$, for all a in A .**

where $<*, *>$ is standard inner product in R^d.

LP for Collision Detection

Given two finite sets A, B in R^d

For each $\boldsymbol{a}$ in A and $\boldsymbol{b}$ in B,

Find $\boldsymbol{x}$ in R^d which minimize *whatever*

Subject to $\langle \boldsymbol{a}, \boldsymbol{x} \rangle > \boldsymbol{0}$, for all $\boldsymbol{a}$ in A

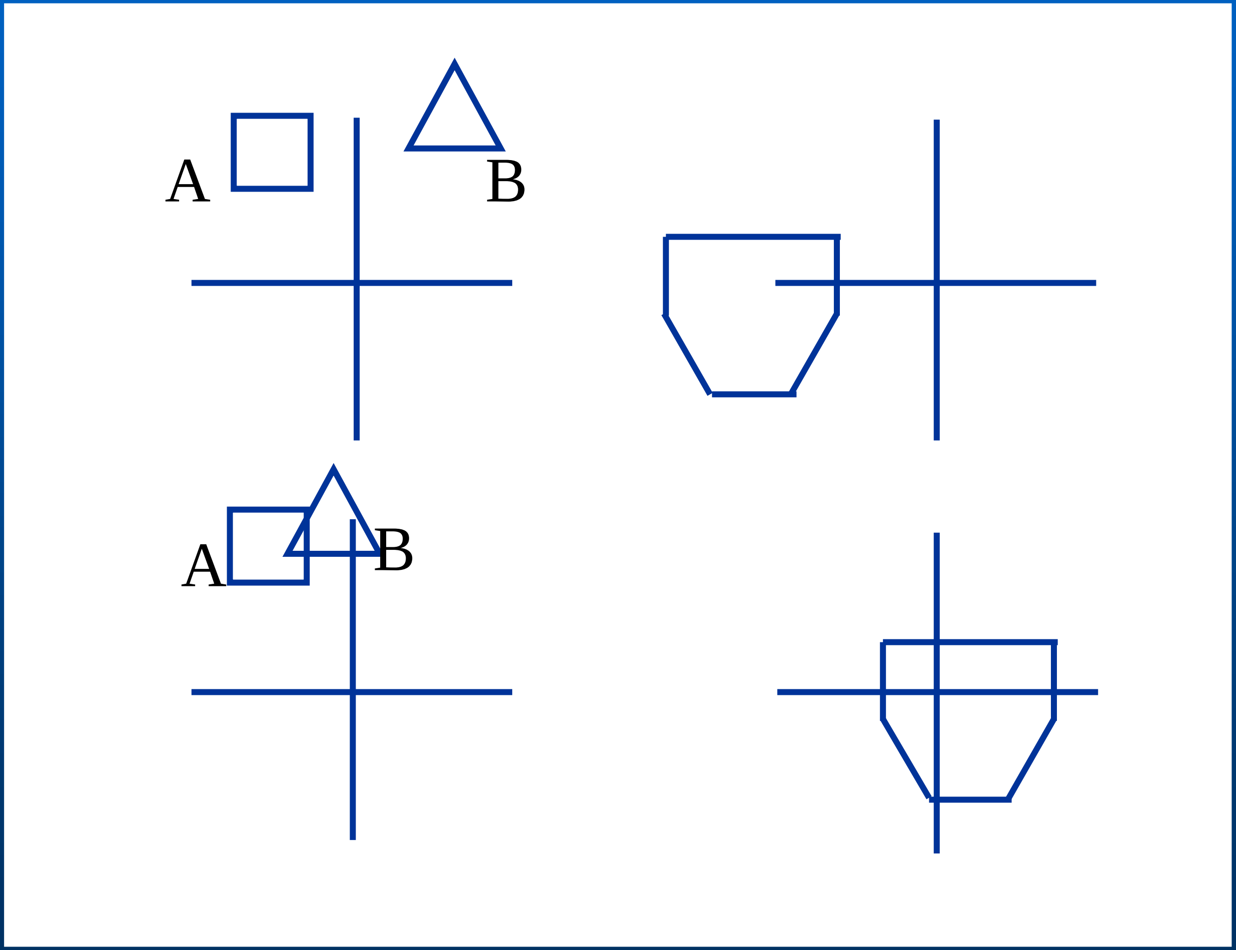
And $\langle \boldsymbol{b}, \boldsymbol{x} \rangle < \boldsymbol{0}$, for all $\boldsymbol{b}$ in B

where d = 2 (or 3).

Minkowski Sums/Differences

- **Minkowski Sum (A, B) = { a + b | a $\in$ A, b $\in$ B }**
- **Minkowski Diff (A, B) = { a - b | a $\in$ A, b $\in$ B }**
- **A and B collide iff Minkowski Difference(A,B) contains the point 0.**

Some Minkowski Differences



Minkowski Difference & Translation

- **Minkowski-Diff(Trans(A, t_1), Trans(B, t_2)) = Trans(Minkowski-Diff(A,B), t_1 - t_2)**

$\Rightarrow$ **Trans(A, t_1) and Trans(B, t_2) intersect iff Minkowski-Diff(A,B) contains point (t_2 - t_1).**

Properties

- **Distance**
  - distance(A,B) = min $_{a \in A,\ b \in B}$ $\| a - b \|_2$
  - distance(A,B) = min $_{c \in \text{Minkowski-Diff}(A,B)}$ $\| c \|_2$
  - if A and B disjoint, c is a point on boundary of Minkowski difference
- **Penetration Depth**
  - pd(A,B) = min$\{ \| t \|_2 \mid A \cap \text{Translated}(B,t) = \emptyset \}$
  - pd(A,B) = min$_{t \notin \text{Minkowski-Diff}(A,B)}$ $\| t \|_2$
  - if A and B intersect, t is a point on boundary of Minkowski difference

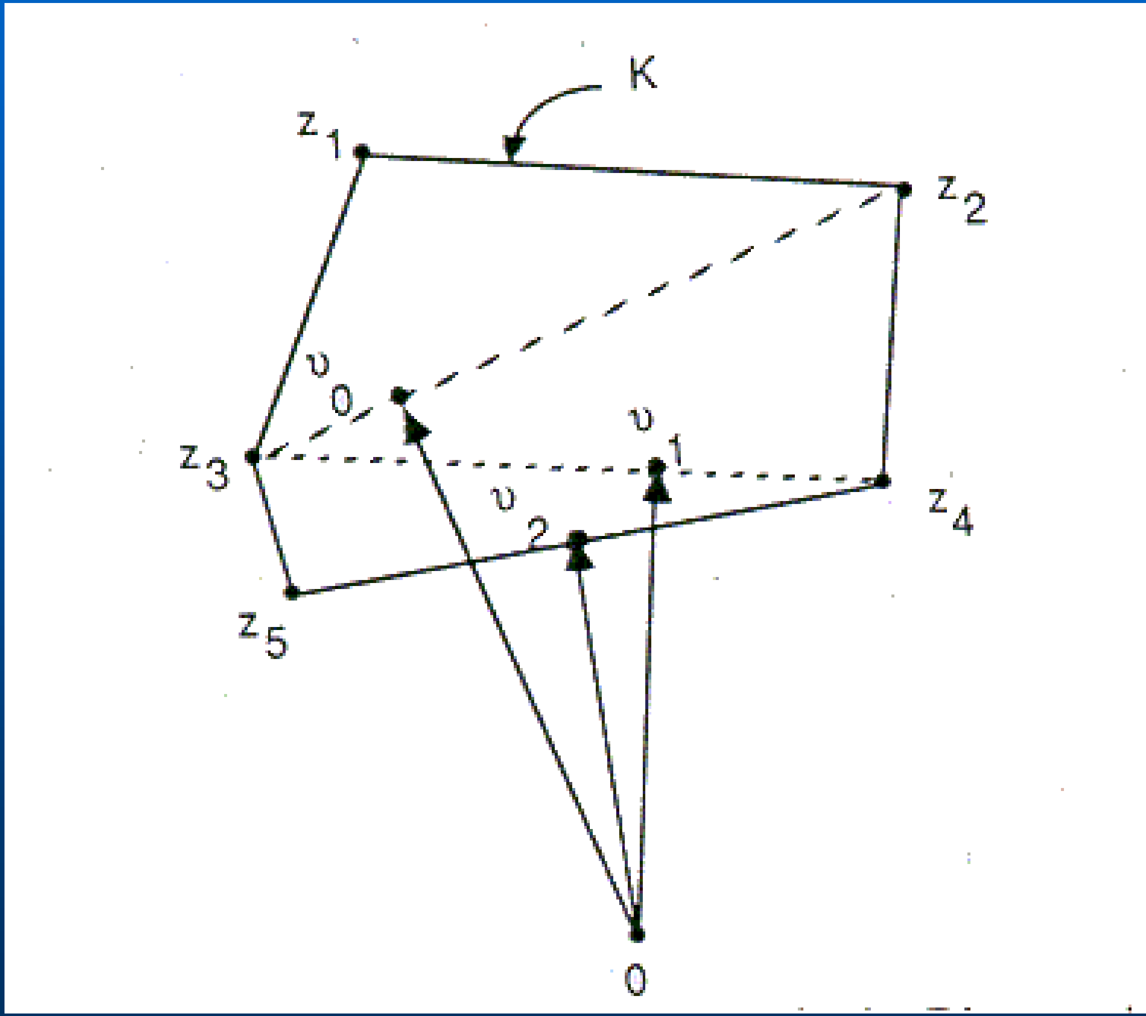
Practicality

- **Expensive to compute boundary of Minkowski difference:**
  - For convex polyhedra, Minkowski difference may take $O(n^2)$
  - For general polyhedra, no known algorithm of complexity less than $O(n^6)$ is known

GJK for Computing Distance between Convex Polyhedra

```
GJK-DistanceToOrigin ( P )   // dimension is m
Initialize P_0 with m+1 or fewer points.
k = 0
while (TRUE) {
        if origin is within CH( P_k ), return 0
        else {
            find x ∈ CH(P_k) closest to origin, and S_k ⊂ P_k s.t. x ∈ CH(S_k)
            see if any point p_-x in P more extremal in direction -x
            if no such point is found, return |x|
            else {
                P_k+1 = S_k ∪ {p_-x}
        k = k + 1
            }
        }
}
```

An Example of GJK



Running Time of GJK

- Each iteration of the while loop requires O(n) time.
- O(n) iterations possible. The authors claimed between 3 to 6 iterations on average for any problem size, making this "expected" linear.
- Trivial O(n) algorithms exist if we are given the boundary representation of a convex object, but GJK will work on point sets - computes CH lazily.

More on GJK

Given A = CH(A')   A' = { $a_1, a_2, \ldots, a_n$ **} and**

B = CH(B')   B' = { $b_1, b_2, \ldots, b_m$ **}**

- **Minkowski-Diff(A,B) = CH(P), P = {**$a - b \mid a \in A', b \in B'$**}**
- **Can compute points of P on demand:**
  - $p_{-x} = a_{-x} - b_x$ **where** a_{-x} **is the point of A' extremal in direction** $-x$**, and** b_x **is the point of B' extremal in direction** x**.**
- **The loop body would take O(n + m) time, producing the "expected" linear performance overall.**

Large, Dynamic Environments

- For dynamic simulation where the velocity and acceleration of all objects are known at each step, use the scheduling scheme (implemented as heap) to prioritize "critical events" to be processed.
- Each object pair is tagged with the estimated time to next collision. Then, each pair of objects is processed accordingly. The heap is updated when a collision occurs.

Scheduling Scheme

- a_{max}: an upper bound on relative acceleration between any two *points* on any pair of objects.
- a_{lin}: relative absolute linear
- α: relative rotational accelerations
- ω: relative rotational velocities
- r: vector difference btw CoM of two bodies
- d: initial separation for two given objects

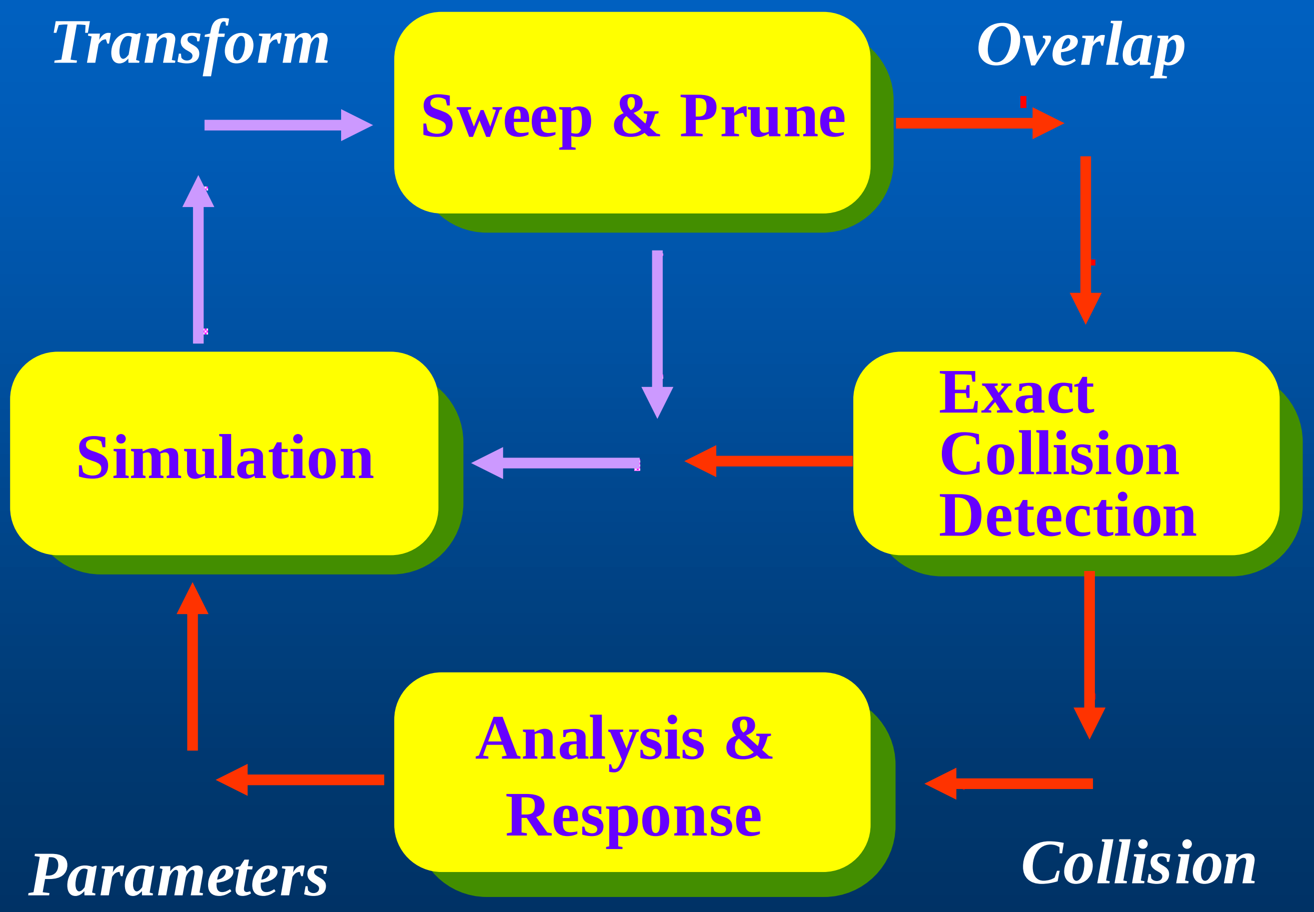
$$a_{max} = | a_{lin} + \alpha \times r + \omega \times \omega \times r |$$

$$v_i = | v_{lin} + \omega \times r |$$

- **Estimated Time to collision:**

$$t_c = \{ (v_i^2 + 2\, a_{max}\, d)^{1/2} - v_i \} / a_{max}$$

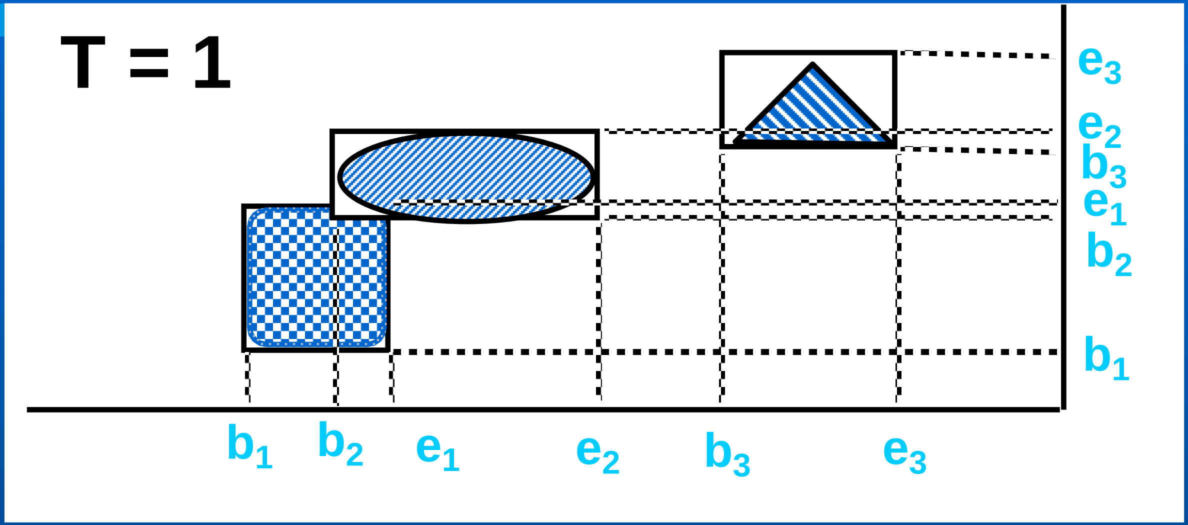
Collide System Architecture

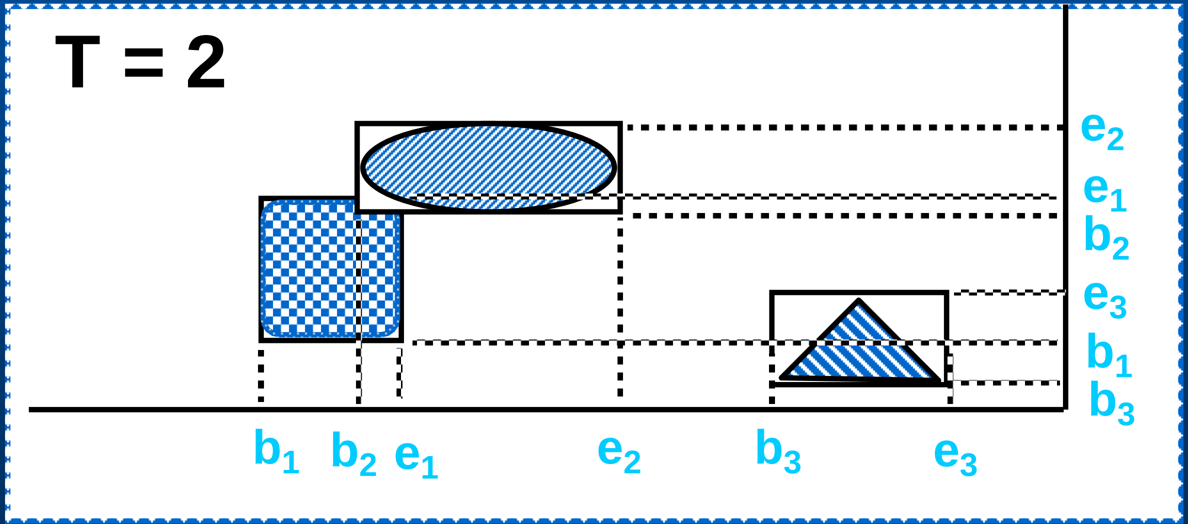


Sweep and Prune

- Compute the axis-aligned bounding box (fixed vs. dynamic) for each object
- Dimension Reduction by projecting boxes onto each *x*, *y*, *z*- axis
- Sort the endpoints and find overlapping intervals
- Possible collision -- only if projected intervals overlap in all 3 dimensions

Sweep & Prune


T = 1
e3
e2
b3
e1
b2
b1
b1 b2 e1 e2 b3 e3


T = 2
e2
e1
b2
e3
b1
b3
b1 b2 e1 e2 b3 e3

Updating Bounding Boxes

- **Coherence** (greedy algorithm)
- **Convexity properties** (geometric properties of convex polytopes)
- **Nearly constant time**, if the motion is relatively "small"

Use of Sorting Methods

- **Initial sort** -- quick sort runs in $O(m \log m)$ just as in any ordinary situation
- **Updating** -- insertion sort runs in $O(m)$ due to coherence. We sort an almost sorted list from last stimulation step. In fact, we look for "swap" of positions in all 3 dimension.

Implementation Issues

- Collision matrix -- basically *adjacency matrix*
- Enlarge bounding volumes with some tolerance threshold
- Quick start polyhedral collision test -- using bucket sort & look-up table