Review

- Particle Dynamics

(see transparencies in class)

Disclaimer

- The following slides reuse materials from SIGGRAPH 2001 Course Notes on Physically-based Modeling (copyright © 2001 by Andrew Witkin at Pixar).

A Newtonian Particle

- Differential equation: f = ma
- Forces can depend on:
  - Position, Velocity, Time

$$\ddot{\mathbf{x}} = \frac{\mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}, t)}{m}$$

Second Order Equations

$$\ddot{\mathbf{x}} = \frac{\mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}, t)}{m}$$

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{v} \\ \dot{\mathbf{v}} = \mathbf{f}/m \end{cases}$$

As discussed in the last lecture, we can transform a second order equation into a couple of first order equations.

$\Leftarrow \Leftarrow \Leftarrow$ as shown here.

Phase (State) Space

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{v} \end{bmatrix}$$

Concatenate **x** and **v** to make a 6-vector: *Position in Phase Space.*

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{bmatrix}$$

Velocity in Phase Space: another 6-vector.

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ \mathbf{f}/m \end{bmatrix}$$

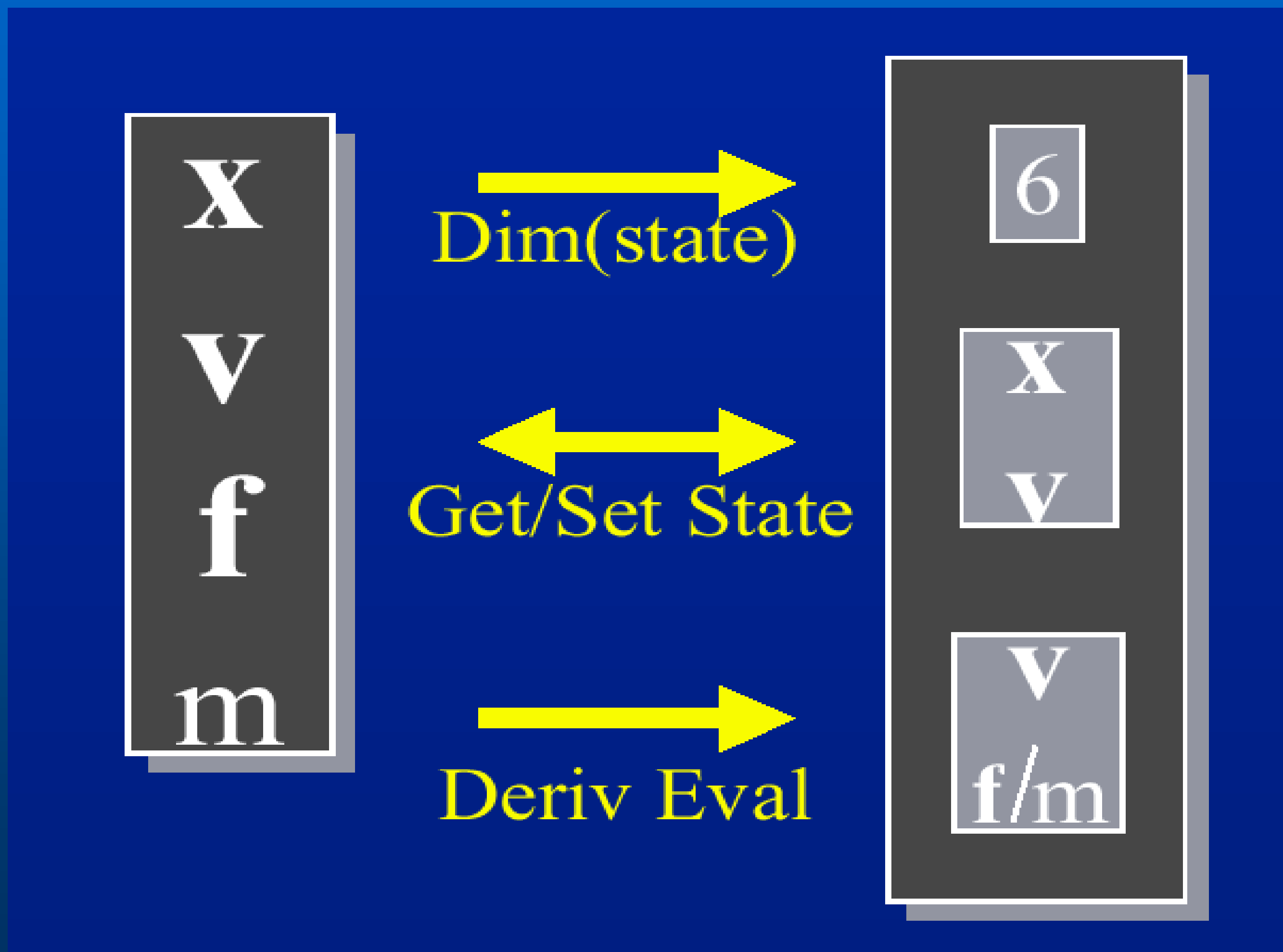
A vanilla 1st-order differential equation.

Particle Structure

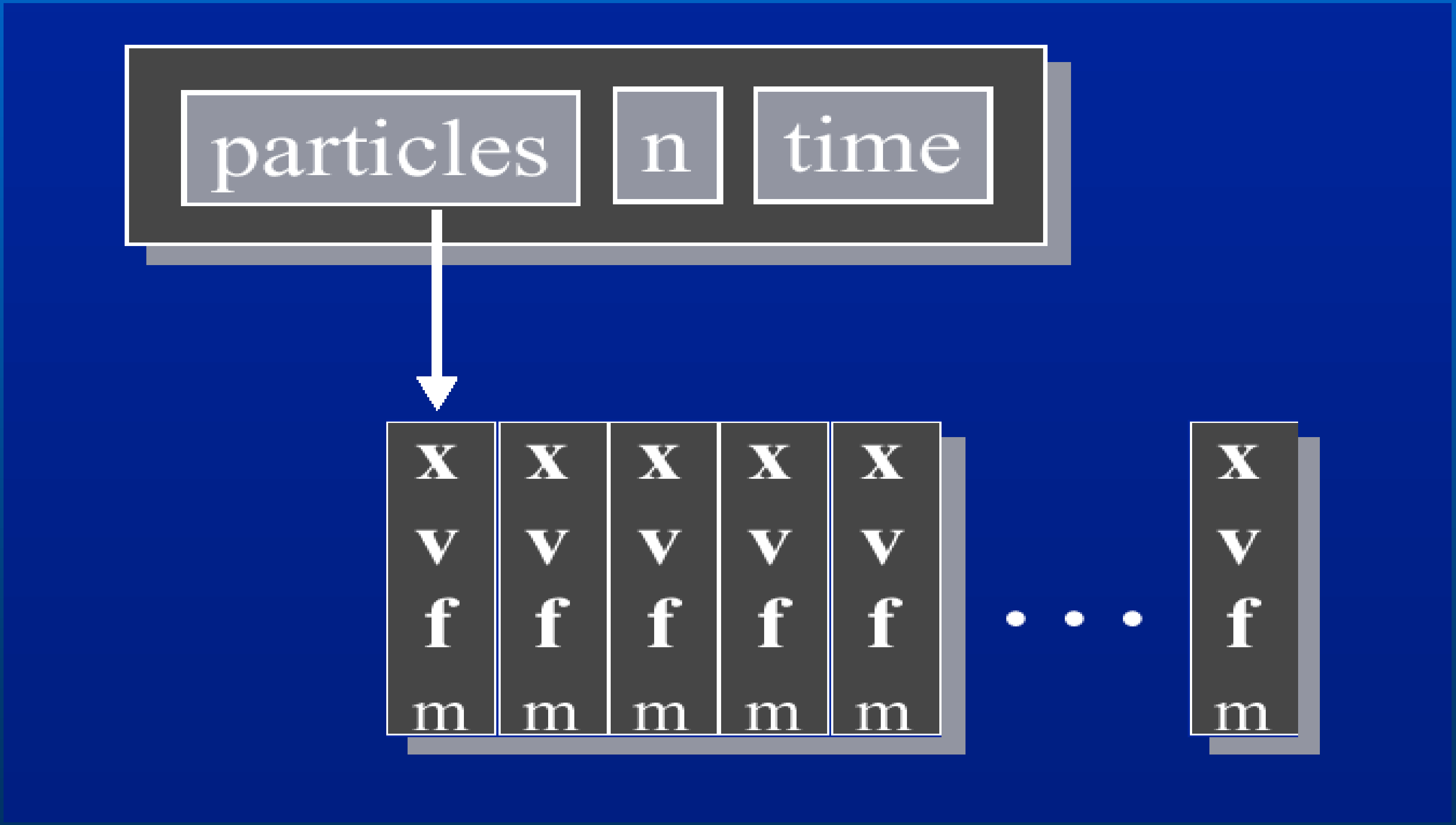
- **x** — Position
- **v** — Velocity
- **f** — Force Accumulator
- **m** — mass

Position in Phase Space

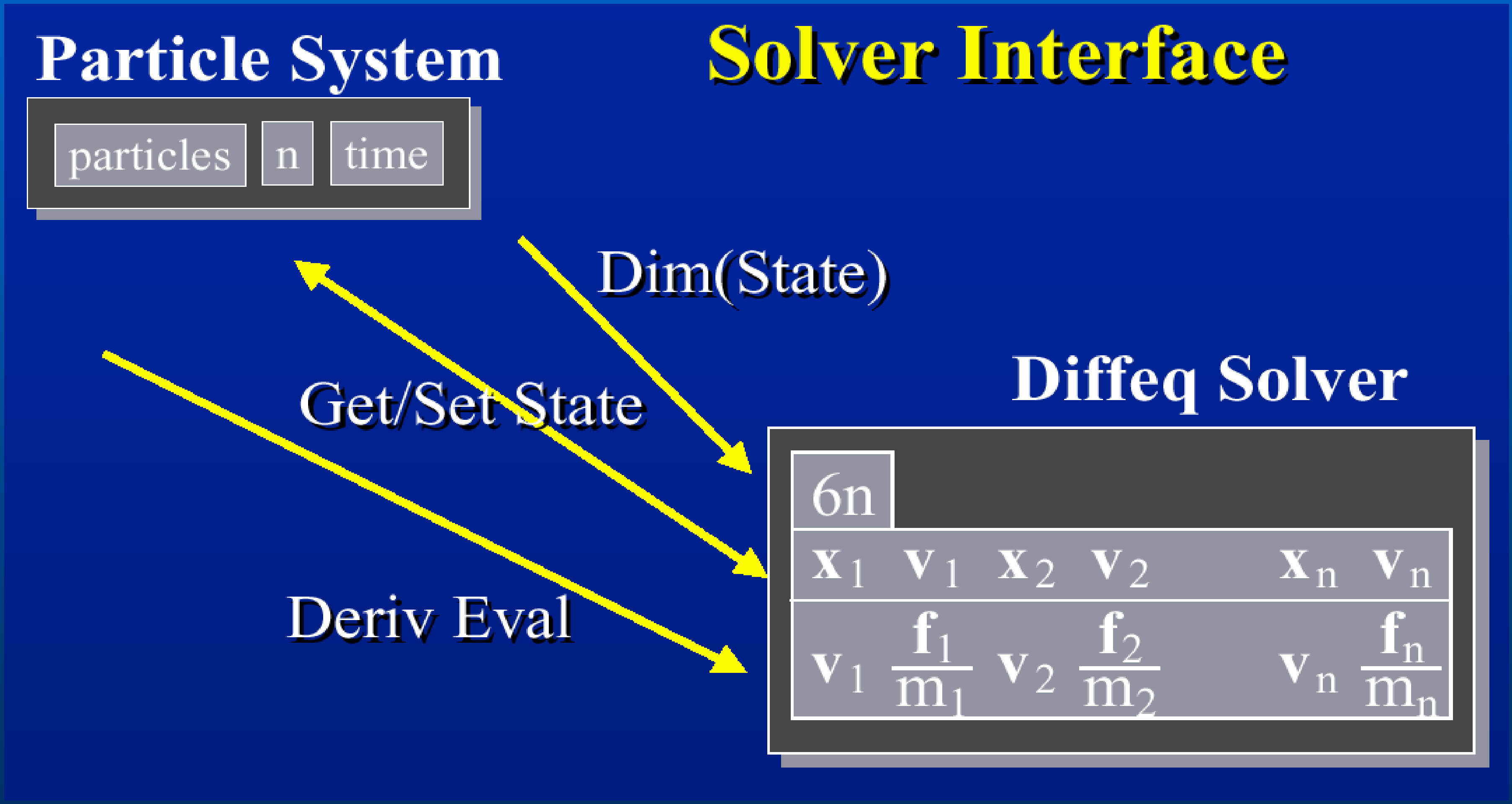
Solver Interface



Particle Systems



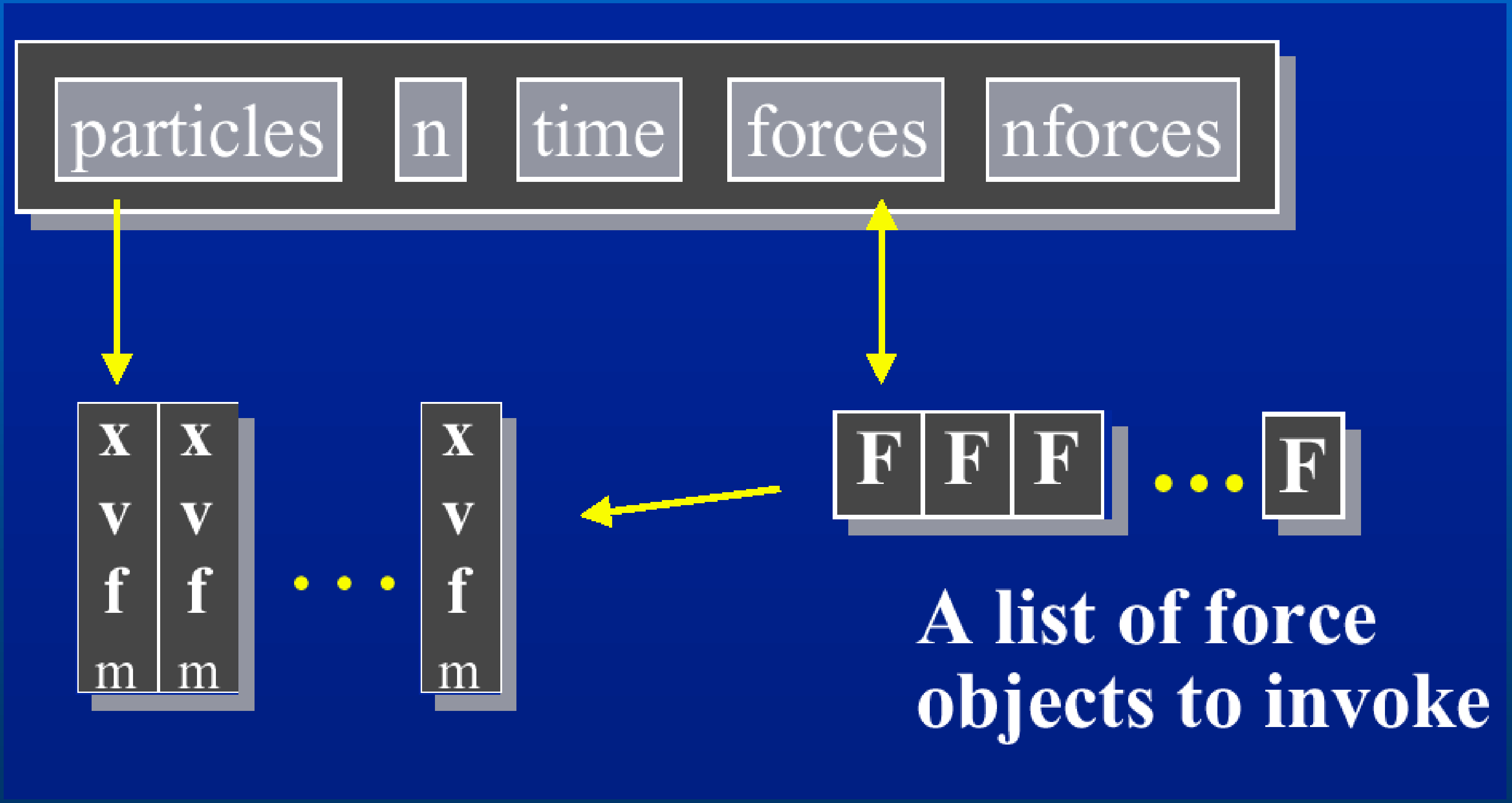
Overall Setup



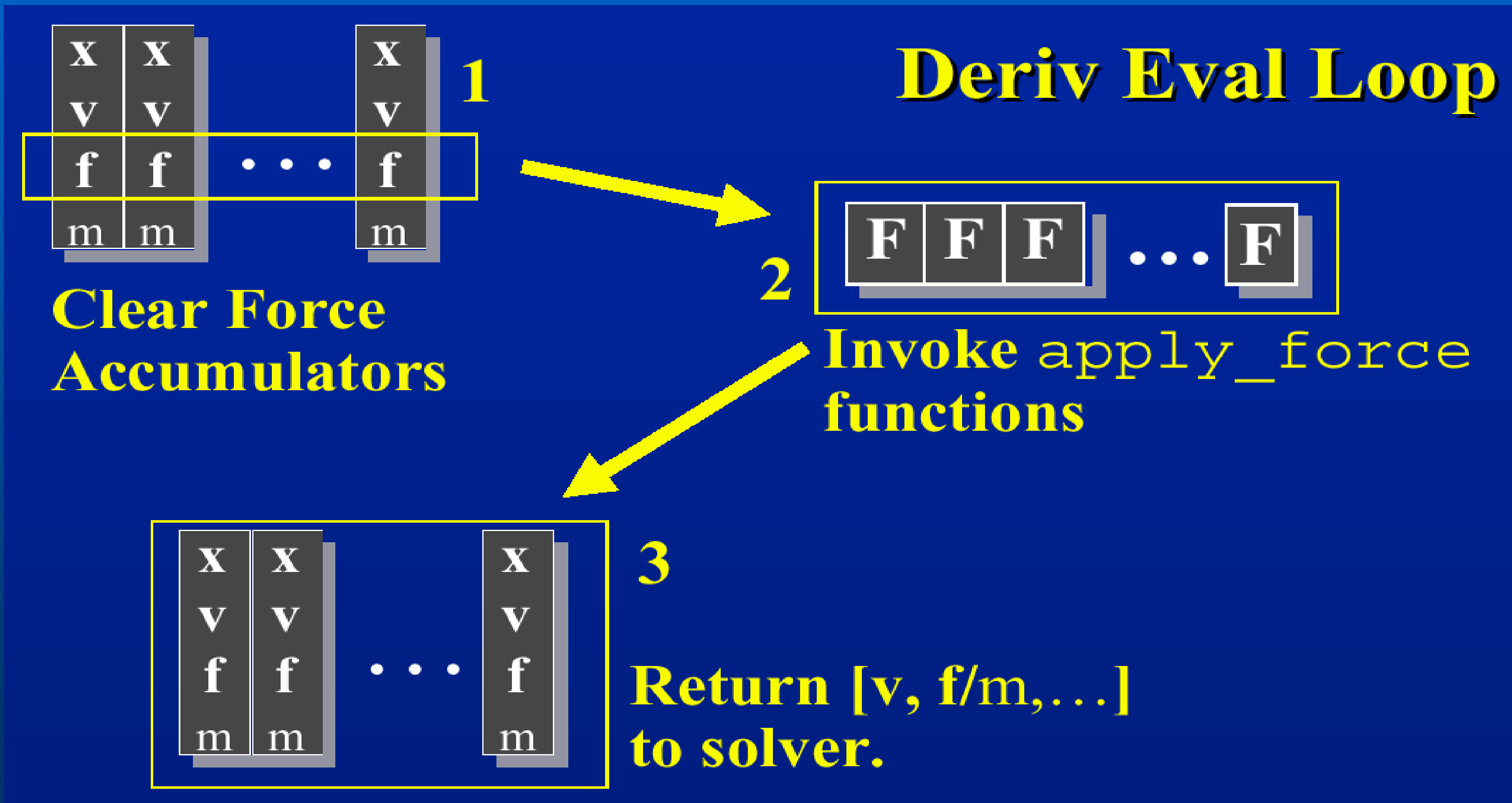
Derivatives Evaluation Loop

- **Clear forces**
  - Loop over particles, zero force accumulators.
- **Calculate forces**
  - Sum all forces into accumulators.
- **Gather**
  - Loop over particles, copying **v** and **f**/m into destination array.

Particle Systems with Forces



Solving Particle System Dynamics



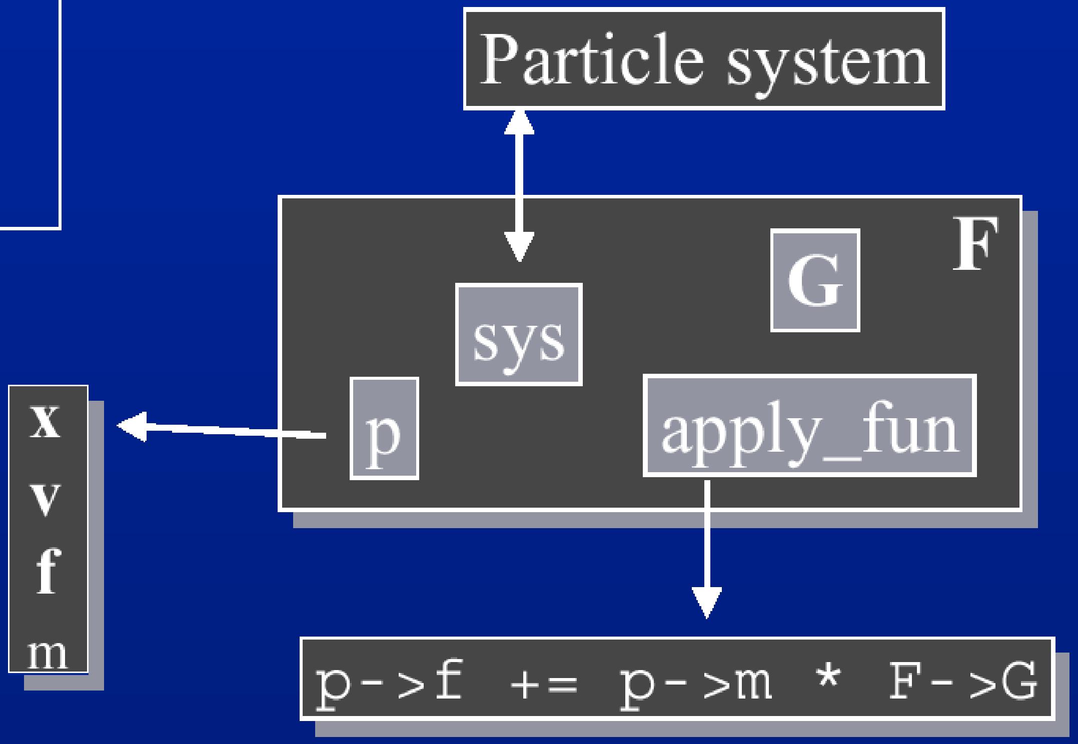
Type of Forces

- Constant — gravity
- Position/time dependent — force fields
- Velocity-Dependent — drag
- n-ary — springs

Gravity

Force Law:

$$\mathbf{f}_{\text{grav}} = m\mathbf{G}$$



Force Fields

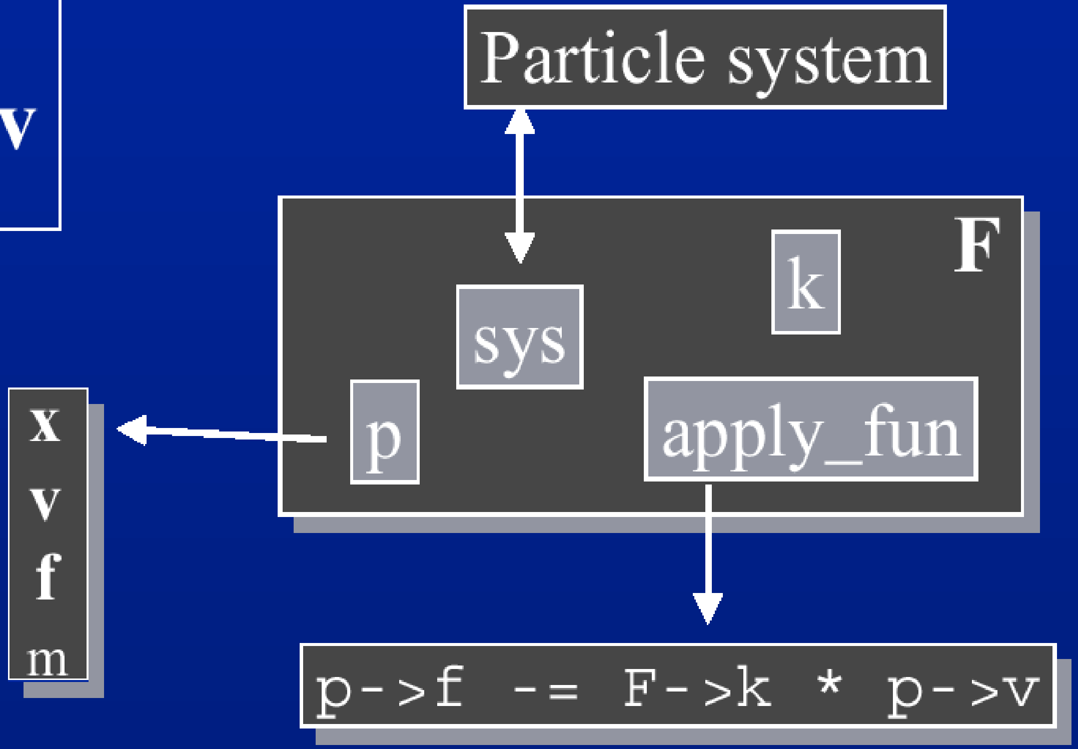
- **Magnetic Fields**
  - the direction of the velocity, the direction of the magnetic field, and the resulting force are all perpendicular to each other. The charge of the particle determines the direction of the resulting force.
- **Vortex (an approximation)**
  - rotate around an *axis of rotation* Θ = *magnitude*/$R^{tightness}$
  - need to specify *center*, *magnitude*, *tightness*
  - R is the distance from center of rotation
- **Tornado**
  - try a translation along the vortex axis that is also dependent on R, e.g. if Y is the axis of rotation, then

$$T(0, -\frac{1}{\sqrt{R^2}}, 0)$$

Viscous Drag

Force Law:

$$\mathbf{f}_{\text{drag}} = -k_{\text{drag}}\mathbf{v}$$



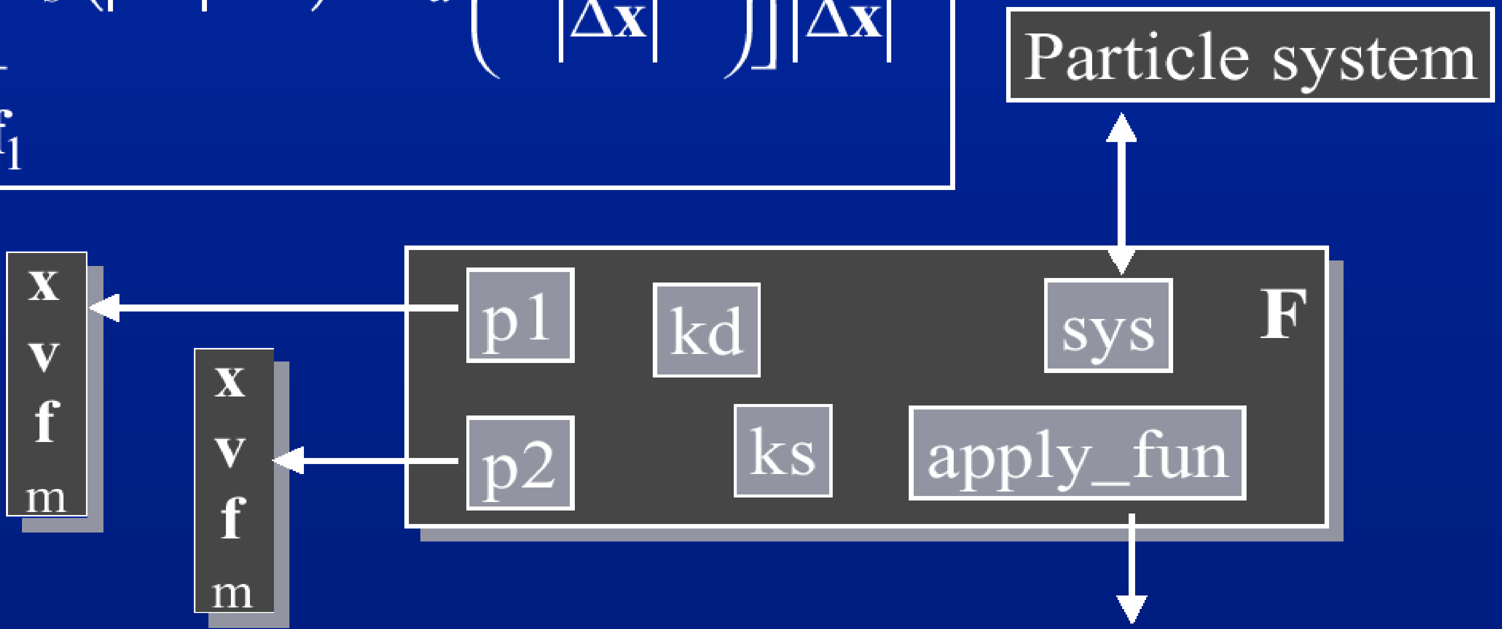
Spring Forces

Damped Spring

Force Law:

$$\mathbf{f}_1 = -\left[k_s \left(|\Delta\mathbf{x}| - r \right) + k_d \left(\frac{\Delta\mathbf{v} \cdot \Delta\mathbf{x}}{|\Delta\mathbf{x}|} \right) \right] \frac{\Delta\mathbf{x}}{|\Delta\mathbf{x}|}$$

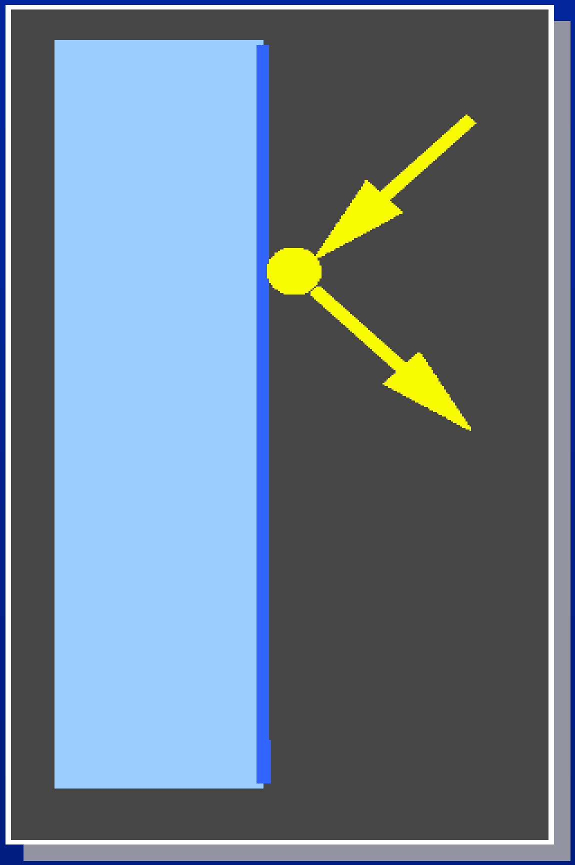
$$\mathbf{f}_2 = -\mathbf{f}_1$$



Collision and Response

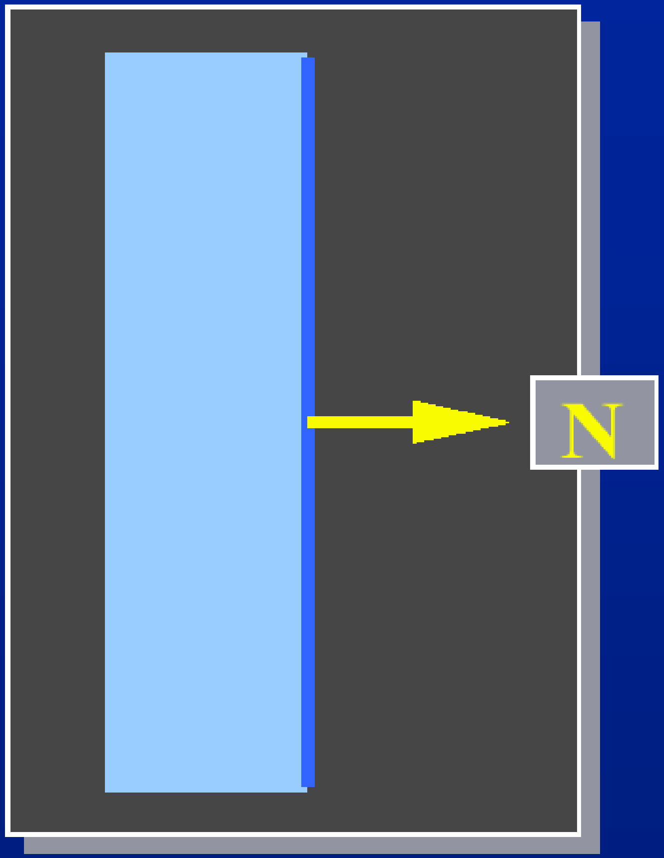
- After applying forces, check for collisions or penetration
- If one has occurred, move particle to surface
- Apply resulting contact force (such as a bounce or dampened spring forces)

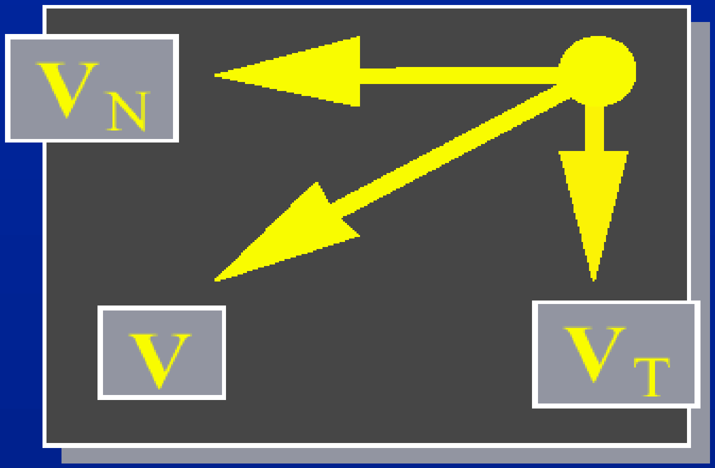
Bouncing off the Wall



- Later: rigid body collision and contact.
- For now, just simple point-plane collisions.
- Add-ons for a particle simulator.

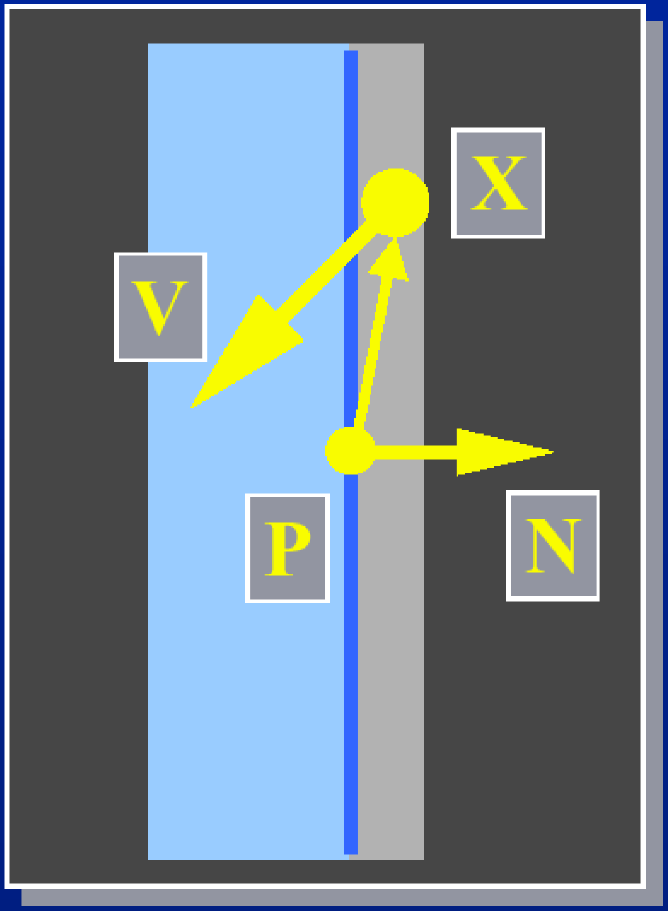
Normal & Tangential Forces





$$V_N = (N \cdot V)N$$
$$V_T = V - V_N$$

Collision Detection

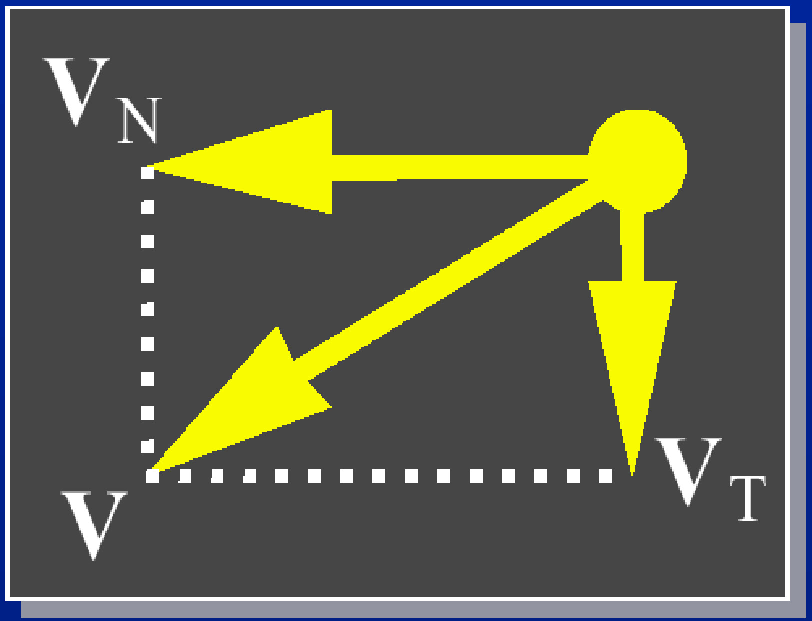


$$(\mathbf{X} - \mathbf{P})\cdot\mathbf{N} < \varepsilon$$

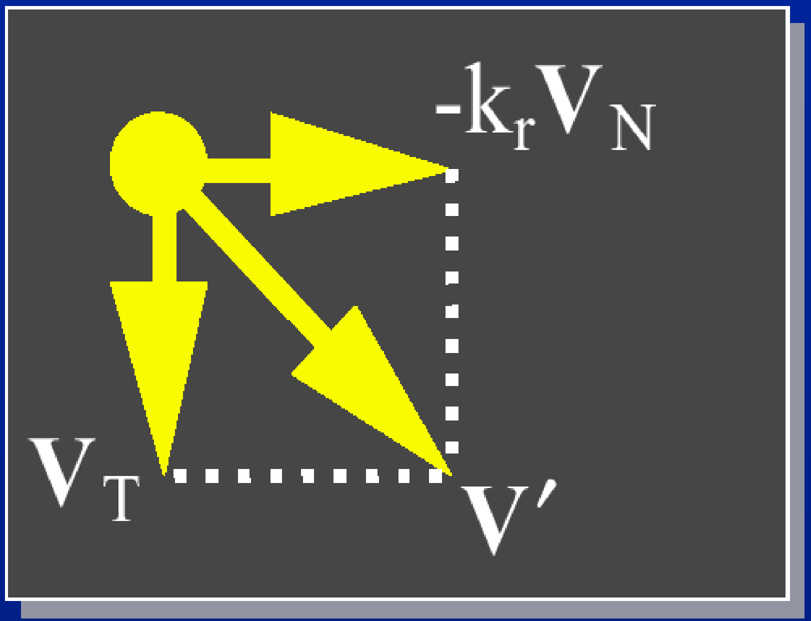
$$\mathbf{N}\cdot\mathbf{V} < 0$$ ***Collision!***

- Within ε of the wall.
- Heading in.

Collision Response



Before

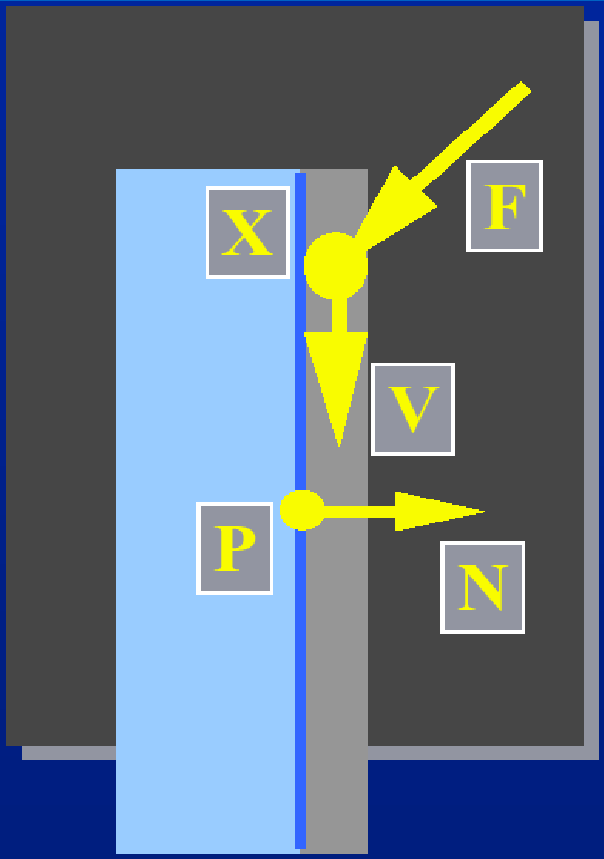


After

$$V' = V_T - k_r V_N$$

(k_r is the coefficient of restitution, $0 \le k_r \le 1$)

Condition for Contact

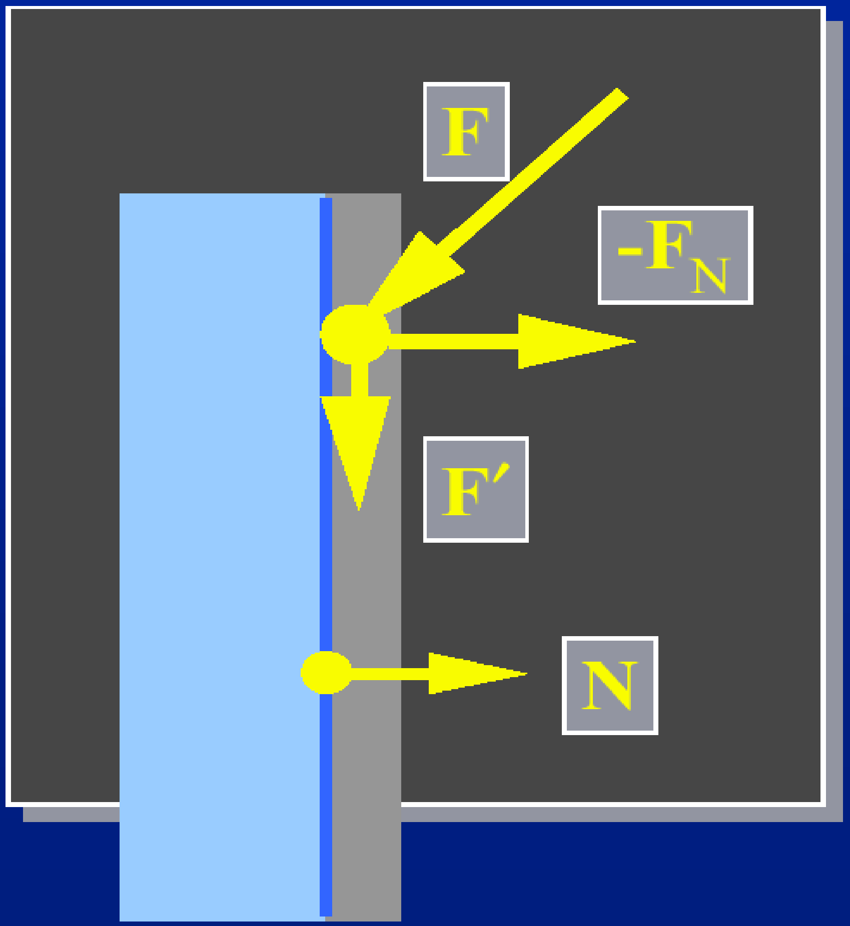


$$|(X - P) \cdot N| < \varepsilon$$

$$|N \cdot V| < \varepsilon$$

- On the wall
- Moving along the wall
- Pushing against the wall

Contact Forces



$$F' = F_T$$

The wall pushes back, cancelling the normal component of F.

$$F_c = -F_N = -(N\cdot F)F$$

(An example of a *constraint force.*)

Friction: $F_f = -k_f\,(-N\cdot F)\,v_t$