Vector Autoregressive

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1 Introduction

Vector Autoregressive (VAR) models are a powerful tool for understanding how multiple variables interact with each other simultaneously. This models capture the relationships among variables without including their current values as predictors. Instead, each variable's past values, or lags, are used to explain its current behavior. This approach allows for a comprehensive analysis of how each variable influences the others over time.

VAR models treat each variable in the system as a function of the lagged values of itself and all other variables in the system. This approach is summarized in the general form of a VAR model:

$$Y_t = A_0 + A_1 Y_{t-1} + \cdots + A_p Y_{t-p} + \varepsilon_t$$

where Y_t is a vector of endogenous variables, A_0 is a vector of constants (intercepts), $A_1, \ldots, A_p$ are matrices of coefficients to be estimated, and ε_t is a vector of error terms

2 Structural Dynamic Models to VAR

A key feature of VAR models is their ability to transform structural dynamic models into a more analyzable form. This transformation involves converting a system of equations with interdependent variables into a VAR model, where each variable is expressed as a linear combination of its own past values and the past values of all other variables in the system. This can be illustrated by the transition from a structural form to the VAR(1) model for two variables y_1 and y_2:

$$By_t = A_0 + A_1 y_{t-1} + \varepsilon_t$$

Through inversion, this leads to the VAR form:

$$y_t = B^{-1} A_0 + B^{-1} A_1 y_{t-1} + B^{-1} \varepsilon_t$$

showing how a structural dynamics models can be captured in a VAR.

3 P-Order VAR Models

When extending VAR models to include more lags, the model is referred to by its order. For instance, a VAR(2) model would include two lags of each variable. This extension is formalized as follows for a VAR(p) model:

$$Y_t = A_0 + A_1 Y_{t-1} + \cdots + A_p Y_{t-p} + \varepsilon_t$$

The inclusion of additional lags allows for a richer dynamic structure, capturing more complex temporal dependencies among the variables

4 Estimation

When we estimate VAR (Vector Autoregressive) models, we face a obstacle: the risk of collinearity among the lagged variables. However, the VAR framework offers a solution by using Ordinary Least Squares (OLS) for efficient estimation, treating each equation separately. The real challenge doesn't lie in estimating the model but rather in interpreting the coefficients, particularly when we're dealing with highly correlated variables. In such cases, our focus often change from discern the specific impact of individual variables to understand the general dynamics.

5 Structural VAR (SVAR)

Identifying the underlying structural model from a VAR framework requires imposing additional restrictions, often based on economic theory. For example, a common method involves employing the Cholesky decomposition of the error covariance matrix to achieve orthogonalization. The Cholesky decomposition ensures that the shocks to the system are uncorrelated, allowing us to interpret impulse response functions more meaningfully. In simpler terms, it allows us to understand how each shock affects all the variables in the system over time in a clearer way.

6 Impulse Response Functions and Forecast Error Variance Decomposition

When we're trying to make sense of VAR models, we have different tools. One is called *impulse response functions*, which basically show us how a sudden shock to one thing affects everything else over time. The other tool is *forecast error variance decomposition.* This one helps us to figure out how much each shock in the system contributes to the ups and downs of our variables. It's more or less like analyze the reasons behind the fluctuations that we observe. By using these tools, we can get a better idea on how different factors interact dynamically and which ones play a bigger role in make the changes we see in each variable.

7 Conclusion

In conclusion, VAR models helps to understand how different things interact over time. They're flexible, which means we can study the interaction of variables in economics and finance without forcing them into rigid cause-and-effect boxes. When we dig into VAR models, we can figure out the underlying structure of the model and what it's telling us about the real world. In addition, tools like impulse response functions and variance decompositions helps us see clearly into the complex web of relationships between variables and how they shape what we observe.

8 Example

In this section we will implement an example of VAR. We will use the same dataset used in the rest of this task: *air traffic.xlsx*. In this analysis, we're focusing on the relationship between two key variables in aviation: the number of passengers and the number of flights. Our goal is to understand how these variables interact over time. Before diving into our modeling approach, it's important to ensure that the passenger count series and the number of flights are stationary. This step is crucial for the reliability of our analysis. By examining the dynamic relationship between passenger numbers and flight frequencies, we aim to gain insights that can inform decision-making in the aviation industry.

You can find the implemented example in the Notebook file.