We are given:

$$P(B) = 0.01, \quad P(\text{Positive} \mid B) = 0.95, \quad P(\text{Negative} \mid \neg B) = 0.90$$

Hence,

$$P(\text{Positive} \mid \neg B) = 1 - 0.90 = 0.10$$

(a) Probability that a person actually has the disease given a positive test

We need to find:

$$P(B \mid \text{Positive})$$

By Bayes' theorem:

$$P(B \mid \text{Positive}) = \frac{P(\text{Positive} \mid B) \cdot P(B)}{P(\text{Positive})}$$

We can compute $P(\text{Positive})$ using the law of total probability:

$$P(\text{Positive}) = P(\text{Positive} \mid B)P(B) + P(\text{Positive} \mid \neg B)P(\neg B)$$

$$P(\text{Positive}) = 0.95 \cdot 0.01 + 0.10 \cdot 0.99 = 0.0095 + 0.099 = 0.1085$$

Now,

$$P(B \mid \text{Positive}) = \frac{0.95 \cdot 0.01}{0.1085} = \frac{0.0095}{0.1085} \approx 0.0876$$

$$\boxed{P(B \mid \text{Positive}) \approx 8.76\%}$$

Interpretation: Even though the test is quite accurate (95% sensitivity and 90% specificity), the disease is rare (1% prevalence), so most positive results are false positives. Hence, the probability that a person who tests positive actually has the disease is only about 8.8%.

(b) Minimum specificity for 50% posterior probability

We want:

$$P(B \mid \text{Positive}) = 0.5$$

Let specificity be $s = P(\text{Negative} \mid \neg B)$, then:

$$P(\text{Positive} \mid \neg B) = 1 - s$$

Using Bayes' theorem:

$$0.5 = \frac{0.95 \cdot 0.01}{0.95 \cdot 0.01 + (1 - s) \cdot 0.99}$$

Multiply both sides by the denominator:

$$0.5\big(0.95 \cdot 0.01 + (1 - s) \cdot 0.99\big) = 0.95 \cdot 0.01$$

$$0.00475 + 0.495(1 - s) \cdot 0.99 = 0.0095$$

Simplify:

$$0.00475 + 0.495(0.99 - 0.99s) = 0.0095$$

$$0.00475 + 0.495(0.99 - 0.99s) = 0.0095$$

$$0.00475 + 0.495 \times 0.99 - 0.495 \times 0.99s = 0.0095$$

Simplify numerically:

$$0.00475 + 0.49005 - 0.49005s = 0.0095$$

$$0.4948 - 0.49005s = 0.0095$$

$$0.49005s = 0.4948 - 0.0095 = 0.4853$$

$$s = \frac{0.4853}{0.49005} \approx 0.9903$$

$$\boxed{\text{Minimum specificity } s \approx 99.03\%}$$

Interpretation: To achieve a 50% chance that a positive result truly indicates disease, the test's specificity must be at least 99%. This highlights how crucial high specificity is when screening for rare diseases.