Piano Tuning Method

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CONTENTS

ABSTRACT

Since the piano string is consider to be a stick rather than a pure ideal string, it contains stiffness and its harmonics will shift in such way that make piano tuning a difficult work. In this work, the method of the optimization algorithm similar to Tunelab®, however, construct and developed all by the author. The algorithm is divided into several models that using various fitting technique to construct model functions, and finally convert to linear regression problem for optimization. Finally, the piano tuning curve is constructed and final tuning frequencies are calculated. In addition, more functions is introduced, such as the different temperament tuning.

Keyword: piano tuning, Tunelab®, inharmonicity, optimization

Project Location

Reference [2]

1 INTRODUCTION

Piano tuning is a difficult work since the harmonics shift that make the piano hard to tune, and tuning process will be a task to highly reduce the audible cacophonous. There are several factors we need to consider, which the rule of harmony is.

- The cacophonous created by its base frequency and audible harmonics; a good tuning will largely reduce the inharmonic for harmonies (the frequency domain will greatly coincide).
- The inner music scales related pitch; the odd pitch tuning will result in the weird sound when playing music scales.

Other famous related works are:

- Tunelab (closed source; has trial version)
- Reyburn CyberTuner (closed source; no trial version)
- Entropy Piano Tuner (open source) [1]

The first two are similar, which represent the old tuning techniques, and my work mostly focus on this algorithm. Since it is closed source, I guessed their tuning method and create a similar solution, and will be shown in this article.

As for Entropy Piano Tuner, it represent the new way of piano tuning, however I heard its demo of tuning, I found that it contain two major deficiencies:

- It violate the second rule of harmony – inner scales sound weird.
- It only consider the sound which at the certain striking level of piano keys, which result in the optimization of keys are based only on sampling striking level.
- The accuracy cannot be too high due to large amount of calculation, it does not achieved an ideal result.

In my work, I will guess the algorithm and model it in the similar way of optimization. Besides, I used more accurate model for inharmonicity coefficients.

In this article, the first part is to introduce the background of knowledge for higher level modeling algorithms. The second part is to introduce my piano modeling and tuning optimization method. Finally, the future work will be introduce and followed a conclusion.

2 TECHNICAL KNOWLEDGE

2.1 Key Names

The left most key name is defined as "A0", where "A" is the note name, 0 is the scale number. "C" is the starting point of one scale. It only allowed sharp in the note, flat is not allowed in this naming format.

A0, A#0, B0, C1, C#1, …, B1, C2, …, B7, C8

There are 88 keys for standard piano.

2.2 Key Numbers

In the real world, the piano key will labeled with numbers when the piano is open and machine part is shown off.

A0 key is labeled to be 1, and "C8" is 88.

However, in my program, "A0" key is labeled as 0 for easier calculation, which is defined as k .

2.3 Functions

Frequency ratio to cents function:

$$\mathrm{Fr}_{\to c}(\gamma) = 1200\log_2(\gamma) \tag{2.1}$$

Where cents is from 12 equal temperament, each half note has 100 point, named cents.

Frequency add cents (pitch) function:

$$\mathrm{F}_{+c}(f, c) = f \cdot 2^{\left(\frac{c}{1200}\right)} \tag{3.1}$$

This function returns the frequency that added the pitch (cents) c.

The ideal frequency for the key k is:

$$\tilde{f}_k = 440 \cdot 2^{\left(\frac{k-48}{12}\right)} \tag{3.2}$$

Where 440Hz is the international standard pitch for “A4”. Other tuning standard will replace this number, 48 is the key number for “A4”.

3 METHOD

3.1 Sampling Piano

Before tuning a piano, we need to sample a piano by recording the piano keys sound audios. This process will roughly or precisely measure the inharmonicity of piano strings (which will talk about later), such that we could model the inharmonicity for the target piano.

The sampling is suggested to measure keys “C1”, “C2”, “C3”, “C4”, “C5” (and probably “C6”; user could record more piano keys such as “A1” ~ “A6” for better result). The piano key sound should be recorded in a quiet environment, which allows more accuracy for later frequency analysis.

In my program, I use fully or almost fully sampled piano for research purposes.

3.2 Audio Processing

Since the real audio may contains the white space at the start or the end, and the sound length varies. I use this method to process my sampled audio:

- Normalize ($\mathrm{N}(x) = x / \max(x)$) the audio file into 1, then, find the peak volume of audio, and start from here.
- Slice these audio pieces into tiny partitions, say 0.1 second is one partition. The maximum number of each partition will be its assumed volume at this time point.
- Select these pieces volume start from some large number to small number – since piano sound is loud from its beginning and decay by the time. Say from 90% to 2% of the sampled sound audio.

3.3 Frequency Analysis

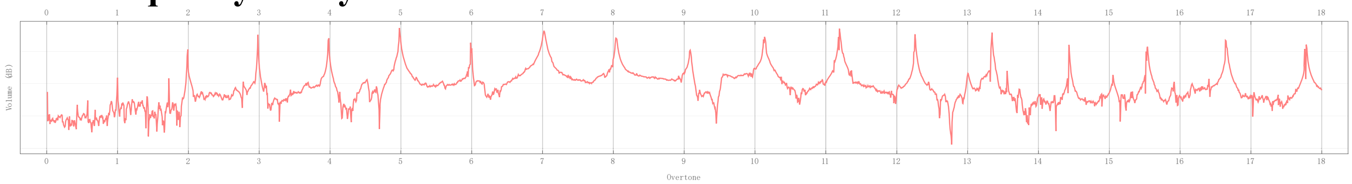

Figure 3-1 "A#0" Key (at Upright Piano Samples) Overtone Plot; Volume at Logarithm Scale

Then, put this audio samples into fourier analysis (FFT algorithm). Then we get the function $\mathrm{G}(\omega)=\mathrm{FFT}(\mathrm{S}(t))$ where $\mathrm{S}(t)$ is the audio function, and $\mathrm{G}(\omega)$ is the frequency domain function. In our work, the frequency domain is converted to the ratio to its ideal fundamental frequency, thus we can see the Figure 3-1, the peaks will always almost lies in the grid by dividing its ideal frequency.

From Figure 3-1, we can see that the higher overtone (right hand side peaks with larger numbers) shifts higher.

It is a problem to capture all these peaks numbers, since some are not clear: the fundamental frequency (at 1), and some has multiple peaks: at 15 ~ 16.

In my work, I use the frequency *Catchup Method* to get octave values for all these peaks.

3.4 Catchup Overtone

From the charactors of these peaks, there are several charactors will be considered:

- From left to right, the gap between two peaks are increasing gradually.
- The largest value of this plot is probably some peak of overtone
- The valid peak should be nearly larger than fundamental frequency position: at 1.
- The peak may be broken into several peaks, we need centralize the targeted position.

From this charactoristics, the *Catchup Method* could be built:

- Analyze the frequency samples which roughly larger than 1 (my program is starting from 0.8), get the peak frequency $f_{k,peak}$ at key number k, and overtone number $peak$.
- Comparing with ideal frequency $\tilde{f}_k$. We can then assume that it is $n=\mathrm{round}\left(f_{k,peak}/\tilde{f}_k\right)$ harmonics. Then, we can know its guessed fundamental frequency is $\hat{f}_k=f_{k,peak}/n$. Then, this should be the step size for catchup method.
- The catchup method is forward (goes to the right), and the backward (goes to the left). If we are in the forward operation, the next guessed target frequency is $\hat{f}_{k,peak+1}=f_{k,peak}+f_k'$, where f_k' is the assumed gap between two peak at this position. In the first try, we set this number to $f_k'=\hat{f}_k$, and this number will be increasing for more right harmonics. Then, we get the around data (in a relatively small area) for guessed target frequency $\hat{f}_{k,peak+1}\pm\delta$. We can find its maximum number these data to be the frequency candidate $\hat{f}_{k,peak+1}^{candidate}$, then we get the data of smaller surround area $\hat{f}_{k,peak+1}^{candidate}\pm\delta'$ where $\delta'<<\delta$. Then, we calculate the weighted average for this smaller area, and the result is the actual frequency of this peak

$f_{k,peak+1} = \int_{\hat{f}-\delta'}^{\hat{f}+\delta'} \omega \cdot \mathrm{G}(\omega) \mathrm{d}\,\omega$, where ω is proportional to frequency. Then, the assumed gap between two peak at this step is updated to be $f_k' = f_{k,peak+1} - f_{k,peak}$.

- Iterate this method for forward catchup to get all higher frequencies.
- If the highest peak is not fundamental frequency, we will perform the backward catchup. Since there are less peaks and the overtone shift will be far less than the right, the assumed targeted gap between two peaks is set to be the assumed fundamental frequency $\hat{f}_k$.

From this method, we can get a overtone (frequency) list for the key k. Which is:

$$k \to \{f_{k,1}, f_{k,2}, \ldots\} \tag{3.3}$$

3.5 Inharmonicity Model

From reference [1], we assume that the piano string is a bar, which follows the partial differential eqution:

$$\ddot{y} \propto -y'' - \varepsilon y'''' \tag{3.4}$$

Where y is the special position of piano string (bar model). The prime is the derivative to spatial domain, and dots is the derivative to time domain.

Then, use the modal analysis and solved the natural frequencies for this string are:

$$f_{k,n} \propto n \cdot f_{k,1}\sqrt{1+B_k \cdot n^2} \Rightarrow f_{k,n} = A_k \cdot n \cdot f_{k,1}\sqrt{1+B_k \cdot n^2} \tag{3.5}$$

Here we have two unknown variables.

Then, we use this function to fit all frequency results at Eq.(3.3). Since A_k value is always almost 1 all the time, we can ignore this number, and focus only on B_k. However in the optimization process, with parameter A_k could achieve much better result, although finally its value is almost 1. We set 0 to be the fundamental frequency is that when $n = 0$ that the equation holds, we will restore this number later.

Then, we can get inharmonicity parameter list $\{\{k, B_k\}\}$.

From my observation, the logarithm of this number has some beautiful properties with the data $\{\{k, \ln(s \cdot B_k)\}\}$, where s is a scaling parameter (I set to 10000).

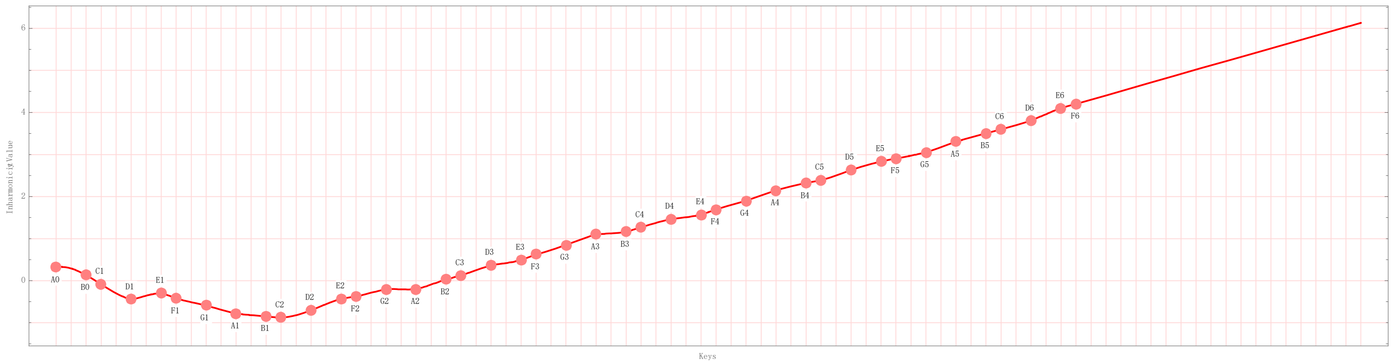

Figure 3-2 Inharmonicity Plot of Grand Piano

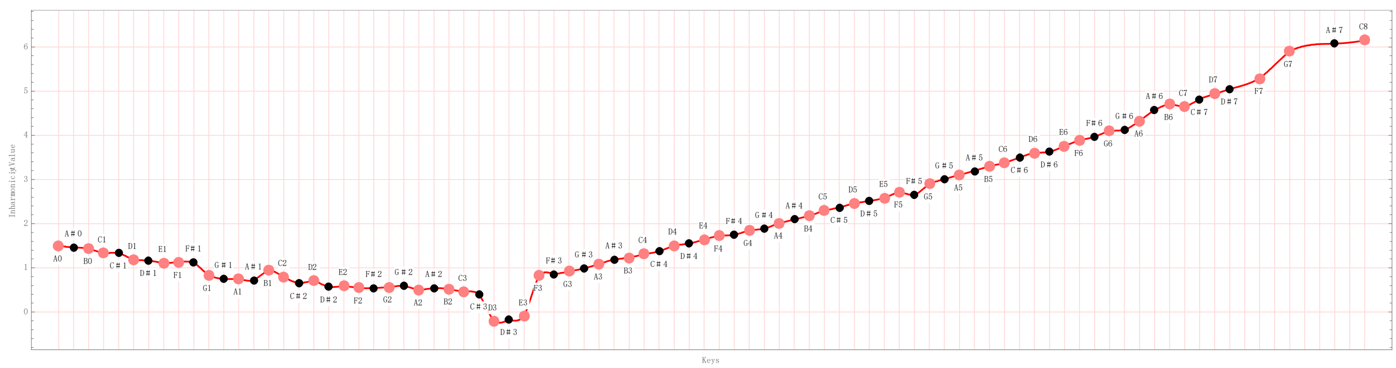

Figure 3-3 Inharmonicity Plot of Upright Piano

From Figure 3-2 and Figure 3-3, we can clearly see the line is divided into 2 parts.

Figure 3-4 Grand Piano String Arrangement

Figure 3-5 Upright Piano String Arrangement

From Figure 3-4 and Figure 3-5, we can clearly see that the string is divided into two parts, the steel string and copper string (may be covered by silver for highly expensive pianos). The upright piano has more copper strings since the steel string cannot goes longer, and the string will become thicker to make the string vibrate slower. From spring vibration formula:

$$\omega = \sqrt{\frac{K}{m}} \tag{3.6}$$

Where ω is proportional to frequency, m is the mass of spring, K is the stiffness of spring.

When m increases, K increase a little bit, ω decreases, then frequency decrease.

Since the piano cannot growing longer, it become thick and more like a stick rather than an ideal string. For higher notes strings, it is too short, and the thickness become relatively larger comparing to its length, thus it is more likely to be a bar.

Thus, from the plot, we can see the inharmonicity increases at two ends, and break at the position of separation of two kinds of strings.

Since grand concert piano is longer, and can have more steel strings, less copper strings, thus the break will become more left side.

The figure of inharmonicity plot also tell us that two separate line are almost linear. In my model, I used the valid sampled points are modeled with interpolation function, and two edges are modeled with linear function, and it is method is shown below.

- We get several samples from one line, and fit in a linear form.
- Get its slope, and build a line which pass the right end point (since I will not wish to have a break for the interpolation function), and add some samples for edges situation to sample pool.
- Similar to the left hand side.
- We use interpolation for these samples of sample pool – "left hand side + samples + right hand side", which is our final model for inharmonicity model function $\mathrm{IH}(k)$.

$$\mathrm{IH}(k) = \ln(s \cdot B_k) \tag{3.7}$$

Thus, we can have the modeled parameter B_k with:

$$B_k = \frac{\mathrm{e}^{\mathrm{IH}(k)}}{s} \tag{3.8}$$

Then, the frequencies $\tau(k,n)$ will be:

$$\tau(k,n) = f_{k,1} \cdot n \cdot \sqrt{1 + B_k \cdot n^2} \tag{3.9}$$

Where $f_{k,1}$ is currently unknown but it will be eliminated, since it is in frequency ratio form.

3.6 Tuning Curve Optimization Model

Similar to Tunelab, I set the tuning optimization method to separate the lower tones (bass) and higher tones (tenor) into tow tuning target optimization method, the separation point k_0 is "C#4/D4". And the default tuning method for bass is to set 6:3. Since 6/3=2 (a/b), this frequency ratio is $\gamma = a/b$, and its corresponding pitch range is $\mathrm{Fr}_{\to\mathrm{c}}(\gamma)$ which is 1200, and 1200 is an octave, it means the tone say "A0"s 6th harmonics will largely match its octave's "A1"s 3rd harmonics.

Here pitch is defined by cents.

The error function ε_k is defined as:

$$\begin{aligned}
\varepsilon_k &= \mathrm{Fr}_{\to\mathrm{c}}\left(\frac{\tau(k,a)}{\tau\left(k + Fr_{\to c}(a/b), b\right)}\right) \\
&= \mathrm{Fr}_{\to\mathrm{c}}\left(\sqrt{\frac{1 + B_k \cdot a^2}{1 + B_{k+Fr_{\to c}(a/b)} \cdot b^2}} \cdot \frac{a}{b} \cdot \left(\frac{f_{k,1}}{f_{k+Fr_{\to c}(a/b),1}}\right)\right) \\
&= \mathrm{Fr}_{\to\mathrm{c}}\left(\sqrt{\frac{1 + B_k \cdot a^2}{1 + B_{k+Fr_{\to c}(a/b)} \cdot b^2}}\right)
\end{aligned} \tag{3.10}$$

We can do this for all bass strings.

For tenor strings, the default tuning method is set to 4:1 (c/d). But this time we count the higher note as the target to calculate.

$$\varepsilon_k = \mathrm{Fr}_{\to\mathrm{c}}\left(\sqrt{\frac{1 + B_{k-Fr_{\to c}(c/d)} \cdot c^2}{1 + B_k \cdot d^2}}\right) \tag{3.11}$$

The combined expression is:

$$\mathrm{E}(k)=\begin{cases}\mathrm{Fr}_{\to c}\left(\sqrt{\dfrac{1+B_k\cdot a^2}{1+B_{k+Fr_{\to c}(a/b)}\cdot b^2}}\right) & k\le k_0\\ \mathrm{Fr}_{\to c}\left(\sqrt{\dfrac{1+B_{k-Fr_{\to c}(c/d)}\cdot c^2}{1+B_k\cdot d^2}}\right) & k>k_0\end{cases} \tag{3.12}$$

From this equation, we can see $\mathrm{E}(k)$ is only a value for calculation at given k.

From this point, we need a function to largely eliminate these errors. The piano tuning curve $\mathrm{C}(k)$ is introduced.

The deviation function $\mathrm{D}(k)$ is:

$$\mathrm{D}(k)=\mathrm{C}(k)-\mathrm{E}(k) \tag{3.13}$$

The cost function $\mathrm{J}(k)$ for optimization is:

$$\mathrm{J}(k)=\sum_k\left(\mathrm{D}(k)\right)^2 \tag{3.14}$$

Which minimize the square error of these functions.

Here I use polynomial for easier calculation:

$$\mathrm{C}(x)=\sum_{i=1}^{n}\chi_i\cdot x^i \tag{3.15}$$

Since $\mathrm{C}(x)$ will pass the fix point, which is “A4” pitch at 440Hz frequency at pitch deviation of 0, thus i is from 1 and $x=k-k_{[A4]}$, where $k_{[A4]}$ is the key number (index) at “A4”, which is 48.

Thus, $\mathrm{J}(k)$ is the second order multi-variable polynomial function, which is very easy to minimize by linear regression method to calculate the fitting parameter $\{\chi_i\}$, and rebuild the functions.

Then, we can bring it to the $\mathrm{D}(k)$ function to calculate its deviations.

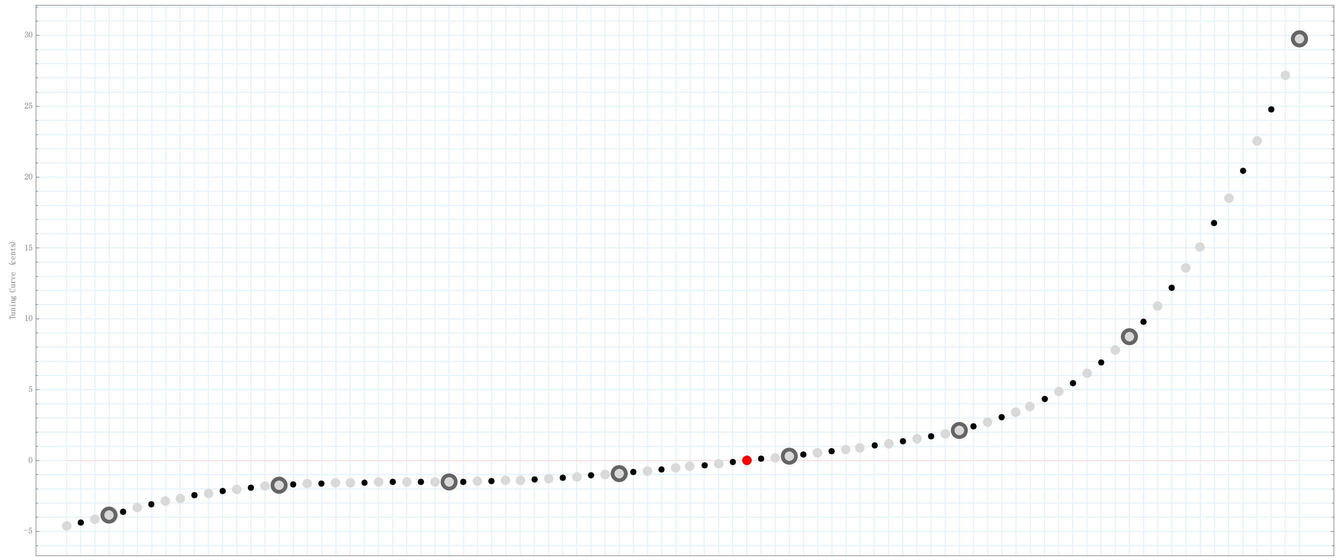

Figure 3-6 C(*k*) for Grand Piano

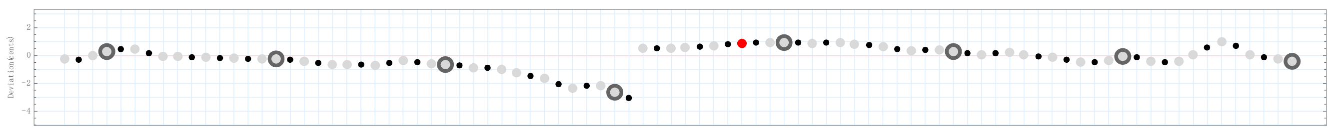

Figure 3-7 D(*k*) for Grand Piano

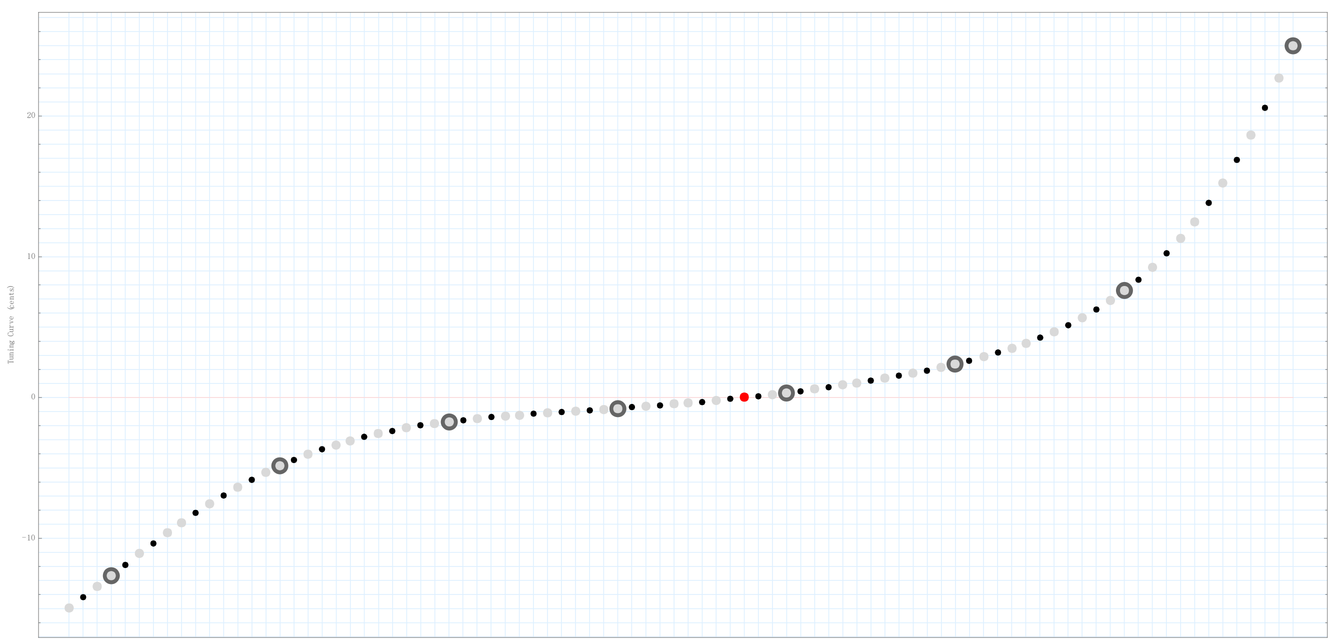

Figure 3-8 C(*k*) for Upright Piano

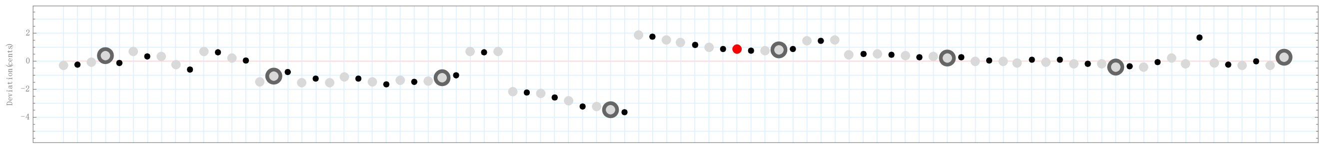

Figure 3-9 D(k) for Upright Piano

The result of two piano is shown above. Horizontal axis is the key number, and the vertical axis the pitch deviation with idea frequencies represented by cents.

From this tuning method, we can see that the bass tuning will consider the deviations from the tenor part, and vice versa. The effect are inner related. Thus this tuning method is theoretically to optimize almost the whole piano keys tuning.

3.7 Temperament Model

With the development of music, various temperament appears and create unique flavor of music. The temperament model is using the pitch deviation tables of different temperament (the unit is cent). We can then create the non 12 equal temperament tuning strategy. The temperament function is defined to be $\mathrm{T}(k)$.

The tuning table such as "Bach - Bradley Lehman" is:

C	C#	D	D#	E	F	F#	G	G#	A	A#	B
5.87	3.91	1.96	3.91	-1.96	7.82	1.96	3.91	3.81	0	3.91	0

Table 3-1 Table for "Bach - Bradley Lehman" Temperament

Where A note will always be 0 since A is the reference frequency and will always keep to 440 Hz (if is standard situation).

This table shows the situation of "C" major.

The other major tuning will follow the rotation of table. For example: if tuning "D" major, the "D" will rotate to current "D" →"C" place, which is rotating left 2 times. However, we will make sure "A" note will always be 0, then, we can subtract the number at "B" → "A" to make it possible.

Then, add these pitch errors to all the notes of tuning, the modified tuning curve is:

$$\mathrm{C}'(k) = \mathrm{C}(k) + \mathrm{T}(k) \tag{3.16}$$

3.8 Creating Tuning Table

The final tuning frequency $\tau(k,n)$ is:

$$f_{k,1} = F_{+c}\left(\hat{f}_k, \mathrm{C}'(k)\right) \tag{3.17}$$

$$\begin{aligned}\tau(k,n) &= f_{k,1} \cdot n \cdot \sqrt{1 + B_k \cdot n^2}\, f \\ &= \mathrm{F}_{+c}\left(\hat{f}_k, \mathrm{C}'(k)\right) \cdot n \cdot \sqrt{1 + B_k \cdot n^2} \\ &= \mathrm{F}_{+c}\left(\hat{f}_k, \mathrm{C}'(k)\right) \cdot n \cdot \sqrt{1 + \frac{\mathrm{e}^{\mathrm{IH}(k)}}{s} \cdot n^2}\end{aligned} \tag{3.18}$$

From Eq.(3.18), we can see only C and IH function is modeled function, other function are basic mathematics functions.

From the modeling, we can get a strategy of piano tuning, then we can convert this strategy into a tuning table, which shows all the frequency of fundamental and its harmonics frequencies, and corresponding deviation to ideal frequencies represented by cents.

The grand and upright piano tuning strategy is shown in Figure 7-1 and Figure 7-2.

The red font is the frequencies recommended for the devices to tune.

4 FUTURE WORK

Although the Entropy piano tuning method is far more advanced than this tuning method theoretically, however the jumpy tuning curve will make the music scales sound weird. Also, it needs huge amount of calculation and the accuracy is low, I prefer to modify the given algorithm to consider every major peaks of each note contribute to the tuning.

Over-pull tuning is implemented experimentally with their tuning apps, and I do not know its method due to its close source reason. And I am still lack of research on this area, thus I will leave it as future work to think about. I know this effect is caused by the experimental result of the percentage that the string pins will loosen and drop the pitch, the tuner will make up the errors of this effect by over pull and tune higher tones.

5 CONCLUSION

This tuning method gives us a solution of piano tuning that works as well as commercial apps Tunelab. The method is presented to optimize the whole piano notes sound.

Future work is given to develop maybe in the future.

6 REFERENCE

[1] Hinrichsen, Haye. "Entropy-based tuning of musical instruments." Revista brasileira de Ensino de Física 34.2 (2012): 1-8.

[2] Github for Piano Tuning Project [https://github.com/RobertBoganKang/piano_tuning]

7 APPENDIX

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A0	27.4266 -4.62¢	54.857 -4.5¢	82.3025 -4.15¢	109.774 -3.55¢	137.284 -2.72¢	164.843 -1.65¢	192.461 -0.34¢	220.152 +1.2¢	247.925 +2.97¢	275.791 +4.98¢	303.762 +7.21¢	331.849 +9.68¢	360.063 +12.37¢	388.414 +15.29¢	416.913 +18.43¢	445.571 +21.79¢
A#0	29.0617 -4.37¢	58.1273 -4.25¢	87.2084 -3.91¢	116.317 -3.33¢	145.464 -2.52¢	174.661 -1.48¢	203.921 -0.21¢	233.254 +1.28¢	262.672 +3.01¢	292.187 +4.95¢	321.809 +7.13¢	351.55 +9.52¢	381.422 +12.14¢	411.435 +14.97¢	441.601 +18.02¢	471.93 +21.29¢
B0	30.7944 -4.11¢	61.5924 -4.01¢	92.4044 -3.71¢	123.241 -3.22¢	154.113 -2.53¢	185.03 -1.64¢	216.004 -0.56¢	247.044 +0.72¢	278.161 +2.19¢	309.365 +3.86¢	340.667 +5.72¢	372.077 +7.77¢	403.606 +10.01¢	435.262 +12.44¢	467.057 +15.05¢	499. +17.85¢
C1	32.6306 -3.84¢	65.2641 -3.76¢	97.9096 -3.53¢	130.576 -3.13¢	163.272 -2.58¢	196.008 -1.86¢	228.791 -0.99¢	261.63 +0.03¢	294.535 +1.22¢	327.515 +2.56¢	360.577 +4.05¢	393.732 +5.7¢	426.988 +7.51¢	460.352 +9.46¢	493.835 +11.57¢	527.445 +13.83¢
C#1	34.5761 -3.58¢	69.1548 -3.52¢	103.744 -3.33¢	138.351 -3.01¢	172.983 -2.56¢	207.649 -1.98¢	242.355 -1.28¢	277.111 -0.45¢	311.922 +0.51¢	346.797 +1.6¢	381.744 +2.81¢	416.769 +4.14¢	451.881 +5.6¢	487.087 +7.19¢	522.394 +8.9¢	557.81 +10.73¢
D1	36.6375 -3.33¢	73.2773 -3.27¢	109.926 -3.11¢	146.592 -2.83¢	183.281 -2.44¢	220.001 -1.94¢	256.758 -1.34¢	293.559 -0.62¢	330.412 +0.21¢	367.324 +1.15¢	404.301 +2.2¢	441.35 +3.35¢	478.478 +4.62¢	515.693 +5.99¢	553.001 +7.47¢	590.408 +9.06¢
D#1	38.8214 -3.09¢	77.6456 -3.03¢	116.481 -2.85¢	155.335 -2.54¢	194.216 -2.12¢	233.132 -1.58¢	272.092 -0.91¢	311.103 -0.13¢	350.174 +0.78¢	389.312 +1.8¢	428.526 +2.94¢	467.823 +4.2¢	507.213 +5.58¢	546.701 +7.08¢	586.297 +8.69¢	626.009 +10.42¢
E1	41.1352 -2.86¢	82.2735 -2.8¢	123.424 -2.61¢	164.596 -2.29¢	205.798 -1.84¢	247.04 -1.26¢	288.331 -0.55¢	329.68 +0.28¢	371.095 +1.24¢	412.587 +2.33¢	454.164 +3.54¢	495.835 +4.88¢	537.609 +6.34¢	579.494 +7.93¢	621.501 +9.64¢	663.638 +11.48¢
F1	43.5865 -2.66¢	87.1758 -2.6¢	130.777 -2.43¢	174.398 -2.14¢	218.047 -1.75¢	261.734 -1.23¢	305.466 -0.61¢	349.253 +0.13¢	393.103 +0.98¢	437.024 +1.94¢	481.025 +3.02¢	525.114 +4.2¢	569.3 +5.5¢	613.592 +6.91¢	657.996 +8.43¢	702.523 +10.06¢
F#1	46.1833 -2.47¢	92.3694 -2.42¢	138.567 -2.26¢	184.783 -2.¢	231.027 -1.64¢	277.308 -1.17¢	323.632 -0.6¢	370.009 +0.07¢	416.447 +0.85¢	462.954 +1.73¢	509.538 +2.71¢	556.208 +3.8¢	602.972 +4.98¢	649.837 +6.27¢	696.812 +7.65¢	743.905 +9.14¢
G1	48.9343 -2.3¢	97.8713 -2.25¢	146.819 -2.11¢	195.786 -1.87¢	244.78 -1.53¢	293.809 -1.1¢	342.882 -0.57¢	392.006 +0.05¢	441.19 +0.77¢	490.441 +1.58¢	539.768 +2.49¢	589.179 +3.49¢	638.681 +4.59¢	688.284 +5.78¢	737.994 +7.06¢	787.82 +8.44¢
G#1	51.8485 -2.15¢	103.7 -2.11¢	155.561 -1.98¢	207.441 -1.76¢	259.347 -1.46¢	311.286 -1.06¢	363.267 -0.59¢	415.297 -0.03¢	467.385 +0.62¢	519.537 +1.36¢	571.762 +2.18¢	624.067 +3.09¢	676.46 +4.08¢	728.949 +5.16¢	781.541 +6.32¢	834.243 +7.56¢
A1	54.9357 -2.02¢	109.874 -1.98¢	164.822 -1.86¢	219.788 -1.66¢	274.779 -1.38¢	329.803 -1.03¢	384.868 -0.59¢	439.98 -0.08¢	495.147 +0.52¢	550.378 +1.19¢	605.678 +1.94¢	661.057 +2.78¢	716.521 +3.68¢	772.078 +4.67¢	827.735 +5.73¢	883.499 +6.88¢
A#1	58.2061 -1.91¢	116.415 -1.87¢	174.634 -1.76¢	232.87 -1.57¢	291.133 -1.3¢	349.429 -0.96¢	407.765 -0.54¢	466.151 -0.04¢	524.593 +0.53¢	583.098 +1.17¢	641.676 +1.9¢	700.332 +2.69¢	759.076 +3.57¢	817.914 +4.51¢	876.853 +5.53¢	935.902 +6.63¢
B1	61.6705 -1.82¢	123.344 -1.78¢	185.027 -1.67¢	246.729 -1.48¢	308.458 -1.22¢	370.221 -0.89¢	432.026 -0.48¢	493.882 0.¢	555.795 +0.55¢	617.774 +1.18¢	679.827 +1.89¢	741.962 +2.66¢	804.187 +3.51¢	866.508 +4.43¢	928.935 +5.43¢	991.474 +6.49¢
C2	65.3404 -1.74¢	130.684 -1.71¢	196.038 -1.6¢	261.411 -1.42¢	326.811 -1.17¢	392.246 -0.84¢	457.725 -0.45¢	523.255 +0.02¢	588.845 +0.56¢	654.503 +1.17¢	720.237 +1.85¢	786.054 +2.6¢	851.963 +3.42¢	917.973 +4.32¢	984.09 +5.28¢	1050.32 +6.31¢
C#2	69.2282 -1.68¢	138.459 -1.64¢	207.703 -1.53¢	276.968 -1.34¢	346.263 -1.07¢	415.598 -0.73¢	484.982 -0.31¢	554.423 +0.19¢	623.932 +0.76¢	693.516 +1.4¢	763.186 +2.12¢	832.949 +2.92¢	902.816 +3.79¢	972.795 +4.74¢	1042.9 +5.76¢	1113.13 +6.86¢
D2	73.3468 -1.63¢	146.697 -1.59¢	220.062 -1.46¢	293.452 -1.25¢	366.879 -0.95¢	440.352 -0.56¢	513.884 -0.09¢	587.484 +0.46¢	661.164 +1.1¢	734.934 +1.83¢	808.806 +2.63¢	882.789 +3.53¢	956.895 +4.51¢	1031.13 +5.57¢	1105.52 +6.71¢	1180.06 +7.94¢
D#2	77.7098 -1.6¢	155.424 -1.55¢	233.156 -1.4¢	310.919 -1.15¢	388.727 -0.8¢	466.593 -0.36¢	544.53 +0.19¢	622.551 +0.83¢	700.671 +1.57¢	778.901 +2.42¢	857.256 +3.35¢	935.748 +4.39¢	1014.39 +5.52¢	1093.2 +6.75¢	1172.18 +8.08¢	1251.35 +9.5¢
E2	82.3319 -1.57¢	164.669 -1.51¢	247.028 -1.35¢	329.424 -1.06¢	411.874 -0.67¢	494.393 -0.16¢	576.997 +0.45¢	659.704 +1.18¢	742.527 +2.02¢	825.484 +2.98¢	908.59 +4.04¢	991.861 +5.21¢	1075.31 +6.49¢	1158.96 +7.89¢	1242.82 +9.39¢	1326.91 +11.¢
F2	87.2286 -1.55¢	174.463 -1.49¢	261.722 -1.31¢	349.022 -1.02¢	436.382 -0.6¢	523.82 -0.07¢	611.353 +0.58¢	698.999 +1.35¢	786.777 +2.24¢	874.703 +3.24¢	962.797 +4.36¢	1051.07 +5.6¢	1139.55 +6.95¢	1228.25 +8.42¢	1317.19 +10.¢	1406.38 +11.69¢
F#2	92.4162 -1.54¢	184.839 -1.47¢	277.29 -1.28¢	369.789 -0.95¢	462.358 -0.5¢	555.016 +0.08¢	647.786 +0.8¢	740.686 +1.64¢	833.739 +2.61¢	926.964 +3.7¢	1020.38 +4.93¢	1114.01 +6.28¢	1207.88 +7.76¢	1302. +9.36¢	1396.39 +11.09¢	1491.08 +12.95¢
G2	97.912 -1.53¢	195.832 -1.46¢	293.784 -1.25¢	391.792 -0.89¢	489.88 -0.4¢	588.072 +0.24¢	686.392 +1.01¢	784.864 +1.93¢	883.511 +2.99¢	982.358 +4.19¢	1081.43 +5.52¢	1180.74 +6.99¢	1280.33 +8.61¢	1380.21 +10.35¢	1480.41 +12.24¢	1580.94 +14.26¢
G#2	103.734 -1.52¢	207.477 -1.45¢	311.254 -1.24¢	415.09 -0.89¢	519.01 -0.4¢	623.04 +0.24¢	727.205 +1.01¢	831.531 +1.92¢	936.041 +2.98¢	1040.76 +4.17¢	1145.72 +5.5¢	1250.93 +6.97¢	1356.43 +8.57¢	1462.25 +10.31¢	1568.39 +12.19¢	1674.89 +14.2¢
A2	109.903 -1.52¢	219.815 -1.45¢	329.763 -1.24¢	439.774 -0.88¢	549.875 -0.39¢	660.092 +0.25¢	770.453 +1.02¢	880.984 +1.94¢	991.712 +3.¢	1102.66 +4.19¢	1213.86 +5.52¢	1325.34 +7.¢	1437.12 +8.61¢	1549.23 +10.35¢	1661.7 +12.23¢	1774.54 +14.25¢
A#2	116.439 -1.51¢	232.888 -1.43¢	349.38 -1.2¢	465.946 -0.8¢	582.618 -0.25¢	699.428 +0.46¢	816.408 +1.32¢	933.589 +2.35¢	1051. +3.52¢	1168.68 +4.86¢	1286.65 +6.34¢	1404.95 +7.98¢	1523.61 +9.78¢	1642.65 +11.72¢	1762.12 +13.82¢	1882.03 +16.06¢
B2	123.363 -1.51¢	246.739 -1.42¢	370.166 -1.15¢	493.682 -0.7¢	617.325 -0.08¢	741.134 +0.73¢	865.145 +1.71¢	989.398 +2.86¢	1113.93 +4.2¢	1238.78 +5.7¢	1363.98 +7.39¢	1489.58 +9.24¢	1615.6 +11.27¢	1742.09 +13.47¢	1869.08 +15.84¢	1996.6 +18.37¢
C3	130.7 -1.49¢	261.414 -1.4¢	392.188 -1.1¢	523.065 -0.61¢	654.091 +0.08¢	785.309 +0.96¢	916.764 +2.04¢	1048.5 +3.31¢	1180.56 +4.77¢	1312.99 +6.43¢	1445.83 +8.28¢	1579.13 +10.31¢	1712.92 +12.54¢	1847.26 +14.95¢	1982.18 +17.55¢	2117.73 +20.34¢
C#3	138.473 -1.48¢	276.963 -1.37¢	415.524 -1.04¢	554.21 -0.48¢	693.073 +0.3¢	832.166 +1.29¢	971.543 +2.51¢	1111.26 +3.94¢	1251.36 +5.6¢	1391.9 +7.47¢	1532.93 +9.55¢	1674.51 +11.85¢	1816.69 +14.36¢	1959.51 +17.08¢	2103.03 +20.01¢	2247.29 +23.14¢
D3	146.708 -1.46¢	293.438 -1.33¢	440.252 -0.96¢	587.215 -0.33¢	734.39 +0.54¢	881.84 +1.67¢	1029.63 +3.04¢	1177.82 +4.65¢	1326.47 +6.52¢	1475.65 +8.62¢	1625.42 +10.97¢	1775.84 +13.56¢	1926.96 +16.38¢	2078.86 +19.44¢	2231.59 +22.74¢	2385.22 +26.26¢
D#3	155.434 -1.43¢	310.892 -1.3¢	466.445 -0.91¢	622.163 -0.25¢	778.117 +0.67¢	934.378 +1.85¢	1091.02 +3.3¢	1248.1 +5.¢	1405.7 +6.96¢	1563.89 +9.17¢	1722.74 +11.64¢	1882.3 +14.36¢	2042.66 +17.33¢	2203.88 +20.54¢	2366.02 +24.¢	2529.16 +27.7¢
E3	164.68 -1.4¢	329.387 -1.26¢	494.201 -0.84¢	659.204 -0.13¢	824.475 +0.86¢	990.094 +2.12¢	1156.14 +3.67¢	1322.7 +5.49¢	1489.84 +7.59¢	1657.65 +9.97¢	1826.2 +12.61¢	1995.58 +15.53¢	2165.85 +18.71¢	2337.1 +22.15¢	2509.4 +25.85¢	2682.82 +29.81¢
F3	174.476 -1.36¢	348.985 -1.2¢	523.625 -0.71¢	698.494 +0.1¢	873.69 +1.23¢	1049.31 +2.69¢	1225.45 +4.46¢	1402.21 +6.56¢	1579.68 +8.97¢	1757.97 +11.69¢	1937.16 +14.73¢	2117.34 +18.07¢	2298.62 +21.71¢	2481.08 +25.65¢	2664.82 +29.89¢	2849.92 +34.42¢
F#3	184.856 -1.32¢	369.751 -1.13¢	554.802 -0.59¢	740.123 +0.32¢	925.833 +1.59¢	1112.05 +3.22¢	1298.88 +5.21¢	1486.44 +7.55¢	1674.85 +10.25¢	1864.22 +13.29¢	2054.67 +16.68¢	2246.29 +20.42¢	2439.21 +24.48¢	2633.53 +28.89¢	2829.36 +33.62¢	3026.8 +38.67¢
G3	195.854 -1.26¢	391.754 -1.06¢	587.837 -0.46¢	784.239 +0.55¢	981.097 +1.96¢	1178.55 +3.77¢	1376.72 +5.98¢	1575.76 +8.58¢	1775.8 +11.57¢	1976.96 +14.94¢	2179.38 +18.7¢	2383.19 +22.83¢	2588.52 +27.34¢	2795.49 +32.21¢	3004.23 +37.44¢	3214.86 +43.02¢
G#3	207.508 -1.2¢	415.071 -0.97¢	622.855 -0.28¢	831.028 +0.88¢	1039.75 +2.49¢	1249.2 +4.56¢	1459.52 +7.09¢	1670.89 +10.06¢	1883.47 +13.48¢	2097.42 +17.34¢	2312.88 +21.63¢	2530.03 +26.35¢	2749.02 +31.49¢	2969.99 +37.04¢	3193.1 +42.99¢	3418.49 +49.34¢
A3	219.855 -1.14¢	439.776 -0.88¢	659.961 -0.1¢	880.607 +1.2¢	1101.91 +3.01¢	1324.07 +5.34¢	1547.28 +8.17¢	1771.73 +11.51¢	1997.62 +15.34¢	2225.12 +19.66¢	2454.44 +24.47¢	2685.76 +29.75¢	2919.25 +35.5¢	3155.09 +41.71¢	3393.47 +48.36¢	3634.56 +55.45¢
A#3	232.938 -1.06¢	465.948 -0.8¢	699.244 0.¢	933.042 +1.33¢	1167.55 +3.19¢	1402.99 +5.57¢	1639.57 +8.47¢	1877.5 +11.89¢	2116.99 +15.82¢	2358.24 +20.25¢	2601.45 +25.18¢	2846.83 +30.59¢	3094.57 +36.47¢	3344.87 +42.83¢	3597.91 +49.64¢	3853.89 +56.89¢
B3	246.8 -0.98¢	493.68 -0.71¢	740.874 +0.12¢	988.619 +1.5¢	1237.15 +3.43¢	1486.7 +5.9¢	1737.5 +8.91¢	1989.79 +12.45¢	2243.79 +16.53¢	2499.72 +21.12¢	2757.81 +26.22¢	3018.28 +31.83¢	3281.34 +37.93¢	3547.21 +44.51¢	3816.1 +51.56¢	4088.2 +59.07¢
C4	261.489 -0.9¢	523.071 -0.59¢	785.024 +0.33¢	1047.63 +1.87¢	1311.16 +4.02¢	1575.9 +6.77¢	1842.11 +10.12¢	2110.08 +14.07¢	2380.06 +18.6¢	2652.32 +23.71¢	2927.13 +29.38¢	3204.74 +35.61¢	3485.4 +42.37¢	3769.36 +49.67¢	4056.87 +57.48¢	4348.15 +65.8¢
C#4	277.052 -0.81¢	554.213 -0.47¢	831.813 +0.56¢	1110.18 +2.26¢	1389.63 +4.65¢	1670.51 +7.7¢	1953.11 +11.42¢	2237.78 +15.79¢	2524.81 +20.82¢	2814.52 +26.47¢	3107.22 +32.75¢	3403.21 +39.63¢	3702.77 +47.11¢	4006.19 +55.17¢	4313.77 +63.78¢	4625.77 +72.95¢
D4	293.542 -0.72¢	587.211 -0.34¢	881.389 +0.78¢	1176.45 +2.65¢	1472.79 +5.26¢	1770.76 +8.6¢	2070.75 +12.67¢	2373.12 +17.46¢	2678.24 +22.95¢	2986.46 +29.13¢	3298.14 +35.98¢	3613.62 +43.49¢	3933.24 +51.65¢	4257.32 +60.42¢	4586.2 +69.8¢	4920.18 +79.77¢
D#4	311.014 -0.62¢	622.169 -0.23¢	933.886 +0.94¢	1246.58 +2.89¢	1560.68 +5.61¢	1876.59 +9.1¢	2194.73 +13.34¢	2515.5 +18.33¢	2839.3 +24.05¢	3166.53 +30.49¢	3497.58 +37.62¢	3832.82 +45.45¢	4172.62 +53.93¢	4517.35 +63.06¢	4867.37 +72.82¢	5223.01 +83.17¢
E4	329.527 -0.52¢	659.213 -0.11¢	989.532 +1.14¢	1320.96 +3.21¢	1653.96 +6.11¢	1989.01 +9.82¢	2326.56 +14.33¢	2667.08 +19.63¢	3011.02 +25.71¢	3358.81 +32.54¢	3710.89 +40.12¢	4067.7 +48.41¢	4429.63 +57.41¢	4797.1 +67.08¢	5170.51 +77.41¢	5550.23 +88.37¢
F4	349.143 -0.42¢	698.475 +0.05¢	1048.57 +1.46¢	1399.99 +3.81¢	1753.3 +7.09¢	2109.07 +11.28¢	2467.84 +16.38¢	2830.16 +22.37¢	3196.57 +29.23¢	3567.59 +36.94¢	3943.74 +45.48¢	4325.53 +54.81¢	4713.44 +64.92¢	5107.95 +75.78¢	5509.52 +87.36¢	5918.6 +99.62¢
F#4	369.926 -0.32¢	740.076 +0.21¢	1111.12 +1.78¢	1483.73 +4.38¢	1858.56 +8.02¢	2236.27 +12.67¢	2617.52 +18.33¢	3002.94 +24.96¢	3393.15 +32.56¢	3788.78 +41.08¢	4190.43 +50.51¢	4598.67 +60.82¢	5014.07 +71.97¢	5437.19 +83.92¢	5868.55 +96.65¢	6308.67 +110.12¢
G4	391.947 -0.21¢	784.156 +0.37¢	1177.41 +2.1¢	1572.5 +4.98¢	1970.19 +9.¢	2371.26 +14.14¢	2776.45 +20.38¢	3186.53 +27.7¢	3602.21 +36.06¢	4024.21 +45.45¢	4453.22 +55.82¢	4889.93 +67.13¢	5334.97 +79.36¢	5788.99 +92.46¢	6252.59 +106.39¢	6726.36 +121.11¢
G#4	415.279 -0.1¢	830.873 +0.55¢	1247.72 +2.52¢	1666.77 +5.78¢	2088.95 +10.34¢	2515.18 +16.15¢	2946.36 +23.21¢	3383.38 +31.47¢	3827.12 +40.92¢	4278.41 +51.49¢	4738.09 +63.16¢	5206.93 +75.88¢	5685.72 +89.6¢	6175.18 +104.27¢	6676.03 +119.83¢	7188.93 +136.25¢
A4	[440. Hz]	880.377 +0.75¢	1322.26 +2.97¢	1766.77 +6.65¢	2215.02 +11.79¢	2668.12 +18.35¢	3127.13 +26.3¢	3593.11 +35.6¢	4067.1 +46.21¢	4550.09 +58.07¢	5043.04 +71.15¢	5546.89 +85.37¢	6062.53 +100.69¢	6590.82 +117.04¢	7132.57 +134.35¢	7688.56 +152.57¢
A#4	466.193 +0.11¢	932.825 +0.93¢	1401.21 +3.37¢	1872.66 +7.43¢	2348.47 +13.07¢	2829.92 +20.27¢	3318.25 +29.¢	3814.69 +39.19¢	4320.43 +50.81¢	4836.61 +63.8¢	5364.35 +78.08¢	5904.71 +93.6¢	6458.7 +110.28¢	7027.31 +128.05¢	7611.45 +146.85¢	8212. +166.59¢
B4	493.945 +0.22¢	988.395 +1.11¢	1484.86 +3.75¢	1984.81 +8.14¢	2489.81 +14.25¢	3001.25 +22.04¢	3520.57 +31.46¢	4049.18 +42.47¢	4588.42 +55.¢	5139.6 +68.99¢	5703.97 +84.36¢	6282.74 +101.03¢	6877.05 +118.93¢	7487.99 +137.98¢	8116.59 +158.09¢	8763.83 +179.19¢
C5	523.35 +0.33¢	1047.27 +1.28¢	1573.47 +4.1¢	2103.65 +8.79¢	2639.48 +15.31¢	3182.62 +23.62¢	3734.68 +33.67¢	4297.22 +45.4¢	4871.77 +58.74¢	5459.78 +73.61¢	6062.67 +89.94¢	6681.76 +107.63¢	7318.34 +126.61¢	7973.61 +146.77¢	8648.72 +168.03¢	9344.72 +190.3¢
C#5	554.506 +0.44¢	1109.69 +1.5¢	1667.57 +4.66¢	2230.16 +9.89¢	2799.45 +17.17¢	3377.37 +26.44¢	3965.83 +37.64¢	4566.67 +50.69¢	5181.66 +65.5¢	5812.5 +81.99¢	6460.81 +100.06¢	7128.15 +119.59¢	7815.97 +140.5¢	8525.65 +162.66¢	9258.48 +185.98¢	10015.7 +210.34¢
D5	587.517 +0.56¢	1175.85 +1.76¢	1767.44 +5.35¢	2364.71 +11.31¢	2970.04 +19.58¢	3585.77 +30.1¢	4214.16 +42.78¢	4857.41 +57.54¢	5517.62 +74.26¢	6196.79 +92.83¢	6896.82 +113.12¢	7619.53 +135.¢	8366.58 +158.35¢	9139.57 +183.04¢	9939.97 +208.94¢	10769.1 +235.91¢
D#5	622.494 +0.67¢	1245.95 +2.01¢	1873.26 +6.03¢	2507.29 +12.67¢	3150.84 +21.88¢	3806.66 +33.59¢	4477.41 +47.69¢	5165.65 +64.05¢	5873.82 +82.56¢	6604.25 +103.07¢	7359.12 +125.44¢	8140.49 +149.5¢	8950.28 +175.11¢	9790.28 +202.11¢	10662.1 +230.35¢	11567.4 +259.7¢
E5	659.556 +0.79¢	1320.23 +2.27¢	1985.4 +6.67¢	2658.37 +13.96¢	3342.42 +24.07¢	4040.72 +36.89¢	4756.35 +52.31¢	5492.24 +70.19¢	6251.21 +90.37¢	7035.89 +112.68¢	7848.76 +136.96¢	8692.16 +163.02¢	9568.23 +190.69¢	10479. +219.8¢	11426.2 +250.17¢	12411.6 +281.65¢
F5	698.826 +0.92¢	1398.93 +2.5¢	2104.14 +7.23¢	2818.23 +15.06¢	3544.91 +25.9¢	4287.78 +39.64¢	5050.32 +56.14¢	5835.84 +75.24¢	6647.49 +96.78¢	7488.23 +120.55¢	8360.81 +146.37¢	9267.83 +174.04¢	10211.7 +203.36¢	11194.5 +234.14¢	12218.3 +266.21¢	13284.9 +299.37¢
F#5	740.438 +1.05¢	1482.32 +2.75¢	2229.99 +7.8¢	2987.71 +16.16¢	3759.68 +27.73¢	4549.97 +42.39¢	5362.48 +59.97¢	6200.93 +80.29¢	7068.83 +103.17¢	7969.49 +128.38¢	8905.97 +155.72¢	9881.11 +184.97¢	10897.5 +215.9¢	11957.6 +248.32¢	13063.6 +282.02¢	14217.3 +316.81¢
G5	784.531 +1.2¢	1570.72 +3.03¢	2363.53 +8.49¢	3167.86 +17.52¢	3988.49 +30.01¢	4830.05 +45.81¢	5696.99 +64.73¢	6593.51 +86.57¢	7523.57 +111.11¢	8490.88 +138.1¢	9498.86 +167.3¢	10550.7 +198.47¢	11649.2 +231.37¢	12797. +265.77¢	13996.5 +301.44¢	15249.9 +338.19¢
G#5	831.257 +1.35¢	1664.5 +3.42¢	2505.67 +9.6¢	3360.63 +19.79¢	4235.08 +32.87¢	5134.52 +51.63¢	6064.18 +72.86¢	7029.02 +97.3¢	8033.64 +124.67¢	9082.31 +154.67¢	10179. +187.02¢	11327.2 +221.42¢	12530.2 +257.59¢	13790.9 +295.26¢	15111.9 +334.19¢	
A5	880.772 +1.52¢	1763.94 +3.98¢	2656.67 +10.94¢	3566.01 +21.49¢	4498.8 +38.45¢	5461.59 +58.55¢	6460.61 +82.49¢	7501.66 +109.97¢	8590.12 +140.62¢	9730.93 +174.09¢	10928.6 +210.94¢	12187.1 +248.14¢	13510.2 +287.95¢	14901. +329.29¢		
A#5	933.247 +1.71¢	1869.31 +4.32¢	2816.6 +12.1¢	3783.37 +22.45¢	4777.62 +42.55¢	5806.99 +64.71¢	6878.64 +91.04¢	7999.26 +121.15¢	9174.98 +154.65¢	10411.4 +191.11¢	11713.5 +230.12¢	13085.9 +271.29¢	14532.4 +314.23¢			
B5	988.86 +1.82¢	1981. +4.79¢	2986.23 +13.35¢	4014.16 +27.43¢	5074.06 +46.77¢	6174.73 +71.01¢	7324.43 +99.73¢	8530.76 +132.52¢	9800.67 +168.86¢	11140.4 +208.28¢	12555.6 +250.31¢	14051.2 +294.51¢	15631.6 +340.46¢			
C6	1047.8 +2.15¢	2099.43 +5.31¢	3166.3 +14.72¢	4259.58 +30.17¢	5390.01 +51.33¢	6567.75 +77.84¢	7802.22 +109.15¢	9102.1 +144.75¢	10475.2 +184.09¢	11928.6 +226.62¢	13468.3 +271.84¢	15100. +319.13¢				
C#6	1110.28 +2.42¢	2225.02 +5.89¢	3357.54 +16.25¢	4520.86 +33.23¢	5727.41 +56.40¢	6988.91 +85.44¢	8316.16 +119.59¢	9719.01 +158.29¢	11206.3 +200.89¢	12785.8 +246.76¢	14464.3 +295.31¢					
D6	1176.5 +2.71¢	2358.27 +6.59¢	3561.06 +18.13¢	4800.17 +37.02¢	6090.2 +62.79¢	7444.78 +94.84¢	8876.4 +132.46¢	10396.3 +174.91¢	12014.4 +221.44¢	13739.5 +271.31¢	15579.2 +323.85¢					
D#6	1246.7 +3.05¢	2499.92 +7.37¢	3779.09 +21.01¢	5103.05 +42.94¢	6489.59 +72.73¢	7955.16 +109.63¢	9514.62 +152.66¢	11181.1 +200.91¢	12966.2 +253.43¢	14879.8 +308.34¢						
E6	1321.12 +3.43¢	2650.22 +8.43¢	4011.06 +24.14¢	5426.54 +49.35¢	6918.14 +83.40¢	8505.48 +125.43¢	10206. +174.11¢	12035. +228.31¢	14005.5 +286.91¢							
F6	1400.02 +3.85¢	2809.4 +9.63¢	4255.96 +26.74¢	5766.43 +54.53¢	7365.7 +91.99¢	9076.29 +137.88¢	10918.1 +190.86¢	12908.1 +249.36¢	15060.9 +312.09¢							
F#6	1483.68 +4.31¢	2978.35 +10.73¢	4516.57 +29.63¢	6129.53 +60.25¢	7846.07 +101.37¢	9692.02 +151.52¢	11689.9 +209.11¢	13858.7 +272.58¢								
G6	1572.4 +4.88¢	3157.68 +11.95¢	4794. +32.84¢	6517.74 +66.56¢	8362.25 +111.67¢	10357.1 +166.42¢	12527.6 +228.94¢	14895.2 +297.48¢								
G#6	1666.49 +5.46¢	3348.08 +13.32¢	5089.5 +36.39¢	6933.13 +73.52¢	8917.54 +122.98¢	11076.5 +182.67¢	13438.5 +250.44¢									
A6	1766.28 +6.17¢	3550.28 +14.83¢	5404.39 +40.32¢	7378.02 +81.19¢	9515.63 +135.30¢	11855.7 +200.36¢	14430.2 +273.71¢									
A#6	1872.13 +6.93¢	3765.06 +16.52¢	5740.15 +44.67¢	7854.93 +89.63¢	10160.6 +148.9¢	12700.9 +219.39¢	15511.5 +298.81¢									
B6	1984.43 +7.78¢	3993.25 +18.39¢	6098.37 +49.47¢	8366.68 +98.9¢	10857.1 +163.68¢	13619.1 +240.43¢										
C7	2103.57 +8.72¢	4235.77 +20.47¢	6480.78 +54.76¢	8916.38 +109.06¢	11610.2 +179.78¢	14618. +262.97¢										
C#7	2230. +9.77¢	4493.59 +22.76¢	6889.31 +60.59¢	9507.47 +120.19¢	12425.5 +197.28¢	15706.2 +287.27¢										
D7	2364.18 +10.92¢	4767.75 +25.29¢	7326.03 +67.¢	10143.8 +132.34¢	13309.4 +216.26¢											
D#7	2506.59 +12.19¢	5059.38 +28.07¢	7793.25 +74.03¢	10829.5 +145.59¢	14269. +236.78¢											
E7	2657.77 +13.58¢	5369.7 +31.12¢	8293.48 +81.73¢	11569.4 +160.01¢	15312.1 +258.92¢											
F7	2818.28 +15.10¢	5700.02 +34.46¢	8829.48 +90.15¢	12368.7 +175.66¢												
F#7	2988.71 +16.74¢	6051.77 +38.14¢	9404.29 +99.34¢	13233.1 +192.61¢												
G7	3169.71 +18.53¢	6426.62 +42.19¢	10022.1 +109.3¢	14171.5 +211.22¢												
G#7	3361.95 +20.47¢	6826.19 +46.61¢	10686.4 +120.6¢	15190.2 +231.4¢												
A7	3566.16 +22.56¢	7252.31 +51.41¢	11401.4 +132.73¢													
A#7	3783.11 +24.8¢	7706.98 +56.71¢	12171.9 +145.94¢													
B7	4013.64 +27.21¢	8192.38 +62.43¢	13003.3 +160.32¢													
C8	4258.61 +29.77¢	8710.88 +68.7¢	13901.5 +175.96¢													

Figure 7-1 Tuning Table for Grand Piano

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A0	27.2634 -14.95¢	54.5391 -14.56¢	81.8635 -13.4¢	109.273 -11.47¢	136.805 -8.77¢	164.493 -5.32¢	192.376 -1.11¢	220.487 +3.83¢	248.861 +9.5¢	277.533 +15.88¢	306.537 +22.96¢	335.905 +30.71¢	365.67 +39.13¢	395.864 +48.18¢	426.516 +57.85¢	457.657 +68.12¢
A#0	28.8972 -14.2¢	57.8069 -13.82¢	86.7667 -12.7¢	115.814 -10.83¢	144.986 -8.22¢	174.319 -4.88¢	203.85 -0.81¢	233.616 +3.97¢	263.653 +9.46¢	293.994 +15.63¢	324.676 +22.48¢	355.731 +29.99¢	387.194 +38.14¢	419.096 +46.91¢	451.47 +56.29¢	484.346 +66.25¢
B0	30.6292 -13.42¢	61.2713 -13.06¢	91.9648 -11.97¢	122.748 -10.16¢	153.659 -7.63¢	184.737 -4.39¢	216.018 -0.45¢	247.54 +4.19¢	279.34 +9.51¢	311.453 +15.51¢	343.917 +22.15¢	376.765 +29.44¢	410.032 +37.36¢	443.751 +45.88¢	477.956 +54.98¢	512.677 +64.66¢
C1	32.4652 -12.64¢	64.9427 -12.31¢	97.4694 -11.33¢	130.082 -9.69¢	162.817 -7.41¢	195.712 -4.48¢	228.802 -0.91¢	262.122 +3.29¢	295.71 +8.11¢	329.598 +13.54¢	363.822 +19.56¢	398.416 +26.18¢	433.413 +33.36¢	468.844 +41.11¢	504.743 +49.39¢	541.14 +58.2¢
C#1	34.4112 -11.86¢	68.8355 -11.53¢	103.312 -10.54¢	137.88 -8.9¢	172.579 -6.6¢	207.448 -3.66¢	242.524 -0.07¢	277.846 +4.15¢	313.453 +8.99¢	349.38 +14.44¢	385.664 +20.5¢	422.342 +27.14¢	459.449 +34.36¢	497.019 +42.14¢	535.087 +50.46¢	573.685 +59.31¢
D1	36.4736 -11.09¢	72.9592 -10.81¢	109.492 -9.96¢	146.109 -8.55¢	182.844 -6.58¢	219.733 -4.06¢	256.811 -0.98¢	294.112 +2.64¢	331.672 +6.8¢	369.525 +11.49¢	407.703 +16.7¢	446.24 +22.43¢	485.168 +28.65¢	524.52 +35.37¢	564.327 +42.57¢	604.619 +50.23¢
D#1	38.6593 -10.33¢	77.3311 -10.05¢	116.053 -9.22¢	154.861 -7.83¢	193.794 -5.88¢	232.888 -3.39¢	272.18 -0.35¢	311.705 +3.22¢	351.501 +7.33¢	391.603 +11.96¢	432.046 +17.11¢	472.865 +22.76¢	514.093 +28.91¢	555.764 +35.54¢	597.912 +42.65¢	640.568 +50.22¢
E1	40.9755 -9.6¢	81.9635 -9.34¢	123.001 -8.55¢	164.126 -7.24¢	205.374 -5.41¢	246.783 -3.06¢	288.39 -0.2¢	330.231 +3.17¢	372.341 +7.04¢	414.757 +11.41¢	457.514 +16.28¢	500.647 +21.6¢	544.189 +27.4¢	588.174 +33.67¢	632.636 +40.38¢	677.607 +47.54¢
F1	43.4298 -8.89¢	86.8731 -8.62¢	130.37 -7.82¢	173.961 -6.48¢	217.685 -4.62¢	261.584 -2.23¢	305.695 +0.69¢	350.06 +4.12¢	394.716 +8.07¢	439.701 +12.52¢	485.055 +17.46¢	530.814 +22.89¢	577.015 +28.8¢	623.696 +35.19¢	670.89 +42.02¢	718.634 +49.31¢
F#1	46.0304 -8.21¢	92.0748 -7.93¢	138.176 -7.15¢	184.375 -5.83¢	230.714 -3.98¢	277.236 -1.62¢	323.982 +1.27¢	370.992 +4.67¢	418.308 +8.57¢	465.971 +12.97¢	514.019 +17.87¢	562.493 +23.23¢	611.43 +29.1¢	660.87 +35.42¢	710.85 +42.19¢	761.406 +49.4¢
G1	48.7857 -7.56¢	97.5825 -7.37¢	146.424 -6.77¢	195.344 -5.78¢	244.375 -4.39¢	293.552 -2.62¢	342.906 -0.45¢	392.472 +2.11¢	442.281 +5.05¢	492.366 +8.36¢	542.759 +12.06¢	593.493 +16.12¢	644.598 +20.55¢	696.106 +25.34¢	748.047 +30.49¢	800.451 +35.98¢
G#1	51.7049 -6.95¢	103.421 -6.77¢	155.181 -6.22¢	207.017 -5.3¢	258.963 -4.02¢	311.051 -2.37¢	363.314 -0.37¢	415.783 +2¢	468.492 +4.72¢	521.472 +7.79¢	574.754 +11.22¢	628.371 +14.99¢	682.352 +19.09¢	736.729 +23.54¢	791.532 +28.31¢	846.79 +33.41¢
A1	54.7977 -6.38¢	109.607 -6.19¢	164.462 -5.63¢	219.398 -4.74¢	274.45 -3.48¢	329.65 -1.81¢	385.035 +0.16¢	440.637 +2.55¢	496.491 +5.21¢	552.63 +8.26¢	609.088 +11.66¢	665.897 +15.4¢	723.089 +19.48¢	780.699 +23.89¢	838.756 +28.63¢	897.292 +33.69¢
A#1	58.0742 -5.84¢	116.16 -5.66¢	174.294 -5.13¢	232.51 -4.25¢	290.845 -3.01¢	349.333 -1.43¢	408.011 +0.5¢	466.912 +2.78¢	526.071 +5.4¢	585.524 +8.36¢	645.304 +11.66¢	705.446 +15.29¢	765.982 +19.24¢	826.946 +23.53¢	888.371 +28.13¢	950.289 +33.06¢
B1	61.5454 -5.33¢	123.107 -5.11¢	184.731 -4.44¢	246.467 -3.35¢	308.361 -1.77¢	370.46 -0.23¢	432.812 +2.08¢	495.463 +5.53¢	558.459 +8.83¢	621.846 +12.56¢	685.67 +16.7¢	749.975 +21.26¢	814.805 +26.22¢	880.204 +31.58¢	946.216 +37.34¢	1012.88 +43.47¢
C2	65.2226 -4.87¢	130.459 -4.68¢	195.754 -4.11¢	261.149 -3.15¢	326.687 -1.82¢	392.411 -0.11¢	458.365 +1.97¢	524.589 +4.43¢	591.127 +7.25¢	658.019 +10.44¢	725.307 +13.99¢	793.033 +17.9¢	861.237 +22.16¢	929.958 +26.77¢	999.236 +31.72¢	1069.11 +37.01¢
C#2	69.118 -4.44¢	138.249 -4.27¢	207.434 -3.78¢	276.71 -2.95¢	346.119 -1.79¢	415.7 -0.3¢	485.492 +1.51¢	555.534 +3.65¢	625.865 +6.11¢	696.523 +8.89¢	767.547 +11.99¢	838.976 +15.4¢	910.846 +19.12¢	983.194 +23.15¢	1056.06 +27.47¢	1129.47 +32.1¢
D2	73.2446 -4.05¢	146.504 -3.87¢	219.823 -3.35¢	293.245 -2.47¢	366.814 -1.25¢	440.576 +0.31¢	514.572 +2.22¢	588.848 +4.48¢	663.447 +7.07¢	738.411 +10¢	813.784 +13.26¢	889.607 +16.85¢	965.923 +20.76¢	1042.77 +25¢	1120.2 +29.55¢	1198.24 +34.41¢
D#2	77.6161 -3.69¢	155.246 -3.53¢	232.93 -3.06¢	310.71 -2.31¢	388.627 -1.25¢	466.721 +0.12¢	545.033 +1.79¢	623.604 +3.76¢	702.473 +6.02¢	781.681 +8.58¢	861.267 +11.44¢	941.271 +14.58¢	1021.73 +18.01¢	1102.69 +21.72¢	1184.18 +25.71¢	1266.24 +29.98¢
E2	82.2469 -3.36¢	164.509 -3.2¢	246.829 -2.74¢	329.253 -1.96¢	411.825 -0.88¢	494.588 +0.52¢	577.587 +2.22¢	660.865 +4.23¢	744.465 +6.54¢	828.431 +9.15¢	912.806 +12.05¢	997.632 +15.26¢	1082.95 +18.75¢	1168.81 +22.53¢	1255.24 +26.6¢	1342.28 +30.94¢
F2	87.1524 -3.06¢	174.32 -2.92¢	261.547 -2.47¢	348.88 -1.72¢	436.363 -0.68¢	524.041 +0.66¢	611.959 +2.3¢	700.162 +4.23¢	788.692 +6.45¢	877.596 +8.96¢	966.915 +11.75¢	1056.69 +14.83¢	1146.98 +18.19¢	1237.8 +21.83¢	1329.22 +25.74¢	1421.26 +29.92¢
F#2	92.3489 -2.8¢	184.714 -2.65¢	277.142 -2.2¢	369.682 -1.46¢	462.38 -0.42¢	555.284 +0.9¢	648.442 +2.55¢	741.9 +4.47¢	835.705 +6.68¢	929.904 +9.18¢	1024.54 +11.97¢	1119.67 +15.04¢	1215.32 +18.39¢	1311.55 +22.02¢	1408.4 +25.92¢	1505.92 +30.09¢
G2	97.8537 -2.56¢	195.725 -2.41¢	293.664 -1.95¢	391.724 -1.19¢	489.956 -0.13¢	588.412 +1.24¢	687.141 +2.9¢	786.196 +4.87¢	885.626 +7.13¢	985.483 +9.68¢	1085.82 +12.53¢	1186.67 +15.67¢	1288.11 +19.09¢	1390.16 +22.8¢	1492.89 +26.79¢	1596.34 +31.06¢
G#2	103.685 -2.35¢	207.389 -2.19¢	311.168 -1.72¢	415.077 -0.94¢	519.174 +0.15¢	623.513 +1.55¢	728.151 +3.26¢	833.143 +5.28¢	938.543 +7.6¢	1044.41 +10.22¢	1150.79 +13.14¢	1257.74 +16.36¢	1365.32 +19.87¢	1473.57 +23.67¢	1582.56 +27.76¢	1692.33 +32.13¢
A2	109.863 -2.16¢	219.743 -2.02¢	329.697 -1.59¢	439.777 -0.87¢	550.038 +0.13¢	660.535 +1.41¢	771.321 +2.97¢	882.45 +4.82¢	993.976 +6.94¢	1105.95 +9.35¢	1218.43 +12.03¢	1331.46 +14.97¢	1445.1 +18.19¢	1559.4 +21.67¢	1674.4 +25.42¢	1790.17 +29.43¢
A#2	116.407 -1.99¢	232.833 -1.84¢	349.34 -1.4¢	465.987 -0.65¢	582.834 +0.39¢	699.94 +1.72¢	817.366 +3.35¢	935.17 +5.28¢	1053.41 +7.49¢	1172.15 +9.99¢	1291.44 +12.77¢	1411.34 +15.84¢	1531.92 +19.19¢	1653.22 +22.82¢	1775.3 +26.72¢	1898.21 +30.89¢
B2	123.339 -1.84¢	246.699 -1.7¢	370.142 -1.26¢	493.73 -0.53¢	617.525 +0.48¢	741.589 +1.79¢	865.984 +3.38¢	990.771 +5.26¢	1116.01 +7.42¢	1241.76 +9.87¢	1368.09 +12.59¢	1495.05 +15.59¢	1622.71 +18.87¢	1751.11 +22.42¢	1880.33 +26.23¢	2010.41 +30.31¢
C3	130.683 -1.71¢	261.387 -1.57¢	392.173 -1.18¢	523.103 -0.48¢	654.238 +0.47¢	785.64 +1.69¢	917.369 +3.18¢	1049.49 +4.94¢	1182.05 +6.96¢	1315.13 +9.25¢	1448.78 +11.8¢	1583.05 +14.61¢	1718.01 +17.67¢	1853.72 +20.99¢	1990.23 +24.57¢	2127.6 +28.39¢
C#3	138.464 -1.59¢	276.948 -1.46¢	415.516 -1.07¢	554.229 -0.42¢	693.15 +0.49¢	832.341 +1.66¢	971.864 +3.08¢	1111.78 +4.76¢	1252.15 +6.7¢	1393.04 +8.89¢	1534.5 +11.32¢	1676.6 +14.01¢	1819.4 +16.94¢	1962.96 +20.12¢	2107.32 +23.54¢	2252.57 +27.2¢
D3	146.706 -1.49¢	293.424 -1.42¢	440.189 -1.21¢	587.036 -0.86¢	734.002 -0.37¢	881.122 +0.26¢	1028.43 +1.02¢	1175.96 +1.92¢	1323.75 +2.96¢	1471.84 +4.14¢	1620.25 +5.46¢	1769.03 +6.91¢	1918.21 +8.5¢	2067.82 +10.22¢	2217.89 +12.07¢	2368.47 +14.06¢
D#3	155.438 -1.39¢	310.889 -1.32¢	466.392 -1.1¢	621.987 -0.74¢	777.712 -0.23¢	933.607 +0.43¢	1089.71 +1.22¢	1246.06 +2.17¢	1402.7 +3.25¢	1559.66 +4.48¢	1716.99 +5.85¢	1874.72 +7.37¢	2032.89 +9.02¢	2191.54 +10.82¢	2350.7 +12.75¢	2510.42 +14.83¢
E3	164.689 -1.31¢	329.393 -1.23¢	494.156 -0.99¢	659.024 -0.6¢	824.04 -0.06¢	989.249 +0.65¢	1154.7 +1.51¢	1320.43 +2.52¢	1486.48 +3.69¢	1652.91 +5.01¢	1819.75 +6.48¢	1987.05 +8.11¢	2154.84 +9.89¢	2323.19 +11.82¢	2492.12 +13.89¢	2661.68 +16.12¢
F3	174.49 -1.23¢	349.019 -1.01¢	523.709 -0.43¢	698.679 +0.56¢	874.049 +1.94¢	1049.94 +3.72¢	1226.46 +5.89¢	1403.75 +8.45¢	1581.9 +11.39¢	1761.04 +14.72¢	1941.29 +18.43¢	2122.75 +22.48¢	2305.55 +26.92¢	2489.78 +31.71¢	2675.57 +36.86¢	2863.02 +42.36¢
F#3	184.873 -1.16¢	369.79 -0.95¢	554.88 -0.34¢	740.275 +0.67¢	926.104 +2.1¢	1112.5 +3.92¢	1299.58 +6.15¢	1487.49 +8.77¢	1676.34 +11.78¢	1866.27 +15.19¢	2057.39 +18.98¢	2249.84 +23.15¢	2443.73 +27.69¢	2639.19 +32.6¢	2836.33 +37.88¢	3035.27 +43.5¢
G3	195.874 -1.09¢	391.797 -0.87¢	587.916 -0.22¢	784.379 +0.86¢	981.332 +2.38¢	1178.92 +4.32¢	1377.29 +6.69¢	1576.58 +9.48¢	1776.95 +12.69¢	1978.52 +16.31¢	2181.44 +20.34¢	2385.86 +24.77¢	2591.9 +29.6¢	2799.7 +34.82¢	3009.4 +40.42¢	3221.13 +46.39¢
G#3	207.529 -1.02¢	415.114 -0.78¢	622.921 -0.09¢	831.118 +1.06¢	1039.87 +2.68¢	1249.34 +4.76¢	1459.7 +7.3¢	1671.11 +10.28¢	1883.73 +13.71¢	2097.72 +17.58¢	2313.24 +21.89¢	2530.44 +26.63¢	2749.49 +31.78¢	2970.53 +37.36¢	3193.72 +43.33¢	3419.2 +49.71¢
A3	219.878 -0.96¢	439.821 -0.7¢	660.024 +0.07¢	880.684 +1.35¢	1101.99 +3.14¢	1324.15 +5.44¢	1547.34 +8.24¢	1771.76 +11.53¢	1997.59 +15.32¢	2225.03 +19.59¢	2454.26 +24.34¢	2685.46 +29.56¢	2918.81 +35.24¢	3154.49 +41.38¢	3392.68 +47.95¢	3633.53 +54.96¢
A#3	232.961 -0.89¢	465.998 -0.61¢	699.341 +0.24¢	933.218 +1.64¢	1167.86 +3.61¢	1403.48 +6.17¢	1640.32 +9.27¢	1878.6 +12.9¢	2118.53 +17.08¢	2360.34 +21.8¢	2604.25 +27.04¢	2850.45 +32.79¢	3099.17 +39.04¢	3350.6 +45.79¢	3604.96 +53.03¢	3862.43 +60.73¢
B3	246.823 -0.83¢	493.73 -0.53¢	740.975 +0.35¢	988.809 +1.83¢	1237.49 +3.9¢	1487.25 +6.54¢	1738.36 +9.78¢	1991.06 +13.59¢	2245.58 +17.97¢	2502.18 +22.92¢	2761.09 +28.28¢	3022.53 +34.27¢	3286.75 +40.78¢	3553.97 +47.81¢	3824.41 +55.33¢	4098.29 +63.34¢
C4	261.51 -0.76¢	523.116 -0.44¢	785.111 +0.52¢	1047.78 +2.12¢	1311.42 +4.36¢	1576.31 +7.22¢	1842.74 +10.71¢	2110.99 +14.82¢	2381.34 +19.53¢	2654.07 +24.85¢	2929.44 +30.75¢	3207.72 +37.22¢	3489.18 +44.25¢	3774.08 +51.84¢	4062.65 +59.95¢	4355.16 +68.58¢
C#4	277.071 -0.69¢	554.251 -0.35¢	831.866 +0.67¢	1110.24 +2.36¢	1389.71 +4.74¢	1670.58 +7.77¢	1953.17 +11.47¢	2237.82 +15.82¢	2524.82 +20.82¢	2814.48 +26.45¢	3107.12 +32.69¢	3403.02 +39.54¢	3702.48 +46.98¢	4005.79 +54.99¢	4313.23 +63.57¢	4625.06 +72.69¢
D4	293.559 -0.62¢	587.25 -0.25¢	881.469 +0.84¢	1176.61 +2.66¢	1473.07 +5.19¢	1771.24 +8.07¢	2071.5 +13.3¢	2374.23 +18.27¢	2679.82 +23.97¢	2988.64 +30.39¢	3301.04 +37.5¢	3617.39 +45.3¢	3938.04 +53.76¢	4263.32 +62.86¢	4593.57 +72.59¢	4929.12 +82.91¢
D#4	311.028 -0.54¢	622.203 -0.14¢	933.962 +1.08¢	1246.74 +3.11¢	1560.98 +5.95¢	1877.11 +9.57¢	2195.55 +13.99¢	2516.74 +19.18¢	2841.07 +25.13¢	3168.98 +31.82¢	3500.85 +39.24¢	3837.08 +47.37¢	4178.06 +56.19¢	4524.15 +65.67¢	4875.74 +75.79¢	5233.16 +86.53¢
E4	329.538 -0.46¢	659.246 -0.02¢	989.631 +1.32¢	1321.2 +3.53¢	1654.46 +6.63¢	1989.91 +10.6¢	2328.04 +15.42¢	2669.34 +21.09¢	3014.29 +27.58¢	3363.36 +34.88¢	3717.02 +42.97¢	4075.71 +51.82¢	4439.88 +61.41¢	4809.96 +71.72¢	5186.36 +82.71¢	5569.49 +94.37¢
F4	349.15 -0.38¢	698.497 +0.11¢	1048.63 +1.56¢	1400.13 +3.99¢	1753.58 +7.37¢	2109.56 +11.69¢	2468.65 +16.95¢	2831.39 +23.13¢	3198.35 +30.2¢	3570.06 +38.14¢	3947.07 +46.93¢	4329.87 +56.55¢	4718.99 +66.96¢	5114.9 +78.14¢	5518.08 +90.05¢	5928.99 +102.66¢
F#4	369.931 -0.29¢	740.074 +0.2¢	1111.06 +1.69¢	1483.53 +4.15¢	1858.1 +7.59¢	2235.39 +11.99¢	2616.03 +17.34¢	3000.62 +23.63¢	3389.75 +30.82¢	3784. +38.9¢	4183.95 +47.84¢	4590.16 +57.61¢	5003.16 +68.19¢	5423.48 +79.55¢	5851.63 +91.65¢	6288.1 +104.46¢
G4	391.95 -0.2¢	784.148 +0.35¢	1177.34 +2¢	1572.27 +4.73¢	1969.67 +8.55¢	2370.27 +13.42¢	2774.8 +19.35¢	3183.96 +26.3¢	3598.45 +34.26¢	4018.95 +43.18¢	4446.11 +53.05¢	4880.6 +63.83¢	5323.02 +75.48¢	5773.99 +87.97¢	6234.09 +101.26¢	6703.89 +115.31¢
G#4	415.28 -0.1¢	830.833 +0.47¢	1247.48 +2.18¢	1666.03 +5.01¢	2087.3 +8.96¢	2512.09 +14.02¢	2941.18 +20.16¢	3375.36 +27.36¢	3815.39 +35.6¢	4262.01 +44.84¢	4715.95 +55.05¢	5177.91 +66.2¢	5648.57 +78.25¢	6128.6 +91.16¢	6618.63 +104.89¢	7119.28 +119.39¢
A4	[440.Hz]	880.33 +0.65¢	1321.98 +2.6¢	1765.93 +5.83¢	2213.16 +10.33¢	2664.63 +16.08¢	3121.29 +23.06¢	3584.07 +31.23¢	4053.89 +40.57¢	4531.63 +51.04¢	5018.15 +62.58¢	5514.29 +75.17¢	6020.85 +88.75¢	6538.61 +103.27¢	7068.31 +118.68¢	7610.65 +134.94¢
A#4	466.193 +0.11¢	932.765 +0.82¢	1400.85 +2.92¢	1871.57 +6.42¢	2346.06 +11.29¢	2825.4 +17.51¢	3310.7 +25.05¢	3803. +33.88¢	4303.36 +43.96¢	4812.78 +55.24¢	5332.23 +67.68¢	5862.66 +81.23¢	6404.98 +95.82¢	6960.06 +111.41¢	7528.74 +127.94¢	8111.8 +145.34¢
B4	493.947 +0.23¢	988.331 +0.99¢	1484.46 +3.28¢	1983.63 +7.09¢	2487.14 +12.39¢	2996.26 +19.15¢	3512.22 +27.35¢	4036.26 +36.94¢	4569.56 +47.87¢	5113.27 +60.1¢	5668.51 +73.56¢	6236.35 +88.2¢	6817.81 +103.96¢	7413.88 +120.76¢	8025.5 +138.56¢	8653.56 +157.27¢
C5	523.355 +0.35¢	1047.23 +1.21¢	1573.17 +3.77¢	2102.73 +8.03¢	2637.44 +13.96¢	3178.79 +21.53¢	3728.25 +30.69¢	4287.28 +41.39¢	4857.25 +53.57¢	5439.52 +67.18¢	6035.38 +82.13¢	6646.09 +98.37¢	7272.82 +115.8¢	7916.71 +134.37¢	8578.82 +153.98¢	9260.18 +174.56¢
C#5	554.516 +0.47¢	1109.62 +1.4¢	1667.09 +4.16¢	2228.68 +8.74¢	2796.13 +15.12¢	3371.14 +23.24¢	3955.4 +33.08¢	4550.53 +44.55¢	5158.1 +57.61¢	5779.64 +72.18¢	6416.6 +88.17¢	7070.39 +105.51¢	7742.33 +124.11¢	8433.67 +143.88¢	9145.61 +164.74¢	9879.28 +186.6¢
D5	587.533 +0.61¢	1175.75 +1.62¢	1766.72 +4.64¢	2362.46 +9.66¢	2965.01 +16.65¢	3576.34 +25.54¢	4198.37 +36.29¢	4833. +48.82¢	5482.02 +63.05¢	6147.18 +78.91¢	6830.15 +96.3¢	7532.5 +115.11¢	8255.75 +135.27¢	9001.31 +156.65¢	9770.52 +179.17¢	10564.6 +202.72¢
D#5	622.52 +0.74¢	1245.81 +1.81¢	1872.16 +5.01¢	2503.87 +10.31¢	3143.19 +17.68¢	3792.32 +27.06¢	4453.43 +38.39¢	5128.59 +51.59¢	5819.82 +66.57¢	6529.06 +83.25¢	7258.14 +101.52¢	8008.81 +121.27¢	8782.74 +142.39¢	9581.47 +164.78¢	10406.5 +188.34¢	11259.1 +212.94¢
E5	659.591 +0.89¢	1320.05 +2.03¢	1983.98 +5.44¢	2653.97 +11.1¢	3332.55 +18.95¢	4022.23 +28.95¢	4725.42 +41.02¢	5444.47 +55.06¢	6181.64 +70.99¢	6939.1 +88.7¢	7718.88 +108.07¢	8522.95 +128.98¢	9353.12 +151.32¢	10211.1 +174.97¢	11098.6 +199.81¢	12016.9 +225.71¢
F5	698.874 +1.04¢	1398.8 +2.34¢	2102.94 +6.25¢	2814.4 +12.71¢	3536.28 +21.68¢	4271.56 +33.08¢	5023.17 +46.81¢	5793.9 +62.76¢	6586.44 +80.8¢	7403.33 +100.81¢	8246.98 +122.64¢	9119.67 +146.14¢	10023.5 +171.16¢	10960.5 +197.57¢	11932.4 +225.22¢	12940.9 +253.96¢
F#5	740.499 +1.2¢	1482.04 +2.42¢	2227.75 +6.07¢	2980.73 +12.11¢	3744.03 +20.51¢	4520.63 +31.19¢	5313.43 +44.06¢	6125.24 +59.03¢	6958.72 +75.99¢	7816.46 +94.82¢	8700.86 +115.39¢	9614.23 +137.57¢	10558.7 +161.22¢	11536.3 +186.22¢	12548.9 +212.44¢	13598.3 +239.73¢
G5	784.607 +1.37¢	1570.64 +2.93¢	2362.35 +7.62¢	3163.95 +15.39¢	3979.57 +26.13¢	4813.22 +39.76¢	5668.77 +56.13¢	6549.91 +75.09¢	7460.16 +96.45¢	8402.81 +120.05¢	9380.96 +145.68¢	10397.5 +173.15¢	11455. +202.27¢	12556. +232.86¢	13702.7 +264.71¢	14897.2 +297.67¢
G#5	831.347 +1.54¢	1664.37 +3.29¢	2504.08 +8.5¢	3355.43 +17.11¢	4223.26 +29.03¢	5112.27 +44.11¢	6026.95 +62.2¢	6971.61 +83.1¢	7950.27 +106.61¢	8966.73 +132.5¢	10024.5 +160.55¢	11126.8 +190.53¢	12276.7 +222.21¢	13476.9 +255.38¢	14729.8 +289.84¢	
A5	880.877 +1.73¢	1763.72 +3.66¢	2654.39 +9.41¢	3558.67 +18.92¢	4482.23 +32.06¢	5430.5 +48.66¢	6408.73 +68.53¢	7421.83 +91.44¢	8474.44 +117.15¢	9570.88 +145.38¢	10715.1 +175.89¢	11910.8 +208.4¢	13161.3 +242.66¢	14469.5 +278.42¢	15838.2 +315.45¢	
A#5	933.365 +1.93¢	1868.99 +4.02¢	2813.6 +10.26¢	3773.87 +20.57¢	4756.25 +34.79¢	5766.99 +52.74¢	6812.01 +74.19¢	7896.92 +98.86¢	9026.95 +126.49¢	10206.9 +156.77¢	11441.3 +189.4¢	12734. +224.1¢	14088.9 +260.57¢	15509.1 +298.54¢		
B5	988.988 +2.14¢	1980.65 +4.49¢	2983. +11.48¢	4003.9 +23.¢	5051. +38.88¢	6131.64 +58.89¢	7252.74 +82.72¢	8420.82 +110.07¢	9641.89 +140.58¢	10921.5 +173.92¢	12264.6 +209.71¢	13675.8 +247.63¢	15159.2 +287.33¢			
C6	1047.94 +2.37¢	2098.92 +4.89¢	3162.06 +12.39¢	4246.29 +24.76¢	5360.31 +41.78¢	6512.38 +63.18¢	7710.33 +88.64¢	8961.45 +117.79¢	10272.4 +150.25¢	11649.5 +185.63¢	13098. +223.53¢	14623.1 +263.57¢				
C#6	1110.41 +2.62¢	2224.48 +5.48¢	3353.16 +13.99¢	4507.18 +27.99¢	5696.9 +47.21¢	6932.14 +71.32¢	8222.12 +99.9¢	9575.36 +132.51¢	10999.6 +168.66¢	12501.9 +207.89¢	14088.5 +249.73¢	15764.9 +293.73¢				
D6	1176.62 +2.89¢	2357.54 +6.05¢	3555.6 +15.47¢	4783.35 +30.94¢	6052.88 +52.15¢	7375.59 +78.67¢	8762.09 +110.02¢	10222.1 +145.66¢	11764.5 +185.04¢	13397. +227.61¢	15126.8 +272.83¢					
D#6	1246.8 +3.18¢	2498.26 +6.42¢	3768.31 +16.06¢	5070.57 +31.89¢	6418.12 +53.58¢	7823.31 +80.69¢	9297.59 +112.72¢	10851.5 +149.09¢	12494.4 +189.25¢	14234.8 +232.62¢						
E6	1321.18 +3.5¢	2648. +7.2¢	3997.32 +18.2¢	5385.54 +36.22¢	6828.33 +60.84¢	8340.4 +91.5¢	9935.23 +127.55¢	11625. +168.31¢	13420.7 +213.07¢	15331.8 +261.15¢						
F6	1400.02 +3.86¢	2806.85 +8.06¢	4240.73 +20.53¢	5721.35 +40.94¢	7267.39 +68.73¢	8896.2 +103.19¢	10623.6 +143.52¢	12463.6 +188.89¢	14428.7 +238.44¢							
F#6	1483.6 +4.24¢	2975.01 +8.79¢	4497.45 +22.29¢	6073.44 +44.33¢	7724.25 +74.27¢	9469.54 +111.31¢	11327.1 +154.53¢	13312.5 +202.97¢	15439.7 +255.69¢							
G6	1572.21 +4.67¢	3153.87 +9.86¢	4773.06 +25.26¢	6456.91 +50.33¢	8230.82 +84.25¢	10118. +125.99¢	12139.2 +174.42¢	14312.5 +228.36¢								
G#6	1666.15 +5.14¢	3342.58 +10.47¢	5059.8 +26.26¢	6847.26 +51.95¢	8732.53 +86.68¢	10740.7 +129.38¢	12894.2 +178.87¢	15212.3 +233.92¢								
A6	1765.76 +5.66¢	3544.82 +12.17¢	5376.64 +31.41¢	7298.97 +62.55¢	9346.67 +104.34¢	11550.9 +155.28¢	13938.7 +213.71¢									
A#6	1871.38 +6.24¢	3760.75 +14.54¢	5721.3 +38.97¢	7803.31 +78.22¢	10052.3 +130.34¢	12507.6 +193.04¢	15202.6 +263.98¢									
B6	1983.39 +6.88¢	3988.6 +16.37¢	6079.99 +44.24¢	8317.93 +88.78¢	10756.3 +147.53¢	13440.9 +217.63¢										
C7	2102.2 +7.59¢	4226.28 +16.58¢	6436.91 +43.¢	8794.92 +85.32¢	11354.9 +141.29¢	14163.7 +208.31¢										
C#7	2228.22 +8.38¢	4483.87 +19.01¢	6847.77 +50.12¢	9395.15 +99.61¢	12192.2 +164.47¢	15294.6 +241.29¢										
D7	2361.91 +9.26¢	4756.94 +21.36¢	7282.45 +56.67¢	10028.2 +112.5¢	13071.8 +185.07¢											
D#7	2503.76 +10.23¢	5046.2 +23.55¢	7740.73 +62.32¢	10691.1 +123.32¢	13985.9 +202.08¢											
E7	2654.29 +11.31¢	5353.76 +25.98¢	8230.6 +68.56¢	11404.7 +135.18¢	14976.8 +220.59¢											
F7	2814.06 +12.5¢	5683.43 +29.43¢	8769.37 +78.33¢	12215.9 +154.15¢												
F#7	2983.65 +13.81¢	6046.66 +36.68¢	9418.05 +101.87¢	13294.9 +200.68¢												
G7	3163.72 +15.26¢	6442.39 +46.43¢	10163.3 +133.71¢	14594. +262.08¢												
G#7	3354.95 +16.86¢	6846.71 +51.81¢	10862.5 +148.9¢	15711.8 +289.85¢												
A7	3558.07 +18.63¢	7267.44 +55.05¢	11555.3 +155.94¢													
A#7	3773.86 +20.56¢	7709.59 +57.3¢	12264. +158.99¢													
B7	4003.17 +22.69¢	8182.64 +60.39¢	13035.1 +164.56¢													
C8	4246.92 +25.02¢	8690.75 +64.69¢	13884.5 +173.85¢													

Figure 7-2 Tuning Table for Upright Piano