

Piano Tuning Method

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ABSTRACT

Since the piano string is consider to be a stick rather than a pure ideal string, it contains stiffness and its harmonics will shift in such way that make piano tuning a difficult work. In this work, the method of the optimization algorithm similar to Tunelab®, however, construct and developed all by the author. The algorithm is divided into several models that using various fitting technique to construct model functions, and finally convert to linear regression problem for optimization. Finally, the piano tuning curve is constructed and final tuning frequencies are calculated. In addition, more functions is introduced, such as the different temperament tuning.

Keyword: piano tuning, Tunelab®, inharmonicity, optimization

Project Location

Reference [2]

1 INTRODUCTION

Piano tuning is a difficult work since the harmonics shift that make the piano hard to tune, and tuning process will be a task to highly reduce the audible cacophonous. There are several factors we need to consider, which the rule of harmony is.

- The cacophonous created by its base frequency and audible harmonics; a good tuning will largely reduce the inharmonic for harmonies (the frequency domain will greatly coincide).
- The inner music scales related pitch; the odd pitch tuning will result in the weird sound when playing music scales.

Other famous related works are:

- Tunelab (closed source; has trial version)
- Keyburn CyberTuner (closed source; no trial version)
- Entropy Piano Tuner (open source) [1]

The first two are similar, which represent the old tuning techniques, and my work mostly focus on this algorithm. Since it is closed source, I guessed their tuning method and create a similar solution, and will be shown in this article.

As for Entropy Piano Tuner, it represent the new way of piano tuning, however I heard its demo of tuning, I found that it contain two major deficiencies:

- It violate the second rule of harmony – inner scales sound weird.
- It only consider the sound which at the certain striking level of piano keys, which result in the optimization of keys are based only on sampling striking level.

In my work, I will guess the algorithm and model it in the similar way of optimization. Besides, I used more accurate model for inharmonicity coefficients.

In this article, the first part is to introduce the background of knowledge for higher level modeling algorithms. The second part is to introduce my piano modeling and tuning optimization method. Finally, the future work will be introduce and followed a conclusion.

2 BACKGROUND KNOWLEDGE

2.1 Key Names

The left most key name is defined as “A0”, where “A” is the note name, 0 is the scale number. “C” is the starting point of one scale. It only allowed sharp in the note, flat is not allowed in this naming format.

A0, A#0, B0, C1, C#1, ..., B1, C2, ..., B7, C8

There are 88 keys for standard piano.

2.2 Key Numbers

In the real world, the piano key will labeled with numbers when the piano is open and machine part is shown off.

A0 key is labeled to be 1, and “C8” is 88.

However, in my program, “A0” key is labeled as 0 for easier calculation, which is defined as k .

2.3 Functions

Frequency ratio to cents function:

$$Fr_{\rightarrow c}(\gamma) = 1200 \log_2(\gamma) \quad (2.1)$$

Where cents is from 12 equal temperament, each half note has 100 point, named cents.

Frequency add cents (pitch) function:

$$F_{+c}(f, c) = f \cdot 2^{\left(\frac{c}{1200}\right)} \quad (3.1)$$

This function returns the frequency that added the pitch (cents) c .

The ideal frequency for the key k is:

$$\tilde{f}_k = 440 \cdot 2^{\left(\frac{k-48}{12}\right)} \quad (3.2)$$

Where 440Hz is the international standard pitch for “A4”. Other tuning standard will replace this number, 48 is the key number for “A4”.

3 METHOD

3.1 Sampling Piano

Before tuning a piano, we need to sample a piano by recording the piano keys sound audios. This process will roughly or precisely measure the inharmonicity of piano strings (which will talk about later), such that we could model the inharmonicity for the target piano.

The sampling is suggested to measure keys “C1”, “C2”, “C3”, “C4”, “C5” (and probably “C6”; user could record more piano keys such as “A1” ~ “A6” for better result). The piano key sound should be recorded in a quiet environment, which allows more accuracy for later frequency analysis.

In my program, I use fully or almost fully sampled piano for research purposes.

3.2 Audio Processing

Since the real audio may contains the white space at the start or the end, and the sound length varies. I use this method to process my sampled audio:

- Normalize ($N(x) = x / \max(x)$) the audio file into 1, then, find the peak volume of audio, and start from here.
- Slice these audio pieces into tiny partitions, say 0.1 second is one partition. The maximum number of each partition will be its assumed volume at this time point.
- Select these pieces volume start from some large number to small number – since piano sound is loud from its beginning and decay by the time. Say from 90% to 2% of the sampled sound audio.

3.3 Frequency Analysis

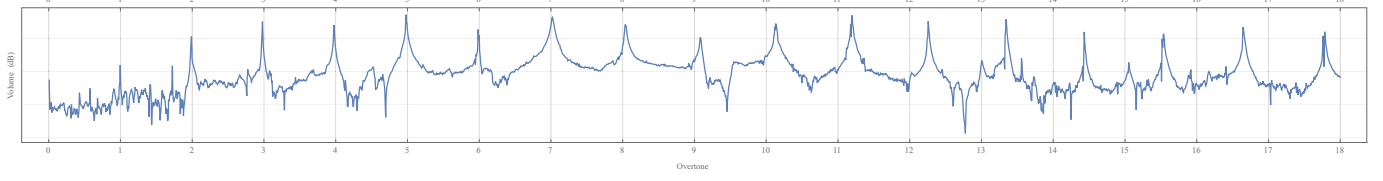


Figure 3-1 “A#0” Key (at Upright Piano Samples) Overtone Plot; Volume at Logarithm Scale

Then, put this audio samples into fourier analysis (FFT algorithm). Then we get the function $G(\omega) = FFT(S(t))$ where $S(t)$ is the audio function, and $G(\omega)$ is the frequency domain function. In our work, the frequency domain is converted to the ratio to its ideal fundamental frequency, thus we can see the Figure 3-1, the peaks will always almost lies in the grid by dividing its ideal frequency.

From Figure 3-1, we can see that the higher overtone (right hand side peaks with larger numbers) shifts higher.

It is a problem to capture all these peaks numbers, since some are not clear: the fundamental frequency (at 1), and some has multiple peaks: at 15 ~ 16.

In my work, I use the frequency *Catchup Method* to get octave values for all these peaks.

3.4 Catchup Overtone

From the charactors of these peaks, there are several charactors will be considered:

- From left to right, the gap between two peaks are increasing gradually.
- The largest value of this plot is probably some peak of overtone
- The valid peak should be nearly larger than fundamental frequency position: at 1.
- The peak may be broken into several peaks, we need centralize the targeted position.

From this characteristics, the *Catchup Method* could be built:

- Analyze the frequency samples which roughly larger than 1 (my program is starting from 0.8), get the peak frequency $f_{k,peak}$ at key number k .
- Comparing with ideal frequency \tilde{f}_k . We can then assume that it is $n = round(f_{k,peak} / \tilde{f}_k)$ harmonics.

Then, we can know its guessed fundamental frequency is $\hat{f}_k = f_{k,peak} / n$. Then, this should be the step size for catchup method.

- The catchup method is forward (goes to the right), and the backward (goes to the left). If we are in the forward operation, the next guessed target frequency is $\hat{f}_{k,peak+1} = f_{k,peak} + f'_k$, where f'_k is the assumed gap between two peak at this position. In the first try, we set this number to $f'_k = \hat{f}_k$, and this number will be increasing for more right harmonics. Then, we get the around data (in a relatively small area) for guessed target frequency $\hat{f}_{k,peak+1} \pm \delta$, we can find its maximum number these data to be the frequency candidate $\hat{f}_{k,peak+1}^{candidate}$, then we get the data of smaller surround area $\hat{f}_{k,peak+1}^{candidate} \pm \delta'$ where $\delta' \ll \delta$. Then, we calculate the weighted average for this smaller area, and the result is the actual frequency of this peak

$f_{k,peak+1} = \int_{\hat{f}-\delta'}^{\hat{f}+\delta'} \omega \cdot G(\omega) d\omega$, where ω is proportional to frequency. Then, the assumed gap between two peak at this step is updated to be $f'_k = f_{k,peak+1} - f_{k,peak}$.

- Iterate this method for forward catchup to get all higher frequencies.
- If the highest peak is not fundamental frequency, we will perform the backward catchup. Since there are less peaks and the overtone shift will be far less than the right, the assumed targeted gap between two peaks is set to be the assumed fundamental frequency \hat{f}_k .

From this method, we can get a overtone (frequency) list for the key k . Which is:

$$k \rightarrow \{f_{k,1}, f_{k,2}, \dots\} \quad (3.3)$$

3.5 Inharmonicity Model

From reference [1], we assume that the piano string is a bar, which follows the partial differential equation:

$$\ddot{y} \propto -y'' - \varepsilon y'''' \quad (3.4)$$

Where y is the special position of piano string (bar model). The prime is the derivative to spatial domain, and dots is the derivative to time domain.

Then, use the modal analysis and solved the natural frequencies for this string are:

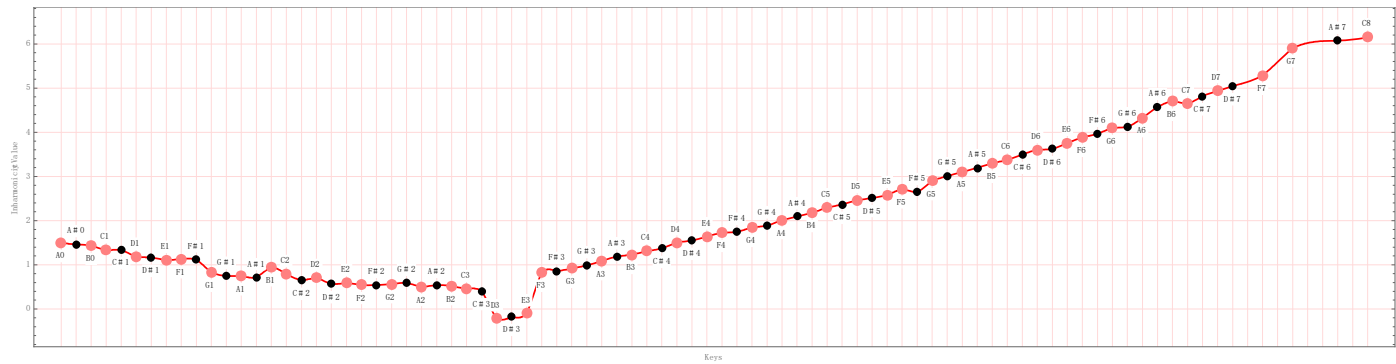
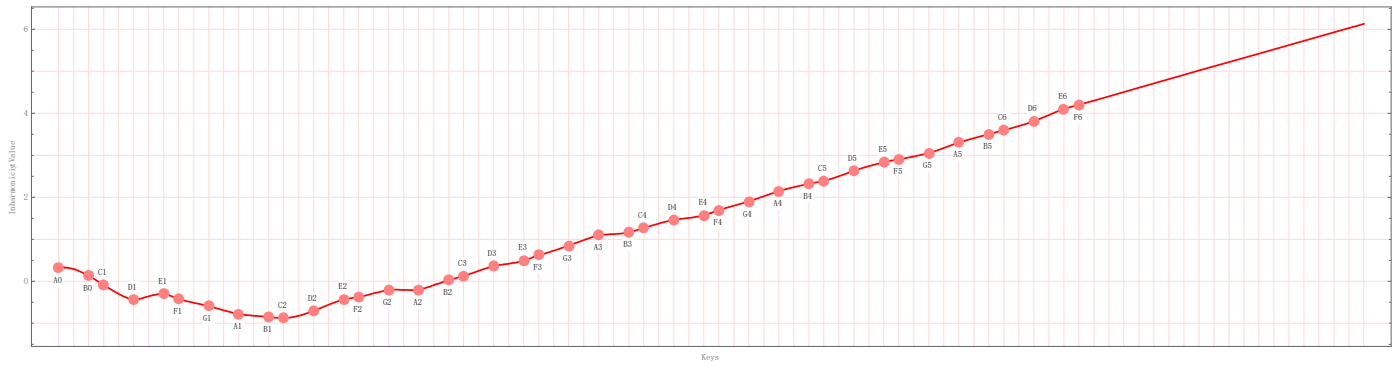
$$f_{k,n} \propto n \cdot f_{k,1} \sqrt{1 + B_k \cdot n^2} \Rightarrow f_{k,n} = A_k \cdot n \cdot f_{k,1} \sqrt{1 + B_k \cdot n^2} \quad (3.5)$$

Here we have two unknown variables.

Then, we use this function to fit all frequency results at Eq.(3.3). Since A_k value is always almost 1 all the time, we can ignore this number, and focus only on B_k . However in the optimization process, with parameter A_k could achieve much better result, although finally its value is almost 1. We set 0 to be the fundamental frequency is that when $n = 0$ that the equation holds, we will restore this number later.

Then, we can get inharmonicity parameter list $\{\{k, B_k\}\}$.

From my observation, the logarithm of this number has some beautiful properties with the data $\{\{k, \ln(s \cdot B_k)\}\}$, where s is a scaling parameter (I set to 10000).



From Figure 3-2 and Figure 3-3, we can clearly see the line is divided into 2 parts.



Figure 3-4 Grand Piano String Arrangement



Figure 3-5 Upright Piano String Arrangement

From Figure 3-4 and Figure 3-5, we can clearly see that the string is divided into two parts, the steel string and copper string (may be covered by silver for highly expensive pianos). The upright piano has more copper strings since the steel string cannot go longer, and the string will become thicker to make the string vibrate slower. From spring vibration formula:

$$\omega = \sqrt{\frac{K}{m}} \quad (3.6)$$

Where ω is proportional to frequency, m is the mass of spring, K is the stiffness of spring.

When m increases, K increase a little bit, ω decreases, then frequency decrease.

Since the piano cannot growing longer, it become thick and more like a stick rather than an ideal string. For higher notes strings, it is too short, and the thickness become relatively larger comparing to its length, thus it is more likely to be a bar.

Thus, from the plot, we can see the inharmonicity increases at two ends, and break at the position of separation of two kinds of strings.

Since grand concert piano is longer, and can have more steel strings, less copper strings, thus the break will become more left side.

The figure of inharmonicity plot also tell us that two separate line are almost linear. In my model, I used the valid sampled points are modeled with interpolation function, and two edges are modeled with linear function, and it is method is shown below.

- We get several samples from one line, and fit in a linear form.
- Get its slope, and build a line which pass the right end point (since I will not wish to have a break for the interpolation function), and add some samples for edges situation to sample pool.
- Similar to the left hand side.
- We use interpolation for these samples of sample pool – “Left hand side + samples + right hand side”, which is our final model for inharmonicity model function $Ih(k)$.

$$Ih(k) = \ln(s \cdot B_k) \quad (3.7)$$

Thus, we can have the modeled parameter B_k with:

$$B_k = \frac{e^{Ih(k)}}{s} \quad (3.8)$$

Then, the frequencies $\tau(k, n)$ will be:

$$\tau(k, n) = f_{k,1} \cdot n \cdot \sqrt{1 + B_k \cdot n^2} \quad (3.9)$$

Where $f_{k,1}$ is currently unknown but it will be eliminated, since it is in frequency ratio form.

3.6 Tuning Curve Optimization Model

Similar to Tunelab, I set the tuning optimization method to separate the lower tones (bass) and higher tones (tenor) into two tuning target optimization method, the separation point k_0 is “C#4/D4”. And the default tuning method for bass is to set 6:3. Since $6/3=2$ (a/b), this frequency ratio is $\gamma = a/b$, and its corresponding pitch range is $Fr_{\rightarrow c}(\gamma)$ which is 1200, and 1200 is an octave, it means the tone say “A0”’s 6th harmonics will largely match its octave’s “A1”’s 3rd harmonics.

Here pitch is defined by cents.

The error function ε_k is defined as:

$$\begin{aligned} \varepsilon_k &= Fr_{\rightarrow c} \left(\frac{\tau(k, a)}{\tau(k + Fr_{\rightarrow c}(a/b), b)} \right) \\ &= Fr_{\rightarrow c} \left(\sqrt{\frac{1 + B_k \cdot a^2}{1 + B_{k+Fr_{\rightarrow c}(a/b)} \cdot b^2}} \cdot \frac{a}{b} \cdot \left(\frac{f_{k,1}}{f_{k+Fr_{\rightarrow c}(a/b),1}} \right) \right) \\ &= Fr_{\rightarrow c} \left(\sqrt{\frac{1 + B_k \cdot a^2}{1 + B_{k+Fr_{\rightarrow c}(a/b)} \cdot b^2}} \right) \end{aligned} \quad (3.10)$$

We can do this for all bass strings.

For tenor strings, the default tuning method is set to 4:1 (c/d). But this time we count the higher note as the target to calculate.

$$\varepsilon_k = Fr_{\rightarrow c} \left(\sqrt{\frac{1 + B_{k-Fr_{\rightarrow c}(c/d)} \cdot c^2}{1 + B_k \cdot d^2}} \right) \quad (3.11)$$

The combined expression is:

$$E(k) = \begin{cases} Fr_{\rightarrow c} \left(\sqrt{\frac{1 + B_k \cdot a^2}{1 + B_{k+Fr_{\rightarrow c}(a/b)} \cdot b^2}} \right) & k \leq k_0 \\ Fr_{\rightarrow c} \left(\sqrt{\frac{1 + B_{k-Fr_{\rightarrow c}(c/d)} \cdot c^2}{1 + B_k \cdot d^2}} \right) & k > k_0 \end{cases} \quad (3.12)$$

From this equation, we can see $E(k)$ is only a value for calculation.

From this point, we need a function to largely eliminate these errors. The piano tuning curve $C(k)$ is introduced.

The deviation function $D(k)$ is:

$$D(k) = C(k) - E(k) \quad (3.13)$$

The cost function $J(k)$ for optimization is:

$$J(k) = \sum_k (D(k))^2 \quad (3.14)$$

Which minimize the square error of these functions.

Here I use polynomial for easier calculation:

$$C(x) = \sum_{i=1}^n \chi_i \cdot x^i \quad (3.15)$$

Since $C(x)$ will pass the fix point, which is “A4” pitch at 440Hz frequency at pitch deviation of 0, thus i is from 1 and $x = k - k_{A4}$, where k_{A4} is the key number (index) at “A4”, which is 48.

Thus, $J(k)$ is the second order multi-variable polynomial function, which is very easy to minimize by linear regression method to calculate the fitting parameter $\{\chi_i\}$, and rebuild the functions.

Then, we can bring it to the $J(k)$ function to calculate its deviations.

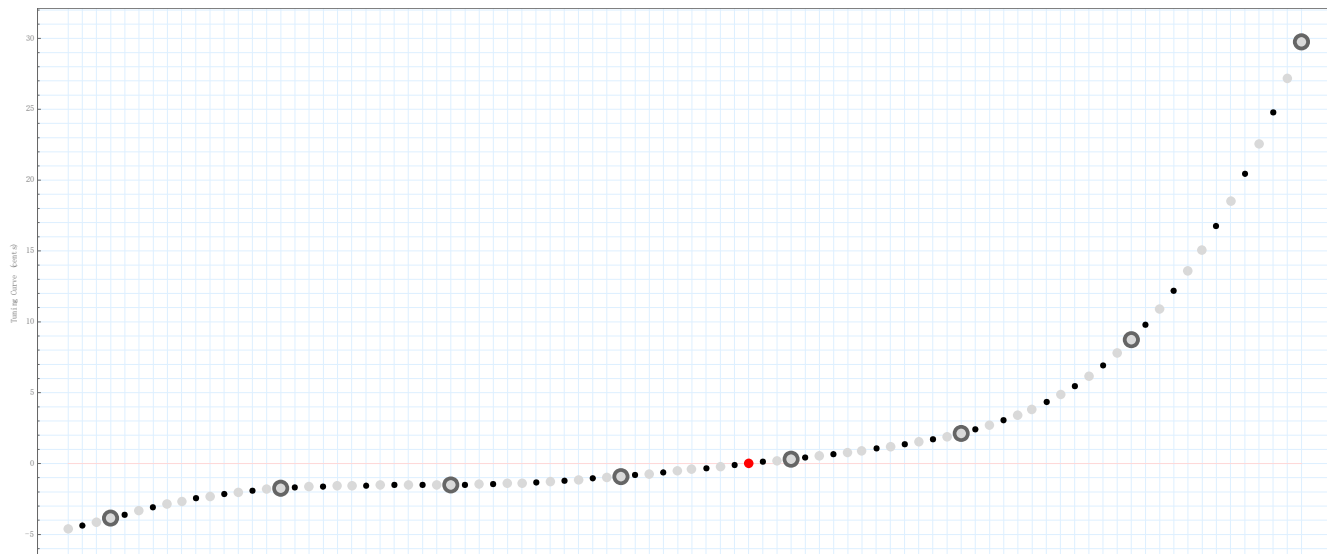


Figure 3-6 $C(x)$ for Grand Piano

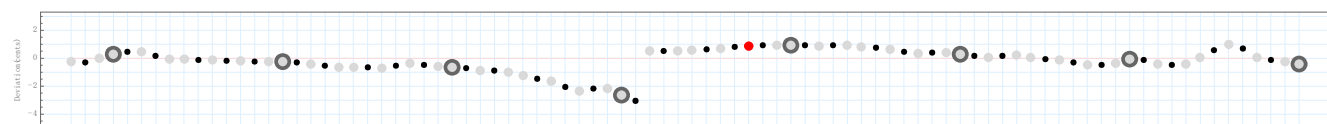


Figure 3-7 $D(x)$ for Grand Piano

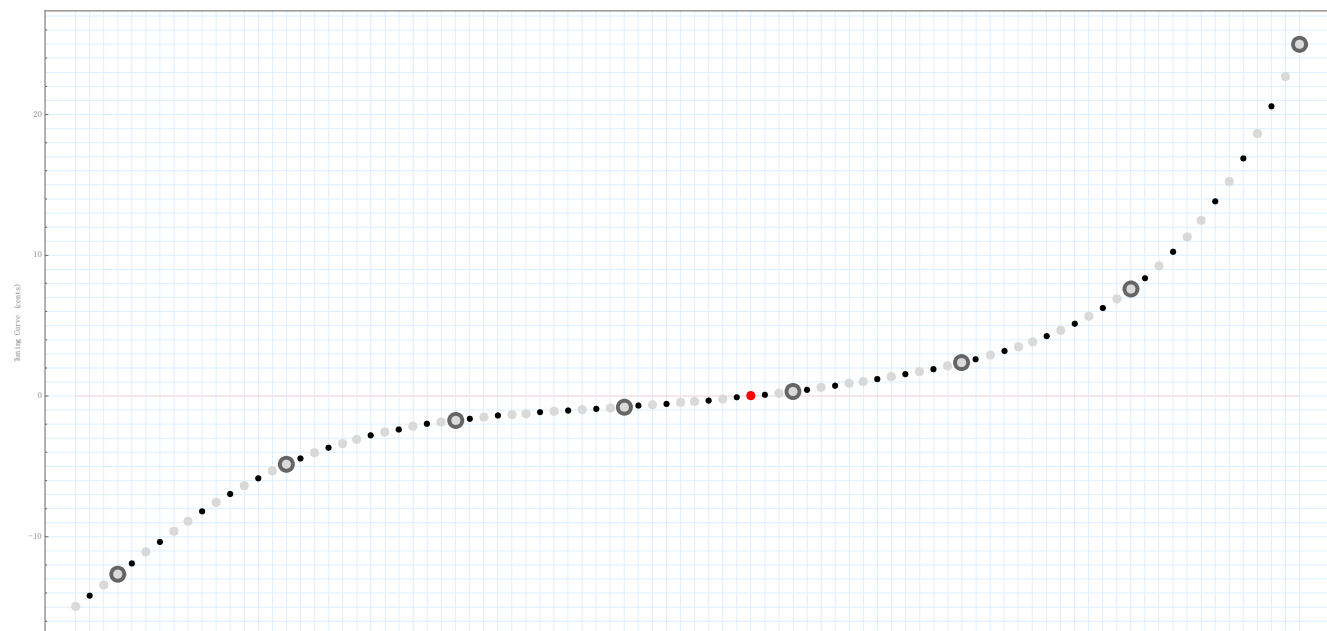


Figure 3-8 $C(x)$ for Upright Piano

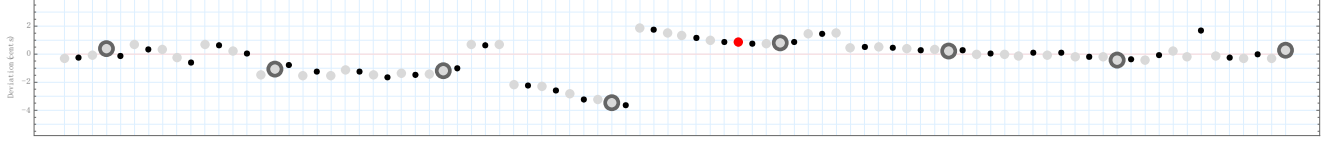


Figure 3-9 $D(x)$ for Upright Piano

The result of two piano is shown above. Horizontal axis is the key number, and the vertical axis the pitch deviation with idea frequencies represented by cents.

From this tuning method, we can see that the bass tuning will consider the deviations from the tenor part, and vice versa. The effect are inner related. Thus this tuning method is theoretically to optimize almost the whole piano keys tuning.

3.7 Temperament Model

With the development of music, various temperament appears and create unique flavor of music. The temperament model is using the pitch deviation tables of different temperament (the unit is cent). We can then create the non 12 equal temperament tuning strategy. The temperament function is defined to be $T(k)$.

The tuning table such as “Bach - Bradley Lehman” is:

C	C#	D	D#	E	F	F#	G	G#	A	A#	B
5.87	3.91	1.96	3.91	-1.96	7.82	1.96	3.91	3.81	0	3.91	0

Table 3-1 Table for “Bach - Bradley Lehman” Temperament

Where A note will always be 0 since A is the reference frequency and will always keep to 440 Hz (if is standard situation).

This table shows the situation of “C” major.

The other major tuning will follow the rotation of table. For example: if tuning “D” major, the “D” will rotate to current “D” \rightarrow “C” place, which is rotating left 2 times. However, we will make sure “A” note will always be 0, then, we can subtract the number at “B” \rightarrow “A” to make it possible.

Then, add these pitch errors to all the notes of tuning, the modified tuning curve is:

$$C'(k) = C(k) + T(k) \quad (3.16)$$

3.8 Creating Tuning Table

The final tuning frequency $\tau(k, n)$ is:

$$f_{k,1} = F_{+c}(\hat{f}_k, C'(k)) \quad (3.17)$$

$$\begin{aligned}
\tau(k, n) &= f_{k,1} \cdot n \cdot \sqrt{1 + B_k \cdot n^2} \\
&= F_{+c}(\hat{f}_k, C'(k)) \cdot n \cdot \sqrt{1 + B_k \cdot n^2} \\
&= F_{+c}(\hat{f}_k, C'(k)) \cdot n \cdot \sqrt{1 + \frac{e^{lh(k)}}{s} \cdot n^2}
\end{aligned} \quad (3.18)$$

From Eq.(3.18), we can see only C and Ih function is modeled function, other function are basic mathematics functions.

From the modeling, we can get a strategy of piano tuning, then we can convert this strategy into a tuning table, which shows all the frequency of fundamental and its harmonics frequencies, and corresponding deviation to ideal frequencies represented by cents.

The grand and upright piano tuning strategy is shown in Figure 7-1 and Figure 7-2.

The red font is the frequencies recommended for the devices to tune.

4 FUTURE WORK

Although the Entropy piano tuning method is far more advanced than this tuning method theoretically, however the jumpy tuning curve will make the music scales sound weird. If I have time, I will implement the entropy tuner with much more smooth functions, this construction is easier than original idea since the optimization at his method is only few parameters if using similar polynomial to optimize the curve to achieve more smooth result with entropy function as cost function.

Over-pull tuning is implemented experimentally with their tuning apps, and I do not know its method due to its close source reason. And I am still lack of research on this area, thus I will leave it as future work to think about. I know this effect is caused by the experimental result of the percentage that the string pins will loosen and drop the pitch, the tuner will make up the errors of this effect by over pull and tune higher tones.

5 CONCLUSION

This tuning method gives us a solution of piano tuning that works as well as commercial apps Tunelab. The method is presented to optimize the whole piano notes sound.

Future work is given to develop maybe in the future.

6 REFERENCE

[1] Hinrichsen, Haye. "Entropy-based tuning of musical instruments." *Revista brasileira de Ensino de Física* 34.2 (2012): 1-8.

[2] Github for Piano Tuning Project [https://github.com/RobertBoganKang/piano_tuning]

7 APPENDIX

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A0	27.4286	54.857	82.3025	109.774	137.284	164.843	192.461	220.152	247.925	275.791	303.762	331.849	360.063	388.414	416.913	445.571
A#0	29.0617	58.1273	87.2084	116.317	145.464	174.661	203.921	233.254	262.672	292.187	321.809	351.55	381.422	411.433	441.601	471.93
B0	30.7944	61.5924	92.4044	123.241	154.113	185.03	216.004	247.044	278.161	309.365	340.667	372.077	403.606	435.262	467.057	499.000
C1	32.6306	65.2641	97.9096	130.576	163.272	196.008	228.791	261.633	294.533	327.515	360.577	393.732	426.988	460.352	493.835	527.445
C#1	34.5761	69.1548	103.744	138.351	172.983	207.649	242.355	277.111	311.922	346.797	381.744	416.769	451.881	487.087	522.394	557.81
D1	36.6375	73.2773	109.926	146.592	183.281	220.01	256.758	293.559	330.412	367.324	404.301	441.35	478.478	515.693	553.001	590.408
D#1	38.8214	77.6456	116.481	155.335	194.216	233.132	272.092	311.103	350.174	389.312	428.526	467.823	507.213	546.701	586.209	626.009
E1	41.1352	82.2735	123.424	164.596	205.798	247.04	288.331	329.68	371.095	412.587	454.164	495.835	537.609	579.494	621.501	663.638
F1	43.5865	87.1758	130.777	174.398	218.047	261.74	305.466	349.253	393.103	437.024	481.025	525.114	569.3	613.592	657.996	702.523
F#1	46.1833	92.3694	138.567	184.783	231.027	277.308	323.632	370.009	416.447	462.954	509.538	556.208	602.972	649.837	696.812	743.905
G1	48.9343	97.8713	146.819	195.786	244.78	293.809	342.882	392.006	441.19	490.441	539.768	589.179	638.681	688.284	737.994	787.82
G#1	51.8485	103.7	155.561	207.441	259.347	311.286	363.267	415.297	467.385	519.537	571.762	624.067	676.46	728.949	781.541	834.243
A1	54.9357	109.874	164.822	219.788	274.779	329.803	384.868	439.98	495.147	550.378	605.678	661.057	716.521	772.078	827.735	883.499
A#1	58.2061	116.415	174.634	232.87	291.133	349.429	407.765	466.151	524.593	583.098	641.676	700.332	759.076	817.914	876.833	935.902
B1	61.6705	123.344	187.027	246.729	308.458	370.221	432.026	492.882	555.795	617.774	679.827	741.962	804.187	866.504	928.935	991.474
C2	65.3494	130.684	196.038	261.411	325.255	392.249	457.725	523.255	588.845	654.503	720.237	786.054	851.963	917.973	984.09	1050.32
C#2	69.2282	138.459	207.703	276.968	346.263	415.598	484.982	554.423	623.932	693.516	763.186	832.949	902.816	972.793	1042.9	1113.13
D2	73.3468	146.697	220.062	293.452	366.879	440.352	513.884	587.484	661.164	734.934	808.806	882.789	956.895	1031.1	1105.52	1180.06
D#2	77.7098	155.424	233.156	310.919	388.727	466.593	544.53	622.551	700.671	778.901	857.256	935.748	1014.39	1093.2	1172.18	1251.35
E2	82.3139	164.669	247.028	329.424	411.874	494.393	576.997	659.704	742.527	825.484	908.59	991.861	1075.31	1158.99	1242.82	1326.91
F2	87.2286	174.463	261.227	343.022	436.382	523.82	611.353	698.999	786.777	874.703	962.797	1051.07	1139.55	1228.25	1317.19	1406.38
F#2	92.4162	184.839	277.29	369.789	462.358	555.016	647.786	740.686	833.739	926.964	1020.38	1114.01	1207.88	1302	1396.39	1491.08
G2	97.912	195.832	293.784	391.792	489.88	588.072	686.392	784.864	883.511	982.358	1081.43	1180.74	1280.33	1380.19	1480.41	1580.94
G#2	103.734	207.477	311.254	415.09	519.01	623.04	727.205	831.531	936.041	1040.76	1145.72	1250.93	1356.43	1462.25	1568.39	1674.89
A2	109.903	219.815	329.763	435.774	549.875	660.092	770.453	880.984	991.712	1102.66	1213.86	1325.34	1437.12	1549.23	1661.7	1774.54
A#2	116.439	232.888	343.38	455.946	569.428	682.618	796.408	910.589	1021.53	1132.84	1246.65	1361.404	1476.93	1593.26	1709.52	1826.63
B2	123.363	246.739	362.186	480.682	597.325	714.134	835.145	958.368	1081.93	1208.78	1336.98	1469.58	1601.6	1732.05	1869.08	1996.6
C3	130.746	261.414	390.168	521.065	654.091	785.309	916.764	1048.513	1180.56	1312.92	1445.83	1579.13	1712.92	1852.18	1992.73	2130.48
C#3	138.473	276.963	415.024	554.21	693.073	832.166	971.543	1111.26	1251.36	1391.9	1532.93	1674.51	1816.69	1959.51	2108.03	2247.29
D3	146.708	293.438	440.252	587.215	734.39	881.84	1029.63	1177.82	1326.12	1475.65	1625.42	1775.84	1925.96	2078.86	2231.09	2385.22
D#3	155.434	310.892	466.445	622.163	778.17	934.378	1091.02	1248.1	1405.7	1563.89	1722.74	1882.3	2042.66	2203.88	2366.02	2529.16
E3	164.68	329.387	494.201	659.294	824.475	990.094	1156.14	1322.7	1489.84	1657.65	1826.2	1995.58	2165.85	2337.1	2509.4	2682.82
F3	174.476	348.985	523.625	698.494	873.69	1049.31	1225.45	1402.21	1579.68	1757.97	1937.16	2117.34	2298.62	2481.06	2664.82	2849.92
F#3	184.856	369.751	554.802	740.123	925.833	1112.05	1298.88	1486.4	1674.85	1864.22	2054.67	2246.29	2439.21	2633.53	2829.36	3026.8
G3	195.854	391.754	587.837	784.239	981.097	1178.55	1376.72	1575.76	1775.8	1976.96	2179.38	2383.19	2588.52	2795.49	3004.23	3214.86
G#3	207.508	415.071	625.855	831.028	1039.75	1249.2	1459.52	1670.89	1883.47	2097.42	2312.88	2530.03	2749.02	2969.99	3193.41	3418.49
A3	219.855	437.676	659.661	880.607	1101.91	1324.07	1547.28	1771.73	2000.67	2225.12	2454.44	2685.76	2919.25	3155.08	3393.47	3634.56
A#3	232.938	465.948	699.244	933.042	1167.55	1402.99	1639.57	1877.5	2116.99	2358.24	2601.45	2846.83	3094.57	3344.87	3597.91	3853.89
B3	246.8	493.68	740.874	988.619	1237.15	1486.7	1737.5	1989.79	2243.79	2499.72	2757.81	3018.28	3281.34	3547.21	3816.1	4088.2
C4	261.899	523.071	785.024	1043.63	1311.16	1575.9	1842.11	2108.108	2380.06	2652.32	2927.13	3204.74	3485.4	3769.36	4056.87	4348.15
C#4	277.052	548.213	831.839	1110.18	1389.63	1670.51	1953.11	2237.78	2524.81	2814.52	3107.22	3403.21	3702.77	4006.19	4313.77	4625.77
D4	293.542	587.211	881.389	1176.79	1472.79	1770.76	2070.75	2372.12	2678.24	2986.46	3298.14	3613.62	3933.24	4257.32	4586.2	4920.18
D#4	311.014	622.169	933.886	1246.58	1560.68	1876.59	2194.73	2515.5	2839.3	3166.53	3497.58	3832.82	4172.62	4517.03	4867.37	5223.01
E4	329.327	659.213	988.532	1320.96	1653.96	1989.01	2326.56	2667.08	3011.02	3358.81	3710.89	4067.7	4429.63	4797.1	5170.51	5550.23
F4	349.143	698.475	1048.57	1399.99	1753.3	2109.07	2467.84	2830.16	3196.57	3567.59	3943.74	4325.53	4713.44	5107.95	5509.52	5918.6
F#4	369.926	740.676	1111.12	1483.73	1858.56	2236.27	2617.52	3002.94	3393.15	3788.78	4190.43	4598.67	5014.07	5437.19	5862.67	6308.67
G4	391.947	784.156	1177.41	1572.5	1970.19	2371.26	2776.45	3186.53	3602.21	4024.21	4453.22	4889.93	5334.97	5788.99	6252.59	6726.36
A4	415.279	830.873	1247.72	1666.77	2088.95	2515.18	2946.36	3383.38	3827.12	4278.41	4738.09	5206.93	5685.72	6175.18	6676.03	7188.93
A#4	440.161	880.377	1322.26	1766.77	2215.02	2668.12	3127.13	3593.11	4067.1	4550.09	5043.04	5546.89	6062.53	6590.82	7132.57	7688.56
B4	466.193	932.825	1401.81	1872.66	2348.47	2829.92	3318.25	3814.69	4320.43	4836.61	5364.35	5904.71	6458.7	7027.31	7611.45	8212.1
C5	493.549	988.395	1484.86	1984.84	2489.81	3001.25	3520.57	4049.18	4588.42	5139.6	5703.97	6282.74	6877.05	7487.99	8116.59	8763.83
C#5	523.35	1047.27	1573.47	2103.65	2639.48	3182.62	3734.68	4297.22	4864.71	5459.78	6062.67	6681.76	7318.34	7973.61	8648.72	9344.72
D5	554.506	1109.69	1657.37	2230.16	2799.45	3360.63	3955.83	4566.67	5181.76	5812.5	6460.81	7128.15	7815.97	8525.65	9258.48	10015.7
D#5	587.517	1175.85	1767.44	2364.71	2970.04	3585.77	4214.16	4857.41	5517.02	6196.79	6896.82	7619.53	8366.58	9139.57	9939.97	10769.1
E5	622.494	1245.95	1873.26	2507.29	3150.84	3806.66	4477.41	5165.65	5873.82	6604.25	7359.12	8140.49	8950.28	9790.28	10662.1	11567.4
F5	659.556	1320.23	1985.4	2658.37	3342.42	4040.72	4756.35	5492.24	6251.21	7035.89	7848.76	8692.16	9568.23	10479.8	11426.2	12411.6
F#5	698.626	1398.93	2124.14	2818.23	3544.91	4287.78	5050.32	5835.84	6647.49	7488.23	8360.61	9267.83	10211.7	11194.5	12218.3	13284.9
G5	740.438	1484.182	2229.99	2987.71	3759.68	4549.97	5362.48	6200.93	7068.83	7969.49	8905.97	9881.1	10897.5	11957.6	13063.6	14217.3
G#5	784.331	1570.17	2363.53	3167.86	3988.49	4830.05	5696.99	6593.51	7523.57	8490.88	9498.86	10550.7	11649.2	12797.7	13966.5	15249.9
A5	831.267	1674.5	2550.67	3360.63	4235.08	5134.52	6064.18	7029.02	8033.64	9124.87	10292.31	11327.2	12530.2	13790.9	15111.9	16548.9
A#5	880.772	1763.94	2656.67	3566.01	4498.8	5461										

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																										
A0	27.2634	-13.592	54.5391	-13.592	81.8635	-13.592	109.272	-13.592	136.805	-13.592	164.403	-13.592	192.376	-13.592	220.487	-13.592	248.801	-13.592	277.533	-13.592	306.537	-13.592	335.905	-13.592	365.67	-13.592	395.657	-13.592	425.957	-13.592	456.514	-13.592	487.337	-13.592	518.427	-13.592	549.782	-13.592	581.403	-13.592	613.291	-13.592	645.446	-13.592	677.869	-13.592	710.561	-13.592	743.531	-13.592	776.779	-13.592	810.306	-13.592	844.113	-13.592	878.201	-13.592	912.571	-13.592	947.224	-13.592	982.161	-13.592	1017.383	-13.592	1052.892	-13.592	1088.689	-13.592	1124.774	-13.592	1161.148	-13.592	1197.811	-13.592	1234.764	-13.592	1272.007	-13.592	1309.54	-13.592	1347.373	-13.592	1385.506	-13.592	1423.944	-13.592	1462.687	-13.592	1501.735	-13.592	1541.088	-13.592	1580.746	-13.592	1620.709	-13.592	1660.977	-13.592	1701.55	-13.592	1742.433	-13.592	1783.616	-13.592	1825.104	-13.592	1866.897	-13.592	1908.995	-13.592	1951.398	-13.592	1994.106	-13.592	2037.119	-13.592	2080.437	-13.592	2124.06	-13.592	2167.987	-13.592	2212.215	-13.592	2256.744	-13.592	2301.574	-13.592	2346.705	-13.592	2392.137	-13.592	2437.87	-13.592	2483.907	-13.592	2530.246	-13.592	2576.885	-13.592	2623.824	-13.592	2671.063	-13.592	2718.602	-13.592	2766.441	-13.592	2814.58	-13.592	2863.027	-13.592	2911.774	-13.592	2960.821	-13.592	3010.168	-13.592	3059.815	-13.592	3109.762	-13.592	3159.909	-13.592	3210.256	-13.592	3260.803	-13.592	3311.55	-13.592	3362.502	-13.592	3413.651	-13.592	3464.9	-13.592	3516.347	-13.592	3567.994	-13.592	3619.841	-13.592	3671.888	-13.592	3724.135	-13.592	3776.582	-13.592	3829.229	-13.592	3882.076	-13.592	3935.123	-13.592	3988.37	-13.592	4041.814	-13.592	4095.451	-13.592	4149.288	-13.592	4203.325	-13.592	4257.562	-13.592	4311.999	-13.592	4366.636	-13.592	4421.473	-13.592	4476.51	-13.592	4531.747	-13.592	4587.184	-13.592	4642.821	-13.592	4698.558	-13.592	4754.495	-13.592	4810.632	-13.592	4866.969	-13.592	4923.506	-13.592	4980.243	-13.592	5037.18	-13.592	5094.317	-13.592	5151.654	-13.592	5209.191	-13.592	5266.928	-13.592	5324.865	-13.592	5383.002	-13.592	5441.339	-13.592	5500.076	-13.592	5559.113	-13.592	5618.45	-13.592	5678.087	-13.592	5737.924	-13.592	5797.961	-13.592	5858.198	-13.592	5918.635	-13.592	5979.272	-13.592	6040.109	-13.592	6101.246	-13.592	6162.583	-13.592	6224.12	-13.592	6285.857	-13.592	6347.794	-13.592	6409.931	-13.592	6472.268	-13.592	6534.805	-13.592	6597.542	-13.592	6660.479	-13.592	6723.616	-13.592	6786.953	-13.592	6850.49	-13.592	6914.227	-13.592	6978.164	-13.592	7042.301	-13.592	7106.638	-13.592	7171.175	-13.592	7235.912	-13.592	7300.849	-13.592	7365.986	-13.592	7431.323	-13.592	7496.86	-13.592	7562.597	-13.592	7628.534	-13.592	7694.671	-13.592	7760.908	-13.592	7827.345	-13.592	7893.982	-13.592	7960.819	-13.592	8027.856	-13.592	8095.093	-13.592	8162.53	-13.592	8230.267	-13.592	8298.2	-13.592	8366.437	-13.592	8434.874	-13.592	8503.511	-13.592	8572.348	-13.592	8641.385	-13.592	8710.622	-13.592	8780.059	-13.592	8849.696	-13.592	8919.533	-13.592	8989.57	-13.592	9059.814	-13.592	9130.251	-13.592	9200.888	-13.592	9271.725	-13.592	9342.762	-13.592	9413.999	-13.592	9485.436	-13.592	9557.073	-13.592	9628.91	-13.592	9700.947	-13.592	9773.184	-13.592	9845.621	-13.592	9918.258	-13.592	9991.095	-13.592	10064.132	-13.592	10137.369	-13.592	10210.806	-13.592	10284.443	-13.592	10358.28	-13.592	10432.317	-13.592	10506.554	-13.592	10580.991	-13.592	10655.628	-13.592	10730.465	-13.592	10805.502	-13.592	10880.739	-13.592	10956.176	-13.592	11031.813	-13.592	11107.65	-13.592	11183.687	-13.592	11260.024	-13.592	11336.661	-13.592	11413.5	-13.592	11490.637	-13.592	11567.974	-13.592	11645.511	-13.592	11723.248	-13.592	11801.185	-13.592	11879.322	-13.592	11957.659	-13.592	12036.196	-13.592	12114.933	-13.592	12193.87	-13.592	12273.014	-13.592	12352.351	-13.592	12431.888	-13.592	12511.625	-13.592	12591.562	-13.592	12671.699	-13.592	12752.036	-13.592	12832.573	-13.592	12913.31	-13.592	12994.247	-13.592	13075.384	-13.592	13156.721	-13.592	13238.258	-13.592	13319.995	-13.592	13401.932	-13.592	13484.069	-13.592	13566.306	-13.592	13648.743	-13.592	13731.38	-13.592	13814.217	-13.592	13897.254	-13.592	13980.491	-13.592	14063.928	-13.592	14147.565	-13.592	14232.402	-13.592	14317.439	-13.592	14402.676	-13.592	14488.113	-13.592	14573.75	-13.592	14659.587	-13.592	14745.624	-13.592	14831.861	-13.592	14918.298	-13.592	15004.935	-13.592	15091.772	-13.592	15178.809	-13.592	15266.046	-13.592	15353.483	-13.592	15441.12	-13.592	15528.957	-13.592	15616.994	-13.592	15705.231	-13.592	15793.668	-13.592	15882.305	-13.592	15971.142	-13.592	16060.179	-13.592	16149.416	-13.592	16238.853	-13.592	16328.49	-13.592	16418.327	-13.592	16508.364	-13.592	16608.601	-13.592	16709.038	-13.592	16809.675	-13.592	16910.512	-13.592	17011.549	-13.592	17112.786	-13.592	17216.223	-13.592	17319.86	-13.592	17423.697	-13.592	17527.734	-13.592	17631.971	-13.592	17736.408	-13.592	17841.045	-13.592	17945.882	-13.592	18050.919	-13.592	18156.156	-13.592	18261.593	-13.592	18367.23	-13.592	18473.067	-13.592	18579.104	-13.592	18685.341	-13.592	18791.778	-13.592	18898.415	-13.592	19005.252	-13.592	19112.289	-13.592	19219.526	-13.592	19326.963	-13.592	19434.6	-13.592	19542.437	-13.592	19650.474	-13.592	19758.711	-13.592	19867.148	-13.592	19975.785	-13.592	20084.622	-13.592	20193.659	-13.592	20302.896	-13.592	20412.333	-13.592	20521.97	-13.592	20631.807	-13.592	20741.844	-13.592	20852.081	-13.592	20962.518	-13.592	21073.155	-13.592	21183.992	-13.592	21295.029	-13.592	21406.266	-13.592	21517.703	-13.592	21629.44	-13.592	21741.381	-13.592	21853.618	-13.592	21966.055	-13.592	22078.692	-13.592	22191.529	-13.592	22304.566	-13.592	22417.803	-13.592	22531.24	-13.592	22645.881	-13.592	22760.718	-13.592	22875.755	-13.592	22990.992	-13.592	23106.429	-13.592	23222.066	-13.592	23337.903	-13.592	23454.04	-13.592	23570.377	-13.592	23686.914	-13.592	23803.651	-13.592	23920.588	-13.592	24037.825	-13.592	24155.262	-13.592	24272.899	-13.592	24390.736	-13.592	24508.773	-13.592	24628.01	-13.592	24747.447	-13.592	24867.084	-13.592	24986.921	-13.592	25106.958	-13.592	25228.195	-13.592	25349.632	-13.592	25471.269	-13.592	25593.106	-13.592	25715.143	-13.592	25837.38	-13.592	25959.817	-13.592	26082.454	-13.592	26205.291	-13.592	26328.328	-13.592	26453.565	-13.592	26578.902	-13.592	26704.439	-13.592	26830.176	-13.592	26956.113	-13.592	27082.25	-13.592	27208.587	-13.592	27335.124	-13.592	27461.861	-13.592	27588.798	-13.592	27715.935	-13.592	27843.272	-13.592	27970.809	-13.592	28098.546	-13.592	28226.483	-13.592	28354.62	-13.592	28482.957	-13.592	28611.494	-13.592	28739.231	-13.592	28867.168	-13.592	28995.305	-13.592	29123.642	-13.592	29252.179	-13.592	29380.916	-13.592	29509.853	-13.592	29638.99	-13.592	29768.327	-13.592	29897.864	-13.592	30027.601	-13.592	30157.538	-13.592	30287.675	-13.592	30417.912	-13.592	30548.349	-13.592	30678.986	-13.592	30809.823	-13.592	30940.86	-13.592	31071.997	-13.592	31203.334	-13.592	31334.871	-13.592	31466.608	-13.592	31598.545	-13.592	31730.682	-13.592	31862.919	-13.592	31995.356	-13.592	32127.993	-13.592	32260.83	-13.592	32393.867	-13.592	32527.004	-13.592	32660.341	-13.592	32793.878	-13.592	32927.615	-13.592	33061.552	-13.592	33195.689	-13.592	33330.026	-13.592	33464.563	-13.592	33599.3	-13.592	33734.237	-13.592	33869.364	-13.592	34014.741	-13.592	34160.358	-13.592	34306.195	-13.592	34454.272	-13.592	34604.599	-13.592	34755.176	-13.592	34906.913	-13.592	35060.8	-13.592	35215.837	-13.592	35371.024	-13.592	35527.361	-13.592	35683.848	-13.592	35840.485	-13.592	35998.272	-13.592	36156.309	-13.592	36314.496	-13.592	36472.833	-13.592	36632.32	-13.592	36791.957	-13.592	36951.744	-13.592	37111.681	-13.592	37271.868	-13.592	37433.305	-13.592	37594.992	-13.592	37756.929	-13.592	37919.116	-13.592	38081.553	-13.592	38244.24	-13.592	38407.177	-13.592	38570.364	-13.592	38733.801	-13.592	38897.488	-13.592	39061.425	-13.592	39225.612	-13.592	39389.949	-13.592	39554.536	-13.592	39719.373	-13.592	39884.46	-13.592	40049.797	-13.592	40215.384	-13.592	40381.221	-13.592	40547.308	-13.592	40713.545	-13.592	40879.932	-13.592	41046.569	-13.592	41213.456	-13.592	41380.593	-13.592	41547.98	-13.592	41715.617	-13.592	41883.494	-13.592	42051.621	-13.592	42219.998	-13.592	42389.625	-13.592	42559.502	-13.592	42729.629	-13.592	42899.906	-13.592	43069.333	-13.592	43238.91	-13.592	43408.647	-13.592	43578.534	-13.592	43748.671	-13.592	43918.958	-13.592	44089.395	-13.592	44259.982	-13.592	44430.719	-13.592	44601.606	-13.592	44772.643	-13.592	44943.83	-13.592	45115.167	-13.592	45286.654	-13.592	45458.391	-13.592	45630.378	-13.592	45802.615	-13.592	45975.002	-13.592	46147.539	-13.592	46319.226	-13.592	46491.063	-13.592	46663.05	-13.592	46835.287	-13.592	47007.764	-13.592	47180.491	-