CS 440
Introduction to Artificial Intelligence

Lecture 12:

Logic and Intro to Probability

25 February 2020

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Example: Reductions

- $\neg\alpha(X) \wedge (\alpha(X) \vee \beta(X)) \rightarrow \beta(X)$
  - $f(X) \wedge (g(X) \vee h(X)) \Leftrightarrow (f(X) \wedge g(X)) \vee (f(X) \wedge h(X))$
- $(\neg\alpha(X) \wedge \alpha(X)) \vee (\neg\alpha(X) \wedge \beta(X)) \rightarrow \beta(X)$
  - $f(X) \wedge \neg f(X) \Leftrightarrow$ false
- false $\vee (\neg\alpha(X) \wedge \beta(X)) \rightarrow \beta(X)$
  - $f(X) \vee g(X) \Leftrightarrow g(X) \vee f(X)$
- $(\neg\alpha(X) \wedge \beta(X)) \vee$ false $\rightarrow \beta(X)$
  - $f(X) \vee$ false $\Leftrightarrow f(X)$
- $\neg\alpha(X) \wedge \beta(X) \rightarrow \beta(X)$
  - $f(X) \wedge g(X) \rightarrow g(X)$
- true

Given list of rules could an AI agent make such a reduction?

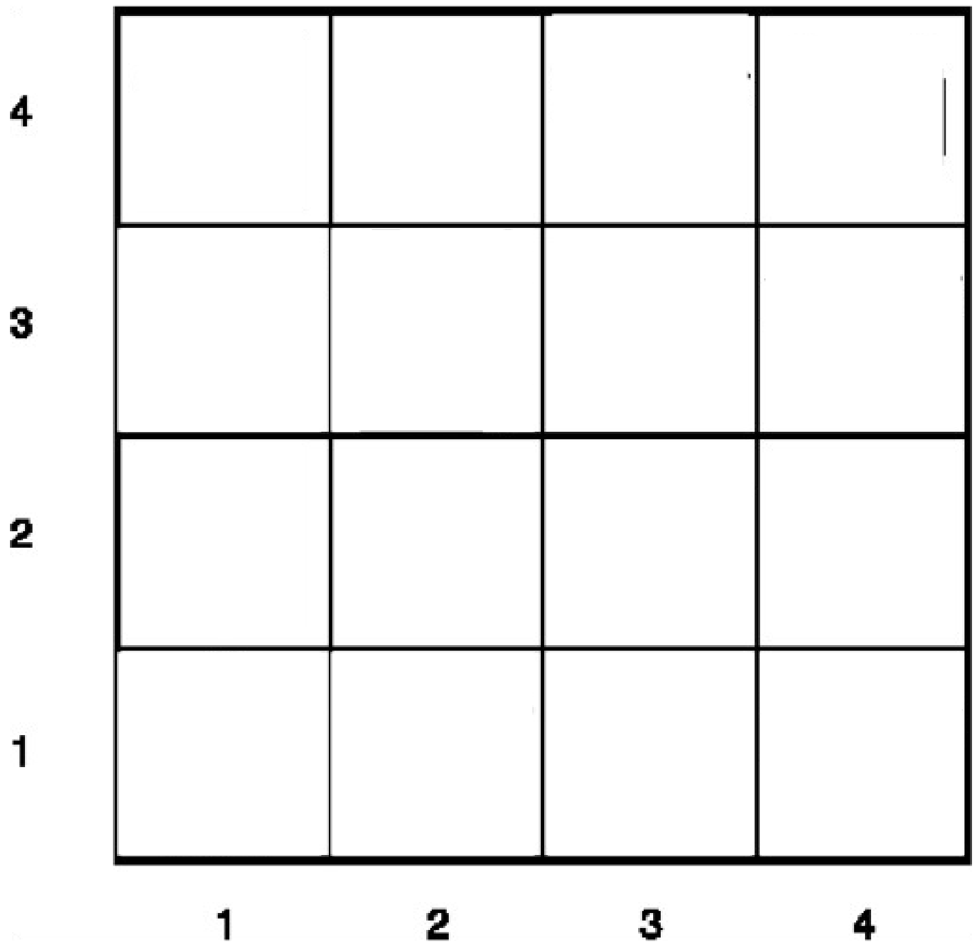
Binary CSP example: Wumpus World

Wumpus 

Agent 

Pit 

Gold 

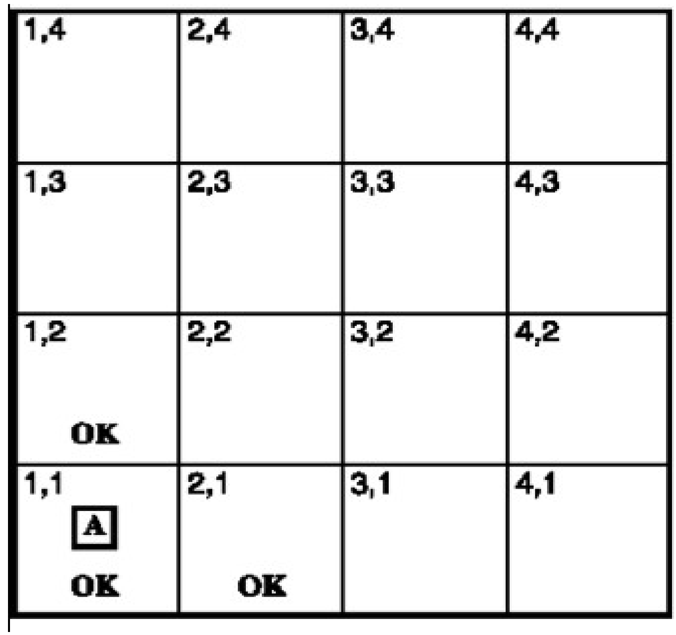


+1000 for gold
-1000 falling into pit
-1 for each action
-10 for using arrow

Formulation

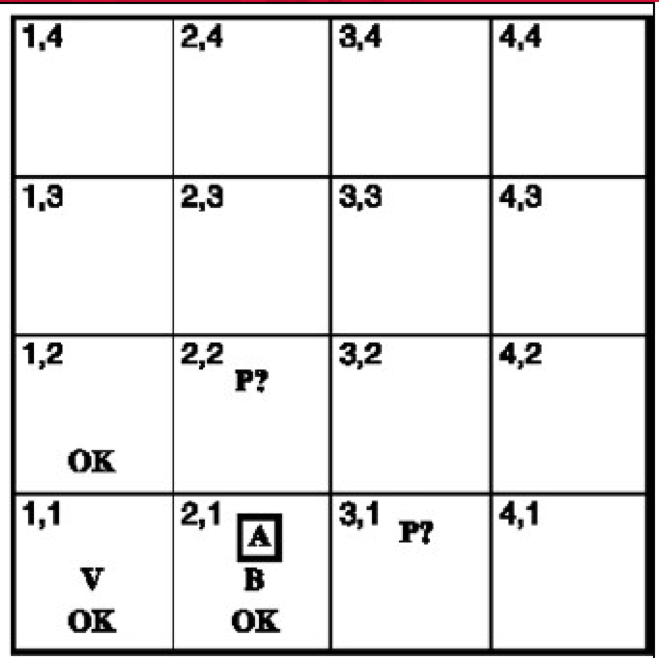
- Variables
  - Defined for each cell
  - $S_{i,j}$, $B_{i,j}$, $P_{i,j}$, $W_{i,j}$, $G_{i,j}$
- Define problem in terms of logical statements
  - $S_{i,j} \Leftrightarrow (W_{i-1,j} \vee W_{i+1,j} \vee W_{i,j-1} \vee W_{i,j+1})$
  - $B_{i,j} \Leftrightarrow (P_{i-1,j} \vee P_{i+1,j} \vee P_{i,j-1} \vee P_{i,j+1})$
- Gold/Pit/Wumpus cannot be in same cell
  - $W_{i-,j} \rightarrow (\neg G_{i-,j} \wedge \neg P_{i-,j})$
  - $G_{i-,j} \rightarrow (\neg W_{i-,j} \wedge \neg P_{i-,j})$
  - $P_{i-,j} \rightarrow (\neg W_{i-,j} \wedge \neg G_{i-,j})$

Wumpus World



- Agent in 1,1
  - No breeze and no smell
- Use Entailment
  - $B_{i,j} \Leftrightarrow (P_{i-1,j} \vee P_{i+1,j} \vee P_{i,j-1} \vee P_{i,j+1}), \neg B_{1,1} \models \neg P_{1,2}, \neg P_{2,1}$
  - $S_{i,j} \Leftrightarrow (W_{i-1,j} \vee W_{i+1,j} \vee W_{i,j-1} \vee W_{i,j+1}), \neg S_{1,1} \models \neg W_{1,2}, \neg W_{2,1}$
- Cells 1,2 and 2,1 are safe

Wumpus World



- Agent in 2,1
  - Breeze and no smell
- Use Entailment
  - $B_{i,j} \Leftrightarrow (P_{i-1,j} \vee P_{i+1,j} \vee P_{i,j-1} \vee P_{i,j+1}),\ B_{2,1} \models P_{2,2} \vee P_{3,1}$
  - $S_{i,j} \Leftrightarrow (W_{i-1,j} \vee W_{i+1,j} \vee W_{i,j-1} \vee W_{i,j+1}),\ \neg S_{2,1} \models \neg W_{2,2}, \neg W_{3,1}$
- Don't move to 2, 2 or 3, 1
  - Go back and explore 1,2 instead

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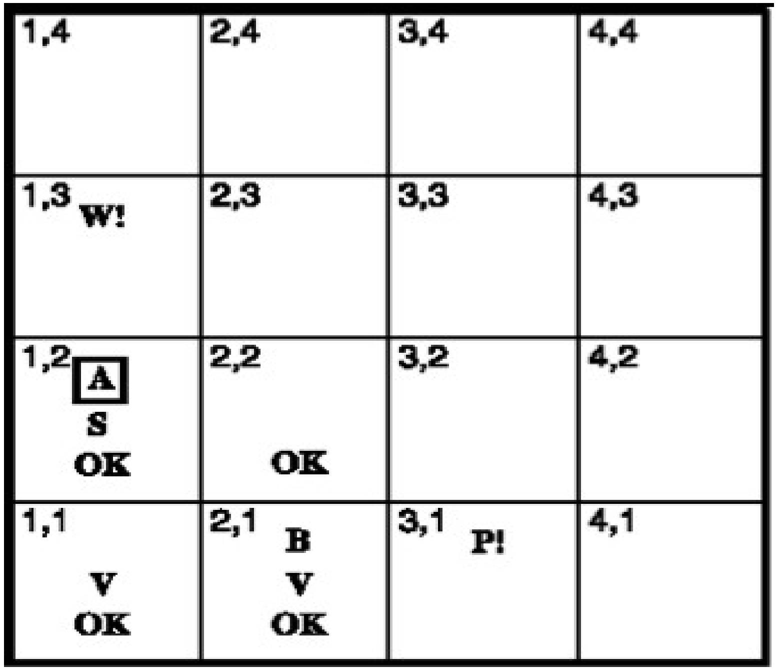
Wumpus World



- Agent in 2,1
  - Smell and no breeze
- $B_{i,j} \Leftrightarrow (P_{i-1,j} \vee P_{i+1,j} \vee P_{i,j-1} \vee P_{i,j+1}),\ \neg B_{2,1} \models \neg P_{1,3},\ \neg P_{2,2}$
- $P_{2,2} \vee P_{3,1},\ \neg P_{2,2} \models P_{3,1}$
- $S_{i,j} \Leftrightarrow (W_{i-1,j} \vee W_{i+1,j} \vee W_{i,j-1} \vee W_{i,j+1}),\ S_{1,2},\ \neg W_{2,2},\ \neg W_{1,1} \models W_{1,3}$
- Cell 2,2 is safe, move there

Wumpus World

1,4	2,4 P?	3,4	4,4
1,3 W!	2,3 $\boxed{A}$ S G B	3,3 P?	4,3
1,2 S V OK	2,2 V OK	3,2	4,2
1,1 V OK	2,1 B V OK	3,1 P!	4,1

- **Agent visits 2,2**
  - **No smell and no breeze**
- $B_{i,j} \Leftrightarrow (P_{i-1,j} \vee P_{i+1,j} \vee P_{i,j-1} \vee P_{i,j+1}),\ \neg B_{2,2} \models \neg P_{2,3},\ \neg P_{3,2}$
- $S_{i,j} \Leftrightarrow (W_{i-1,j} \vee W_{i+1,j} \vee W_{i,j-1} \vee W_{i,j+1}),\ \neg S_{2,2} \models \neg W_{2,3},\ \neg W_{3,2}$
- **Cell 3,2 and 2,3 are safe**
  - Move to 2,3
  - Find the gold there!!!!!

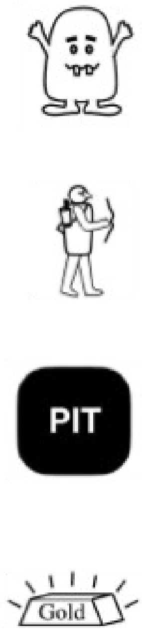
Wumpus World

Wumpus

Agent

Pit

Gold



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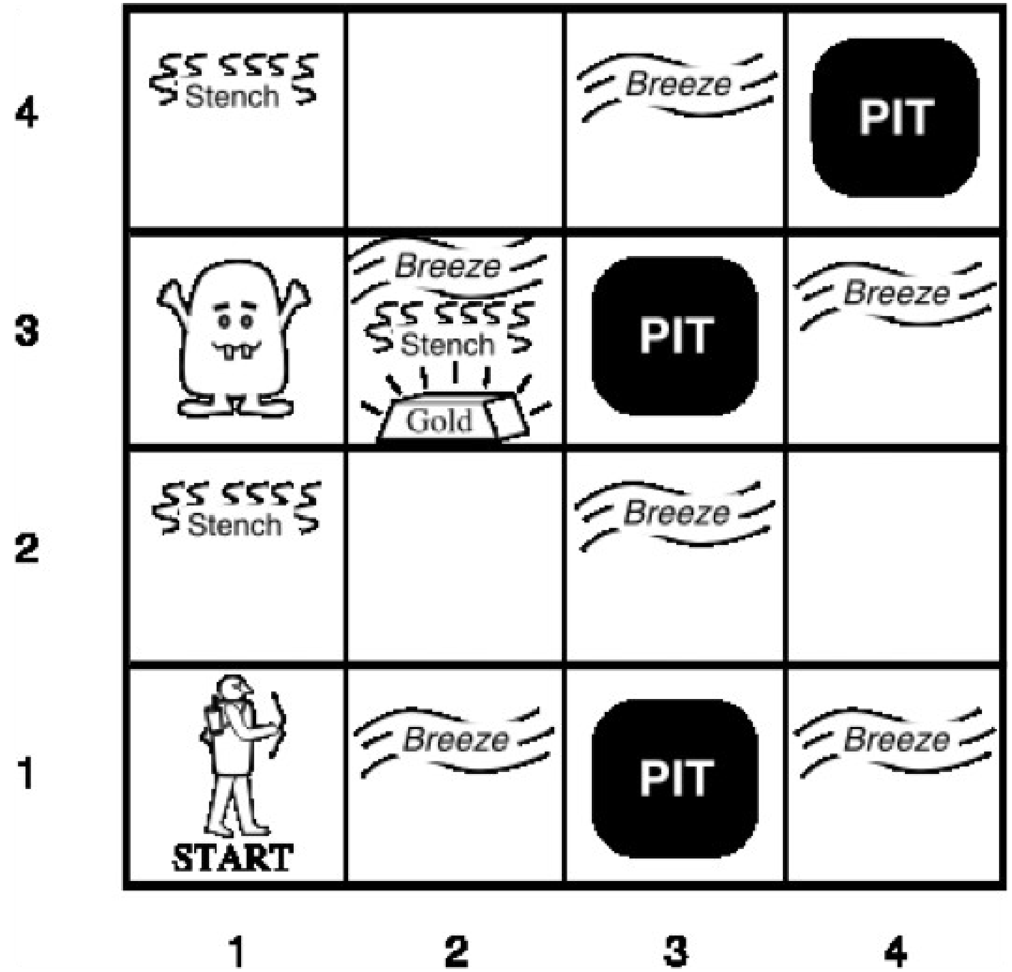
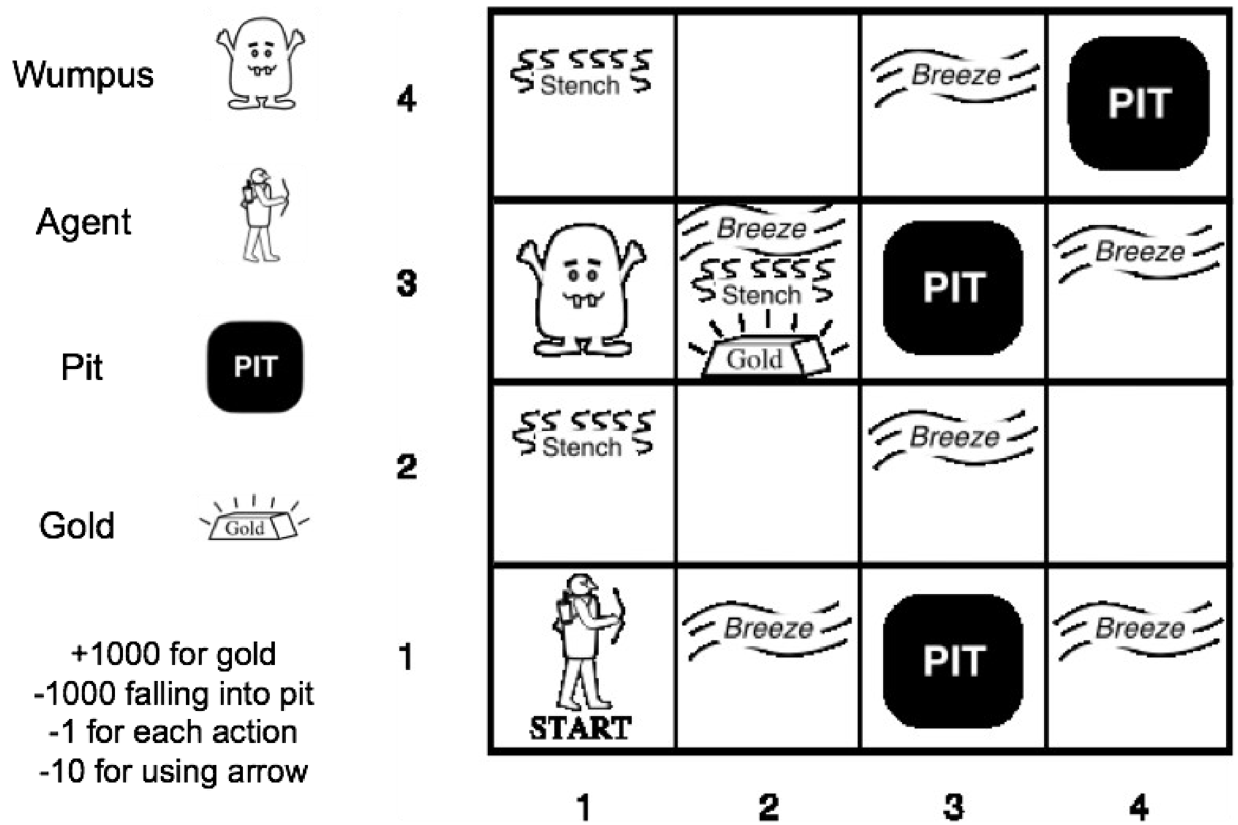


Discussion

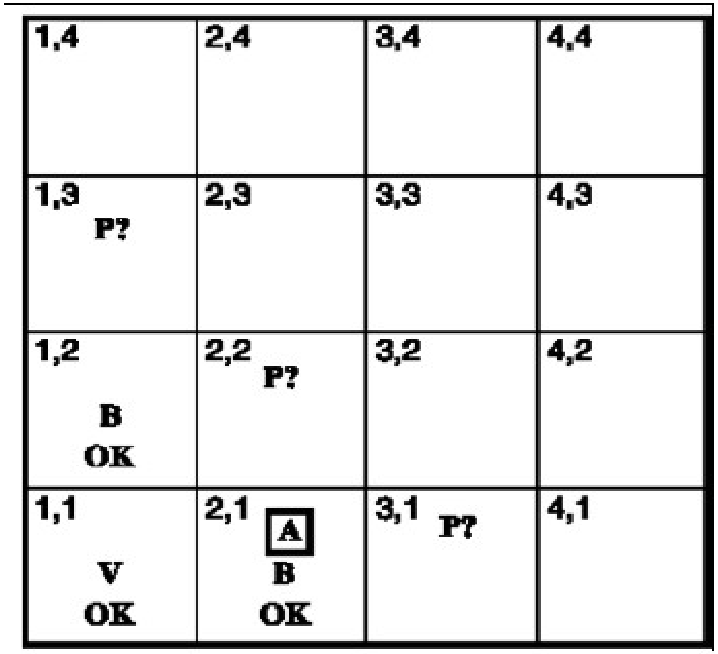


- How would you automate planning in environment like Wumpus World?
- What is the major difficulty?

Summary of First Third of Class

- **End of section on deterministic reasoning**
  - **Problem Formalization**
    - **State/Action/Transition/Observation/ect.**
  - **Local Search**
    - **Hill Climbing**
    - **Gradient descent**
  - **Search**
    - **BFS/DFS/SPF**
    - **Heuristics**
    - **A* Search**
  - **Adversarial Search**
    - **Minimax Search**
    - **Alpha-Beta Pruning**
  - **Decision Trees**
    - **Constructing Decision Trees**
  - **Constraint Satisfaction**
    - **Backtracking Algorithm**
  - **Logic**

- Thus far we have assumed environment is completely observable
  - Agent know state of environment after each step
  - Agent can generate state from environment
- What to do when environment not observable
  - Example Wumpus world



Intro to Probability

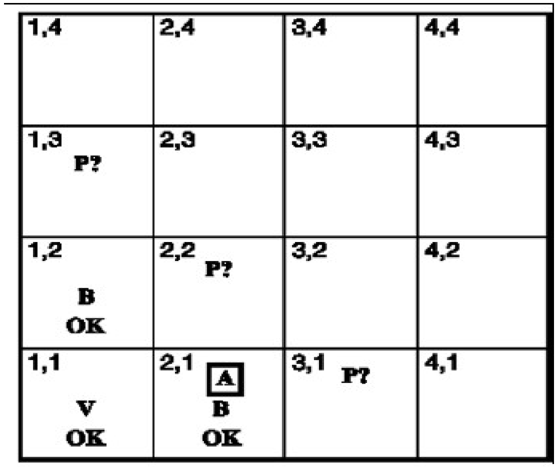
- Probability
  - Likelihood something will happen
    - Examples
      - Probability you will get in a car accident
      - Probability you will have a heart attack
      - Probability you will win the lottery
      - Expected time for next bus to arrive
      - What factors impact probability of each of these?
  - Likelihood of an unobserved event
    - Examples
      - If I have a card in my hand what is the likelihood it is an ace
      - What is the likelihood you have cancer
        - What observations can help improve accuracy of this?

Applications of Probability

- Games
  - Cards games: Poker, Blackjack, Magic
  - Games with dice: D&D, Monopoly
  - Roulette, slot machines
- Medicine
  - Likelihood you will contract a disease
  - Likelihood a cure will be effective
- Insurance
  - Likelihood you will need to file a claim

- Computer Hardware
  - Likelihood a component will fail (mean time to failure)
  - Supercomputers: MTF proportional to number of components
    - MTF measured in days or even hours

Probability



- **Notation: p(x)**
  - **Wumpus World p($P_{3,1}$)**
  - **p(car_accident)**
- **Hint: Think about probabilities in terms of of total population**
  - **p(car_accident)**
    - **Proportion of drivers that have gotten in a car accident**
      - **number of drivers who have gotten in car accident over total number of drivers**
      - **$n_{accident\_drivers}/n_{drivers}$**
  - **What are some limitations of this and who would we address them?**
    - **Hint think about a driver with a lot of accidents vs a driver with a clean record**

Probability Given Observation

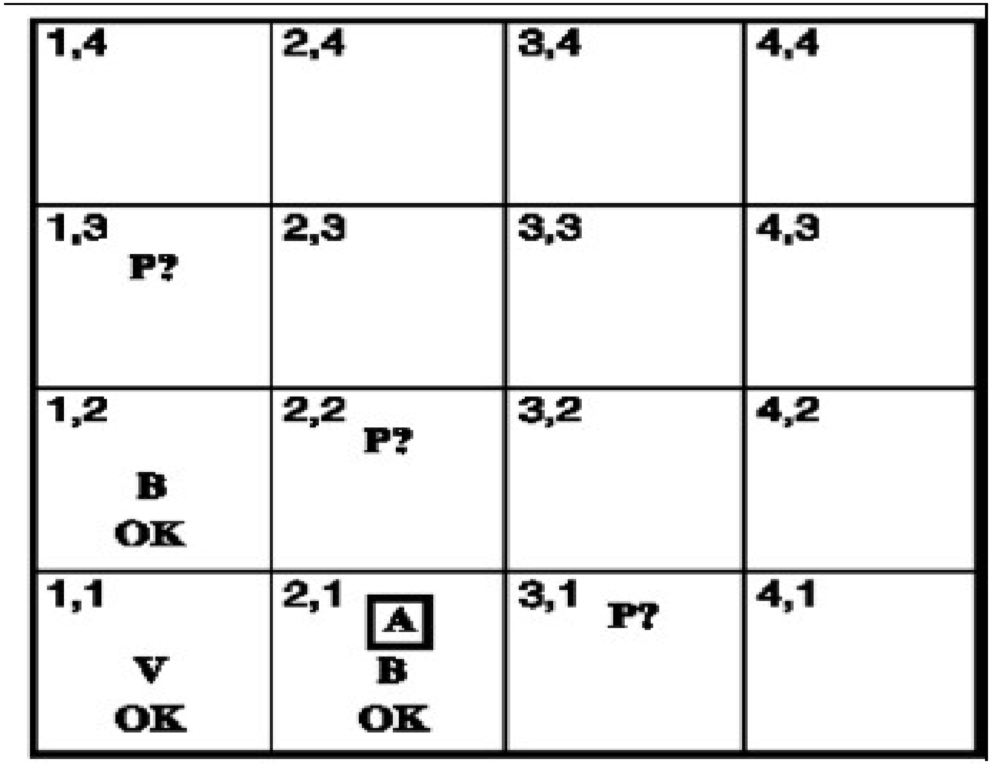
- Notation: p(x|y)
  - Probability of x given y
  - Example: probability you will get in a car accident given that you have a clean record
    - p(accident|clean_record)
- Compute proportion for population which condition holds
  - p(accident|clean_record) = $n_{\text{accident clean record}}/n_{\text{clean record}}$
    - Number of drivers with clean record that got into accident over total number of drivers with clean record
- More observations you can make - more accurate your predicition

Example

	Number	Lung Cancer
Smokers	300	105
Non-smokers	1700	20
Total	2000	125

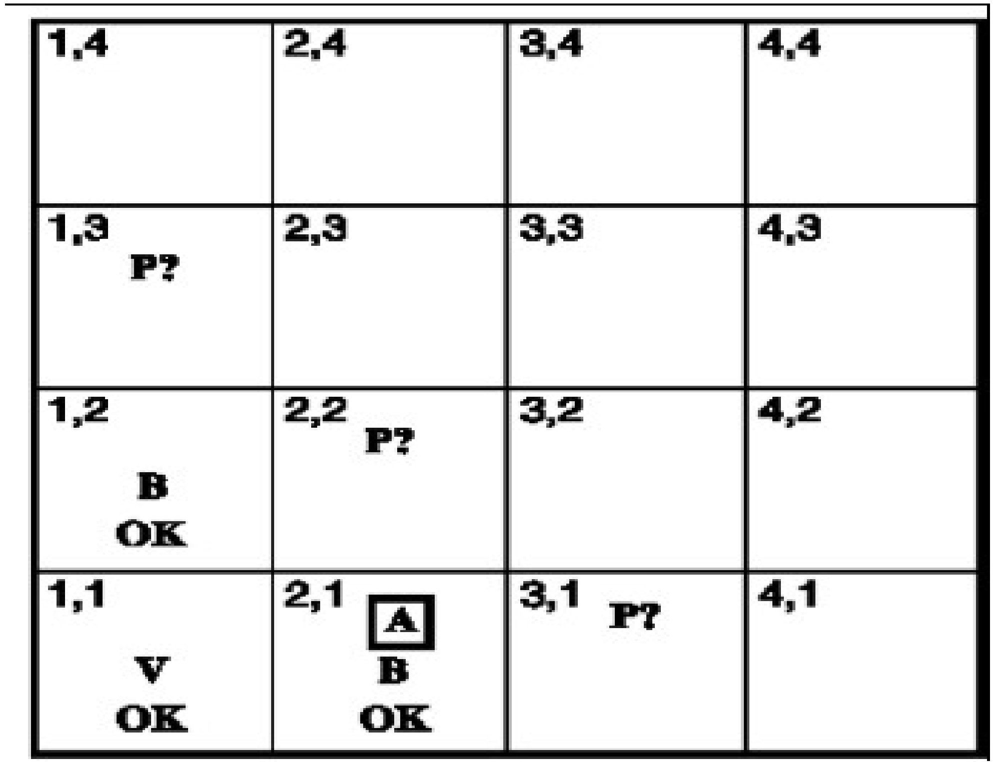
- **What is p(lung_cancer)?**
- **What is p(smoker)?**
- **What is p(lung _cancer|smoker) ?**
- **What is p(lung _cancer|non-smoker) ?**

Example: Wumpus World



- What is $p(P_{1,3})$, $p(P_{2,2})$ and $p(P_{3,1})$?

Example: Wumpus World



- What is $p(P_{1,3})$, $p(P_{2,2})$ and $p(P_{3,1})$?
  - Assume all valid states of Wumpus World are equally likely
  - $p(P_{1,3} | B_{1,2} \wedge B_{2,1})$
    - How can we compute this?

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Example: Wumpus World

1,4	2,4	3,4	4,4
1,3 P?	2,3	3,3	4,3
1,2 B OK	2,2 P?	3,2	4,2
1,1 V OK	2,1 A B OK	3,1 P?	4,1

- What is $p(P_{1,3})$, $p(P_{2,2})$ and $p(P_{3,1})$?
  - Assume all valid states of Wumpus World are equally likely
  - $p(P_{1,3} | B_{1,2} \wedge B_{2,1})$
    - Number of states with $B_{1,2} \wedge B_{2,1} \wedge P_{1,3}$ over number of states with $B_{1,2} \wedge B_{2,1}$