

Teacher Section:

At step 0 there is a .1 probability Bob has a deadline and a .9 probability he doesn't have a deadline. After the transition the probability is as follows.

n = total population

.1 n = population bob is inititall sick

.9 n = population that bob is not initially sick

.1 n = population that bob doesn't have deadline at initial step

.9 n = population that bob has deadline at initial step

Transition

.18 n = population where bob has deadline at step 1

(1-.18) n = population where bob has no deadline at step 1

.17 n = population where bob is sick at step 1

(1-.17) n = population where bob is not sick at step 1

Observe that bob is in office at step 1.

Translation from step 0 to step 1:

$p(\text{deadline } 1) = p(\text{deadline } 0) * p(\text{deadline } 1 \mid \text{deadline } 0) + p(\text{no deadline } 0) * p(\text{deadline } 1 \mid \text{not deadline } 0)$

$$.1 * .9 + .9 * .1 = .18$$

$p(\text{sick } 1) = p(\text{sick } 0) * p(\text{sick } 1 \mid \text{sick } 0) + p(\text{Not sick } 0) * p(\text{sick } 1 \mid \text{not sick } 0)$

$$.1 * .8 + .9 * .1 = .08 + .09 = .17$$

Now John observes that Bob is in the office at step 1.

Total population is n

$n_{\text{bob_office}} =$

$n * p(\text{sick, deadline}) * p(\text{office} \mid \text{sick, deadline})$

$+ n * p(\text{sick, no deadline}) * p(\text{office} \mid \text{sick, no deadline})$

$+ n * p(\text{not sick, deadline}) * p(\text{office} \mid \text{not sick, deadline})$

$+ n * p(\text{not sick, not deadline}) * p(\text{office} \mid \text{not sick, not deadline})$

$$n_{\text{bob_office}} = n * .17 * .18 * .6 + n * .17 * (1 - .18) * .1 + .18 * (1 - .17) * .9 + n * (1 - .18) * (1 - .17) * .7$$

$n_{\text{deadline_office}} =$

$n * p(\text{sick, deadline}) * p(\text{office} \mid \text{sick, deadline})$

$+ n * p(\text{not sick, deadline}) * p(\text{office} \mid \text{not sick, deadline})$

$$p(\text{Deadline 1}) = n_{\text{deadline_office}} / n_{\text{bob_office}}$$

$$n_{\text{sick_office}} = n * p(\text{sick, deadline}) * p(\text{office} | \text{sick, deadline}) + n * p(\text{sick, no deadline}) * p(\text{office} | \text{sick, no deadline})$$

$$p(\text{Sick 1} | \text{office 1}) = n_{\text{sick_office}} / n_{\text{bob_office}}$$

$$(.17 * .18 * .6 + .17 * (1 - .18) * .1) / (.17 * .18 * .6 + .17 * (1 - .18) * .1 + .18 * (1 - .17) * .9 + (1 - .18) * (1 - .17) * .7)$$

Ant Example

Observed Variable
Position of Ant

Unobserved Variable
Position of food

Ant moves in direction of food with probability of .3

Moves away from food with probability of .1

.2 probability that ant will remain in same space

Initial state

```
.25          .25
-----
-----
-- A --
-----
-----
.25          .25
```

Step 1:

Ant moves Up

```
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-- A --
-----
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$p(u|up)$
 $p(ur|up)$
 $p(l|up)$
 $p(lr|up)$

$$n_up = (1-.2)/4$$

cases

food ul

$$n_up=n*p(ul)*p(up|ul) =n*.25*.3$$

food ur

$$n_up=n*p(ur)*p(up|ur) =n*.25*.3$$

food ll

$$n_up=n*p(ll)*p(up|ll) =n*.25*.1$$

food lr

$$n_up=n*p(lr)*p(up|lr) =n*.25*.1$$

$$p(ul)=n*p(ul)*p(up|ul) / (n*p(ul)*p(up|ul) + n*p(ur)*p(up|ur) + n_up=n*p(ll)*p(up|ll) + n_up=n*p(lr)*p(up|lr))$$

$$p(ul)=.25*.3/ (.25*3+.25*3+.25*1+.25*1)$$

$$p(ur)=.25*.3/ (.25*3+.25*3+.25*1+.25*1)$$

$$p(ll)=.25*.1/ (.25*3+.25*3+.25*1+.25*1)$$

$$p(lr)=.25*.1/ (.25*3+.25*3+.25*1+.25*1)$$