CS 440
Introduction to Artificial Intelligence

Lecture 18:

Hidden Markov Models (cont.)

March 26, 2020

Project 1

- Average grades
  - Task 1: 5
  - Task 2: 9.9
  - Task 3: 9.9
  - Task 4: 14.4
  - Task 5: 9.8
  - Task 6: 19.4
  - Task 7: 19.65
  - Task 8: 9.3
  - Total: 96.1
- Will be releasing grades soon
- People who have not done demo yet should keep bugging us

RUTGERS

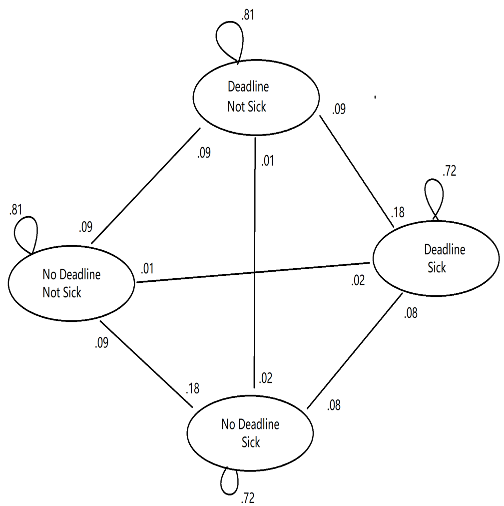
Project 2 and Midterm

- **Any questions about Midterm or Project 2?**

RUTGERS

Hidden Markov Model: Example

- Consider the following model
  - If Bob has a paper deadline at time step i there is a .9 probability he will have a paper deadline at step i+1
  - If Bob has a no deadline at time step i there is a .1 probability he will have a paper deadline at step i+1
  - If Bob is sick at time step i there is a .8 probability he will be sick at step i+1
  - If Bob is not at time step i there is a .1 probability he will be sick at step i+1
- If we know the values of deadline and sick this is a Markov Chain



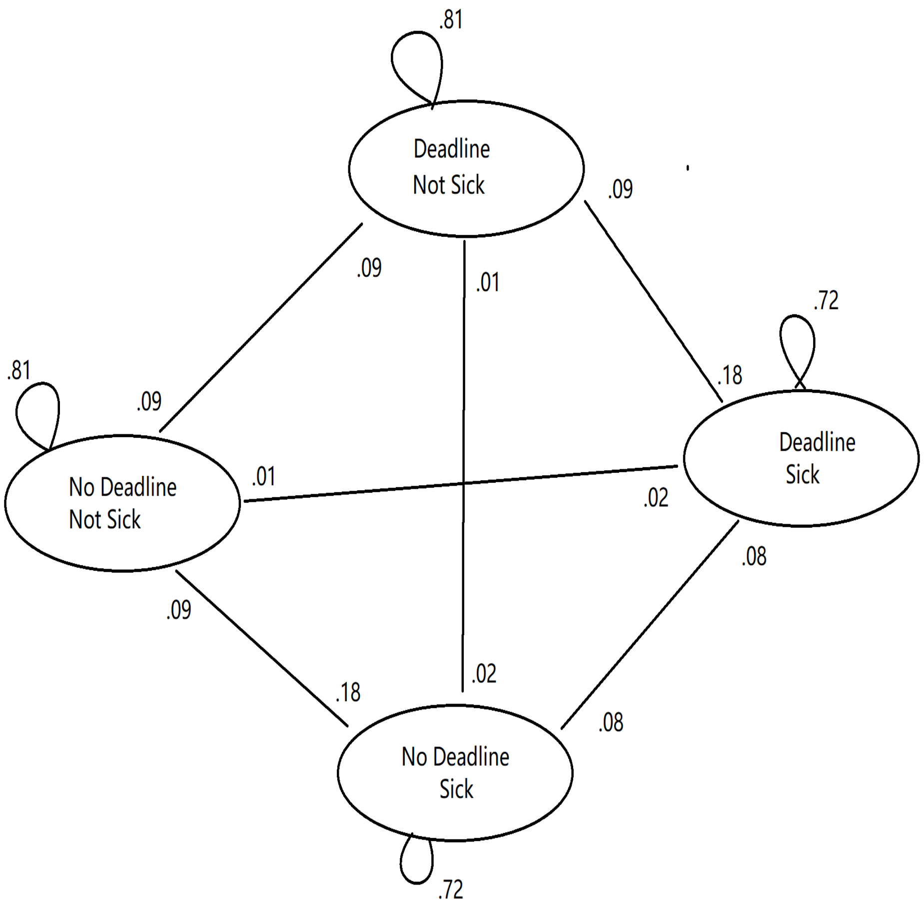
Hidden Markov Model: Example

	Not Sick	Sick
Paper	.9	.6
No Paper	.7	.1

- One an ordinary day Bob has a .7 probability of being in his office
- If Bob is busy working on a paper deadline he has a .9 probability of being in his office
- If he is sick and there is no paper deadline he has a .1 probability of being in his office
- If he is sick but there is a paper deadline he has a .6 probability of being in his office
- John does not know if Bob has a paper deadline of if he is sick
  - He only knows if Bob is in his office
  - Can he infer the probability Bob has a paper deadline or is sick based on these observations?
  - Can he predict if Bob will be in his office?

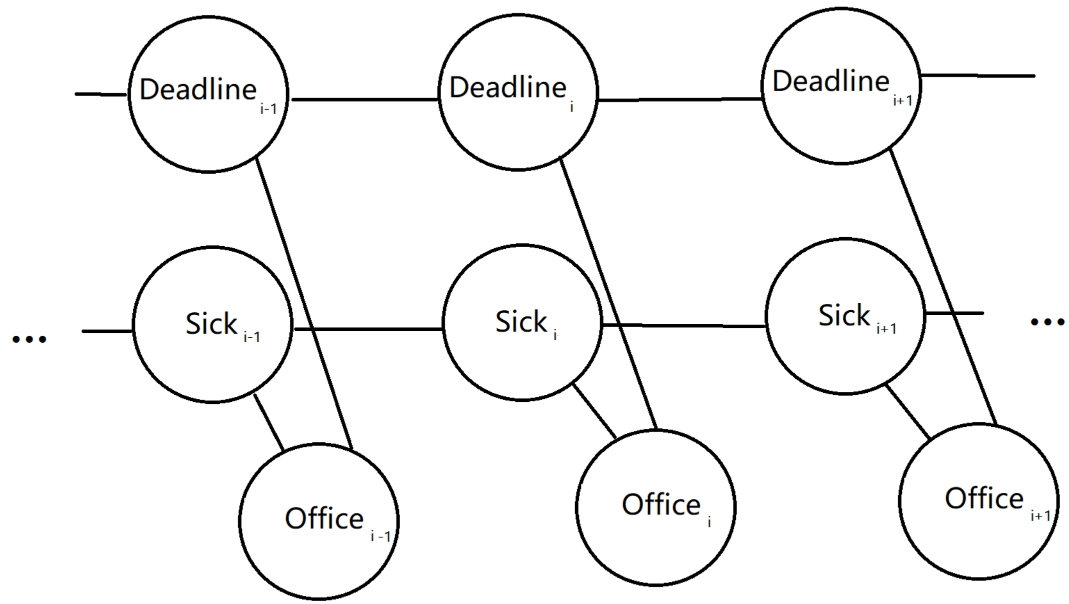
RUTGERS

Hidden Markov Model: Example



	Not Sick	Sick
Paper	.9	.6
No Paper	.7	.1

Model as Bayesian Infinite Network



- At each step Deadline and Sick are dependent on values in previous step
- Office depends on values of Deadline and Sick for that step
- Need to know probabilities of Deadline and Sick at initial step
  - Let $p(\text{Deadline}_0) = .1$
  - Let $p(\text{Sick}_0)=.1$
- At step i
  - Know value of Office_i
  - Compute $p(\text{Deadline}_i)$ from $p(\text{Deadline}_{i-1})$ and Office_i
  - Compute $p(\text{Sick}_i)$ from $p(\text{Sick}_{i-1})$ and Office_i

Hidden Markov Model: Solution

- At each step
  - Apply transition
    - Compute p($Deadline_i$) given p($Deadline_{i-1}$)
    - Compute p($Sick_i$) given p($Sick_{i-1}$)
  - Incorporate Observation
    - p($Deadline_i$|$Office_i$)
    - p($Sick_i$|$Office_i$)

	Initial	Transition 1	Observation 1
Observation			In office
p(Deadline)	.1	.18	.136
p(Sick)	.1	.17	.04

Hidden Markov Model: Solution

- At each step
  - Apply transition
    - Compute p($Deadline_i$) given p($Deadline_{i-1}$)
    - Compute p($Sick_i$) given p($Sick_{i-1}$)
  - Incorporate Observation
    - p($Deadline_i$|$Office_i$)
    - p($Sick_i$|$Office_i$)

	Step i-1	Step i	Step i+1
Observation	o_{i-1}	o_i	o_{i+1}
p(Deadline)	p($Deadline_{i-1}$| $Deadline_{i-2}$, o_{i-1})	p($Deadline_i$| $Deadline_{i-1}$, o_i)	p($Deadline_{i+1}$| $Deadline_i$, o_{i+1})
p(Sick)	p($Sick_{i-1}$| $Sick_{i-2}$, o_{i-1})	p($Sick_i$|$Sick_{i-1}$, o_i)	p($Sick_{i+1}$|$Sick_i$,$o_{i+!}$)

Rutgers

Hidden Markov Model: General

	Step i-1	Step i	Step i+1
Observation	o_{i-1}	o_i	o_{i+1}
p(X)	$p(X_{i-1})$ $p(x_{i-1,0}), p(x_{i-1,1}), \ldots$	$p(X_i)$ $p(x_{i,0}), p(x_{i,1}), \ldots$	$p(X_{i+1})$ $p(x_{i+1,0}), p(x_{i+1,1}), \ldots$

- Let X be the set of variable settings and $x \in X$ be one of those settings
  - Example
    - $X = \{Sick \wedge Deadline, Sick \wedge \neg Deadline, \neg Sick \wedge Deadline, \neg Sick \wedge \neg Deadline\}$
    - $x = Sick \wedge \neg Deadline$
- Intermediate step
  - $p(x_i') = \sum_{xi-1 \in X} p(x_{i-1}) * p(x_i' | x_{i-1})$
    - $p(x_i' | x_{i-1})$ = transition probability
- $p(x_i) = \sum_{xi' \in X} p(x_i') * p(x_i | x_i', o_i) \; / \; \sum_{xi' \in X} p(o_i | x_i')$
  - Bayes theorem

RUTGERS

Hidden Markov Model Example 2

- **Food at corner of grid**
  - **Don't know where food is**
  - **And has .3 probability of moving towards food and .1 probability of moving away**

		.1		
	.1		..3	
		.3