CS 440
Introduction to Artificial Intelligence

Lecture 13:

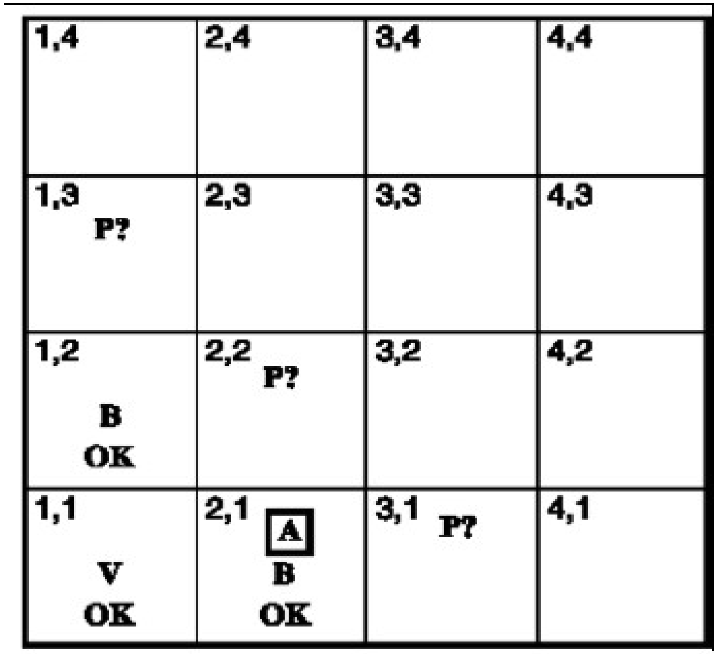
Probability and Bayesian Networks

27 February 2020

Summary of First Third of Class

- **End of section on deterministic reasoning**
  - **Problem Formalization**
    - **State/Action/Transition/Observation/ect.**
  - **Local Search**
    - **Hill Climbing**
    - **Gradient descent**
  - **Search**
    - **BFS/DFS/SPF**
    - **Heuristics**
    - **A* Search**
  - **Adversarial Search**
    - **Minimax Search**
    - **Alpha-Beta Pruning**
  - **Decision Trees**
    - **Constructing Decision Trees**
  - **Constraint Satisfaction**
    - **Backtracking Algorithm**
  - **Logic**

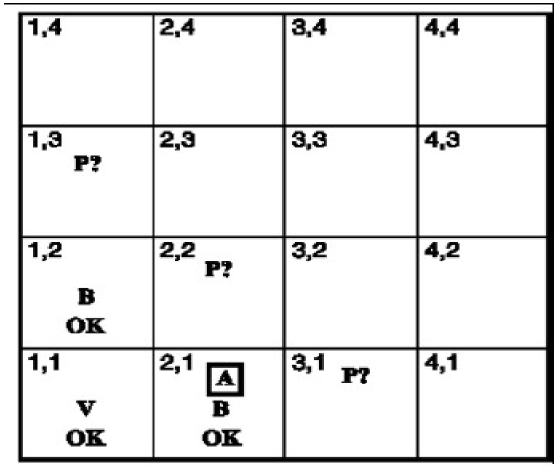
- Thus far we have assumed environment is completely observable
  - Agent know state of environment after each step
  - Agent can generate state from environment
- What to do when environment not observable
  - Example Wumpus world



Intro to Probability

- Probability
  - Likelihood something will happen
    - Examples
      - Probability you will get in a car accident
      - Probability you will have a heart attack
      - Probability you will win the lottery
      - Expected time for next bus to arrive
      - What factors impact probability of each of these?
  - Likelihood of an unobserved event
    - Examples
      - If I have a card in my hand what is the likelihood it is an ace
      - What is the likelihood you have cancer
        - What observations can help improve accuracy of this?

Probability



- Notation: p(x)
  - Wumpus World $p(P_{3,1})$
  - p(car_accident)
- Hint: Think about probabilities in terms of of total population
  - p(car_accident)
    - Proportion of drivers that have gotten in a car accident
      - number of drivers who have gotten in car accident over total number of drivers
      - $n_{accident\_drivers}/n_{drivers}$
  - What are some limitations of this and who would we address them?
    - Hint think about a driver with a lot of accidents vs a driver with a clean record

Probability Given Observation

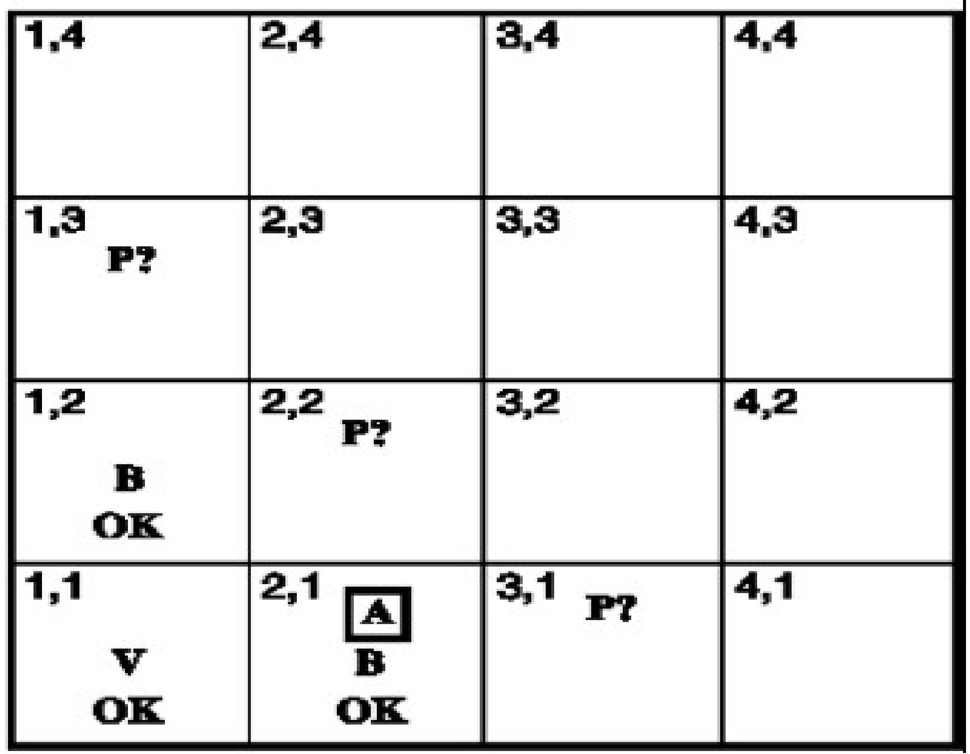
- **Notation: p(x|y)**
  - **Probability of x given y**
  - **Example: probability you will get in a car accident given that you have a clean record**
    - **p(accident|clean_record)**
- **Compute proportion for population which condition holds**
  - **p(accident|clean_record) = $n_{\text{accident clean record}}/n_{\text{clean record}}$**
    - **Number of drivers with clean record that got into accident over total number of drivers with clean record**
- **More observations you can make - more accurate your predicition**

Example

	Number	Lung Cancer
Smokers	300	105
Non-smokers	1700	20
Total	2000	125

- What is p(lung_cancer)?
- What is p(smoker)?
- What is p(lung _cancer|smoker) ?
- What is p(lung _cancer|non-smoker) ?

Example: Wumpus World



- What is $p(P_{1,3})$, $p(P_{2,2})$ and $p(P_{3,1})$?
  - Assume all valid states of Wumpus World are equally likely
  - $p(P_{1,3} | B_{1,2} \wedge B_{2,1})$
    - Number of states with $B_{1,2} \wedge B_{2,1} \wedge P_{1,3}$ over number of states with $B_{1,2} \wedge B_{2,1}$

Bayes' Theorem

- **Alice has lung cancer, what is the probability she is a smoker?**
  - **Given p(smoker), p(cancer) and p(cancer | smoker)**
  - **Can you compute p(smoker|cancer)?**

Bayes' Theorem

- Alice has lung cancer, what is the probability she is a smoker?
  - Given p(smoker), p(cancer) and p(cancer|smoker)
  - Can you compute p(smoker|cancer)?
  - We know that p(smoker|cancer) equal to the number of people who are smokers with cancer over the total number of people with cancer.
    - p(smoker|cancer) = $n_{smokers,cancer}/n_{cancer}$
    - But we don't know $n_{smokers,cancer}$ or n_{cancer}
  - What can we do?

Bayes' Theorem

- **Alice has lung cancer, what is the probability she is a smoker?**
  - **Given p(smoker), p(cancer) and p(cancer|smoker)**
  - **Can you compute p(smoker|cancer)?**
  - **We know that p(smoker|cancer) equal to the number of people who are smokers with cancer over the total number of people with cancer.**
    - **p(smoker|cancer) = $n_{smokers,cancer}/n_{cancer}$**
    - **But we don't know $n_{smokers,cancer}$ or n_{cancer}**
  - **Define n to be the size of our total population**
    - **We don't know what n is**
    - **n_{cancer} = n*p(cancer)**
    - **$n_{smokers}$ = n*p(smoker)**
    - **$n_{smokers,cancer} = n_{smokers}$ * p(cancer|smoker) = n*p(smoker)*p(cancer|smoker)**
    - **p(smoker|cancer) = p(smoker)*p(cancer|smoker) / p(cancer)**

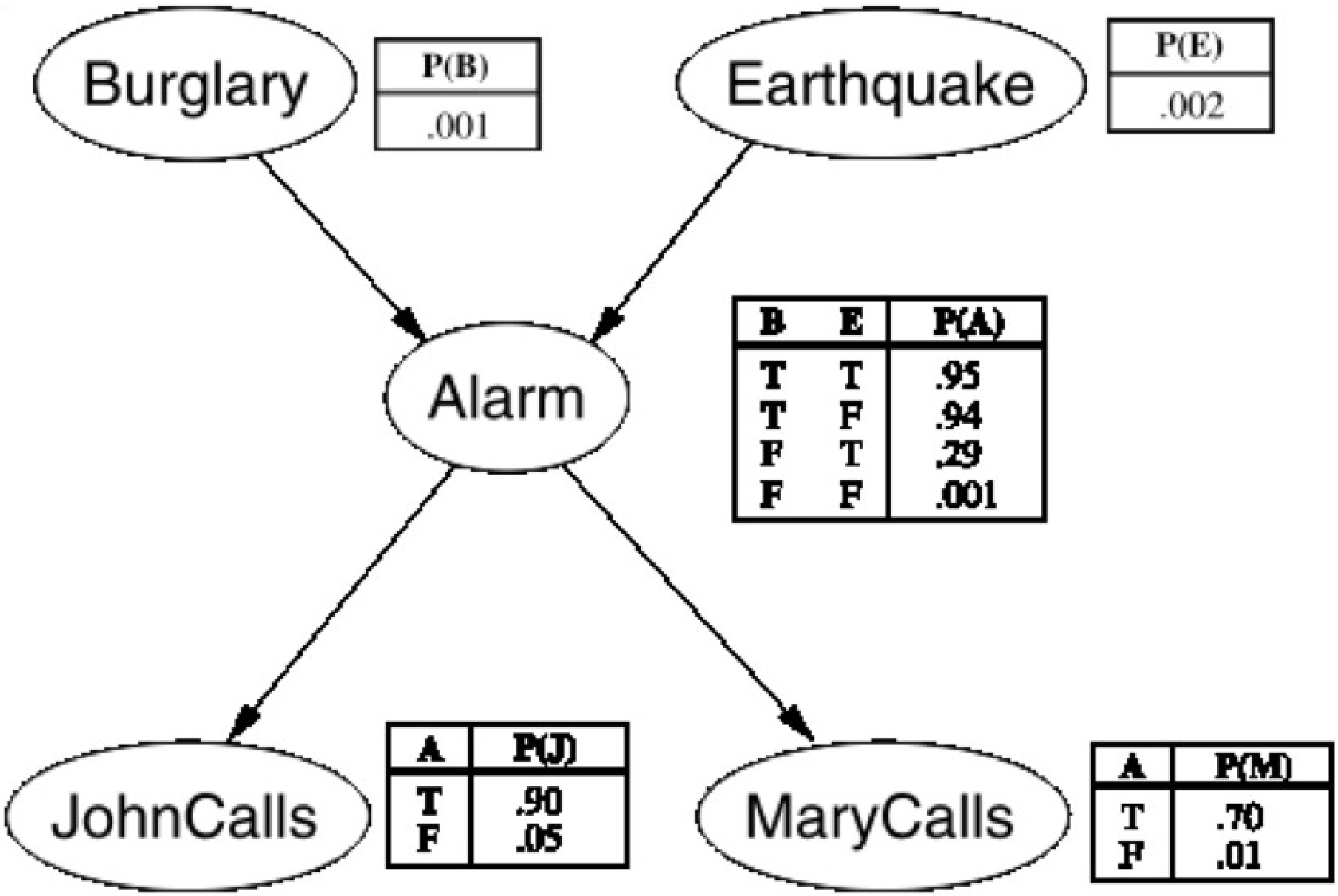
Bayes' Theorem

- **Congratulations, you just derived Bayes' Theorem!!!!**
    - **p(y|x) = p(y)*p(x|y) / p(x)**

Bayes' Theorem Example

- Lets say you are traveling and ask 2 of your neighbors (John and Mary) to watch your house
- There is a $P(B) = .001$ probability there will be a burglary while you are away
- There is also a $P(E) = .002$ probability there will be an earthquake
- Both a burglary and an earthquake may trigger your home alarm
  - $P(A|B \wedge \neg E) = .94$ probability that a burglary will trigger your alarm
  - $P(A|E \wedge \neg B) = .29$ probability an earthquake will trigger your alarm
  - $P(A|B \wedge E)=.95$ probability your alarm will go off if both a burglary and an earthquake occur
  - $P(A|\neg E \wedge \neg B)=.001$ probability alarm will go off if there is no
- There is a $P(J|A)=.9$ probability John will call if the alarm goes off
  - There is also a $P(J|\neg A)=.05$ probability John will call if there is no alarm
- There is a $P(M|A)=.7$ probability Mary will call if the alarm goes off
  - There is also a $P(M|\neg A)=.01$ probability Mary will call if there is no alarm

Example Bayesian Network



P(B)
.001

P(E)
.002

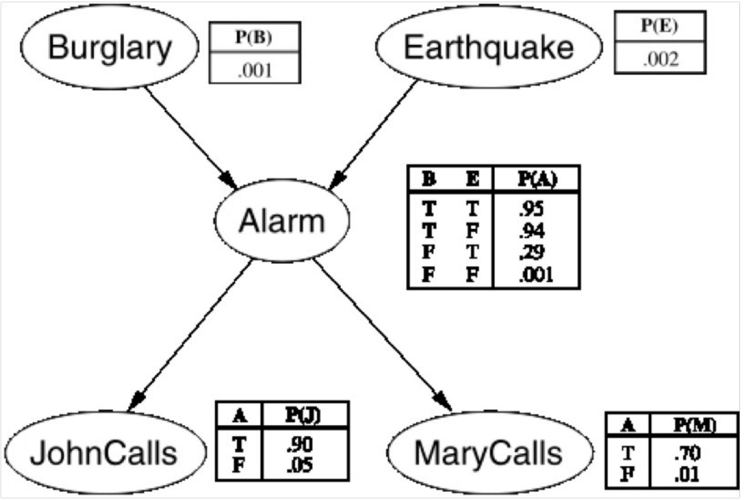
B	E	P(A)
T	T	.95
T	F	.94
F	T	.29
F	F	.001

A	P(J)
T	.90
F	.05

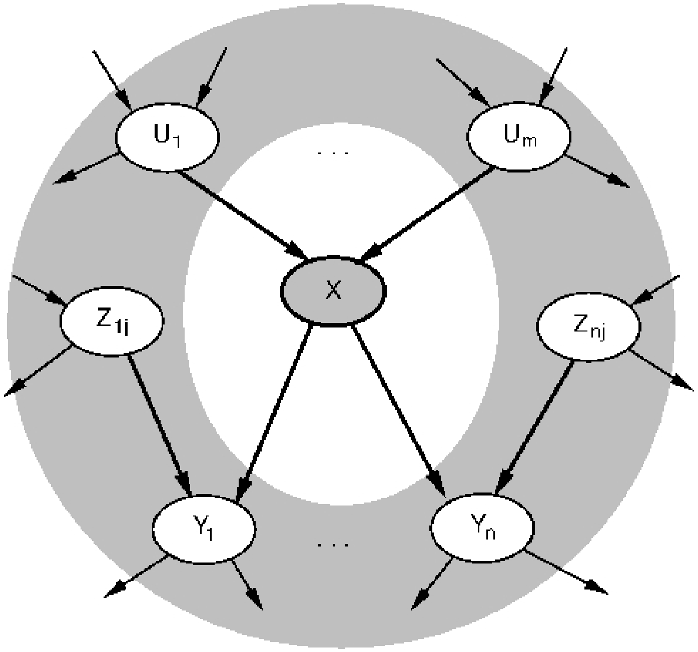
A	P(M)
T	.70
F	.01

Cond. Independence Properties

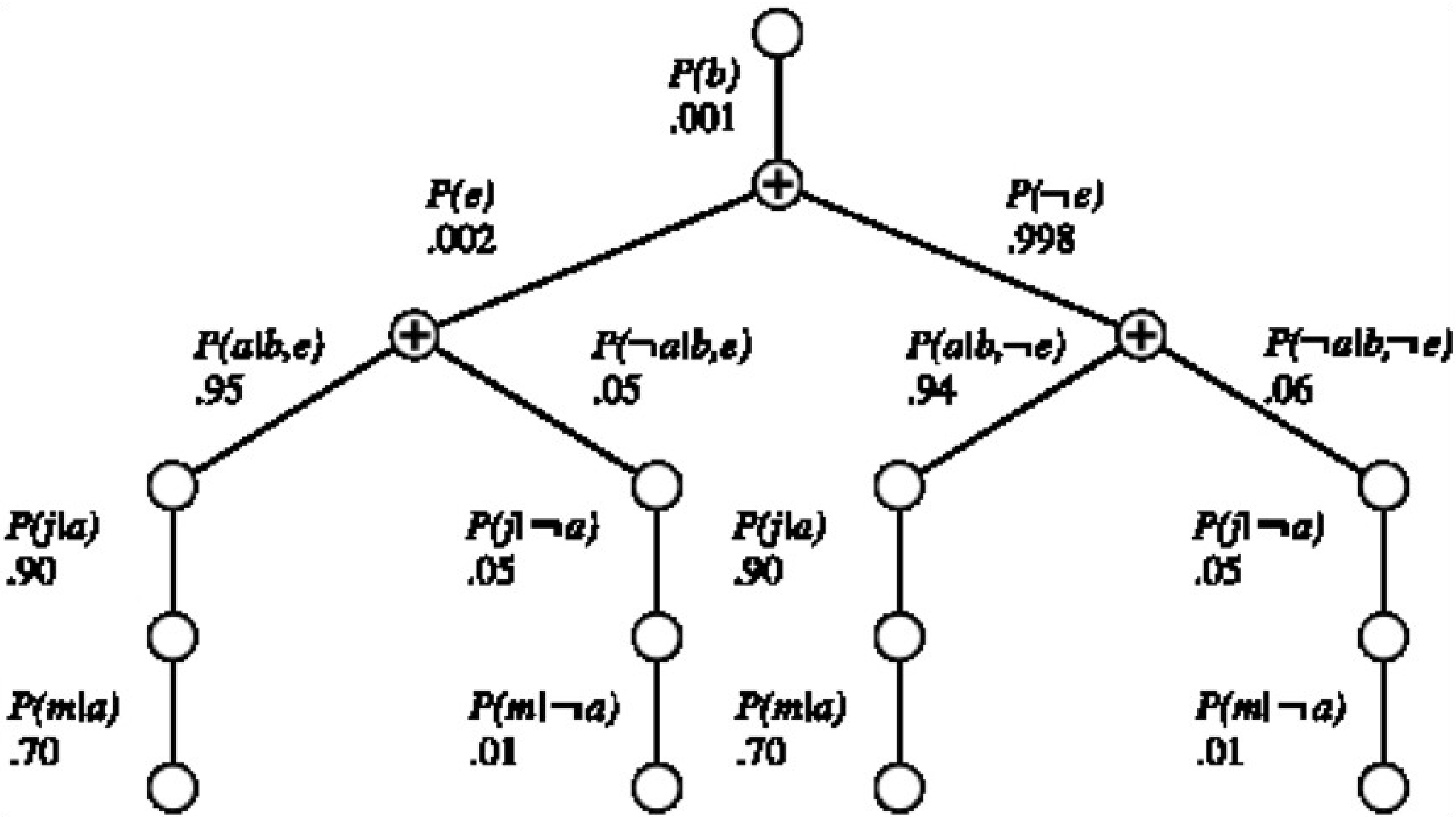
- A node is conditionally independent of its non-descendants, given its parents:
  - e.g., JohnCalls is independent from Burglary and Earthquake given an Alarm



- A node is conditionally independent of all other nodes in the network given its:
  - Markov blanket: which encompasses the parents, children and children's parents.
    - e.g., Burglary is independent of JohnCalls and MaryCalls, given Alarm and Earthquake



Enumeration Tree



Bayes' Theorem Example

- Assume you are given the following
  - P(c) probability someone in construction business
  - P(a) probability someone is exposed to asbestos
  - P(a|c) probability someone exposed to asbestos given they work in construction
  - P(s) probability someone is a smoker
  - P(l) probability someone develops lung cancer
  - P(l|a) probability someone develops ling cancer given they were exposed to asbestos
  - p(l|s) probability of developing lung cancer given that someone is a smoker
- What is the probability someone who has lung cancer works in construction?
- What is the probability a smoker who works in construction will develop lung cancer?
- What is the probability someone who has lung cancer is both a smoker and works in construction?