Assignment 3

Question 1 [25 points]: Assume you are interested in buying a used vehicle C_1. You are also considering of taking it to a qualified mechanic and then decide whether to buy it or not. The cost of taking it to the mechanic is \$100. C_1 can be in good shape (quality q^+) or bad one (quality q^-). The mechanic might help to indicate what shape the vehicle is in. C_1 costs \$3,000 to buy and its market value is \$4,000 if in good shape; if not, \$1,400 in repairs will be needed to make it in good shape. Your estimate is that C_1 has a 70% chance of being in good shape. Assume that the utility function depends linearly on the vehicle's monetary value.

a) Calculate the expected net gain from buying C_1, given no test.

b) We also have the following information about whether the vehicle will pass the mechanic's test:

$$P(\text{pass}(c_1)|q^+(c_1)) = 0.8$$
$$P(\text{pass}(c_1)|q^-(c_1)) = 0.35$$

Use Bayes' theorem to calculate the probability that the car will pass/fail the test and hence the probability that it is in good/ bad shape given what the mechanic will tell you.
[Hint: Compute the four probabilities: $P(q^+|\text{Pass})$, $P(q^-|\text{Pass})$, $P(q^+|\neg\text{Pass})$, $P(q-|\neg\text{Pass})$]

c)
What is the best decision given either a pass or a fail? What is the expected utility in each case?
[Hint: Use the probabilities from the previous question.]

d) What is the value of optimal information for the mechanic's test? Will you take C_1 to the mechanic or not?
[Hint: You can easily answer this based on the answers from questions a) and c).]

Question 2 [30 points]: Consider an ant in a 1 by 5 grid that contains an ant and a piece of food. The piece of food is located in an unknown cell that is selected at random. The ant's position and movement are fully observable and the ant's behavior is defined as follows.

1. If the food is located to the left of the ant, the ant will move left with a .5 probability, move right with a .2 probability and remain in the same cell with a .3 probability.

2. If the food is located to the right of the ant, the ant will move right with a .5 probability, move left with a .2 probability and remain in the same cell with a .3 probability.

3. If the food is located in the same cell as the ant, the ant will move right with a .25 probability, move left with a .25 probability and remain in the same cell with a .5 probability.

4. If the ant tries to move to the right while in the rightmost cell or to the left while in the leftmost cell it will remain in the same cell.

The ant starts in cell 2 and its first three moves are a move to the right, a second move to the right and a move to the left (so that it is in cell 3).

a) Given these three moves, what is the probability that the food is located in each of the grid cells?

b) What is the probability that the ant will select each move (left, right, stay in cell) as its fourth move.

Question 3 [25 points]: One method for approximate inference in Bayesian Networks is the Markov Chain Monte Carlo (MCMC) approach. This method depends on the important property that a variable in a Bayesian network is independent from any other variable in the network given its Markov Blanket.

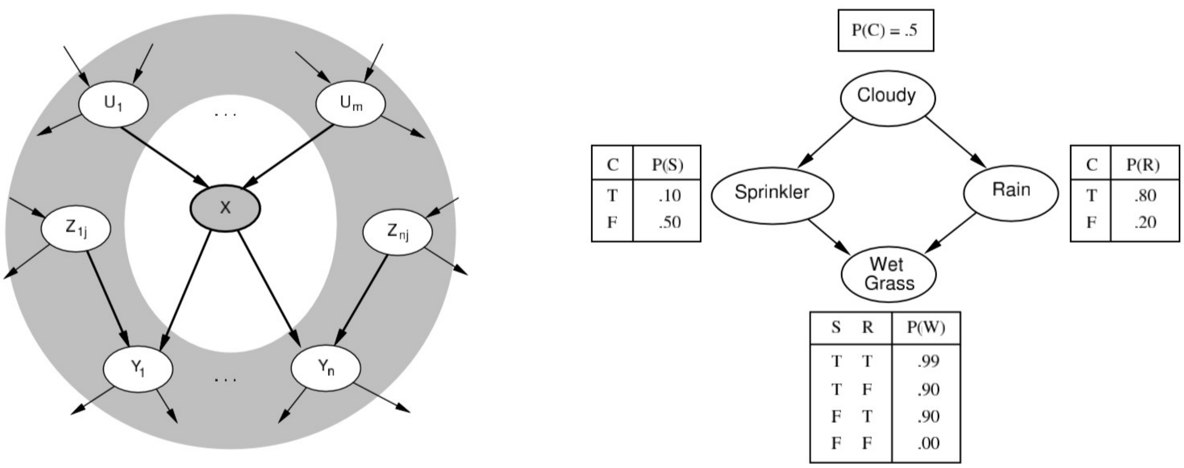

Figure 3: (left) The Markov Blanket of variable X (right) The Rain/Sprinkler network.

a) Prove that

$$P(X \mid MB(X)) = \alpha\, P(X \mid U_1,\dots,U_m) \prod_{Y_i} P(Y_i \mid Z_{i1} \dots)$$

where MB(X) is the Markov Blanket of variable X.

b) Consider the query

P(Rain|Spronkler = true, WetGrass = true)

in the Rain/Sprinkler network and how MCMC would answer it. How many possible states are there for the approach to consider given the network and the available evidence variables?

c) Calculate the transition matrix Q that stores the probabilities $P(y \rightarrow y')$ for all the states y,y'. If the Markov Chain has n states, then the transition matrix has size $n \times n$ and you should compute n^2 probabilities.

[Hint: Entries on the diagonal of the matrix correspond to self-loops, i.e., remaining in the same state. Such transitions can occur by sampling either variable. Entries where one variable is different between y and y', must sample that one variable. Entries where two or more variables change cannot occur, since in MCMC only one variable is allowed to change at each transition.]

Problem 4 [20 points]: Consider the *k* knights problem. This is the problem of putting *k* knights on a *n* by *n* chessboard in such a way that no two knights are attacking each-other.

a) Discuss how you would formulate this as a constraint satisfaction problem. What are the variables in this formalization and what are their possible values.

b) What constraints are there on the relationship between settings of these variables. [hint: Think of how we specified the constraints for wumpus world.]

c) Describe how you you would use the CSF algorithm presented in class to determine the maximum number of knights you can place on a *n* by *n* board