CS 440
Introduction to Artificial Intelligence

Lecture 15:

Markov Processes

5 March 2020

Discussion

- We computed probability of a call given that there is a burglary
  - P(call|burglary)
- How could we compute probability that there is a burglary given that you received a call?
  - P(burglary|call)
- One option - use Bayes theorem
  - P(burglary|call) = P(burglary) * P(call|burglary) / P(call)
  - Compute P(call) given enumeration tree

Bayesian Network: Formal Definition

- Network used for burglary problem and example of a neural network.
- Nodes correspond to variables
- Connections indicate cause and effect
  - Example burglary effects probability alarm will go off
- Each node stores probability of variable given all possible settings of parents
  - Non-discrete variables need function mapping all possible settings of parents to probability distribution of child

Dependence

- Given a Bayesian network
- Observe a variable in the network
- What other variables are effected by observation?
  - Variables whose probability is effected by observation

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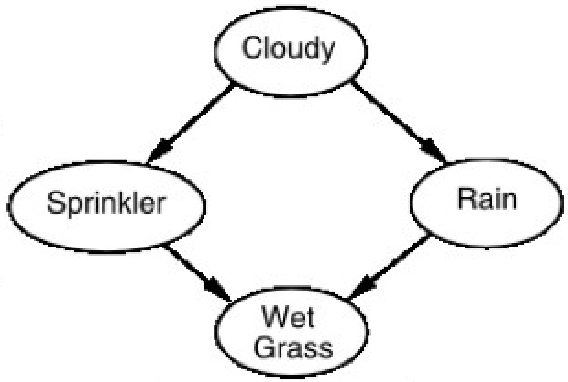
Multiply Connected Network

P(C) = .5

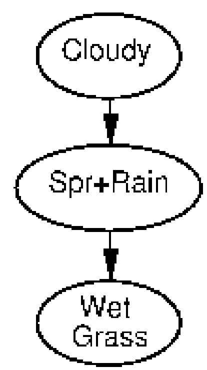


C	P(S)
T	.10
F	.50

C	P(R)
T	.80
F	.20

S	R	P(W)
T	T	.99
T	F	.90
F	T	.90
F	F	.00

P(C) = .5



C	P(S+R=x) TT	TF	FT	FF
T	.08	.02	.72	.18
F	.10	.40	.10	.40

S+R	P(W)
T T	.99
T F	.90
F T	.90
F F	.00

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Bayes' Theorem Example

- Assume you are given the following
  - P(c) probability someone in construction business
  - P(a) probability someone is exposed to asbestos
  - P(a|c) probability someone exposed to asbestos given they work in construction
  - P(s) probability someone is a smoker
  - P(l) probability someone develops lung cancer
  - P(l|a) probability someone develops ling cancer given they were exposed to asbestos
  - p(l|s) probability of developing lung cancer given that someone is a smoker
- What is the probability someone who has lung cancer works in construction?
- What is the probability a smoker who works in construction will develop lung cancer?
- What is the probability someone who has lung cancer is both a smoker and works in construction?

Markov Processes

- Consider an environment
  - Environment may transition to different states
    - Due to actions selected by agent
    - Due to things outside of agent's control
- Example: Autonomous car
  - State of road changes based on what agent does as well as what other agents do
    - Agent may turn, change lanes, accelerate/decelerate, ect.
    - Other cars may move, change lanes, cut you off, ect.
- Very complicated
  - Impossible to predict exactly
- Given state of environment possible to estimate future state
  - Give you position of cars with current speed
    - Predict likely position of cars after certain amount of time has passed

Markov Processes

- Markovian assumption
  - Future state only depend on current state
  - Only matters where cars currently are on road
    - Doesn't matter what maneuvers they took to get there
  - Assumption makes solving problems a lot easier
    - Only need to keep track of current state
      - As opposed to history of previous states
    - Only need to reason over current state

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Types of Markov Processes

- Discrete vs continuous
- Passive vs active
  - Active process: The agent's actions influence process
- Observable state vs partially observable state
  - Observable state: agent can observe state directly
  - Partially observable state:
    - Example: Wumpus world

	Observable State	Hidden State
Passive	Markov Chain	Hidden Markov Model
Active	Markov Decision Process (MPD)	Partially Observable Markov Decision Process (POMPD)

Markov Chain

- May be discrete or continuous
- Passive - state of environment does not depend of actions of agent
- Fully observable
- May be discrete or continuous
  - Discrete Markov chains can be represented as finite state machines
- Example: Bit flip
  - String of bits
    - State: string of 1s and 0s
  - At each time step each bit has a p probability of flipping

Solutions to Markov Chain

- Given an environment in a known state
  - What could the environment look like after n steps?
  - Probability of being in each state
- Solve inductively
  - Assume we know the probability you are in each state s at step i
    - $p_{s,i}$ for all $s \in S$
  - Compute probability for each state at step i+1
    - $p_{s',i+1}$ for all $s' \in S$
  - Probability we will transition from state s to s' at step i+1 equal to probability we are in state s at step i times the transition probability from s to s'
    - $p_{s',i+1} = p(s'|s) * p_{s,i}$
  - Probability we will be in state s' at step i+1
    - $p_{s',i+1} = \sum_{s' \in S} p(s'|s) * p_{s,i}$

Example: Bit Flipping

- Example: Bit flip
  - String of bits 2
    - States 00, 01, 10, 11
    - Transitions: Each bit has a p probability of flipping

T =

	00	01	10	11
00	$(1-p)^2$	$p(1-p)$	$p(1-p)$	p^2
01	$p(1-p)$	$(1-p)^2$	p^2	$p(1-p)$
10	$p(1-p)$	p^2	$(1-p)^2$	$p(1-p)$
11	p^2	$(1-p)^2$	$(1-p)^2$	$(1-p)^2$

  - Probability at step n

n	00	01	10	11
0	1.0	0	0	0
1	$(1-p)^2 =$ .81	$p(1-p) =$ .09	$p(1-p) =$ .09	$p^2 =$ .01
2	.6724	.1476	.1476	.0324

Can we formulate as a matrix multiplication problem?

Solve By Matrix Multiplication

- Let $P_i = \{P_{s1,i}, P_{s2,i}, P_{s3,i}, \ldots\}$
- Let $P_{i+1} = \{P_{s1,i+1}, P_{s2,i+1}, P_{s3,i+1}, \ldots\}$
- Let T be a transition function
- $P_{i+1} = T * P_i^T$
- Compute P_{i+1} by repetitively multiplying by T
  - $P_{i+1} = T^{i+1} * P_0^T$
- Can use parallel computing to expedite these computations

Markov Decision Problem

- May be discrete or continuous
- Active - agents actions effect state
- Fully observable
- May be discrete or continuous
- Formalization
  - State space S
  - Set of actions A
  - Transition function T(s,a)
    - Result of taking action a while in state s
    - Returns a probability distribution over S
      - probability that taking action a in state s will yield each $s' \in S$
  - Reward function R
    - Could define reward of being in a state, R(s)
    - Could define reward of performing action while in state R(s,a)
    - Could be reward of performing action that ends up in a particular state R(a,s')
- Immediate objective: Determine the best action to take given your current state
  - Action that maximizes expected future reward

Markov Chains Example

- **Robot in a grid with noisy actions**
  - **Robot can choose Left/Right/Up/Down**
  - **Actions may bring robot to wrong cell**