Read in the following dictionary:

```
28.0
              -4.00x_1 - 2.00x_2
                                          +4.00x_4 +2.00x_5 -1.00x_6 -5.00x_7
x_8
      24.0
              +2.00x_1 -2.00x_2 -3.00x_3 -2.00x_4 -7.00x_5 -7.00x_6 -1.00x_7
x_9
      -8.0
              +2.00x_1 + 4.00x_2 + 1.00x_3 - 5.00x_4 - 3.00x_5 - 10.00x_6 + 8.00x_7
x_{10}
      47.0
              +5.00x_1 - 7.00x_2 - 6.00x_3 - 5.00x_4 + 3.00x_5 + 9.00x_6 - 8.00x_7
x_{11}
              +3.00x_1 +5.00x_2 -8.00x_3 -7.00x_4 -10.00x_5 +6.00x_6 +3.00x_7
      0.0
x_{12}
      -1.0
              -5.00x_1 + 2.00x_2 - 6.00x_3 + 4.00x_4 + 1.00x_5
x_{13}
      0.0
              -3.00x_1 + 2.00x_2 + 3.00x_3
                                                                -1.00x_{6}
 z
```

0.1 Initialization Phase: Dual Problem Solving

New Objective in primal was changed to :

$$\max \sum_{j=1}^{7} -x_j$$

Primal variable x_j corresponds to dual variable y_j for j = 1, ..., 13 Dual Dictionary (with objective changed is):

```
+4.00y_8 -2.00y_9 -2.00y_{10} -5.00y_{11} -3.00y_{12} +5.00y_{13}
y_1
            +2.00y_8 +2.00y_9 -4.00y_{10} +7.00y_{11} -5.00y_{12} -2.00y_{13}
y_2
     1.0
                      +3.00y_9 -1.00y_{10} +6.00y_{11} +8.00y_{12} +6.00y_{13}
     1.0
y_3
            -4.00y_8 +2.00y_9 +5.00y_{10} +5.00y_{11} +7.00y_{12} -4.00y_{13}
     1.0
y_4
     1.0
           -2.00y_8 +7.00y_9 +3.00y_{10} -3.00y_{11} +10.00y_{12} -1.00y_{13}
y_5
     1.0
           +1.00y_8 +7.00y_9 +10.00y_{10} -9.00y_{11} -6.00y_{12}
y_6
     1.0
           +5.00y_8 +1.00y_9 -8.00y_{10} +8.00y_{11} -3.00y_{12} -5.00y_{13}
     -0
           -28.00y_8 - 24.00y_9 + 8.00y_{10} - 47.00y_{11}
```

Initialization succeeded in finding final dual dictionary with 2 pivots

```
0.75
                +2.75y_8 -2.25y_9 +0.25y_7 -7.00y_{11} -2.25y_{12} +6.25y_{13}
 y_1
       0.5
                -0.50y_8 +1.50y_9 +0.50y_7 +3.00y_{11} -3.50y_{12} +0.50y_{13}
y_2
      0.875
                -0.62y_8 +2.88y_9 +0.12y_7 +5.00y_{11} +8.38y_{12} +6.62y_{13}
y_3
      1.625
                -0.88y_8 +2.62y_9 -0.62y_7 +10.00y_{11} +5.12y_{12} -7.12y_{13}
y_4
      1.375
                -0.12y_8 +7.38y_9 -0.38y_7
                                                            +8.88y_{12} - 2.88y_{13}
 y_5
                +7.25y_8 \ +8.25y_9 \ -1.25y_7 \ +1.00y_{11} \ -9.75y_{12} -6.25y_{13}
       2.25
y_6
                +0.62y_8 +0.12y_9 -0.12y_7 +1.00y_{11} -0.38y_{12} -0.62y_{13}
y_{10}
      0.125
               -23.00y_8 - 23.00y_9 - 1.00y_7 - 39.00y_{11} - 3.00y_{12} - 4.00y_{13}
 z
       1.0
```

Primal Dictionary is:

```
23.0
              -2.75x_1 + 0.50x_2 + 0.62x_3 + 0.88x_4 + 0.12x_5 - 7.25x_6 - 0.62x_{10}
x_8
       23.0
              +2.25x_1 - 1.50x_2 - 2.88x_3 - 2.62x_4 - 7.38x_5 - 8.25x_6 - 0.12x_{10}
x_9
              -0.25x_1 - 0.50x_2 - 0.12x_3 \ + 0.62x_4 \ + 0.38x_5 + 1.25x_6 + 0.12x_{10}
       1.0
x_7
       39.0
              +7.00x_1 -3.00x_2 -5.00x_3 -10.00x_4
                                                                  -1.00x_6 - 1.00x_{10}
x_{11}
              +2.25x_1 +3.50x_2 -8.38x_3 -5.12x_4 -8.88x_5 +9.75x_6 +0.38x_{10}
       3.0
x_{12}
              -6.25x_1 - 0.50x_2 - 6.62x_3 + 7.12x_4 + 2.88x_5 + 6.25x_6 + 0.62x_{10}
x_{13}
              -0.75x_1 - 0.50x_2 - 0.88x_3 - 1.62x_4 - 1.38x_5 - 2.25x_6 - 0.12x_{10}
```

Primal Dictionary with original objective is:

```
23.0
             -2.75x_1 + 0.50x_2 + 0.62x_3 + 0.88x_4 + 0.12x_5 - 7.25x_6 - 0.62x_{10}
      23.0
             +2.25x_1 - 1.50x_2 - 2.88x_3 - 2.62x_4 - 7.38x_5 - 8.25x_6 - 0.12x_{10}
x_9
             -0.25x_1 - 0.50x_2 - 0.12x_3 + 0.62x_4 + 0.38x_5 + 1.25x_6 + 0.12x_{10}
x_7
             +7.00x_1 -3.00x_2 -5.00x_3 -10.00x_4
      39.0
                                                                 -1.00x_6 - 1.00x_{10}
x_{11}
             +2.25x_1 +3.50x_2 -8.38x_3 -5.12x_4 -8.88x_5 +9.75x_6 +0.38x_{10}
x_{12}
      3.0
      4.0
              -6.25x_1 - 0.50x_2 - 6.62x_3 + 7.12x_4 + 2.88x_5 + 6.25x_6 + 0.62x_{10}
x_{13}
      0.0
              -3.00x_1 + 2.00x_2 + 3.00x_3
                                                                 -1.00x_{6}
 z
```

1 Optimization Phase Simplex

Starting Dictionary is:

```
-2.75x_1 + 0.50x_2 + 0.62x_3 + 0.88x_4 + 0.12x_5 - 7.25x_6 - 0.62x_{10}
x_8
x_9
      23.0
             +2.25x_1 -1.50x_2 -2.88x_3 -2.62x_4 -7.38x_5 -8.25x_6 -0.12x_{10}
       1.0
             -0.25x_1 - 0.50x_2 - 0.12x_3 + 0.62x_4 + 0.38x_5 + 1.25x_6 + 0.12x_{10}
x_7
      39.0
             +7.00x_1 -3.00x_2 -5.00x_3 -10.00x_4
                                                                -1.00x_6 -1.00x_{10}
x_{11}
       3.0
             +2.25x_1 +3.50x_2 -8.38x_3 -5.12x_4 -8.88x_5 +9.75x_6 +0.38x_{10}
x_{12}
              -6.25x_1 - 0.50x_2 - 6.62x_3 + 7.12x_4 + 2.88x_5 + 6.25x_6 + 0.62x_{10}
       4.0
x_{13}
       0.0
             -3.00x_1 + 2.00x_2 + 3.00x_3
```

 x_2 enters and x_7 leaves

```
24.0
             -3.00x_1 - 1.00x_7 + 0.50x_3 + 1.50x_4 + 0.50x_5 - 6.00x_6 - 0.50x_{10}
x_8
             +3.00x_1 + 3.00x_7 - 2.50x_3 - 4.50x_4 - 8.50x_5 - 12.00x_6 - 0.50x_{10}
x_9
x_2
       2.0
              -0.50x_1 - 2.00x_7 - 0.25x_3 + 1.25x_4 + 0.75x_5 + 2.50x_6 + 0.25x_{10}
             +8.50x_1 +6.00x_7 -4.25x_3 -13.75x_4 -2.25x_5 -8.50x_6 -1.75x_{10}
      33.0
x_{11}
             +0.50x_1 - 7.00x_7 - 9.25x_3 - 0.75x_4 - 6.25x_5 + 18.50x_6 + 1.25x_{10}
x_{12}
      10.0
x_{1\underline{3}}
       3.0
              -6.00x_1 + 1.00x_7 - 6.50x_3 + 6.50x_4 + 2.50x_5 + 5.00x_6 + 0.50x_{10}
       4.0
              -4.00x_1 - 4.00x_7 + 2.50x_3 + 2.50x_4 + 1.50x_5 + 4.00x_6 + 0.50x_{10}
 z
```

 x_3 enters and x_{13} leaves

```
24.2307692308
                          -3.46x_1 -0.92x_7 -0.08x_{13} +2.00x_4 +0.69x_5 -5.62x_6 -0.46x_{10}
x_8
      18.8461538462
                          +5.31x_1 +2.62x_7 +0.38x_{13} -7.00x_4 -9.46x_5 -13.92x_6 -0.69x_{10}
x_9
x_2
      1.88461538462
                          -0.27x_1 -2.04x_7 + 0.04x_{13} + 1.00x_4 + 0.65x_5 + 2.31x_6 + 0.23x_{10}
      31.0384615385
                         +12.42x_1 +5.35x_7 +0.65x_{13} -18.00x_4 -3.88x_5 -11.77x_6 -2.08x_{10}
x_{11}
      5.73076923077
                          +9.04x_1 -8.42x_7 + 1.42x_{13} - 10.00x_4 - 9.81x_5 + 11.38x_6 + 0.54x_{10}
x_{12}
      0.461538461538
                          -0.92x_1 +0.15x_7 -0.15x_{13} +1.00x_4 +0.38x_5 +0.77x_6 +0.08x_{10}
x_3
      5.15384615385
                          -6.31x_1 -3.62x_7 -0.38x_{13} +5.00x_4 +2.46x_5 +5.92x_6 +0.69x_{10}
```

 x_4 enters and x_{12} leaves

```
25.3769230769
                         -1.65x_1 -2.61x_7 +0.21x_{13} -0.20x_{12} -1.27x_5 -3.34x_6 -0.35x_{10}
x_8
      14.8346153846
                         -1.02x_1 + 8.51x_7 - 0.61x_{13} + 0.70x_{12} - 2.60x_5 - 21.89x_6 - 1.07x_{10}
x_9
      2.45769230769
                         +0.63x_1 -2.88x_7 +0.18x_{13} -0.10x_{12} -0.33x_5 +3.45x_6 +0.28x_{10}
x_2
      20.7230769231
                         -3.85x_1 + 20.51x_7 - 1.91x_{13} + 1.80x_{12} + 13.77x_5 - 32.26x_6 - 3.05x_{10}
x_{11}
      0.573076923077
x_4
                         +0.90x_1 -0.84x_7 +0.14x_{13} -0.10x_{12} -0.98x_5 +1.14x_6 +0.05x_{10}
      1.03461538462
                          -0.02x_1 -0.69x_7 -0.01x_{13} -0.10x_{12} -0.60x_5 +1.91x_6 +0.13x_{10}
x_3
                         -1.79x_1 -7.83x_7 +0.33x_{13} -0.50x_{12} -2.44x_5 +11.62x_6 +0.96x_{10}
      8.01923076923
```

 x_6 enters and x_{11} leaves

```
x_8
     23.2324749642
                         -1.26x_1 - 4.73x_7 + 0.41x_{13} - 0.39x_{12} - 2.69x_5 + 0.10x_{11} - 0.04x_{10}
     0.77217453505
                         +1.59x_1 - 5.40x_7 + 0.68x_{13} - 0.52x_{12} - 11.94x_5 + 0.68x_{11} + 1.00x_{10}
x_9
     4.67131616595
                         +0.22x_1 -0.69x_7 -0.02x_{13} +0.09x_{12} +1.14x_5 -0.11x_{11} -0.04x_{10}
x_2
     0.64234620887
                         -0.12x_1 + 0.64x_7 - 0.06x_{13} + 0.06x_{12} + 0.43x_5 - 0.03x_{11} - 0.09x_{10}
x_6
                         +0.77x_1 -0.12x_7 +0.07x_{13} -0.04x_{12} -0.49x_5 -0.04x_{11} -0.05x_{10}
     1.30436337625
                         -0.25x_1 + 0.52x_7 - 0.12x_{13} + 0.01x_{12} + 0.22x_5 - 0.06x_{11} - 0.05x_{10}
x_3
     2.26001430615
     15.4803290415
                         -3.17x_1 - 0.44x_7 - 0.36x_{13} + 0.15x_{12} + 2.52x_5 - 0.36x_{11} - 0.14x_{10}
```

 x_5 enters and x_9 leaves

```
23.0582420545
                             -1.61x_1 - 3.51x_7 + 0.25x_{13} - 0.27x_{12} + 0.23x_9 - 0.05x_{11} - 0.26x_{10}
x_8
x_5
     0.0646723447594
                             +0.13x_1 -0.45x_7 +0.06x_{13} -0.04x_{12} -0.08x_9 +0.06x_{11} +0.08x_{10}
                             +0.38x_1 -1.21x_7 +0.04x_{13} +0.04x_{12} -0.10x_9 -0.04x_{11} +0.05x_{10}
       4.74529460515
x_2
                             -0.06x_1 + 0.44x_7 - 0.03x_{13} + 0.04x_{12} - 0.04x_9 - 0.01x_{11} - 0.06x_{10}
x_6
      0.669948377949
       1.27235873831
                             +0.70x_1+0.11x_7+0.05x_{13}-0.01x_{12}+0.04x_9-0.06x_{11}-0.10x_{10}
x_{4}
       2.27411608471
                             -0.22x_1 + 0.43x_7 - 0.11x_{13} - 0.00x_{12} - 0.02x_9 - 0.05x_{11} - 0.03x_{10}
                             \overline{-2.84x_1} \, \overline{-1.58x_7 \, -0.22x_{13}} \, +0.04x_{12} \, -0.21x_9 \, -0.22x_{11} \, +0.08x_{10}
       15.6429890865
```

 x_{10} enters and x_6 leaves

```
20.0503059143
x_8
                           -1.33x_1 - 5.50x_7 + 0.41x_{13} - 0.44x_{12} + 0.39x_9 - 0.02x_{11} + 4.49x_6
      1.01767505099
                           +0.04x_1 +0.18x_7 +0.01x_{13} +0.01x_{12} -0.13x_9 +0.05x_{11} -1.42x_6
x_5
x_2
      5.37049626105
                          +0.32x_1 -0.80x_7 +0.01x_{13} +0.08x_{12} -0.13x_9 -0.05x_{11} -0.93x_6
      11.4031271244
                           -1.06x_1 + 7.53x_7 - 0.59x_{13} + 0.63x_{12} - 0.61x_9 - 0.11x_{11} - 17.02x_6
x_{10}
      0.188987083617
                          +0.80x_1 -0.61x_7 +0.10x_{13} -0.07x_{12} +0.10x_9 -0.05x_{11} +1.62x_6
x_4
       1.91910265126
                           -0.18x_1 + 0.19x_7 - 0.09x_{13} - 0.02x_{12} + 0.00x_9 - 0.04x_{11} + 0.53x_6
x_3
       16.4983004759
                           -2.92x_1 - 1.02x_7 - 0.26x_{13} + 0.09x_{12} - 0.26x_9 - 0.23x_{11} - 1.28x_6
```

 x_{12} enters and x_4 leaves

```
18.9523809524
                        -6.00x_1 -1.95x_7 -0.19x_{13} +5.81x_4 -0.19x_9 +0.29x_{11} -4.90x_6
x_8
      1.04081632653
                        +0.14x_1 +0.10x_7 +0.02x_{13} -0.12x_4 -0.12x_9 +0.04x_{11} -1.22x_6
x_5
      5.56462585034
                        +1.14x_1 -1.42x_7 +0.12x_{13} -1.03x_4 -0.03x_9 -0.10x_{11} +0.73x_6
x_2
      12.9977324263
                        +5.71x_1 +2.38x_7 +0.28x_{13} -8.44x_4 +0.23x_9 -0.56x_{11} -3.38x_6
x_{10}
x_{12}
      2.52154195011
                        +10.71x_1 - 8.14x_7 + 1.37x_{13} - 13.34x_4 + 1.32x_9 - 0.70x_{11} + 21.58x_6
                         -0.43x_1 +0.38x_7 -0.12x_{13} +0.30x_4 -0.03x_9 -0.03x_{11} +0.04x_6
      1.86167800454
x_3
                        -2.00x_1 -1.71x_7 -0.14x_{13} -1.14x_4 -0.14x_9 -0.29x_{11} +0.57x_6
      16.7142857143
```

 x_6 enters and x_5 leaves

```
14.78333333333
                             -6.57x_1 -2.36x_7 -0.27x_{13} +6.30x_4 +0.30x_9 +0.12x_{11} +4.01x_5
x_8
x_6
             0.85
                             +0.12x_1 +0.08x_7 +0.02x_{13} -0.10x_4 -0.10x_9 +0.03x_{11} -0.82x_5
       6.18333333333
                             +1.23x_1 -1.36x_7 + 0.13x_{13} -1.10x_4 -0.10x_9 - 0.08x_{11} -0.59x_5
x_2
       10.1277777778
                             +5.32x_1 +2.10x_7 +0.22x_{13} -8.10x_4 +0.57x_9 -0.67x_{11} +2.76x_5
x_{10}
                            +13.23x_1 - 6.34x_7 + 1.73x_{13} - 15.50x_4 - 0.83x_9 + 0.02x_{11} - 17.62x_5
       20.86111111111
x_{12}
                             -0.42\underline{x_1} \ +0.38\underline{x_7} \ -0.12\underline{x_{13}} \ +0.30\underline{x_4} \ -0.03\underline{x_9} \ -0.03\underline{x_{11}} \ -0.03\underline{x_5}
       1.89444444444
x_3
 z
             17.2
                             -1.93x_1 -1.67x_7 -0.13x_{13} -1.20x_4 -0.20x_9 -0.27x_{11} -0.47x_5
```

Final Dictionary Solution: 17.2 Num Pivots: 9