hw2

September 5, 2018

1 Week 2 Homework

1.1 Question 1

(a) $\sum_{n=1}^{3} x_i = x_1 + x_2 + x_3$

(b) $\sum_{t=1}^{3} 2^t w_{2t} = 2w_2 + 4w_4 + 8w_6$

(c) $\sum_{i=1}^{3} \sum_{j=1}^{i} x_{ij} = x_{11} + x_{21} + x_{22} + x_{31} + x_{32} + x_{33}$

(d) $\sum_{i=1}^{3} \sum_{j=2}^{4} (x_i + y_{ij}) = 3x_1 + y_{12} + y_{13} + y_{14} + 3x_2 + y_{22} + y_{23} + y_{24} + 3x_3 + y_{32} + y_{33} + y_{34}$

(e) $\sum_{k=1}^{3} (2k+1) x_{k+1} = 3x_2 + 5x_3 + 7x_4$

(f) $\sum_{n=3}^{5} \sum_{m=n+1}^{n+3} x_n y_m = x_3y_4 + x_3y_5 + x_3y_6 + x_4y_5 + x_4y_6 + x_4y_7 + x_5y_6 + x_5y_7 + x_5y_8$

1.2 Question 2

(a) The dimension of each vector is **3**.

(b) $\mathbf{x} + 2\mathbf{y} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} + 2 \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ 8 \end{bmatrix}$

(c) $\|\mathbf{x} - \mathbf{y}\| = \left\| \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} - \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \right\| = \left\| \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \right\| = \sqrt{(-1)^2 + 0^2 + 2^2} = \sqrt{5}$

(d) $\mathbf{x}^{\mathbf{T}}(\mathbf{x} + \mathbf{y}) = \begin{bmatrix} 2\,1\,4 \end{bmatrix} \left(\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \right) = \begin{bmatrix} 2\,1\,4 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 6 \end{bmatrix} = 36$

(e) $x_2 y_1 = 1 \times 3 = 3$

1.3 Question 3

To determine if each set is convex, we'll use the provided definition: A set $X \in \mathbf{R}^n$ is convex if $\forall x, y \in X$ and $\lambda \in [0,1]$ it holds $\lambda x + (1-\lambda) y \in X$.

(a) The set $\{(x_1, x_2) \mid x_1^2 + x_2^2 \geq 0\}$ is **convex**.

Applying the definition, we have a tuple of the form $(\lambda x_1 + (1-\lambda)y_1, \lambda x_2 + (1-\lambda)y_2)$. The square of each term is ≥ 0 therefore the sum is ≥ 0. Thefore the result is a member of X.

(b) The set $\{x \mid 3 - x^2 = 0\}$ is **convex**.

This set has 2 members: $\sqrt{3}$ and $-\sqrt{3}$. Applying the definition we have $\lambda\sqrt{3} + (1-\lambda)(-\sqrt{3})$. This result is a member of the set when $\lambda = \frac{1}{2}$.

(c) The set $\{(x_1, x_2) \mid \frac{x_1}{x_2+2} \leq 3,\ x_2 \geq -1\}$ is **convex**.

This can be rewritten as the set $\{(x_1, x_2) \mid x_1 - 3x_2 \leq 6, x_2 \geq -1$. Applying the defnition when get the following:
$\lambda(x_1 - 3x_2) + (1-\lambda)(y_1 - 3y_2)$. This is ≤ 6 given the defintion of the set.

1.4 Question 4

A program is convex if its objective function is convex and the contraints are convex.

(a) The problem $min\ x_1^2 + x_2^2\ s.t.\ (x_1, x_2) \in$ $\mathbf{R}^2$ **is a convex program**.

The objective was shown to be convex in Q3. There are no constraints to consider since all of $\mathbf{R}^2$ is considered.

(b) The problem $max\ 3x_1 + 2x_2\ s.t.\ x_1^2 + x_2^2 \leq 10$ **is a convex program**.

The objective is a line and therefore is convex. The constraint is a filled circle and therefore is convex.

(c) The problem $min \sum_{i=1}^{n} 2^i (x_i)^{2i}\ s.t.\ \sum_{i=1}^{n} x_i \geq 10$ is **not a convex program**.

The objective is a polynomial of order 2n and is not convex.

1.5 Question 5

A quantity y is known to depend upon another quantity x. A set of n data pairs $\{y_i, x_i\}$ where i = 1..n has been collected.

(a) The optimization model is $min\ \sum_{i=1}^{n} |y_i - (ax_i + b)|$. This is a non-linear optimization program since the absolute function is not linear. All variables (a,b) are continuous. It is a convex program since each measurement is evaluated with a linear function that is convex.

(b) The optimization model is $min\ max(y_i - (ax_i + b))$. This is a non-linear optimization program since the max function is not linear. All variables (a,b) are continuous. It is a convex program since each measurement is evaluated with a function that is convex.

(c) The optimization model is $min\ max(y_i - (ax_i^2 + bx_i + c))$. This is a non-linear optimization program since the max function is not linear. All variables (a,b, c) are continuous. It is not a convex program since each measurement is evaluated with a function that is not convex.