hw11

October 31, 2018

1 HW 11

1.1 Question 1

Given the following LP:

$$\begin{aligned} \max\ & x_{12} + x_{22} + x_{23} \\ \text{s.t. } & x_{11} + x_{23} \leq 12 \\ & x_{11} + x_{12} + x_{13} = 20 \\ & x_{21} + x_{22} + x_{23} = 20 \\ & x_{11} + x_{21} = 10 \\ & x_{12} + x_{22} = 20 \\ & x_{13} + x_{23} = 10 \\ & x_{ij} \geq 0, \forall i \in \{1,2\} j \in \{1,2,3\} \end{aligned}$$

Consider the first constraint $x_{11} + x_{23} \leq 12$ as the "complicating" constraint (i.e. the $Dx \leq b$ constraint) and consider the remaining constraints including nonnegativity constraints as the "easy" constraints, which define a polyhedron P. That is, P is defined by the above five equality constraints and the nonnegativity constraints.

1) Argue that the polyhedron P defined above is bounded. P is bounded since each varaiable $(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23})$ is bounded above by the given constraints equality constraints and bounded below by the given non-negativity costraints.

2) Since P is bounded, we can use the extreme point representation for P. Specify c, D, and b for this problem. $c = [0\ 1\ 0\ 0\ 1\ 1]^T$
$D = [1\ 0\ 0\ 0\ 0\ 1]$
$b = [12\ 20\ 20\ 10\ 20\ 10]^T$

3) Construct the restricted master problem using these two extreme points given the following two extreme points of the polyhedron P: $x^1 = (x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}) = (10, 10, 0, 0, 10, 10)$
and
$x^2 = (x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}) = (0, 10, 10, 10, 10, 0)$
To get the RMP, we'll first calculate the individual parts:
$c^T x^1 = 30\ Dx^1 = 20$
$c^T x^2 = 20\ Dx^2 = 0$

The RMP becomes:

$$\begin{aligned} \min\ & 30\lambda_1 + 20\lambda_2 \\ \text{s.t. } & 20\lambda_1 = 12 \\ & \lambda_1 + \lambda_2 = 1 \\ & \lambda_1, \lambda_2 \geq 0 \end{aligned}$$

4) Find the optimal solution of the restricted master problem, which can be solved by hand. In the prior section, we can see that $\lambda_1 = \frac{3}{5}$ which implies $\lambda_2 = \frac{2}{5}$.

5a) Find the basis matrix for the optimal solution of the restricted master problem and compute the dual variables using: $B = \begin{bmatrix} 20 & 0 \\ 1 & 1 \end{bmatrix}$

$c_B = [30\ 20]^T$

$[\hat{y}^T \hat{r}] = c_B^T B^{-1} = [20\ 20] \begin{bmatrix} \frac{1}{20} & 0 \\ \frac{-1}{20} & 1 \end{bmatrix} = [\frac{1}{2}\ 20]$

5b) Form the dual problem of the restricted master problem, and compute the optimal dual variables using Complementary Slackness The dual is of the form:

$$\begin{aligned} \max\ & 12\lambda_1 + \lambda_2 \\ \text{s.t. } & 20\lambda_1 + \lambda_2 \leq 30 \\ & \lambda_2 \leq 20 \\ & \lambda_1, \lambda_2 \geq 0 \end{aligned}$$

From here we can see $\lambda_2 = 20$ which gives a $\lambda_1 = \frac{1}{2}$ which agrees with the above.

6) Using the dual variables computed in the previous part, formulate the subproblem that maximizes the reduced cost of the restricted master problem. $\hat{Z} = \min\ (c^T - \hat{y}^T D)x - \hat{r}$

$c^T - \hat{y}^T D = [0\ 1\ 0\ 0\ 1\ 1] - \frac{1}{2}[1\ 0\ 0\ 0\ 0\ 1] = [\frac{-1}{2}\ 1\ 0\ 0\ 1\ \frac{1}{2}]$

$$\begin{aligned} \min\ & \frac{-1}{2}x_{11} + x_{12} + x_{22} + \frac{1}{2}x_{23} - 20 \\ \text{s.t. } & x_{11} + x_{12} = 20 \\ & x_{22} + x_{23} = 20 \\ & x_{11} = 10 \\ & x_{12} + x_{22} = 20 \\ & x_{23} = 10 \\ & x_{ij} \geq 0 \end{aligned}$$

$\hat{Z} = -5 + 10 + 10 + 5 - 20 = 0$

This cost is non-negative therefore we should terminate.

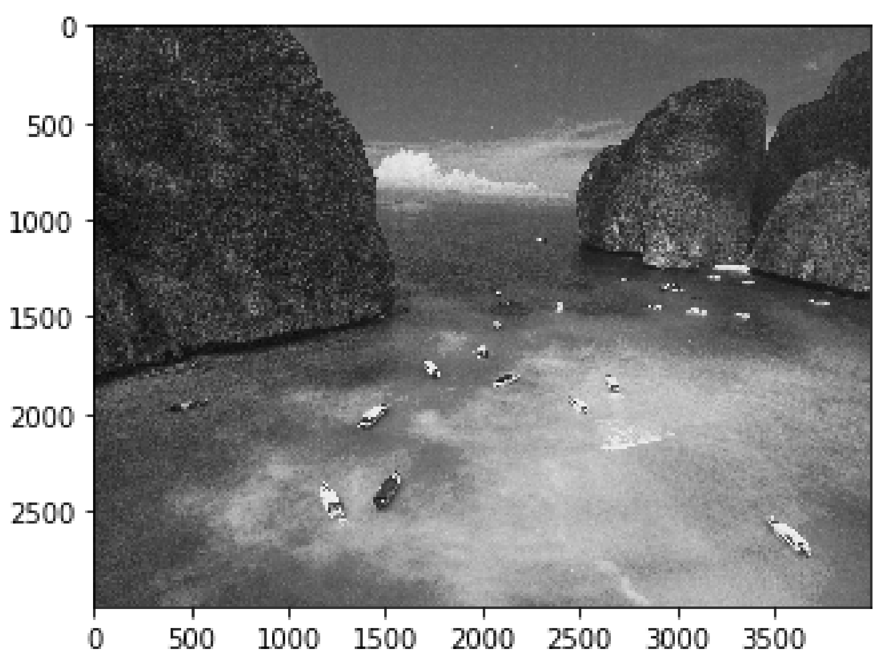
7) The polytope P has an interesting interpretation. Think about x_{ij} as the amount of product shipped from warehouse i to city j. Use this to interpret the five equality constraints as flow conservation constraints.

2 Question 2

Use SVD to compress the clown image.

Note that python is being used instead of matlab as stated in the HW assignment. This is becuase all other assignments are in python.

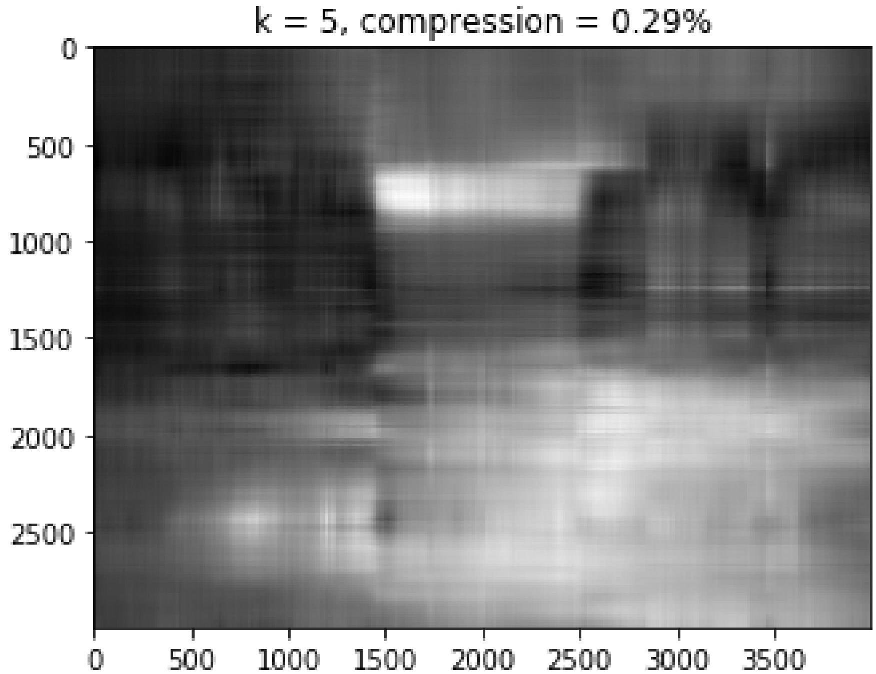
In [12]:
```
import matplotlib.pyplot as plt
import numpy as np
from PIL import Image

#open image and convert to np matrix
img = Image.open('image.jpg').convert('L')
plt.figure(figsize=(6, 4))
plt.imshow(img);

imgmat = np.array(img.getdata())
imgmat.shape = (img.size[1], img.size[0])
imgmat = np.matrix(imgmat)
```

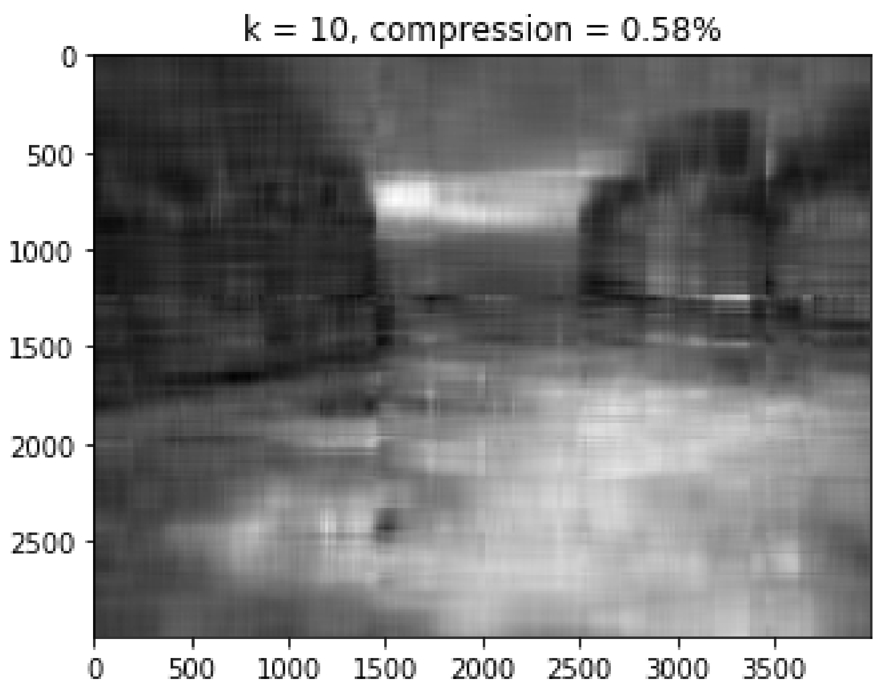


In [13]:
```
U, sigma, V = np.linalg.svd(imgmat)
```

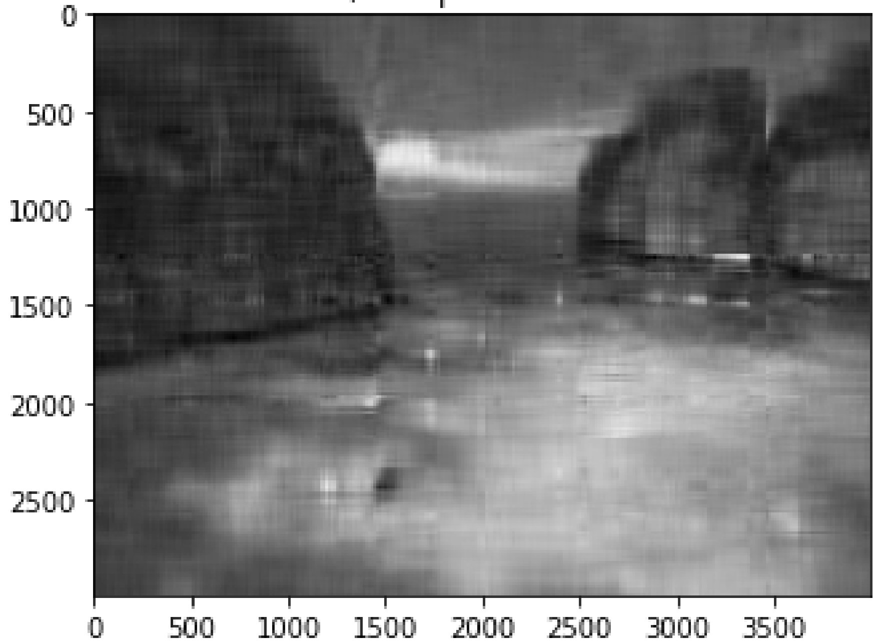
In [14]:
```
cmp = (img.size[0] + img.size[1]) / (img.size[0]*img.size[1])

for k in [5, 10, 15, 20, 25, 100]:
    newimg = U[:, :k] * np.diag(sigma[:k]) * V[:k, :]
    plt.figure(figsize=(6, 4))
```

```
plt.imshow(newimg, cmap='gray')
title = "k = {}, compression = {}%".format(k, round(100.*cmp*k,2))
plt.title(title)
plt.show()
```

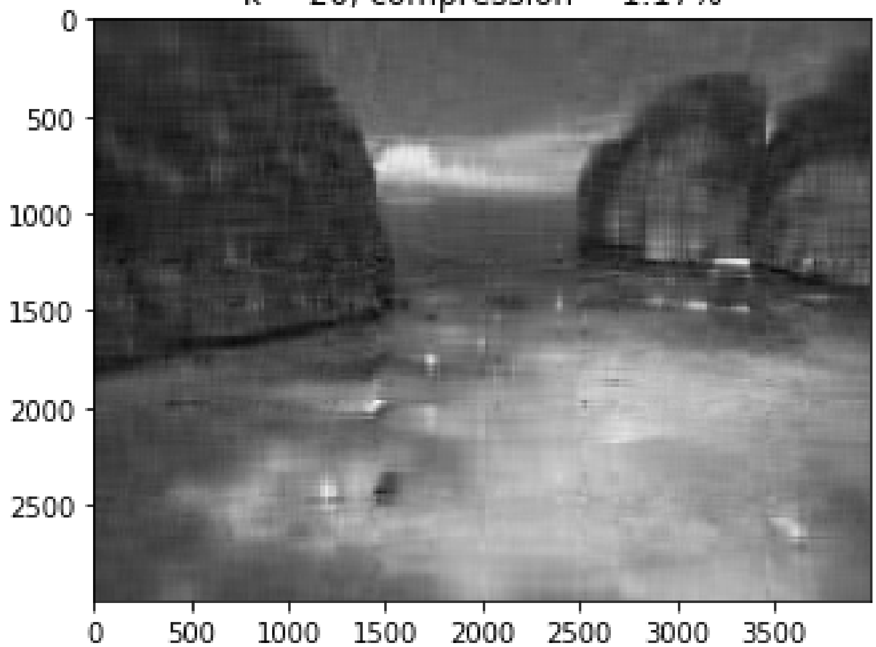


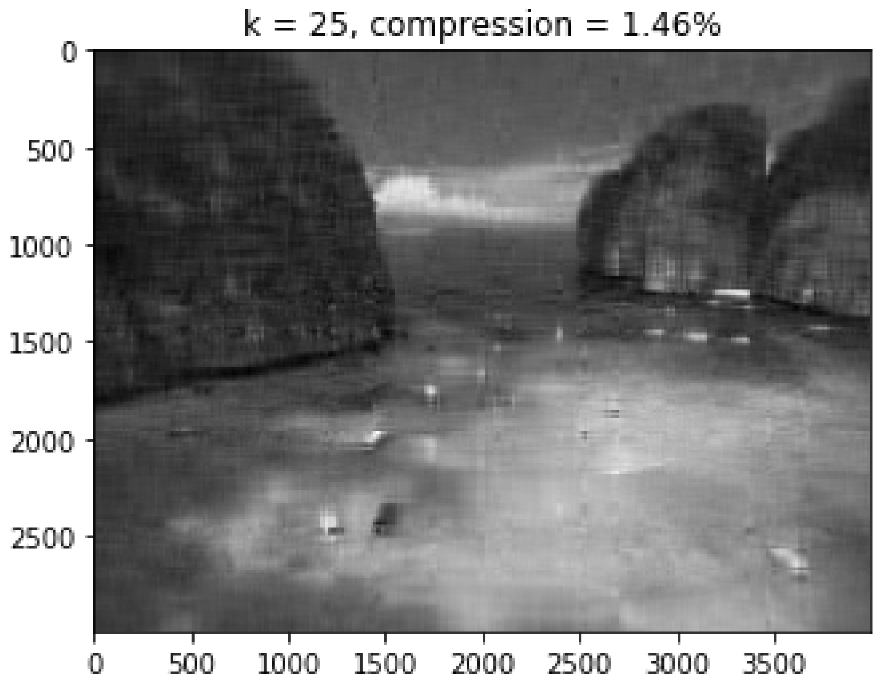


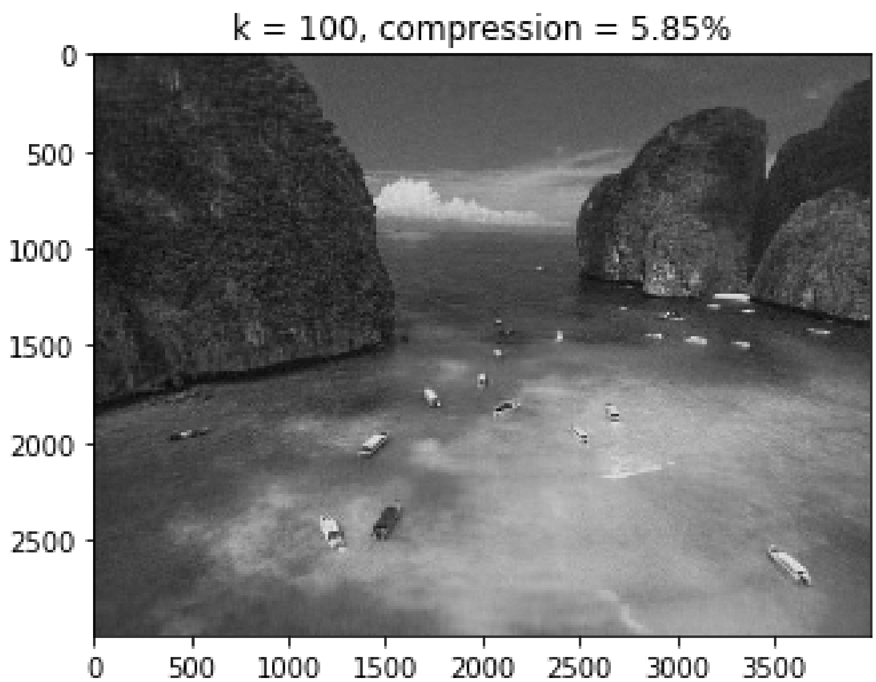
k = 15, compression = 0.88%



k = 20, compression = 1.17%




k = 25, compression = 1.46%
0
500
1000
1500
2000
2500
0
500
1000
1500
2000
2500
3000
3500


k = 100, compression = 5.85%
0
500
1000
1500
2000
2500
0
500
1000
1500
2000
2500
3000
3500