hw3

September 13, 2018

1 Week 3 Homework

1.1 Question 1

For each of the following cases, give an example demonstrating that problem **P** may have an optimal solution, and an example demonstrating that **P** may not have an optimal solution, or argue that such an example does not exist.

$P : min\ f(x)\ s.t.\ x \in X, X \subseteq \mathbf{R}^n$

(a) The function f is discontinuous and the set X is compact. Given that the set is compact, we know that it is bounded and closed. Therefore, **P** will not have a solution if the discontinuity exists where the minimum would be, and **P** will have a solution if the discontinuity exists elsewhere.

(b) The function f is continuous and the set X is not closed. If the set is not closed, then we may have an interval such as $-1 < x \leq 1$. If $f(x) = x$ then **P** does not have a solution since we can always choose a smaller x. However, if $f(x) = x^2$ then **P** does have a solution at $x = 0$.

(c) The function f is convex and the set X is not bounded. Given an unbounded set, **P** will have a solution for some convex functions but not others. For example, let X be a set that is not bounded below. **P** would have not a solution for $f(x) = x$. However, **P** would have a solution for $f(x) = x^2$.

(d) The function f is convex and the set X is compact. Given a compact set and a convex function, **P** will always have a soution. There are no examples of where **P** does not have a solution.

(e) The function f is linear and the set X is not closed. If X is not closed, and $f(x) = a * x + b$, then whether or not **P** has a solution depends on the constant a and the boundry on which X is not closed. For example, if $a > 0$, and X includes the lower bound on its interval, then **P** has a solution. However, if $a > 0$ and X does not include the lower bound of the interval, then **P** does not have a solution.

(f) The function f is linear and the set X is compact. Given a compact set and a linear function, **P** will always have a soution. There are no examples of where **P** does not have a solution.

1.2 Question 2

For each of the statements below, state whether it is true or is false.

(a) An optimization problem with a discontinuous objective function and a closed and bounded feasible region can never have an optimal solution. This is false. The discontinuity doesn't necessarily impact the optimial value.

(b) Consider the optimization problem $min\ f(x)\ s.t.\ g(x) \leq 0$**. Suppose the current optimal objective value is** v**. Now, if I change the right-hand-side of the constraint to 1 and resolve the problem, the new optimal objective value will be less than or equal to** v**.** This is true. Increasing the upper bound on the constraint still includes the original constraint, therefore the optimal value would only stay the same or improve.

(c) Consider an optimization problem $(P)\ :\ min\ f(x)\ s.t.\ x \in X$**, where** X **is a non-empty closed convex set. Suppose that the problem** (P) **has the property that every local optimal solution is also globally optimal then** $f(x)$ **must be a convex function.** This is false. Let $f(x) = sin(x), 0 \leq x \leq 4\pi$. Each local optimimum at a multiple of $\frac{3\pi}{2}$ is also a global optimum, but $sin(x)$ is not convex.

(d) The problem $(P) : min\ x + y\ subject\ to\ x^2 + y^2 \leq 4$ **is a convex optimization problem.** This is true becuase the objective function is convex and the constraint is convex.

(e) If I solve an optimization problem, then add a new constraint to it and solve it again, the solution must change. This is false. Adding a constraint doesn't necessarily change the feasible region, and doesn't necessarily change the optimal solution.

(f) Consider the following optimization problem: $min[f(x)]^2\ s.t.\ x \in X$ **where** $f(x)$ **is a non-convex function and** X **is a non-empty set. Suppose at a feasible solution** $x \in X$ **the objective value is 0, then** x **must be an optimal solution.** This is true. The minimum value of the square of any (real) valued function is 0. The non-convexivity of $f(x)$ is not a problem since we are not concerned with whether or not x^* is a global optimum.

(g) If I maximize a univariate convex function over a closed interval then there has to be an optimal solution which is one of the end points. This is true if the convex function is not a constant (e.g. $f(x) = n$ for some arbitrary number n.