hw7

October 3, 2018

1 Homework 7

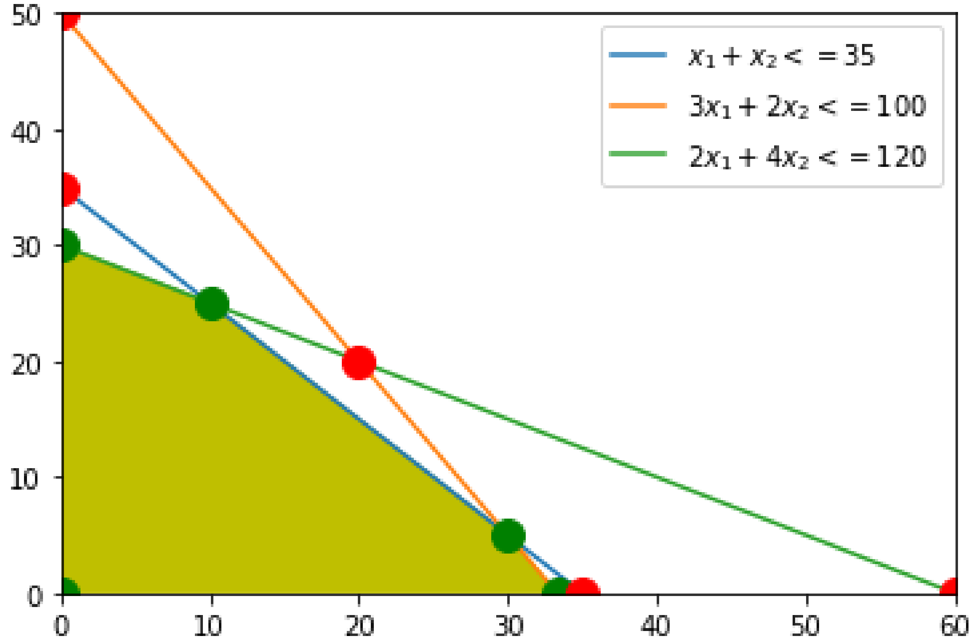
1.0.1 (a) Consider the following linear program:

$$
\begin{aligned}
& & & (1)\\
\min & & -2x_1 - 3x_2 & \quad (2)\\
\text{s.t.} & & x_1 + x_2 \leq 35 & \quad (3)\\
& & 3x_1 + 2x_2 \leq 100 & \quad (4)\\
& & 2x_1 + 4x_2 \leq 120 & \quad (5)\\
& & x_1, x_2 \geq 0 & \quad (6)\\
& & & (7)
\end{aligned}
$$

Below we plot the constraints of this linear program.

In [7]:
```
import matplotlib.pyplot as plt
import numpy as np

# x1-values for our plot
xmax = 60
ymax = 50
x1 = np.arange(0, xmax, 0.1)

# the constraints to plot
c1 = -x1 + 35.
c2 = -3./2.*x1 + 100. / 2.
c3 = -2./4.*x1 + 120. / 4.

# plot the constraints
plt.xlim(0, xmax)
plt.ylim(0, ymax)
plt.plot(x1, c1, x1, c2, x1, c3, label='Feasible Region')
plt.legend([r'$x_1 + x_2 <= 35$', r'$3x_1 + 2x_2 <= 100$', r'$2x_1 + 4x_2 <= 120$']);

# fill in the feasable region (using a polygon)
xp = [0, 0, 10, 30, 100./3.]
yp = [0, 30, 25, 5, 0]
```

```python
plt.fill(xp ,yp, color='y');

#plot basic solutions (green = feasible, red = not-feasible)
plt.plot(xp, yp, 'or', markersize=12, color='green');
plt.plot([0,0,20,35,60], [35,50,20,0,0], 'or', markersize=12, color='red');
```



1.0.2 (b) Transform it into a standard form LP.

The objective is already a minimization, so the objective function does not need to be transformed. Each of the 3 constraints are of the same type of inqequality, therefore we just need to add a slack variable for each (x_3, x_4, x_5) constant and change the inequality to an equals.

$$
\begin{aligned}
\text{min} \quad & -2x_1 - 3x_2 && (8)\\
\text{s.t.} \quad & x_1 + x_2 + x_3 = 35 && (9)\\
& 3x_1 + 2x_2 + x_4 = 100 && (10)\\
& 2x_1 + 4x_2 + x_5 = 120 && (11)\\
& x_1, x_2, x_3, x_4, x_5 \geq 0 && (12)\\
& && (13)
\end{aligned}
$$

In matrix format this yields the following.

$x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{bmatrix}^T$

$c = \begin{bmatrix} -2 & 3 & 0 & 0 & 0 \end{bmatrix}^T$

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 3 & 2 & 0 & 1 & 0 \\ 2 & 4 & 0 & 0 & 1 \end{bmatrix}$$
$$b = \begin{bmatrix} 35 & 100 & 120 \end{bmatrix}^T$$

1.0.3 (c) Use the procedure discussed in lecture to find *all* basic solutions.

We have 3 constraints therefore we need to pick 3 columns to determine a solution. To find all basic solutions we need $C(5,3) = 10$. This is too many iterations to perform by hand, so we'll use python to iterate over the column combinations.

```
In [6]: import itertools as it
        import numpy as np
        from scipy import linalg

        c = np.array([-2., -3., 0., 0., 0.])
        A = np.array([[1., 1., 1., 0., 0.],[3., 2., 0., 1., 0.],[2., 4., 0., 0., 1.]])
        b = np.array([35., 100., 120.]).T

        #obtain column combinations
        n = range(5)
        idx = [list(i) for i in it.combinations(n, 3)]
        print('column combo count:', len(idx))
        print('column combo sample:', idx[:3])
        print('')
        #iterate over each column combo and check solution
        for i in idx:
            B = A[:, i]
            BI = linalg.inv(B)
            xb = BI.dot(b)
            print('xb =', ['x{}'.format(ii+1) for ii in i], '=', xb)
            print('xn =', ['x{}'.format(ii+1) for ii in n if ii not in i], '=', '[0, 0]')
            print('feasible:', all(xb > 0))
            #x = np.array([99 if ii in i else 0. for ii in i])
            x = np.zeros(5)
            for ii in range(3):
                x[i[ii]] = xb[ii]
            cost = c.dot(x)
            print('cost =', round(cost,2))
            print('-'*100)
```

```
column combo count: 10
column combo sample: [[0, 1, 2], [0, 1, 3], [0, 1, 4]]

xb = ['x1', 'x2', 'x3'] = [20. 20. -5.]
xn = ['x4', 'x5'] = [0, 0]
feasible: False
cost = -100.0
```

```
--------------------------------------------------------------------------------
xb = ['x1', 'x2', 'x4'] = [10. 25. 20.]
xn = ['x3', 'x5'] = [0, 0]
feasible: True
cost = -95.0
--------------------------------------------------------------------------------
xb = ['x1', 'x2', 'x5'] = [30.  5. 40.]
xn = ['x3', 'x4'] = [0, 0]
feasible: True
cost = -75.0
--------------------------------------------------------------------------------
xb = ['x1', 'x3', 'x4'] = [ 60. -25. -80.]
xn = ['x2', 'x5'] = [0, 0]
feasible: False
cost = -120.0
--------------------------------------------------------------------------------
xb = ['x1', 'x3', 'x5'] = [33.33333333  1.66666667 53.33333333]
xn = ['x2', 'x4'] = [0, 0]
feasible: True
cost = -66.67
--------------------------------------------------------------------------------
xb = ['x1', 'x4', 'x5'] = [35. -5. 50.]
xn = ['x2', 'x3'] = [0, 0]
feasible: False
cost = -70.0
--------------------------------------------------------------------------------
xb = ['x2', 'x3', 'x4'] = [30.  5. 40.]
xn = ['x1', 'x5'] = [0, 0]
feasible: True
cost = -90.0
--------------------------------------------------------------------------------
xb = ['x2', 'x3', 'x5'] = [ 50. -15. -80.]
xn = ['x1', 'x4'] = [0, 0]
feasible: False
cost = -150.0
--------------------------------------------------------------------------------
xb = ['x2', 'x4', 'x5'] = [ 35.  30. -20.]
xn = ['x1', 'x3'] = [0, 0]
feasible: False
cost = -105.0
--------------------------------------------------------------------------------
xb = ['x3', 'x4', 'x5'] = [ 35. 100. 120.]
xn = ['x1', 'x2'] = [0, 0]
feasible: True
cost = 0.0
--------------------------------------------------------------------------------
```

1.0.4 (d) Among all the basic solutions you found, which basic solutions are feasible, thus are basic feasible solutions?

Feasible solutions are annototed in the output above with "feasible: True". 5 of the 10 solutions are marked feasible. This corresponds to the 5 corners of the feasible region shown in part (a).

1.0.5 Which basic solution are infeasible?

5 of the 10 solutions are marked as not-feasible. These 5 solutions represent the intersection of constraints outside of the feasible region shown in part (a).

1.0.6 Locate each basic solution on the graph you draw in part (a)

Feasible and not-feasible solutions are shown in the graph in part (a). Feasible solutions are shown in green. Not-feasible solutions are shown in red.

The optimal solution appears to be -95 $(x_1, x_2) = (10, 25)$.

For fun, we can also solve the original (non-standard) LP with cvxpy.

```
In [5]: import cvxpy as cp
        import numpy as np

        #setup variables and coeffcients
        x = cp.Variable(2, 1)
        c = np.array([-2., -3.])
        A = np.array([[1.,1.],[3.,2.],[2.,4.]])
        b = np.array([35., 100., 120.])

        #setup objective and constraints
        objective = cp.Minimize(c*x)
        constraints = [A*x <= b, x >= 0.]

        # solve
        prob = cp.Problem(objective, constraints)
        result = prob.solve()

        # display optimal value of variables
        print('The solution status is', prob.status)
        print('The optimal value is ', round(result))
        print('The optimal [x1, x2] is ', [round(xx,2) for xx in x.value])
```

```
The solution status is optimal
The optimal value is  -95.0
The optimal [x1, x2] is  [10.0, 25.0]
```