Name: ____________________

SSC329C Final
Fall 2013

Note: Closed book. You are allowed one sheet (two sides) of notes. No explicit worked problems. Just notes. Formulae, definitions, algorithms are fine.

1. Compute

   (a) (3 points) $\begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -2 & 2 \end{pmatrix} =$

   (b) (3 points) $\begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix} =$

   (c) (2 points) $\begin{pmatrix} 1 & -2 & 0 & 1 \\ -1 & 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -2 & 2 \\ 1 & -2 \\ 0 & 2 \end{pmatrix} =$

   (d) (2 points) How are the results in (a), (b), and (c) related?

2. Consider

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 3 & -1 & 0 \\ 0 & -1 & 1 \end{pmatrix}, \quad \text{and} \quad B = \begin{pmatrix} -2 & 2 \\ -1 & 1 \\ 3 & -3 \end{pmatrix}.$$

(a) (7 points)
Solve for X the equation $AX = B$. (If you have trouble figuring out how to approach this, you can "buy" a hint for 3 points.)

(b) (3 points) In words, describe a way of solving this problem that is different from the approach you used in part a).

(c) (3 bonus points) In words, describe yet another way of solving this problem that is different from the approach in part a) and b).

3. Compute the inverses of the following matrices

(a) (4 points) $A = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}$. $A^{-1} =$

(b) (3 points) $B = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$. $B^{-1} =$

(c) (3 points) $C = BA$ where A and B are as in parts (a) and (b) of this problem.
$C^{-1} =$

4. Consider the vectors $a_0 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $a_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$.

(a) (6 points) Compute the projection of $b = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ onto the space spanned by the vectors a_0 and a_1. (The numbers may not work out very nicely. Set the problem up first, plug in the numbers, then move on and solve other problems. Then come back and solve this one. There may be fractions.)

(b) (4 points) Compute the linear least-squares solution to find an approximate solution to $Ax = b$. (The numbers may not work out very nicely. Set the problem up first, plug in the numbers, then move on and solve other problems. Then come back and solve this one. There may be fractions.)

5. Consider the vectors $a_0 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $a_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$.

(a) (6 points) Compute orthonormal vectors q_0 and q_1 so that q_0 and q_1 span the same space as a_0 and a_1.

(b) (4 points) Let $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 2 \end{pmatrix}$. Give Q and R such that $A = QR$ and R is upper triangular.

6. Consider the matrix $A = \begin{pmatrix} 1 & -1 & -2 & 1 \\ -1 & 3 & 2 & 0 \end{pmatrix}$.

(a) (3 points) Give a basis for the column space of A.

(b) (3 points) Give a basis for the row space of A.

(c) (6 points) Find two linearly independent vectors in the null space of A.

7. Consider $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$.

(a) (4 points) Show that $x = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is one of its eigenvectors.

(b) (4 points) Find *ONE* eigenvalue of this matrix.

8. (a) (4 points) Give all eigenvalues and corresponding eigenvectors of $A = \begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix}$.

(b) (3 points) Let λ be an eigenvalue of A and $Ax = \lambda x$ so that x is an eigenvector corresponding to A. Show that $(A - \mu I)x = (\lambda - \mu)x$.

(c) (3 points) Give all eigenvalues and corresponding eigenvectors of $B = \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}$. (You can save yourself some work if you are clever...)

9. Let $L = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$, $U = \begin{pmatrix} 1 & -2 & -1 & 1 \\ 0 & 0 & 2 & 1 \end{pmatrix}$, and $b = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$. Let $A = LU$.

(a) (5 points) Solve $Lx = b$.

(b) (5 points) Find a specific (particular) solution of $Ax = b$.

(c) (1 points) Is b in the column space of A? Yes/No

(d) (1 points) Is b in the column space of L? Yes/No

(e) (5 points) Find two linearly independent solutions to $Ax = 0$.

(f) (3 points) Give a general solution to $Ax = b$.