There are 6 integrable irreducible representations for G2 at level 3

: See the fusion graphs of G2 at that level.

Integrable fundamental weights for G2 at level 3 :

(list, classical dimensions, quantum dimensions)

{{0, 0}, {0, 1}, {1, 0}}

{1, 7, 14}

$\left\{1, \frac{3}{2}+\frac{\sqrt{21}}{2}, \frac{3}{2}+\frac{\sqrt{21}}{2}\right\}$

$\mathcal{E}_3$

A quantum subgroup of $A_3(G2)$ ie of G2 at level 3

Altitude is 4+3 = 7 , so

Central charge : 6

Quantum cardinal of E : $\frac{3}{2}\left(7+\sqrt{21}\right)$

Quantum cardinal of A :

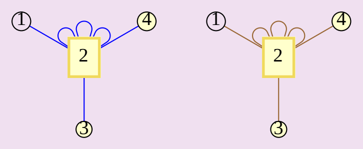
Q-dimension of Frobenius algebra F (or of A/E) :

$\frac{1}{2}\left(7+\sqrt{21}\right)$

Induction rules : E1 ,

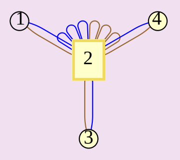
$$E1 = \begin{pmatrix} 1 & . & . & . \\ . & 1 & . & . \\ . & 1 & 1 & 1 \\ . & 1 & . & . \\ . & 1 & . & . \\ 1 & 1 & . & . \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & . & . & . & . & 1 \\ . & . & . & . & . & . \\ . & . & 2 & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 1 & . & . & . & . & 1 \end{pmatrix}$$

The fusion graph of E3(G2)



Blue lines: multiplication by the generator (0,1)

Brown lines: multiplication by the generator (1,0)



Both generators together

Quantum groupoid :

Size of the 6 blocks :

Total horizontal dimension : 61

Size of the 8 dual blocks :

Total vertical dimension : 69

Modified vertical dimension (4 → 4/3) : 61

Bialgebra dimension : 717 = $3^1\, 239^1$

Out[602]=

Simple objects : 4 (list, quantum dimensions) :

{1, 2, 3, 4}

$\left\{1, \frac{1}{2}\left(3+\sqrt{21}\right), 1, 1\right\}$

Modular vertices : 3

(position in list, induction, exponents (back shifted to 0), conformal weights of G2 mod 1) :

α = {1, 3, 4}

. Modular invariant : $Z = \sum_\alpha |\chi_\alpha|^2$

1 : F = $\chi[1] = \sigma[0] + \sigma[5]$

3 : $\chi[3] = \sigma[2]$

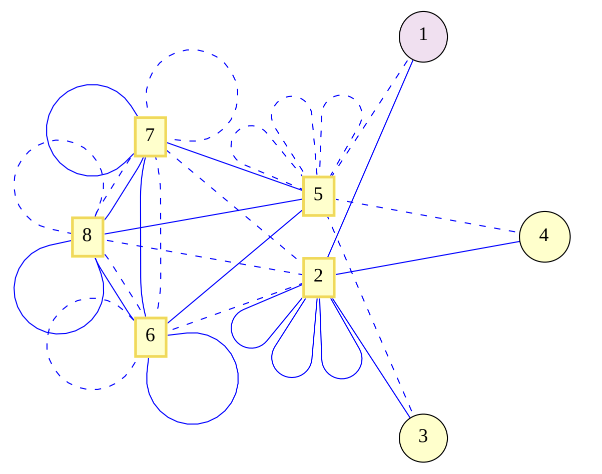
4 : $\chi[4] = \sigma[2]$

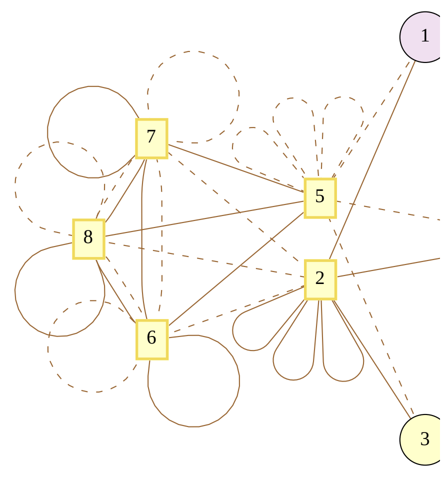
{{0, 0}, {0, 2}, {0, 2}, {1, 1}}

$\left\{0, \frac{2}{7}, \frac{2}{3}, \frac{4}{7}, \frac{1}{7}, 0\right\}$

The graph of quantum symmetries (Ocneanu graph)

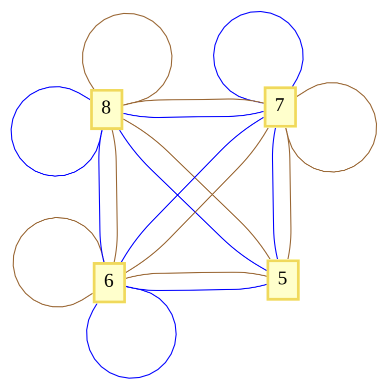
The Ocneanu graph of E3(G2)
Left chiral part : full lines
Right chiral part: dashed lines

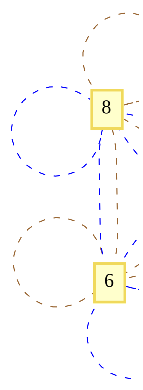


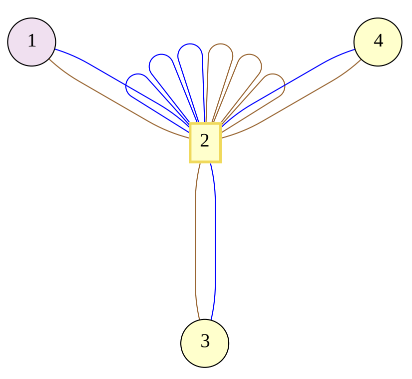


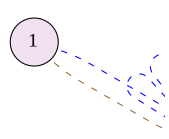
Chiral subgraphs relative to the generator (0,1)   Chiral subgraphs relative to the ge

The Ocneanu graph of E3(G2)
Left chiral part : full lines
Right chiral part: dashed lines
Generator (0,1): blue
Generator (1,0): brown









Left chiral subgraphs relative to generators (0,1) and (1,0)   Right chiral subgraphs relat
The fusion graph E3(G2) appears with its module M3(G2)   The fusion graph E3(G2) appe

Remark:
There are two conformally exceptional quantum subgroups of G2, namely E3(G2) and E4(G2).
The quantum subgroup E3(G2) can be obtained from the conformal embedding of G2, at level 3, in E6.
E3(G2) shares many features with the quantum subgroup of A1=SU(2) described by the graph D4=D4(A1), itself an orbifold of A5=A4(A1).
Because of this, it would not be unreasonable to rename E3(G2) and call it D3(G2).
The other conformally exceptional quantum subgroup of G2, namely E4(G2) is a kind of analogue of D6=D8(A1).
The quantum subgroup E4(G2) could also be renamed D4(G2).

Details about E3(G2) and E4(G2) can be found in <<Exceptional quantum subgroups for the rank two Lie algebras B2 and G2>>, arXiv:1001.5416 , by R. Coquereaux, R. Rais and H. Tahrie, to appear in the Journal of Mathematical Physics.