Reformulation of Reduction-to-Pole by Reduction-to-Equator Operators

R Nate Crummett, Mengli Zhang, and Yaoguo Li

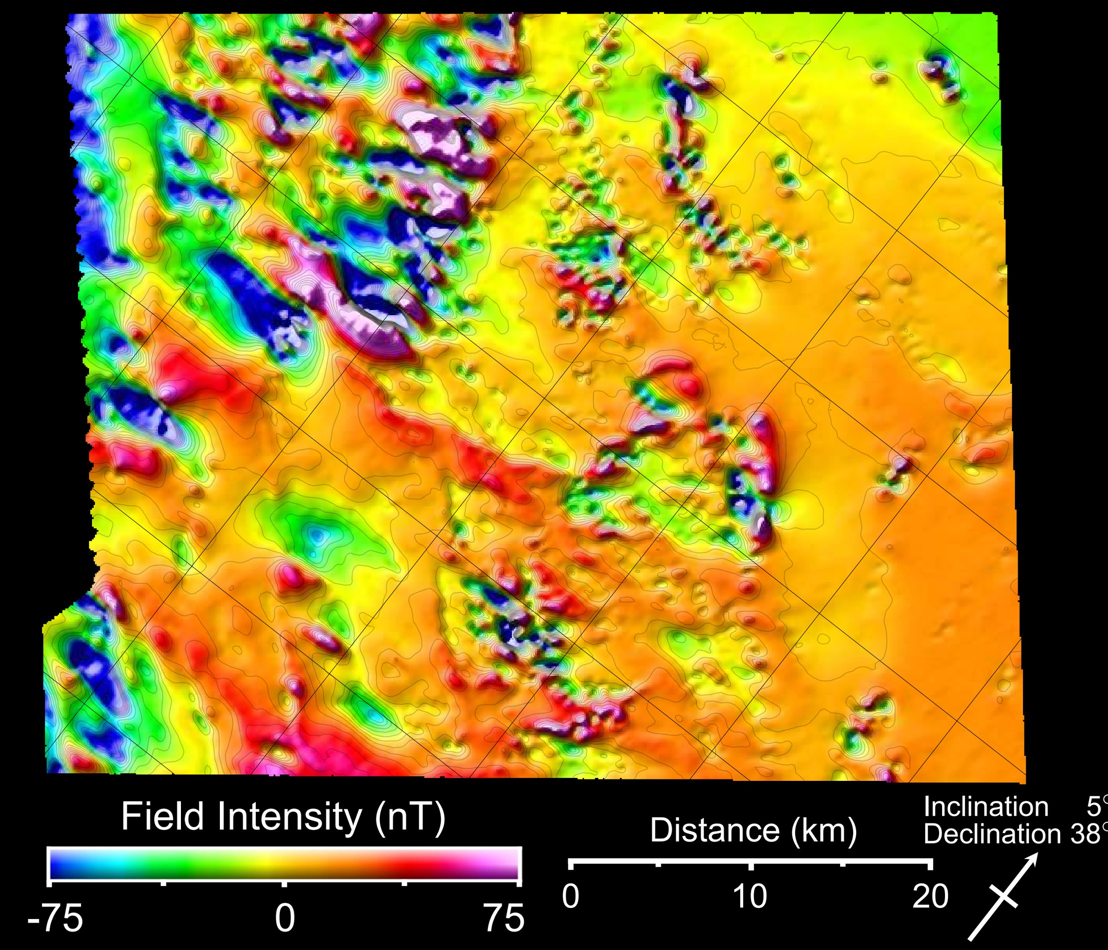
Laplace's Equation

for the Magnetic Monopole U

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0$$

Reduction to the pole…	a second vertical derivative of U
Reduction to the equator…	a second horizontal derivative of U
The remaining term…	a *rotated* second horizontal derivative of U!

Total-Field Anomaly

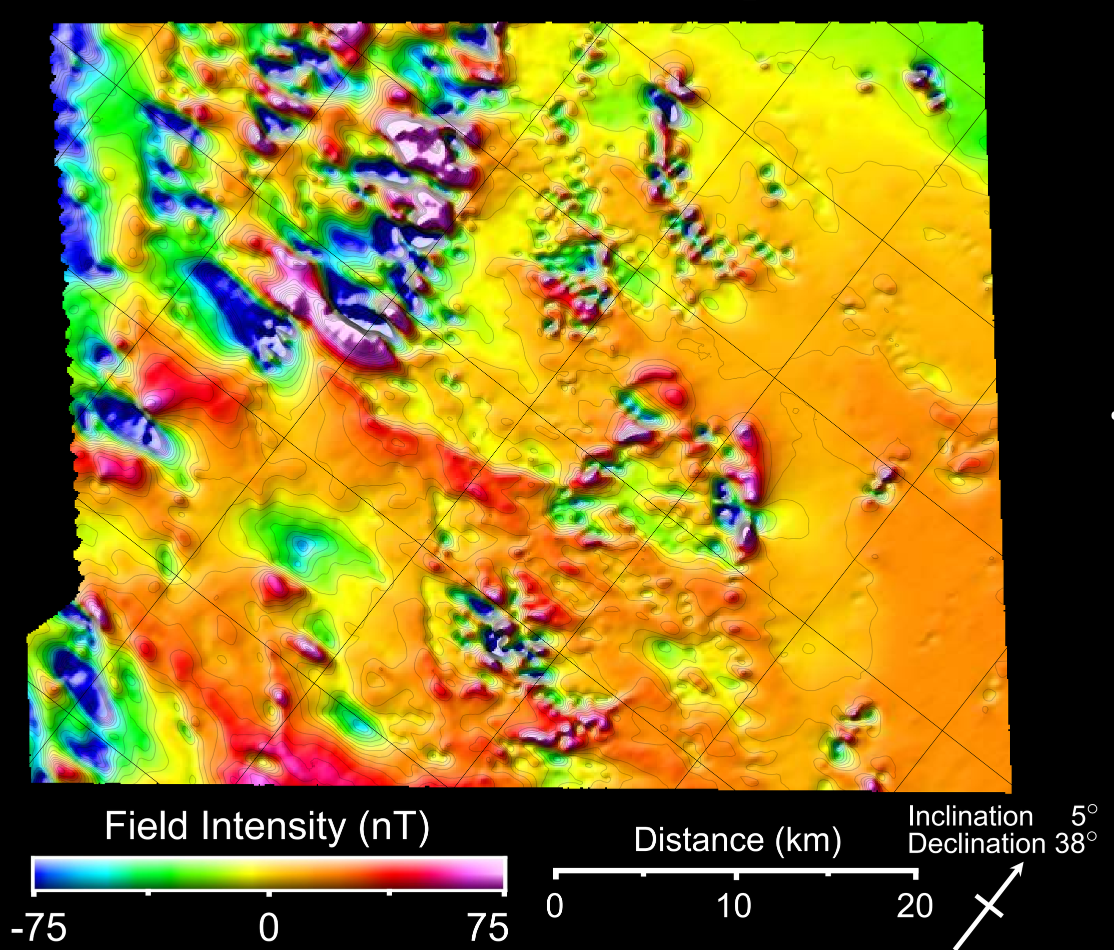


Two Horizontal Projections Instead of One Vertical Projection

Rather than stabilizing the reduction-to-pole operator, we stabilize a rotated reduction-to-equator and reconstruct the reduction-to-pole field with Laplace's equation

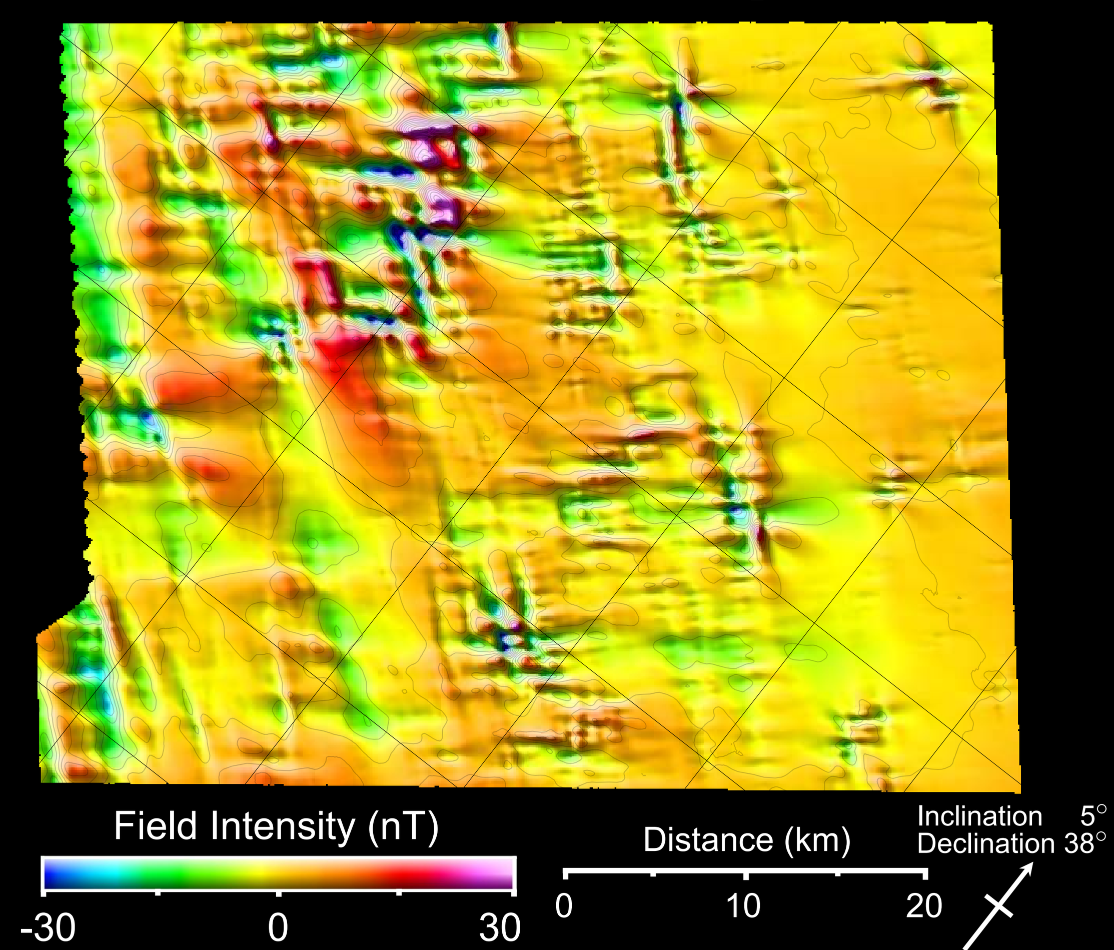
Reduction-to-Equator

$-$



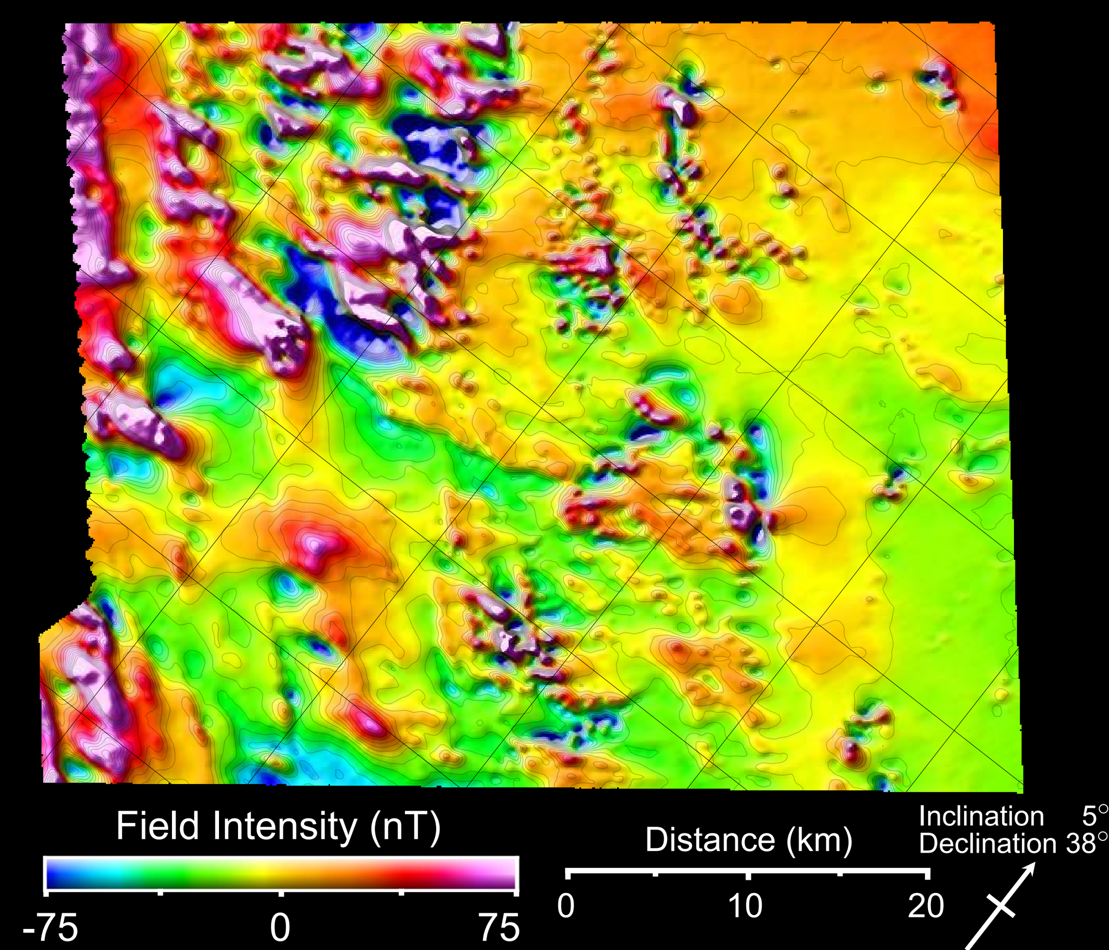
Rotated Reduction-to-Equator

$-$

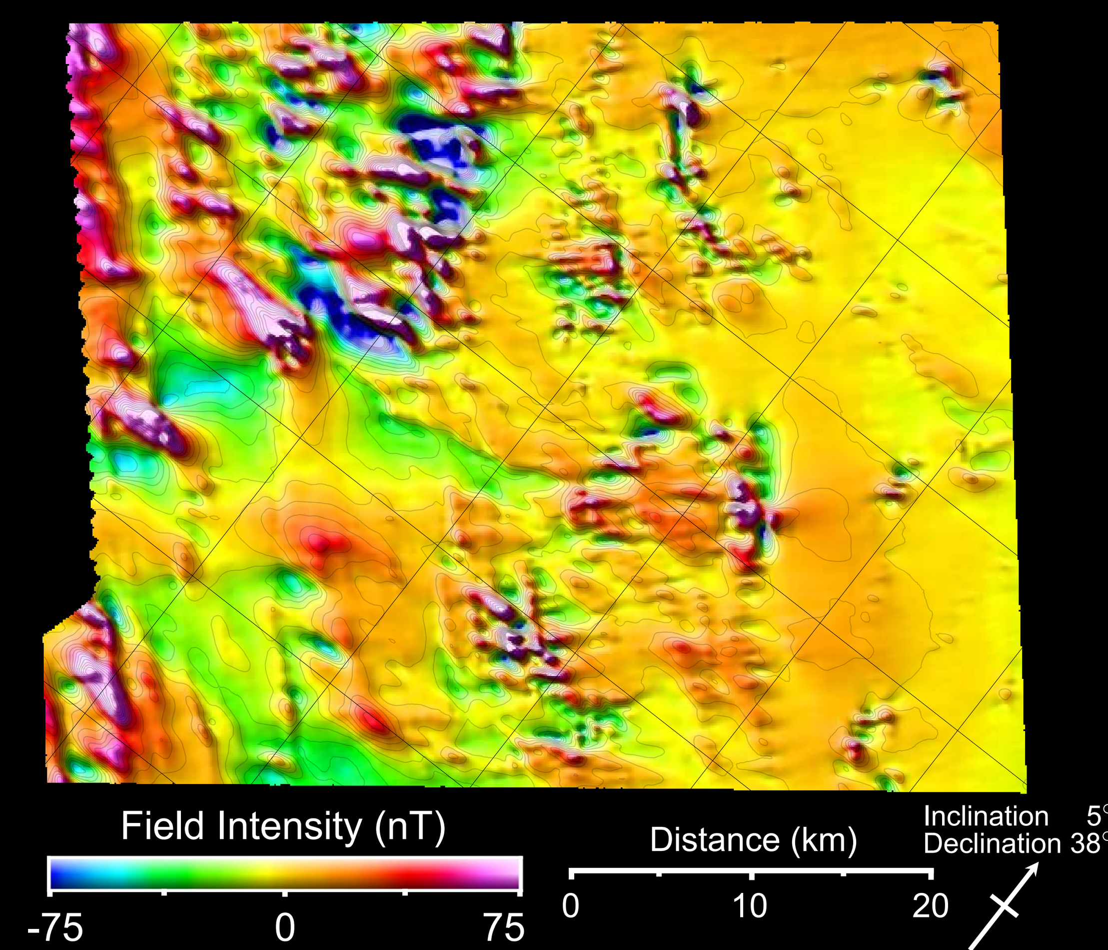


Reformulated Reduction-to-Pole

$=$



Direct Reduction-to-Pole



Direct Method

$$R = (G_{rtp}^{-1} G_{rtp}^{-H} + \beta I)^{-1} G_{rtp}^{-H} T$$

R	Fourier transform of the reduction-to-pole
T	Fourier transform of the total-field anomaly
G_{rtp}	reduction-to-pole transfer function
β	regularization parameter

Reformulated Method

$$R = -G_{rte} T - (G_{per}^{-1} G_{per}^{-H} + \beta I)^{-1} G_{per}^{-H} T$$

G_{rte}	reduction-to-equator transfer function
G_{per}	rotated reduction-to-equator transfer function

Only Half The Problem Requires Stabilization

Conclusions

The reduction-to-pole reformulation enhances amplitude recovery. The reformulation is grounded in potential theory and adds no additional computational or cognitive cost. Our approach can augment existing filter-based stabilization strategies to improve their amplitude recovery.

Teck Resources Limited provided data, preprocessing, and constructive feedback for this project. Thank you.
The Geo-Multiphysics Research Consortium sponsored this research.

Li & Oldenburg, 2001
"Stable reduction to the pole at the magnetic equator". GEOPHYSICS 66, no. 2: 571–78.

Blakely, 1995
Potential Theory in Gravity and Magnetic Applications. Cambridge University Press.