MA2101 Class test

7th October 2024

Solve all problems. Justify your answers.

1. Find all first-order partial derivatives (i.e. find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and, if meaningful, $\frac{\partial f}{\partial z}$) of the following functions:

   (a) $f(x, y)) = \frac{x^2}{x^2+y^2}$

   (b) $f(x, y, z) = x^2y^3z\sin(1/(1 + x^2))$

2. Find the directional derivative $D_{\gtrsim}$ of the function $f(x, y) = 5x^2y - 4xy^3$ in the direction of the following vectors:

   (a) $\langle 3, -4\rangle$

   (b) $\langle 1, 2\rangle$

3. In each part, evaluate $\alpha_p(\mathbf{v}_p)$, where α is a 1-form, p a point in $\mathbb{R}^3$, and $\mathbf{v}_p$ a vector in $T_p(\mathbb{R}^3)$:

   (a) $\alpha = xy\,dx + y^2\,dy - z\,dz$, $p = (1, 0, 1)$, and $\mathbf{v}_p = \langle 2, 1, 0\rangle$.

   (b) $\alpha = x^2\,dx + y^2\,dy + z^2\,dz$, $p = (2, 1, 1)$, and $\mathbf{v}_p = \langle 1, 1, 1\rangle$.

4. For each function f, find its differential:

   (a) $f(x, y) = x^2y^3$

   (b) $f(x, y, z) = x + y^2 + z^3$