Kinetic Energy and Angular Momentum of a Rotating Solid Cylinder

Consider a solid cylinder of mass m and radius a rotating with an angular velocity ω about an axis along its length, i.e., parallel to the axis of symmetry. We aim to find the kinetic energy and angular momentum about this axis.

Kinetic Energy (KE)

The kinetic energy of a rotating body is expressed as:

$$KE = \frac{1}{2} I \omega^2$$

where I denotes the moment of inertia of the body about the axis of rotation, and ω represents the angular velocity.

For a solid cylinder rotating about its axis, the moment of inertia I is:

$$I = \frac{1}{2} m a^2$$

Therefore, the kinetic energy of the cylinder is calculated as follows:

$$KE = \frac{1}{2} \left(\frac{1}{2} m a^2 \right) \omega^2 = \frac{1}{4} m a^2 \omega^2$$

Angular Momentum (L)

The angular momentum L of a rotating body about the axis of rotation is given by:

$$L = I \omega$$

For our solid cylinder, utilizing the previously determined moment of inertia I, the angular momentum is:

$$L = \frac{1}{2} m a^2 \omega$$

These formulas provide the kinetic energy and angular momentum of the cylinder in terms of its mass m, radius a, and angular velocity ω.