Extension of a Spring in Circular Motion

Problem Statement

A particle of mass 1 kg is attached to one end of an elastic spring of natural length 1 m and modulus of elasticity 50 N. The other end is fastened to a point on a smooth horizontal table. If the spring and particle describe circles on the table at 60 revolutions per minute, find the extension of the spring.

Solution

The centripetal force required for circular motion is provided by the force exerted by the spring. The centripetal force F is given by $F = \frac{mv^2}{r}$, and the force by the spring follows Hooke's Law $F = kx$, where k is the spring constant and x is the extension of the spring.

Given:

- Mass of the particle $m = 1$ kg,
- Spring constant $k = 50$ N/m,
- Revolutions per minute $n = 60$.

First, we convert the revolutions per minute to angular velocity in radians per second:

$$\omega = 2\pi \times \frac{n}{60} \,\text{rad/s} = 2\pi \,\text{rad/s}$$

The radius of the motion r is equal to the extension x since the natural length does not provide any force. The centripetal force is therefore:

$$m(\omega x)^2 = kx$$

Solving for x gives us:

$$x = \frac{m\omega^2}{k}$$

Substituting the values we have:

$$x = \frac{(1\,\text{kg})(2\pi\,\text{rad/s})^2}{50\,\text{N/m}}$$

$$x = \frac{(1)(4\pi^2)}{50}\,\mathrm{m}$$
$$x = \frac{4\pi^2}{50}\,\mathrm{m}$$

Conclusion

The extension x of the spring is calculated to be $\frac{4\pi^2}{50}$ meters when the particle describes circles at a rate of 60 revolutions per minute.