MP170/180 Problem Sheet 2 (For Tutorial discussion) One-Dimensional Kinematics II

1. Evaluate the first and second derivatives of

$$f(t) = \sqrt{1 + 2t^2}$$

Hint: Use the chain rule $\frac{df}{dt} = \frac{df}{du}\frac{du}{dt}$ and write $u = 1 + 2t^2$.

2. Use a change of variable, or a suitable substitution, to evaluate the following indefinite integral

$$\int \frac{t}{(1+t^2)^2}\, dt$$

3. Two cars A and B are initially at rest side by side. A starts off on a straight track with acceleration of 2 m/s^2. Five seconds later, B starts off on a parallel track to A with acceleration of 3.125 m/s^2.

   (a) Calculate the distance travelled by A after 5 s.

   (b) Calculate the time taken by B to catch up with A.

   (c) Find the velocities of A, B at that time.

4. A ball is thrown vertically upwards with an initial velocity of 30 m/s. One second later, another ball is thrown upwards with an initial velocity of u m/s. If they collide after a further 2 s, find the initial velocity u of the second ball.

5. A particle travels a distance of 300 m in a straight line at an average speed of 4 m/s, going from rest with constant acceleration a_1 for 10 s, then moving with constant speed and then coming to rest with a constant retardation a_2 for the last 20 s of the motion.
   Using the constant acceleration formalism for each stage of the motion, find what must be the values of a_1 and a_2.
   Deduce, also, how far the particle travels while accelerating, while travelling at a constant velocity and while decelerating.

6. A car has maximum acceleration a_1 and maximum retardation a_2 and the maximum speed is V. Find the mimimum time taken to travel a distance s from rest to rest along a straight road when (1) $s < s_0$ (2) and $s > s_0$ where

$$s_0 = \frac{V^2}{2}(\frac{1}{a_1} + \frac{1}{a_2})$$