Velocity of a Hollow Sphere Rolling Down an Inclined Plane

Question: A hollow uniform sphere is released from rest on an inclined plane at angle 30° to the horizontal. Assuming that the sphere rolls without slipping, find its velocity after it has rolled 6 m down the plane. You may take $g \approx 10\,\mathrm{m/s}^2$.

Solution

To find the velocity of the hollow sphere, we use the principle of conservation of energy. The initial gravitational potential energy of the sphere is converted into translational and rotational kinetic energy as it rolls down the plane.

Given:

- The inclination angle of the plane, $\theta = 30^\circ$.
- The distance rolled down the plane, $d = 6\,\mathrm{m}$.
- The acceleration due to gravity, $g = 10\,\mathrm{m/s}^2$.

The height h from which the sphere rolls down can be calculated as $h = d \sin\theta$. Therefore, the initial potential energy (U) is:

$$U = mgh = mgd\sin\theta$$

The sphere rolls without slipping, so its motion includes both translational and rotational kinetic energy at the bottom of the incline. The translational kinetic energy (K_{trans}) is given by:

$$K_{\text{trans}} = \frac{1}{2}mv^2$$

The rotational kinetic energy (K_{rot}) for a hollow sphere is:

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

where $I = \frac{2}{3}mr^2$ for a hollow sphere, and the relation between v and ω is $v = r\omega$, giving:

$$K_{\text{rot}} = \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{3}mv^2$$

By conservation of energy, $U = K_{\text{trans}} + K_{\text{rot}}$:

$$mgd \sin\theta = \frac{1}{2}mv^2 + \frac{1}{3}mv^2$$

Simplifying for v, we have:

$$v = \sqrt{\frac{2gd\sin\theta}{\frac{5}{3}}}$$

Substituting the given values:

$$v = \sqrt{\frac{2 \times 10 \times 6 \sin 30^\circ}{\frac{5}{3}}} = \sqrt{\frac{60}{\frac{5}{3}}} = \sqrt{36} = 6\,\text{m/s}$$

Therefore, the velocity of the hollow sphere after rolling 6 m down the inclined plane is $6\,\text{m/s}$.