# MP170/180 Problem Sheet 10
# Ordinary Differential Equations

1. The evolution of a population of unicellular bacteria is assumed to follow the Malthusian model (unlimited nutrients and no growth inhibitors). Starting from 1 bacterium, and assuming that each bacterium divides on average every 20 minutes, how long will it take for the bacterial population to cover the Earth's surface with a layer of one meter?

2. A population of size $P$ (in millions) varies in time $t$ (in years) according to a logistic equation of the form
$$\frac{dP}{dt} = \frac{1}{20}(4P - P^2)$$
   (a) If the initial population size is $P(0) = 1$ find the population for all time.
   (b) What size does the population have as $t \to \infty$?
   (c) How long does it take for the population to reach 3 million?

3. Solve the following first order differential equations using the integrating factor method:
   (a) $\dfrac{dy}{dx} = -4y \qquad$ where $\quad y(0) = -2.$
   (b) $\dfrac{dx}{dt} + x = t \qquad$ where $\quad x(1) = 0.$
   (c) $\dfrac{dx}{dt} - 3x = \sin(t) \qquad$ where $\quad x(0) = 0.$
   (d) $t\dfrac{dx}{dt} - 2x = t \qquad$ where $\quad x(1) = 1.$

4. Coffee at a temperature of 85° C is cooling in a room at constant temperature of 20° C. It takes 1 minute for the temperature to reach 75° C.
   Based on Newton's law of cooling, find
   (a) the coffee's temperature after 3 minutes;
   (b) the time it takes for the coffee to reach a temperature of 45°.