Ladder Slipping Problem

Problem Statement

A uniform ladder of length $2L$ and mass m rests at an angle of 60° to the horizontal against a rough vertical wall and rough horizontal floor with coefficients of friction $\frac{1}{2}$ and $\frac{1}{4}$, respectively. A man of mass $8m$ climbs the ladder. We are to find the position of the man when the ladder slips.

Solution

The conditions for equilibrium, before the ladder slips, are:

1. The sum of the vertical forces must be zero, yielding the normal force at the floor:
$$N_{\text{floor}} = mg + 8mg$$
2. The sum of the horizontal forces must be zero, giving the normal force at the wall:
$$N_{\text{wall}} = F_{\text{friction, floor}} = \mu_{\text{floor}} N_{\text{floor}}$$
3. The sum of the torques about the base of the ladder must be zero:
$$8mgx\sin(60^\circ) + mg(L\sin(60^\circ)) = F_{\text{friction, wall}}(2L\sin(60^\circ))$$

Substituting the known values into the torque equation, we can solve for x, the man's position on the ladder:

$$8mgx\sin(60^\circ) + mg(L\sin(60^\circ)) = \mu_{\text{wall}} N_{\text{wall}}(2L\sin(60^\circ))$$

$$8mgx\sin(60^\circ) = \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)(mg + 8mg)(2L\sin(60^\circ)) - mg(L\sin(60^\circ))$$

Solving for x, we find the position where the man causes the ladder to slip.

Conclusion

By calculating x, we determine the point at which the man's weight will cause the ladder to slip, given the coefficients of friction at the wall and the floor.