Newton's Law of Cooling Problem

Problem Statement

Coffee at a temperature of 85°C is cooling in a room at constant temperature of 20°C. It takes 1 minute for the temperature to reach 75°C. Based on Newton's law of cooling, find:

(a) the coffee's temperature after 3 minutes;

(b) the time it takes for the coffee to reach a temperature of 45°.

Solution

Newton's Law of Cooling is given by the differential equation:

$$\frac{dT}{dt} = -k(T - T_{\text{ambient}})$$

Part (a)

We use the initial condition $T(0) = 85$°C to find the constant k. When $T = 75$°C, we solve for k:

$$\frac{dT}{dt} = -k(75 - 20)$$

Since the temperature decreases by 10 degrees in 1 minute, we have:

$$-k \times 55 = \frac{-10}{1}$$

$$k = \frac{10}{55}$$

Using this k, the solution to the differential equation is:

$$T(t) = T_{\text{ambient}} + (T(0) - T_{\text{ambient}})e^{-kt}$$

$$T(3) = 20 + (85 - 20)e^{-\frac{10}{55} \times 3}$$

Part (b)

To find the time it takes for the coffee to reach 45°C, we set $T(t) = 45$°C and solve for t:

$$
\begin{aligned}
45 &= 20 + (85 - 20)e^{-\frac{10}{55}t} \\
25 &= 65e^{-\frac{10}{55}t} \\
e^{-\frac{10}{55}t} &= \frac{25}{65} \\
-\frac{10}{55}t &= \ln\left(\frac{25}{65}\right) \\
t &= -\frac{55}{10}\ln\left(\frac{25}{65}\right)
\end{aligned}
$$

Conclusion

We have calculated the coffee's temperature after 3 minutes and the time it takes to reach 45°C using Newton's Law of Cooling.