Bacterial Population Growth

Problem Statement

Calculate how long it will take for a population of unicellular bacteria, starting from 1 bacterium and dividing every 20 minutes, to cover the Earth's surface with a layer of one meter deep according to the Malthusian model.

Solution

The growth of the bacterial population is described by the Malthusian model of exponential growth:

$$P(t) = P_0 e^{rt}$$

where $P(t)$ is the population at time t, P_0 is the initial population, and r is the growth rate.

Given the doubling time $T_d = 20$ minutes, the growth rate r is:

$$r = \frac{\ln(2)}{T_d}$$

To find the time t when the population reaches a certain size, we rearrange the growth equation:

$$t = \frac{\ln(P(t)/P_0)}{r}$$

The Earth's surface area is approximately 510.1×10^6 km^2, and we wish to cover it with a layer 1 meter deep. Converting the area to m^2:

$$\text{Surface area} = 510.1 \times 10^6 \times 10^6 \text{ m}^2$$

Assuming each bacterium occupies a volume of $1 \mu m^3$, or 1×10^{-18} m^3, the number of bacteria needed to cover the Earth with a 1-meter layer is:

$$N = \frac{\text{Surface area}}{\text{Bacterium volume}}$$

$$N = \frac{510.1 \times 10^{12}}{1 \times 10^{-18}}$$

The initial population $P_0 = 1$. We now solve for t:

$$t = \frac{\ln(N/P_0)}{r}$$

$$t = \frac{\ln(510.1 \times 10^{30})}{\frac{\ln(2)}{20}}$$

This will give us t in minutes. To convert t to more practical units, such as days or years, we will use appropriate conversion factors.

Conclusion

By calculating t, we will determine the time required for the bacterial population to cover the Earth's surface with a layer one meter deep, assuming the Malthusian growth model with no limitations on resources or space.