Banked Curved Road Problem

Problem Statement

A car travels on a rough banked curved road of radius 200 m with velocity v where the road is inclined at an angle of $15°$ to the horizontal. The coefficient of friction between the road and the car is $\frac{1}{4}$. Find the value of v if:

(a) the car is just about to slip up the road;

(b) the car is just about to slip down the road.

Solution

To solve for v, we balance the forces acting on the car. The gravitational force mg has a component $mg\sin(\theta)$ acting down the incline, and a component $mg\cos(\theta)$ acting normal to the incline. The normal force N is equal to $mg\cos(\theta)$, and the maximum frictional force f is μN, where μ is the coefficient of friction.

Part (a)

For the car just about to slip up the road, the frictional force f acts down the incline, opposing the centripetal force. The centripetal force needed to keep the car in circular motion is given by:

$$f + N\sin(\theta) = \frac{mv^2}{r}$$

The frictional force is $f = \mu N = \mu mg\cos(\theta)$. Substituting N and f we get:

$$\mu mg\cos(\theta) + mg\cos(\theta)\sin(\theta) = \frac{mv^2}{r}$$

$$(\mu + \sin(\theta))\, g\cos(\theta) = \frac{v^2}{r}$$

Solving for v gives us:

$$v = \sqrt{(\mu + \sin(\theta))\, g\cos(\theta) \cdot r}$$

Part (b)

For the car just about to slip down the road, the frictional force f acts up the incline, contributing to the centripetal force. The equation now becomes:

$$N\sin(\theta) - f = \frac{mv^2}{r}$$

$$mg\cos(\theta)\sin(\theta) - \mu mg\cos(\theta) = \frac{mv^2}{r}$$

$$(\sin(\theta) - \mu)\, g\cos(\theta) = \frac{v^2}{r}$$

Solving for v gives us:

$$v = \sqrt{(\sin(\theta) - \mu)\, g\cos(\theta) \cdot r}$$

Values

Plugging in $\theta = 15°$, $\mu = \frac{1}{4}$, $g = 9.8\,\mathrm{m/s^2}$, and $r = 200\,\mathrm{m}$, we can compute the specific values for v in both scenarios.