Problem Sheet 11 - MP170/180

Rigid Body Motion

1. Show that the centre of mass of a conical shell of radius a and height h is located at a distance of $h/3$ from the base along the axis of symmetry.

2. A two dimensional solid is made by welding the edge of a disk of radius a to the mid-point of the edge of a square plate of side $2a$. Assuming that the disk and square are of the same constant density, find the centre of mass of the solid.

3. A solid cylinder of mass m and radius a rotates with angular velocity ω about an axis along its length (i.e. parallel to the axis of symmetry). Find the kinetic energy and angular momentum about this axis.

4. A solid sphere of mass m and radius a can rotate freely about a point A on its edge. The sphere is held initially at rest with the line OA through A and the centre of the sphere O horizontal and is released under gravity. Find the angular velocity of the system when OA first becomes vertical.

5. A hollow uniform sphere is released from rest on an inclined plane at angle 30^o to the horizontal. Assuming that the sphere rolls without slipping, find its velocity after it has rolled 6 m down the plane. You may take $g \approx 10\text{m/s}^2$.