Problem Sheet 12 - MP170/180

Damped and Forced Simple Harmonic Motion

1. Solve the differential equation of damped harmonic motion

$$\frac{d^2x}{dt^2} + b\frac{dx}{dt} + x = 0$$

for (i) $b = \sqrt{5}$, (ii) $b = 2$ and (iii) $b = \sqrt{3}$.

In each case find the complete solution subject to the initial condition $x(0) = 0$, $\dot{x}(0) = 1$, and sketch the solution. Comment on the type of damping observed.

2. A 2 kg mass is attached to a spring of stiffness $k = 8$ N/m. The mass moves on a **rough** horizontal table with frictional force $-10v$ where v is the velocity. At $t = 0$ the mass is held at rest where the spring is extended by 3 m from its natural length. Show that for $t \geq 0$ the subsequent extension of the spring is given by

$$x = 4\exp(-t) - \exp(-4t).$$

3. The same spring system of the previous question is placed at rest on a **smooth** table. At $t = 0$ it is then subjected to a driving force $2\cos(2t)$. Describe the subsequent behaviour of the system.

4. Find the general solution to
$$\ddot{x} + 4x = t - 1.$$

5. Find the general solution to
$$\ddot{x} + \dot{x} + x = 2\sin t.$$