Center of Mass of a Composite Solid

A two-dimensional solid is composed of a disk of radius a welded to the midpoint of the edge of a square plate of side $2a$. Both the disk and the square have uniform density. To find the center of mass of this composite solid, we consider the center of mass of each part separately.

Given:

- The side of the square is $2a$, thus its area is $4a^2$.
- The radius of the disk is a, thus its area is πa^2.
- Let ρ represent the constant density of both the square and the disk.
- Therefore, the mass of the square (m_s) is $4a^2\rho$ and the mass of the disk (m_d) is $\pi a^2\rho$.

The center of mass of the square is at its geometric center, located at (a, a), assuming the square is aligned with its side along the x-axis.

The center of mass of the disk, when the disk is welded to the midpoint of the square's edge, is located at $(2a, a)$, as the disk's center is a distance a (its radius) from the square's edge.

The total mass of the solid is $M = m_s + m_d = 4a^2\rho + \pi a^2\rho$.

The center of mass of the solid can be found by:

$$x_{CM} = \frac{m_s x_s + m_d x_d}{M} \quad \text{and} \quad y_{CM} = \frac{m_s y_s + m_d y_d}{M}$$

where $(x_s, y_s) = (a, a)$ for the square and $(x_d, y_d) = (2a, a)$ for the disk.

Substituting the values:

$$x_{CM} = \frac{4a^2\rho \cdot a + \pi a^2\rho \cdot 2a}{4a^2\rho + \pi a^2\rho} \quad \text{and} \quad y_{CM} = \frac{4a^2\rho \cdot a + \pi a^2\rho \cdot a}{4a^2\rho + \pi a^2\rho}$$

Simplifying, we find:

$$x_{CM} = \frac{4a^3\rho + 2\pi a^3\rho}{4a^2\rho + \pi a^2\rho} \quad \text{and} \quad y_{CM} = \frac{4a^3\rho + \pi a^3\rho}{4a^2\rho + \pi a^2\rho}$$

Given that ρ, a, and π are constants, the expressions for x_{CM} and y_{CM} can be further simplified to determine the exact location of the center of mass in terms of a and numerical coefficients.

This calculation shows how the center of mass for a composite object can be found by considering the mass and center of mass of each component part.