Robert Davidson

ST1112: Statistics

70% Exam
30% Continuous Assessment (3 parts)

Contents

1 Descriptive Statisitcs

1.1 Sampling the mean

In **probability** we consider the underlying process which has some randomness or uncertainity, and we try to figure out what happens

In **statistics** we consider the data that we have, and we try to figure out what the underlying process is. The basic aim to infer the population from the sample.

Example Consider a jar of red and green jelly beans

A probabilist starts by knowing the proportion of red and green jelly beans in the jar, and then tries to figure out the probability of drawing a red jelly bean.
A statistician starts by drawing a sample of jelly beans from the jar, and then tries to figure out the proportion of red and green jelly beans in the jar.

Definition : Central Limit Theorem

Sample means follow a normal distrubution, centered on the popular mean, with a standard deviation equal to population standard deviation divided by the square root of the sample size.

$$\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

Definition : Standard Error

The standard error is the variability in the sampling distrubution.
The standard error describes the typical difference between the sample measurement and the population parameter.

$$SE = \frac{\sigma}{\sqrt{n}}$$

Definition : Estimate σ

Often the value of the population standard deviation is unknown, and hence the standard error of the mean is unknown.
We can estimate the value of the standard error using the sample standard deviation (s) as an unbiased estimator of the population standard deviation (σ).

$$\sigma_{\bar{X}} = \frac{s}{\sqrt{n}}$$

2 Interential Statistics - Interval Estimation

2.1 Confidence Intervals for a mean

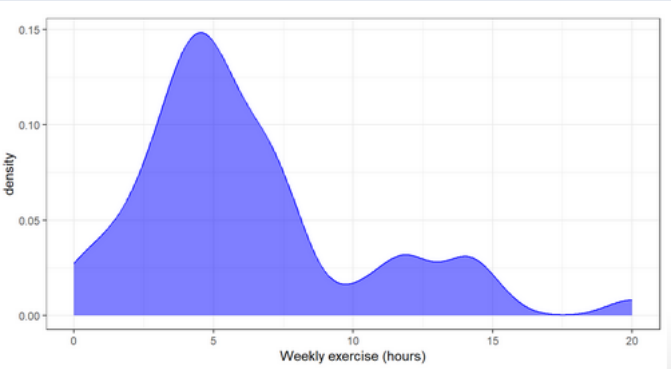
Example The student newspaper wants to know how many students are exercising per week on average

- Take a **sample from this population**
- Esimate the **population paramater** using the **sample statistic**

```
st1112_data %>%
    select(exercise) %>%
    summarise(n = n(),
        mean = mean(exercise,na.rm=TRUE),
        sd = sd(exercise,na.rm=TRUE))

##    n mean   sd
## 1 54 6.19 4.11

st1112_data %>%
    ggplot(aes(x = exercise)) +
    labs(x = "Weekly exercise (hours)") +
    geom_density(colour = "blue",
                 fill="blue",alpha=0.5)+theme_bw()
```



But a new survery on another 54 students would lead to a different estimate, so which should we report back to the newspaper? If we sample data from the population, there is uncertainity in our estimate of the population mean. The standard error of the mean is a measure of this uncertainity. In our example, the standard error of the mean is:

$$SE = \frac{4.11}{\sqrt{54}} = 0.56$$

We use the Central Limit Theorem to provide a range of values that will capture 95% of the sample means

Definition Confidence Interval for $n > 30$

For a large sample size, $n > 30$, a Confidence Interval for the population mean is given by:

$$\bar{X} \pm Z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

The distrubutional cut-off $Z_{\frac{\alpha}{2}}$ allows us to adapt the Confidence Interval to the level of confidence we require, i.e. a 95% Confidence Interval has a $Z_{\frac{\alpha}{2}}$ value of 1.96.