MA2287: Complex Analysis Exam Notes

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Contents

1 Question 1:

1.1 Sketch the region in the complex plane determined by the inequality

- $|z-4| > 3|z+4|$ 2023 Q1(a)
- $\{z \in \mathbb{C} : |2z-1| < 2|2z-i|\}$ 2022 Q1(a), 2021 Q1(d), 2017 Q1(a), 2016 Q1(a)

1.2 Determine all solutions to roots of unity

- $z^6 - 1 = 0$ and factorize $x^6 - 1$ as a product of linear and quadratic factors 2023 Q1(b),
- $z^4 = -81i$ and find a polynomial $p(z)$ with complex coefficients with root w and $p(\overline{w}) \neq 0$ 2022 Q1(b), 2018 Q1(b)
- $z^6 - 1 = 0$ and factorize $x^6 - 1$ as a product of linear and quadratic factors 2021 Q1(c)
- $z^3 = 1 + i$, let $n \in \mathbb{N}$ and $w \neq 1$ be an n-th root of unity. Prove $1 + w + w^2 + \ldots + w^{n-1} = 0$ 2017 Q1(b)

1.3 Determine and sketch the image under the mapping

- $w = e^z$, $\{z \in \mathbb{C} : \pi/4 \leq \mathrm{Im}(z) \leq \pi/2\}$ 2023 Q1(c), 2021 Q1(a), 2017 Q1(d)
- $w = \mathrm{Log}(z)$, $\{z : |z| > 1, 0 \leq \mathrm{Arg}(z) \leq \pi/2\}$ 2022 Q1(d), 2018 Q1(d)

1.4 Find z where the function is 0

- $\cos(z) = \frac{e^{iz} + e^{-iz}}{2}$ 2022 Q1(d)

1.5 Calculate principal value Log(z)

- z $= -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$ and prove e^z is the inverse function of Log(z) 2022 Q1(c), 2018 Q1(c), 2017 Q1(c)

1.6 Prove the following

- Define the complex conjugate ($\overline{w}$) and prove if w is a zero of a polynomial $p(z) = a_0 + a_1 z + \ldots + a_n z^n$ then $\overline{w}$ is also a zero of $p(z)$ 2021 Q1(b), 2018 Q1(a), 2016 Q1(b)
- Define the complex exponential function e^z and prove Eulers Formula $e^{i\theta} = \cos(\theta) + i\sin(\theta)$ 2017 Q1(b)