Mathematical Methods II

Exams:

70% Exam

30% Continuous Assessment (3 parts)

Contents

1 Week 1: Intro to Laplace Transforms

1.1 Preliminary: Exponential Functions

Recall the following facts:

1. $e^t = \exp(t) = 1 + \frac{t^1}{1!} + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots = \sum_{i=0}^{\infty} \frac{t^i}{i!}$.
2. $e^0 = 1$.
3. As $t \to \infty$, $e^t \to \infty$; as $t \to -\infty$, $e^t \to 0$.
4. $\frac{d}{dt} e^t = e^t$, and $\frac{d}{dt} e^{st} = s\, e^{st}$.
5. $\displaystyle\int e^t\, dt = e^t + C$, and $\displaystyle\int e^{st}\, dt = \frac{1}{s} e^{st} + C$.
6. $e^{t_1} \cdot e^{t_2} = e^{t_1 + t_2}$.

1.2 Laplace Transforms

Definition

Consider a function $f(t)$ for $t > 0$.

We define the Laplace transform of $f(t)$ as

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt.$$

Note

We can also write $\mathcal{L}\{f(t)\}$ as $F(s)$.

Alternatively,

$$\mathcal{L}\{f(t)\} = \lim_{R \to \infty} \int_0^R e^{-st} f(t)\, dt.$$

Recalling that

$$\int_0^1 st^2\, dt = s\left[\frac{t^3}{3}\right]_0^1 = \frac{s}{3}$$

we see that $\mathcal{L}\{f(t)\}$ is just a function of s.