Group Scheduling for Block Diagonal Digital Precoder in Multi-user MIMO System

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Abstract—**Beam division multiple access (BDMA) has recently been proposed for massive multiple-input multiple-output (MIMO) systems by simultaneously transmitting multiple users' data streams via different beams. In our previous work, single-path propagation channel model has been investigated by opportunistically selecting users to suppress the multiuser interference. Similarly, for multipath channel model, the different paths of each user can be chosen opportunistically. Furthermore, the block diagonal precoding is proposed and the number of RF chains can be significantly reduced by applying the Time Division Duplex(TDD) or switches. Simulation results confirm the effectiveness of proposed block diagonal precoding algorithm.**

I. INTRODUCTION

To meet the ever-increasing demand of higher user data rates, it is envisioned that the next-generation cellular systems will be equipped with massive antenna arrays [1]. Capitalizing on the large number of antennas at the base-station (BS), beam division multiple access (BDMA) has recently been proposed to transmit multiple users' data streams via different beams [2], [3]. In contrast to the more conventional multiple access schemes such as Code Division Multiple Access (CDMA) or Orthogonal Frequency Multiple Division Access (OFDMA) that multiplex users in code, time and frequency domains, BDMA separates users in the beam space by transmitting data to different users in orthogonal beam directions. In [2], BDMA was first proposed to decompose the multiuser multiple-input multiple-output (MU-MIMO) system into multiple single-user MIMO channels by multiplexing multiple users' data onto non-overlapping beams. More recently, joint user scheduling and beam selection for BDMA was formulated under the Lyapunov-drift optimization framework before the optimal user-beam scheduling policy was derived in a closed form [3].

In the meantime, hyrbid digital and analog beamforming has also been developed for millimeter wave (mmWave) massive MIMO transmissions by dividing the procoding process into two steps, namely analog and digital precoding [4], [5]. More specifically, the transmitted signals are first precoded digitally using a smaller number of radio frequency (RF) chains followed by the analog precoding implemented with a much larger number of low-cost phase shifters. As a result, the hybrid analog-digital precoding architecture requires significantly less RF chains as compared to the fully digital precoding in which every available antenna element is supported by one RF chain.

However, the number of RF chains is lower bounded by the transmitted the number of data streams. In our proposed system, the users are grouped to several clusters. We assume the symbol
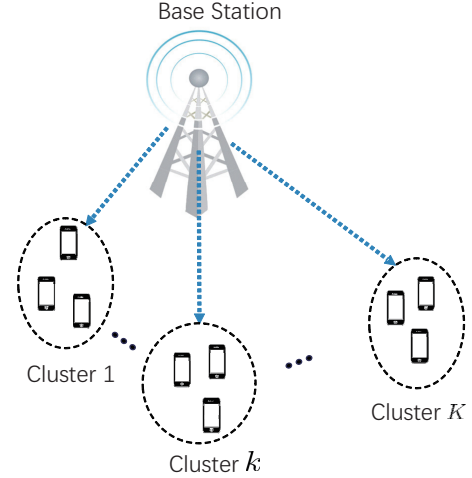

Fig. 1. Group scheduling to reduce the number of RF chains and eliminate the intra-cluster interference

duration is much larger than time delay thus the received symbols of different users are same and the channel gain is constant. Compared to serve each cluster separately, the interference will increase since each user has to decode received signal from other clusters. Leveraging the scheduling of users, we can firstly use analog precoding to eliminate the inter-cluster interference between cluster and then implement digital precoding to suppress the intra-cluster interference. The simulation results show that our proposed algorithm can efficiently reduce the number of RF chains without introducing large intra-cluster interference.

Notation: Vectors and matrices are denoted by boldface letters. $\boldsymbol{A}^T$ and $\boldsymbol{A}^H$ denote transpose and conjugate transpose of $\boldsymbol{A}$, respectively. $\boldsymbol{A}^{\dagger}$ being the pseudo inverse of $\boldsymbol{A}$ while $||\boldsymbol{A}||$ and $|\boldsymbol{A}|$ stand for the Frobenius norm and determinant of $\boldsymbol{A}$, respectively. $\boldsymbol{A}(i,j)$ denotes the i row, j column element of $\boldsymbol{A}$; $|\mathcal{I}|$ is the cardinality of the enclosed set $\mathcal{I}$; Finally, $\mathbb{E}[\cdot]$ and $\Re\{\cdot\}$ denote the expectation and real part of a random variable.

II. SYSTEM MODEL

There are N_{tot} users under considered base station, and N_U of them will be selected to serve. We consider a multi-user mmWave MIMO system shown in Fig. 2, in which a transmitter equipped with N_{RF} RF chains and N_T antennas transmits N_U data streams to N_U receivers with N_R receive antennas. Following the same assumption commonly employed in the literature [6], we assume
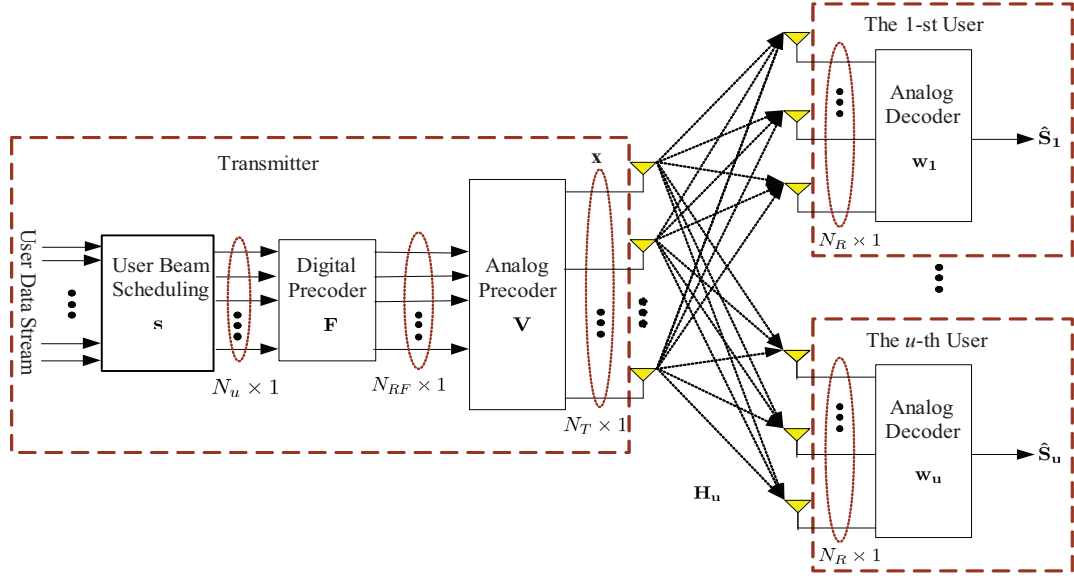
Fig. 2. Block diagram of the hybrid precoding system under consideration

only one data stream is designated to each scheduled receiver. We use $s(n)$ to denote the n-th block of N_U data to be transmitted with $\mathbb{E}\left[ss^H\right] = \frac{1}{N_U}I_{N_U}$. In the sequel, we concentrate on a single block and omit the temporal index n for notational simplicity.

Our basic idea is to group the users into several clusters and then the inter-cluster interference can be minimized by analog precoding with group-scheduling and the intra-cluster interference can be eliminated by digital precoding.

The N_U users are divided into K clusters and each cluster has M_k users. Obviously, we have

$$\sum_{k=1}^{K} M_k = N_U, \quad 0 < M_k \le N_U \tag{1}$$

And the digital precoder is given by

$$F = \begin{bmatrix} \mathcal{F}_1 & \cdots & 0 & 0 \\ \vdots & \mathcal{F}_2 & \vdots & \vdots \\ 0 & \cdots & \ddots & 0 \\ 0 & \cdots & 0 & \mathcal{F}_K \end{bmatrix}, \quad F_k \in \mathcal{C}^{N_{RF} \times M_k} \tag{2}$$

Correspondingly, the analog precoder is also divided into K parts

$$V = [\mathcal{V}_1, \mathcal{V}_2, \cdots, \mathcal{V}_K], \quad \mathcal{V}_k \in \mathcal{C}^{N_T \times M_k} \tag{3}$$

Also, the data streams S is divided into K clusters

$$s = \left[s_1^T, s_2^T, \cdots, s_K^T\right]^T, \quad s_k \in \mathcal{C}^{M_j \times 1} \tag{4}$$

We use f_{ku}, v_{ku} and s_{ku} to represent the digital precoding, analog precoding and data stream for u-th user in k-th cluster.

The resulting precoded signal x of dimension $N_T \times 1$ can be expressed as

$$x = V \cdot F \cdot s = \sum_{k=1}^{K} \mathcal{V}_k \mathcal{F}_k s_k \tag{5}$$

The precoded signal x is then broadcast to N_U users. The signal received by the u-th user is given by

$$y_{ku} = \underbrace{H_{ku} \mathcal{V}_k f_{ku} s_{ku}}_{\text{Desired Signal}} + \underbrace{H_{ku} \mathcal{V}_k \sum_{\substack{i=1 \\ i \ne u}}^{M_K} f_{ki} s_{ki}}_{\text{Intra-cluster Interference}}$$

$$+ \underbrace{H_{ku} \sum_{\substack{j=1 \\ j \ne k}}^{K} \mathcal{V}_j \mathcal{F}_j s_j}_{\text{Inter-cluster Interference}} + \underbrace{n_{ku}}_{\text{Noise}} \tag{6}$$

where $H_{ku} \in \mathbb{C}^{N_R \times N_T}$ is the MIMO channel matrix between the transmitter and the u-th receiver [5]. Furthermore, n_u is complex additive white Gaussian noise with zero mean and variance equal to σ^2.

Assuming the receivers are all low-cost terminals that perform analog beamforming only in decoding, the decoded signal by the u-th user in k-th cluster denoted by $\hat{s}_u$ is given as

$$\hat{s}_{ku} = w_{ku}^H H_{ku} \mathcal{V}_k f_{ku} s_{ku} + w_{ku}^H \tilde{n}_{ku}, \tag{7}$$

where w_u of dimension $N_R \times 1$ is the analog beamforming vector employed by the u-th receiver with the power constraint of $|w_u|^2 = 1$ and

$$\tilde{n}_u = H_{ku} \mathcal{V}_k \sum_{\substack{i=1 \\ i \ne u}}^{M_K} f_{ki} s_{ki} + H_{ku} \sum_{\substack{j=1 \\ j \ne k}}^{K} \mathcal{V}_j \mathcal{F}_j s_j + \underbrace{w_{ku}^H n_{ku}}_{\text{Noise}} \tag{8}$$

Note that the first term in Eq. (7) stands for the desired signal while the second term is the sum of its own receiver noise and interference from intra-cluster users and other clusters' users.

A. Channel Model

As shown in [7], the mmWave wireless channel can be well modeled by the Saleh-Valenzuela model. Following the same approach developed in [8], we assume that each scatter only contributes one single propagation path. As a result, the u-th user's channel model can been modeled as:

$$\boldsymbol{H}_u = \sqrt{\frac{N_T N_R}{L_u}} \sum_{l=1}^{L_u} \alpha_{u,l} \cdot \boldsymbol{a}_R(\phi_{u,l}^r, \theta_{u,l}^r) \cdot \boldsymbol{a}_T^H(\phi_{u,l}^t, \theta_{u,l}^t), \quad (9)$$

where L_u is the number of scatters of the u-th user's channel. Furthermore, $\alpha_{u,l}$, $\theta_{u,l}^r/\phi_{u,l}^r$ and $\theta_{u,l}^t/\phi_{u,l}^t$ are the complex path gain, azimuth/elevation angles of arrival(AoA) and azimuth/elevation angles of departure(AoD) of the l-th path of the u-th user, respectively. Finally, $\boldsymbol{a}$ is the array response vector. For an uniform planar array (UPA) of size $P \times Q$ considered in this work, the array response vector $\boldsymbol{a}$ is given by [8]

$$\boldsymbol{a}(\phi, \theta) = \frac{1}{\sqrt{N_T}} \left[1, e^{jkd(\sin\phi\sin\theta + \cos\theta)}, \cdots, \right.$$
$$e^{jkd(p\sin\phi\sin\theta + q\cos\theta)}, \cdots,$$
$$\left. e^{jkd((P-1)\sin\phi\sin\theta + (Q-1)\cos\theta)} \right]^T, \quad (10)$$

where $k = \frac{2\pi}{\lambda}$ is the wavenumber while d is the distance between two adjacent antennas.

B. Problem Formulation

For notational simplicity, we denote by $\boldsymbol{g}_{ku}^H$ the effective array gain of the u-th user in k-th cluster with

$$\boldsymbol{g}_{ku}^H = \boldsymbol{w}_{ku}^H \boldsymbol{H}_{ku} \boldsymbol{\mathcal{V}}_k. \quad (11)$$

And the effective array gain of u-th user from other clusters is given by

$$\boldsymbol{g}_{ju}^H = \boldsymbol{w}_{ku}^H \boldsymbol{H}_{ku} \boldsymbol{\mathcal{V}}_j. \quad (12)$$

Then, the channel capacity of the u-th user is given by

$$R_{ku} = \log \left(1 + \frac{\frac{P}{N_U}|\boldsymbol{g}_{ku}^H \boldsymbol{f}_{ku}|^2}{\frac{P}{N_U}\sum_{\substack{i=1 \\ i\neq u}}^{M_k}(|\boldsymbol{g}_{ku}^H \boldsymbol{f}_{ki}|^2 + \sum_{\substack{j=1 \\ j\neq k}}^{K}||\boldsymbol{g}_{ju}^H \boldsymbol{F}_j||^2) + \sigma^2} \right). \quad (13)$$

Subsequently, the system average capacity that is a function of $\boldsymbol{V}$ and $\boldsymbol{F}$ can be computed as

$$R_{avg} = \frac{1}{KN_U} \sum_{k=1}^{K} \sum_{u=1}^{N_U} R_{ku}. \quad (14)$$

Then the number of required RF chains can be reduced to $N_{RF} = \max\{M_k\}_{k=1}^K$.

Finally, for the given K, the optimal design of the digital and analog precoding matrices can be formulated as

$$P_1: \max_{\boldsymbol{W}, \boldsymbol{V}, \boldsymbol{F}, \boldsymbol{p}} \quad R_{avg} \quad (15)$$

$$s.t. \quad C_1 : \mathrm{diag}(\boldsymbol{v}_{ku} \boldsymbol{v}_{ku}^H) = \frac{\boldsymbol{I}_{N_T}}{\sqrt{N_T}};$$

$$C_2 : \mathrm{diag}(\boldsymbol{w}_{ku} \boldsymbol{w}_{ku}^H) = \frac{\boldsymbol{I}_{N_R}}{\sqrt{N_R}};$$

$$C_3 : \mathrm{tr}(\boldsymbol{F}\boldsymbol{F}^H) \leq N_U;$$

$$C_4 : \sum_{u=1}^{N_U} p_u = P_{tot};$$

$$C_5 : \boldsymbol{F} = \mathrm{diag}(\boldsymbol{\mathcal{F}}_1, \boldsymbol{\mathcal{F}}_2, \cdots, \boldsymbol{\mathcal{F}}_K);$$

$$C_6 : \boldsymbol{V} = [\boldsymbol{\mathcal{V}}_1, \boldsymbol{\mathcal{V}}_2, \cdots, \boldsymbol{\mathcal{V}}_K];$$

$$C_7 : \max\{M_k\}_{k=1}^K \leq N_{RF}$$

III. PROPOSED BLOCK HYBRID BEAMFORMING FOR RF CHAINS REDUCTION

In this section, the power of users will be uniformly allocated

$$p_u = \frac{P_{tot}}{N_U}, \quad u = 1, 2, \cdots, N_U \quad (16)$$

Thus, the constraint C_3 in Eq. (15) will be temporarily removed. We will firstly consider a conventional case that $N_{RF} \geq N_U$. Then the new block hybrid precoding structure will be proposed for $N_{RF} < N_U$.

A. Conventional hybrid beamforming by zero-forcing

Considering a case of $K = 1$ ($N_U < N_{RF}$), the problem will be simplified to conventional hybrid beamforming precoding problem, i.e. $\boldsymbol{V} = \boldsymbol{\mathcal{V}}_1$, $\boldsymbol{F} = \boldsymbol{\mathcal{F}}_1$. For notational simplicity, we will ignore the subscript k.

With the assumption that array response vectors corresponding to distinct beams are asymptotically orthogonal with infinite number of antennas at transmitter

$$\lim_{N \to +\infty} \boldsymbol{a}_T^H(\phi_{i,l}^t, \theta_{i,l}^t) \cdot \boldsymbol{a}_T(\phi_{j,p}^t, \theta_{j,p}^t) = \delta(i-j)\delta(l-p), \quad (17)$$

For multi-path channel model as shown in Eq. (9), We need to select the beams with least interference. Based on the idea of BDMA, the analog precoder is solved by

$$\{\boldsymbol{w}_u^*, \boldsymbol{v}_u^*\}_{u=1}^{N_U} = \arg\max \frac{||\boldsymbol{w}_u^H \boldsymbol{H}_u \boldsymbol{v}_u||_F^2}{\sum_{i=1, i\neq u}^{N_U} ||\boldsymbol{w}_u^H \boldsymbol{H}_u \boldsymbol{v}_i||_F^2} \quad (18)$$

$$s.t. \quad \boldsymbol{v}_u \in \{\boldsymbol{a}_T(\phi_{u,l}^t, \theta_{u,l}^t)\}_{l=1}^{L_u}$$

$$\boldsymbol{w}_u \in \{\boldsymbol{a}_R(\phi_{u,l}^r, \theta_{u,l}^r)\}_{l=1}^{L_u} \quad (19)$$

Assuming that the transmitter has perfect channel state information (CSI), then all AoA and AoD information, i.e. $\{\phi_u^t, \theta_u^t, \phi_u^r, \theta_u^r\}$, is perfectly known to the transmitter. As a result, the optimization problem in Eq. (15) can be simplified as

$$P_2: \max_{\boldsymbol{F}} \quad R_{avg} \quad (20)$$

$$s.t. \quad \mathrm{tr}(\boldsymbol{F}\boldsymbol{F}^H) \leq N_U;$$

We denote by $\hat{s} = [\hat{s}_1, \hat{s}_2, \cdots, \hat{s}_{N_U}]^T$ the estimated signal vector. Recalling Eq. (7), $\hat{s}$ can be expressed as [8]

$$\hat{s} = G \cdot F \cdot s + \xi, \tag{21}$$

where $G = [g_1, g_2, \cdots, g_{N_U}]^H$ is of dimension $N_U \times N_{RF}$ and $\xi = \left[w_1^H \tilde{n}_1, w_2^H \tilde{n}_2, \cdots, w_{N_U}^H \tilde{n}_{N_U}\right]^T$. [8] proposed a zero-forcing approach to solve Eq. (20) by setting

$$F_{ZF} = G^\dagger = G^H(GG^H)^{-1}, \tag{22}$$

with $N_{RF} \geq N_U$.

To satisfy the power constraint, power normalization is performed on each f_u derived from $F_{ZF} = [f_{ZF,1}, f_{ZF,2}, \cdots, f_{ZF,N_U}]$ as

$$f_{ZF,u}^* = \frac{f_{ZF,u}}{||V \cdot f_{ZF,u}||}. \tag{23}$$

B. Block Diagonal Digital Precoder

In general, the number of RF chains is constant but not the number of users. In this section, we will use less RF chains to serve users more than N_{RF} by taking the advantage of user scheduling. For the assumption of Eq. (17), although the infinite antennas can't be practical, the residual interference of difference users can be minimized by

$$\{\mathcal{W}_k^*, \mathcal{V}_k^*\}_{k=1}^K = \arg\max \frac{||w_{ku}^H H_{ku} \mathcal{V}_k||_F^2}{\sum_{j=1, j\neq k}^{N_U} ||w_{ku}^H H_{ku} \mathcal{V}_j||_F^2} \tag{24}$$

$$s.t. \quad \mathcal{V}_k \in \{v_u^*\}_{u=1}^{N_U},$$
$$\mathcal{W}_k^* \in \{w_u^*\}_{u=1}^{N_U}, \quad k = 1, 2, \cdots, K \tag{25}$$

and the following deduction can be given

$$\mathcal{W}_k^* H_{ku} \mathcal{V}_j^* \approx 0 \quad \text{for} \quad k \neq j \tag{26}$$

Then the Eq. (8) can be simplified as

$$\tilde{n}_{ku} \approx g_{uk} \sum_{\substack{i=1 \\ i\neq u}}^{M_k} f_{ik} s_{ik} + w_{uk} n_{uk} \tag{27}$$

Where the inter-cluster interference term is eliminated by Eq. (26).

Compared to Eq. (21), the only difference is that the number of considered users is reduced from N_U to M_k with K clusters. The number of required RF chains is then reduce to $N_{RF} = \max\{M_k\}_{k=1}^K$. Then the digital precoder can be solved by

$$\mathcal{F}_k = \mathcal{G}_k^\dagger = \mathcal{G}_k^H(\mathcal{G}_k \mathcal{G}_k^H)^{-1}. \tag{28}$$

Where $\mathcal{G}_k = [g_{k1}, g_{k2}, \cdots, g_{kM_k}]$.

C. Greedy group scheduling algorithm

To solve the Eq. (24), although global optimal solution of group clustering problem by exhaustive searching, the computational cost is also very huge. Considering the K-means algorithm to minimize the summation of Euclidean distance, we will propose a similar algorithm to cluster the users with least interference.

The algorithm can be summarized as Algorithm 1. The users index is represented as $\mathcal{I}$.

Algorithm 1 Greedy clustering algorithm for block hybrid beamforming system

Input:
All user index set: $\mathcal{X}$
Selected user index set : $\mathcal{I}_k = \emptyset$, $k = 1, 2, \cdots, K$
Number of clusters: K
Analog precoding matrix solved by Eq. (18): V^*
Procedures:
Initialization: Randomly assign a user index with k-th cluster x_k corresponding to $\mathcal{I}_1$, *i.e.* $\mathcal{I}_1 \leftarrow x_k^*$, $\mathcal{I} \leftarrow \mathcal{I}_1$ and $\mathcal{X} \setminus x_k^*$,
while $2 \leq k \leq K$ **do**
  for x_k in $\mathcal{X}$ **do**
    $p(x_k) = ||v_{kx}^H v_{\mathcal{I}}||^2$
  end for
  Find the user index x_k^* with minimum $p(x_k)$ in k cluster
  Update $\mathcal{I}_k \leftarrow x_k^*$, $\mathcal{I} \leftarrow \mathcal{I}_k$ and $\mathcal{X} \setminus x_k^*$
end while
for x_k in $\mathcal{X}$ **do**
  while $1 \leq k \leq K$ **do**
    inter-cluster interference: $p(x_k) = \sum_{k=1, j\neq k}^K ||v_{kx} v_{\mathcal{I}_j}||^2$
  end while
  Find the user index x_k^* with minimal $p(x_k)$
  Update $\mathcal{I}_k \leftarrow x_k^*$, $\mathcal{I} \leftarrow \mathcal{I}_k$ and $\mathcal{X} \setminus x_k^*$
end for

IV. SIMULATION RESULTS

In this section, we use computer simulation to compare the performance of sum-rate capacity for RF chains-reduction algorithm. Unless specified otherwise, we consider a transmitter equipped with an 12×12 UPA (*i.e.* $N_T = 144$) and $N_{RF} = 8$ RF chains serving $N_U = 16$ users each equipped with a 8×8 UPA (*i.e.* $N_R = 64$). The channels are multi-path with the azimuth AoAs/AoDs being uniformly distributed over $[0, 2\pi]$ and the elevation AoAs/AoDs being uniformly distributed in $[-\pi/2, \pi/2]$, respectively. For each computer experiment, we compute the average over 500 realizations.

We firstly compare the two proposed algorithm for RF chains reduction with conventional algorithm. As shown in Fig. 3, the proposed algorithms have good performance compared with zero-forcing and even better than 'single user' in the interval of SNR<10.

In the Fig. 4, we can see that the sum-rate capacity will increase as more RF chains added.

Finally, we study the multiuser gain provided by opportunistic scheduling. By varying the number of active users, we investigate the sum-rate capacity improvement. Fig. 3 shows that PAPRA-OHBF can significantly benefit from the increase of active users as the number of active users grows from 6 to 20. However, the benefit of having more active users diminishes as the number of active users increases beyond 30. In contrast, the non-opportunistic analog beamforming is not capable of reaping multiuser gains. As a result, the curve labelled "Analog Beamforming" does not change much with the number of active users.
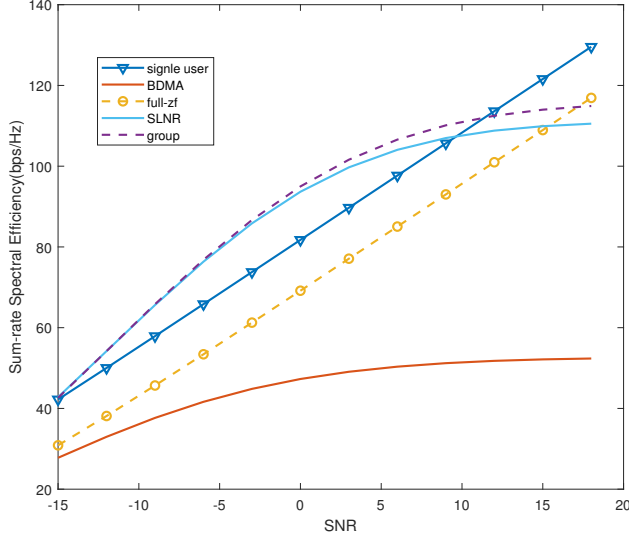
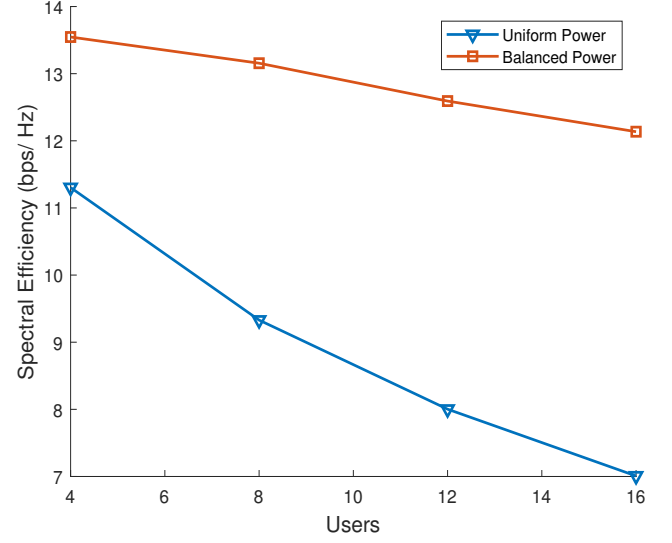Fig. 3. Sum-rate capacity comparison with different algorithm.
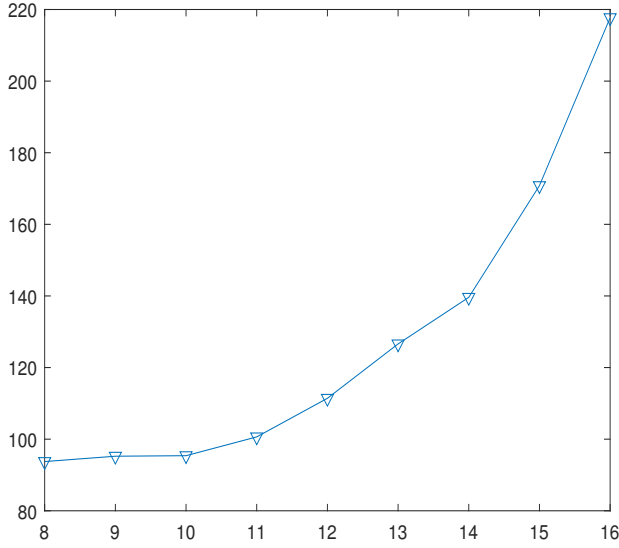

Fig. 5. Power allocation.

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Fig. 4. Different number of RF chains.

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