Proof for The Matrix of Uplink Is The Transpose of Downlink

1 In BS-to-BS Matrix

We assume N base stations, and a common channel. Let L_i denote the user number of cell i. We define a geometry as show in Fig. 1, where g_{ikj} denotes the gain for user k in cell i to base station in cell j.

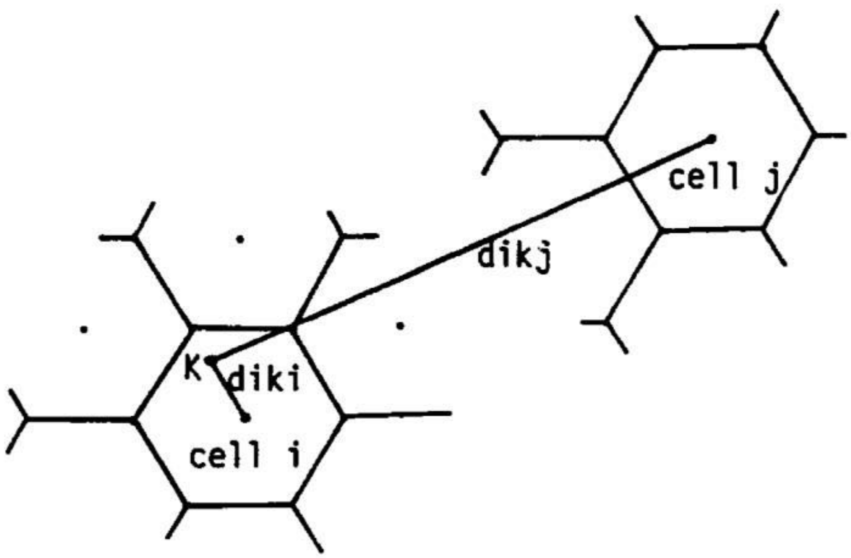

Figure 1: Interference Geometry

1.1 SIR in Uplink

Let p_i^U denote the received power by the BS of the i-th cell from any one users in this cell, so that the power vector is

$$\boldsymbol{P}^U = \left[p_1^U, p_2^U, \cdots, p_N^U\right]^T.$$

The signal to interference ration can be expressed as:

$$\gamma_i^U = \frac{p_i^U}{\sum\limits_{j=1}^{N} p_j^U \sum\limits_{k=1}^{L_j} \frac{g_{jki}}{g_{jkj}} - p_i^u}. \tag{1}$$

Thus,

$$G^U = [G^U_{ij}];$$

where

$$G^U_{ij} = \sum_{k=1}^{L_j} \frac{g_{jki}}{g_{jkj}}. \tag{2}$$

1.2 SIR in Downlink

Then we assumed p^D_{ik} as the transmit power at BS of cell i to the k-th user in the same cell. So that the desired signal of which user received is

$$p_{desired} = p^D_{ik} \times g_{iki}.$$

And Q_i represents the total transmit power for the i-th BS. Thus, we can get that

$$Q_i = \sum_{k=1}^{L_i} p^D_{ik}. \tag{3}$$

so that the power vector in the downlink is

$$\boldsymbol{Q} = \left[Q_1, Q_2, \cdots, Q_N\right]^T.$$

The SIR at the user k in cell i can be expressed as:

$$\gamma^D_{ik} = \frac{p_{desired}}{\sum\limits_{j=1}^{N} Q_j g_{ikj} - p_{desired}} = \frac{p^D_{ik} g_{iki}}{\sum\limits_{j=1}^{N} Q_j g_{ikj} - p^D_{ik} g_{iki}} \tag{4}$$

By the global power control algorithm, we should set $\gamma^D_i = \gamma^D_{ik}$ (i=1,2,...,N). Generally, the SIR values are different form cell to cell, and within a given cell, the SIR will be different between uplink and downlink. Also by the global PCA, we need to balance the values of SIR in the downlink for each cell. Setting all $\gamma^D_i$to be equal leads to an eigenvalue equation problems, which is same like uplink. Hence,

$$\begin{aligned}
\gamma^D_i &= \frac{\sum p_{desired}}{\sum\limits_{j=1}^{N} Q_j \sum\limits_{k=1}^{L_i} g_{ikj} - \sum p_{desired}} \\
&= \frac{\sum p^D_{ik} g_{iki}}{\sum\limits_{j=1}^{N} Q_j \sum\limits_{k=1}^{L_i} g_{ikj} - \sum p^D_{ik} g_{iki}} \\
&= \frac{\sum p^D_{ik}}{\sum\limits_{j=1}^{N} Q_j \sum\limits_{k=1}^{L_i} \frac{g_{ikj}}{g_{iki}} - \sum p^D_{ik}}
\end{aligned}$$

By (3) we can get the SIR values of downlink is

$$\gamma^D_i = \frac{Q_i}{\sum\limits_{j=1}^{N} Q_j \sum\limits_{k=1}^{L_i} \frac{g_{ikj}}{g_{iki}} - Q_i} \tag{5}$$

Thus,

$$G^D = [G_{ij}^D];$$

where

$$G_{ij}^D = \sum_{k=1}^{L_i} \frac{g_{ikj}}{g_{iki}}. \tag{6}$$

1.3 result

Now, we list the matrix of uplink and downlink,

$$\boldsymbol{G}^U = \begin{bmatrix} \sum_{k=1}^{L_1} \frac{g_{1k1}}{g_{1k1}} & \sum_{k=1}^{L_2} \frac{g_{2k1}}{g_{2k2}} & \cdots & \sum_{k=1}^{L_N} \frac{g_{Nk1}}{g_{NkN}} \\ \sum_{k=1}^{L_1} \frac{g_{1k2}}{g_{1k1}} & \sum_{k=1}^{L_2} \frac{g_{2k2}}{g_{2k2}} & \cdots & \sum_{k=1}^{L_N} \frac{g_{Nk2}}{g_{NkN}} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{k=1}^{L_1} \frac{g_{1kN}}{g_{1k1}} & \sum_{k=1}^{L_2} \frac{g_{2kN}}{g_{2k2}} & \cdots & \sum_{k=1}^{L_N} \frac{g_{NkN}}{g_{NkN}} \end{bmatrix}. \tag{7}$$

$$\boldsymbol{G}^D = \begin{bmatrix} \sum_{k=1}^{L_1} \frac{g_{1k1}}{g_{1k1}} & \sum_{k=1}^{L_1} \frac{g_{1k2}}{g_{1k1}} & \cdots & \sum_{k=1}^{L_1} \frac{g_{1kN}}{g_{1k1}} \\ \sum_{k=1}^{L_2} \frac{g_{2k1}}{g_{2k2}} & \sum_{k=1}^{L_2} \frac{g_{2k2}}{g_{2k2}} & \cdots & \sum_{k=1}^{L_2} \frac{g_{2kN}}{g_{2k2}} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{k=1}^{L_N} \frac{g_{Nk1}}{g_{NkN}} & \sum_{k=1}^{L_N} \frac{g_{Nk2}}{g_{Nk2}} & \cdots & \sum_{k=1}^{L_N} \frac{g_{NkN}}{g_{NkN}} \end{bmatrix}. \tag{8}$$

The equations (7) and equation (8) reveals that

$$G^U = \left[G^D\right]^T.$$