Centralized Power Control in Cellular Radio Systems

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***Abstract*— This paper describes a centralized power control scheme for cellular mobile radio systems. The power for the mobiles in the scheme proposed here is computed based on signal strength measurements. All the mobiles using the same channel in this scheme will attain a common carrier-to-interference ratio. The proposed scheme is analyzed and shown to have an optimal solution.**

I. Introduction

Transmitter power is an important resource in cellular mobile radio systems. When used effectively it can increase system capacity and quality of communications. A commonly used measure of the quality of communications is the carrier-to-interference ratio (CIR) at the receiver. The central idea in power control schemes is to maximize the minimum CIR in each of the channels in the system. Centralized power control (CPC) schemes require some sort of central controller which has knowledge about all the radio links in the system. Therefore, they are not easy to implement. However, CPC schemes help in the design of distributed power control schemes that are easy to implement.

Early work on power control [1]–[3] centered on schemes which kept the received power at a constant level. These schemes have the advantage that the dynamic range requirements for the receiver are smaller, resulting in better adjacent channel protection. These schemes indicate an increase of system capacity by a factor roughly equal to two, relative to schemes with constant transmitted power. In [4] a CPC scheme that achieves the same CIR in all the radio links is proposed for a nonfading spread-spectrum system.

A CPC scheme is derived in [5] that has an optimal solution. This scheme was used to construct a distributed power control scheme [6], which converged to the optimal solution. In these schemes the CIR is controlled directly instead of controlling the received power.

In our work, we propose a CPC scheme that has the same optimal solution as the schemes in [5] and [6]. It lays the foundation for a distributed power control scheme [7] that converges to the same optimal solution. We analyze the scheme for the uplink (radio link from mobile to base). The same procedure is applicable to the downlink (radio link from base to mobile). Our scheme computes transmitter power to affect the CIR directly. That is, each mobile is assigned a transmitter power that adjusts its CIR to a common value.

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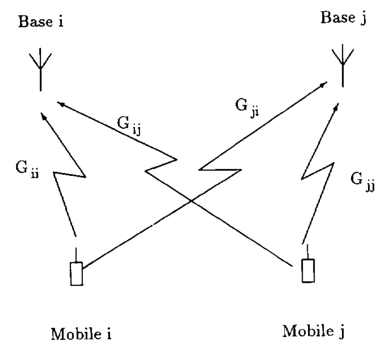

Fig. 1. Gain of the communication link between the ith base and the jth mobile.

In Section II we present the system model. The proposed power control scheme is presented and analyzed in Section III.

II. System Model

We consider a cellular radio system with a finite channel (a frequency or a time slot) set of size N. The channels are reused in the system according to an arbitrary channel assignment scheme. M denotes the number of mobiles using a given channel. The transmitter power of mobile i is denoted by P_i. The M dimensional vector $\boldsymbol{P}$ is the transmitter power vector for the mobiles in the given channel. We consider centralized power control for a given channel assignment.

Fading is not accounted for in our model, though the proposed CPC scheme is still applicable when the fading factor is included. It is assumed that the system is interference limited, and, therefore, noise is ignored. A mobile i uses the base station i which is closest to it for communication. The gain of the communication link between the ith base and the jth mobile is denoted by G_{ij}. This is illustrated in Fig. 1. All the G_{ij}'s are greater than zero. The carrier to interference ratio (CIR) of mobile i at its base station i is then given by

$$\gamma_i = \frac{P_i G_{ii}}{\sum_{\substack{j=1 \\ j \neq i}}^{M} P_j G_{ij}}, \qquad 1 \leq i \leq M. \tag{1}$$

We define the quantity γ^- presented in [5] here again as

$$\gamma^- = \max_{\boldsymbol{P} \geq \boldsymbol{o}} \min_{1 \leq i \leq M} \{\gamma_i\} \tag{2}$$

where the notation $\boldsymbol{P} \geq \boldsymbol{o}$ is used to denote that all the elements of the M dimensional transmitter power vector $\boldsymbol{P}$

are nonnegative. We also define the quantity γ^+ as

$$\gamma^+ = \min_{\boldsymbol{P}\geq \boldsymbol{o}} \max_{1\leq i\leq M}\{\gamma_i\}. \tag{3}$$

The next section describes a CPC scheme that achieves the same CIR for all the mobiles in each of the channels in the system.

III. Centralized Power Control

In this section, we describe a centralized power control scheme that maximizes in a given channel the minimum of the CIR's of all the mobiles using that channel. The CPC scheme is described and analyzed here for the uplinks. For the downlinks a similar procedure can be followed.

Equation (1) can be rewritten as

$$\gamma_i = \frac{P_i}{\sum_{j=1}^{M} A_{ij}P_j} \tag{4}$$

where

$$A_{ij} = \begin{cases} \frac{G_{ij}}{G_{ii}}; & \text{if } i \neq j \\ 0; & \text{if } i = j \end{cases} \tag{5}$$

Note that G_{ij}/G_{ii} is always greater than 0. Let us define $\boldsymbol{A}$ as an $M \times M$ matrix that has A_{ij} as its elements. The $M \times M$ matrix $\boldsymbol{A}$ has a few important properties which are described by the theorem and lemma that follow. The theorem is originally due to O. Perron and G. Frobenius [8]–[12] and is stated here without proof.

Theorem 1: Let $\boldsymbol{A}$ be an $M \times M$ irreducible nonnegative matrix with eigenvalues $\{\lambda_i\}_{i=1}^{M}$. Then:

1) $\boldsymbol{A}$ has a positive real eigenvalue λ^* with $\lambda^* = \max\{|\lambda_i|\}_{i=1}^{M}$.
2) λ^* above has an associated eigenvector $\boldsymbol{P}^*$ with strictly positive entries.
3) λ^* has algebraic multiplicity equal to 1.
4) All eigenvalues λ of $\boldsymbol{A}$ other than λ^* satisfy $|\lambda| < |\lambda^*|$ if and only if there is a positive integer k with all entries of $\boldsymbol{A}^k$ strictly positive.
5) The minimum real λ such that the inequality
$$\lambda \boldsymbol{P} \geq \boldsymbol{A}\boldsymbol{P} \tag{6}$$
has solutions for $\boldsymbol{P} \geq \boldsymbol{o}$ is $\lambda = \lambda^*$.
6) The maximum real λ such that the inequality
$$\lambda \boldsymbol{P} \leq \boldsymbol{A}\boldsymbol{P} \tag{7}$$
has solutions for $\boldsymbol{P} \geq \boldsymbol{o}$ is $\lambda = \lambda^*$.

The notation $\boldsymbol{A} \geq \boldsymbol{B}$ is used to imply that each element in matrix $\boldsymbol{A}$ is greater than or equal to the corresponding element in matrix $\boldsymbol{B}$.

We now present the lemma for the matrix $\boldsymbol{A}$.

Lemma 1: $\boldsymbol{A}$ is an irreducible nonnegative matrix.

Proof: The matrix $\boldsymbol{A}$ by definition has all the elements on its main diagonal equal to 0 and all other elements greater than 0. An $M \times M$ matrix $\boldsymbol{B}$ is reducible if there exists a permutation matrix $\boldsymbol{Q}$ such that

$$\boldsymbol{Q}\boldsymbol{B}\boldsymbol{Q}^T = \begin{pmatrix} \boldsymbol{C} & \boldsymbol{D} \\ \boldsymbol{O} & \boldsymbol{E} \end{pmatrix} \tag{8}$$

with $\boldsymbol{C}$ $r \times r$ and $\boldsymbol{E}$ $(M-r)\times(M-r)$ for $1 \leq r \leq M-1$. $\boldsymbol{O}$ is a matrix with all its entries equal to zero. Matrix $\boldsymbol{A}$ would be reducible only if it had at least one row with more than one 0 element. So $\boldsymbol{A}$ is an irreducible nonnegative matrix.

In the centralized power control (CPC) scheme proposed here it is assumed that a central controller has knowledge about the signal strengths in all the radio links. In other words the central controller has knowledge of matrix $\boldsymbol{A}$. It then computes the eigenvector of the matrix $\boldsymbol{A}$ as $\boldsymbol{P}^*$. The mobiles would then use the elements of $\boldsymbol{P}^*$ as their transmitter powers to achieve γ^* in the system. For the CPC scheme we have the following proposition.

Proposition 1: There exists a unique γ^* given by

$$\gamma^* = \gamma^- = \gamma^+ \tag{9}$$

that may be achieved by all the M mobiles using a given channel and is given by

$$\gamma^* = \frac{1}{\lambda^*} \tag{10}$$

where λ^* is the largest real eigenvalue of the matrix $\boldsymbol{A}$. The power vector $\boldsymbol{P}^*$ that achieves γ^* is the eigenvector corresponding to λ^*.

Proof: Let us define the quantities

$$\gamma_{\min} = \min_{1\leq i\leq M}\{\gamma_i\} \quad \text{and} \quad \gamma_{\max} = \max_{1\leq i\leq M}\{\gamma_i\}. \tag{11}$$

We have

$$\gamma_i \geq \gamma_{\min}, \quad i = 1, \cdots, M. \tag{12}$$

Using (4) we can rewrite the above as

$$\frac{1}{\gamma_{\min}} P_i \geq \sum_{j=1}^{M} A_{ij}P_j, \quad i = 1, \cdots, M. \tag{13}$$

The above can be expressed in matrix notation as

$$\lambda \boldsymbol{P} \geq \boldsymbol{A}\boldsymbol{P} \tag{14}$$

where

$$\lambda = \frac{1}{\gamma_{\min}}. \tag{15}$$

From Theorem 1 and Lemma 1 it follows that the minimum real λ such that the above set of inequalities will hold true is λ^*, which is the largest eigenvalue of $\boldsymbol{A}$ that is positive and has $\boldsymbol{P}^*$ as the corresponding eigenvector.

$$\boldsymbol{G} = \begin{pmatrix} 1.0 \times 10^{-4} & 4.82253 \times 10^{-9} & 3.57346 \times 10^{-10} \\ 1.52416 \times 10^{-8} & 6.25 \times 10^{-6} & 3.50128 \times 10^{-9} \\ 7.67336 \times 10^{-10} & 2.44141 \times 10^{-8} & 1.23457 \times 10^{-6} \end{pmatrix}.$$

We also have

$$\gamma_i \leq \gamma_{\max}, \quad i = 1, \cdots, M. \tag{16}$$

Following the same procedure as above, we have

$$\lambda \boldsymbol{P} \leq \boldsymbol{A}\boldsymbol{P} \tag{17}$$

where

$$\lambda = \frac{1}{\gamma_{\max}}. \tag{18}$$

Again from Theorem 1 and Lemma 1, the maximum real λ such that the above set of inequalities hold true is λ^*. So we have a γ^* that is achievable by all mobiles and $\gamma^* = \gamma^- = \gamma^+$. □

It also follows from Theorem 1 and Lemma 1 that $\lambda^* > 0$ and, therefore, $\gamma^* > 0$. From the above proof for Proposition 1 we see that γ^* is achieved for the system with

$$\lambda^* \boldsymbol{P}^* = \boldsymbol{A}\boldsymbol{P}^* \tag{19}$$

which implies that all the mobiles in this case will have the same CIR, γ^*.

The following example will illustrate the proposed CPC scheme. Consider three mobiles using the same channel with link gains given by the $\boldsymbol{G}$ matrix given at the bottom of the previous page.

Using equal transmitter powers, we obtain the CIR's of the three mobiles using (1) as 42.85 dB, 25.23 dB, and 16.90 dB, respectively. We next use transmitter powers according to the proposed CPC scheme (namely the elements of the eigenvector $\boldsymbol{P}^*$ of the corresponding $\boldsymbol{A}$ matrix) as 1.79189×10^{-3}, 8.67402×10^{-2} and 5.11641×10^{-1}, respectively. The corresponding CIR's of the three mobiles are all equal to 24.74 dB. We see an improvement of 7.84 dB in the minimum CIR.

IV. Conclusions

A centralized power control scheme which computes transmitter powers so as to have a common CIR for all the receivers has been presented. It is analyzed and shown to maximize the minimum of the CIR's and also to minimize the maximum of the CIR's for all the mobiles using the same channel.

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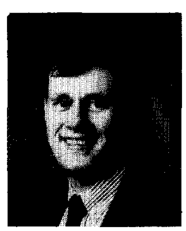

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