

Appendix for SIR-Based Power Control Used in CDMA Systems

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1 Derivation for Eigenvalue problem

In this system, the N users are distributed in K cells randomly as shown in Fig.1. The users are grouped by calculating the minimal distance on the center of each cell.

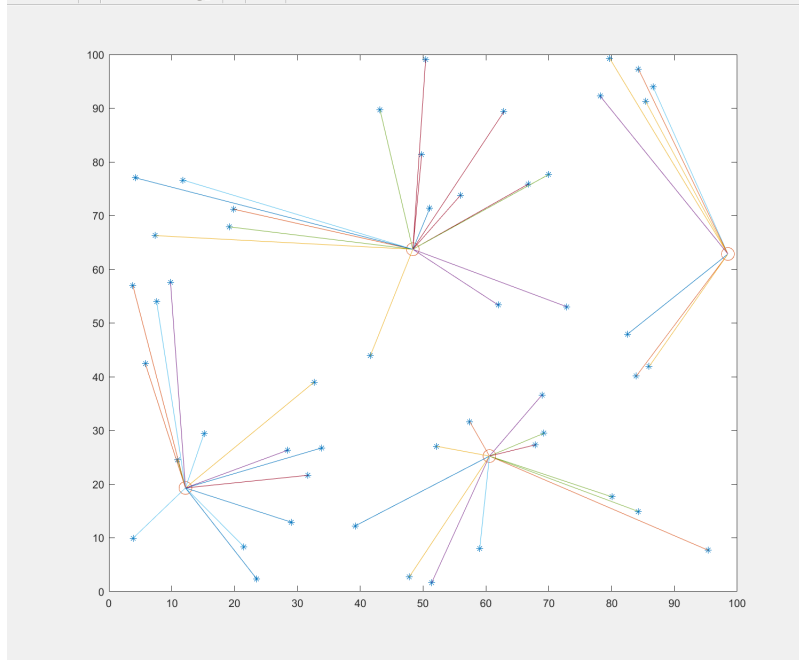


Figure 1: An example with 50 users and 4 cells.

The pathloss of each user is given by

$$L_{uk} = d_{uk}^3 \quad (1)$$

where d_{uk} is the distance from u -th user to k -th center.

The SIR γ_1 of 1st user in 1st cell can be represented as

$$\begin{aligned}\gamma_1 &= \frac{P_1/L_{11}}{\sum_{u \neq 1}^N P_u/L_{u1}} \\ &= \frac{P_1/L_{11}}{\sum_{u=1}^N P_u/L_{u1} - P_1/L_{11}} \\ &= \frac{P_1}{L_{11} \sum_{u=1}^N P_u/L_{u1} - P_1}\end{aligned}\tag{2}$$

From Eq. (2), we have

$$\frac{\gamma+1}{\gamma}P_1 = L_{11} \sum_{u=1}^N P_u/L_{u1}\tag{3}$$

$$= L_{11}[1/L_{11}, 1/L_{21}, \dots, 1/L_{N1}] \begin{bmatrix} P_1 \\ \vdots \\ P_N \end{bmatrix}\tag{4}$$

$$= [1, L_{11}/L_{21}, \dots, L_{11}/L_{N1}] \begin{bmatrix} P_1 \\ \vdots \\ P_N \end{bmatrix}\tag{5}$$

By applying the balanced SIR algorithm on all users, the problem is formulated as

$$\frac{\gamma+1}{\gamma}\mathbf{p} = \mathbf{G}\mathbf{p}\tag{6}$$

$$\frac{1}{\gamma}\mathbf{p} = (\mathbf{G} - \mathbf{I})\mathbf{p}\tag{7}$$

where

$$\mathbf{G} = \begin{bmatrix} 1 & L_{11}/L_{21} & \cdots & L_{11}/L_{N1} \\ L_{2k_2}/L_{1k_2} & 1 & \cdots & L_{2k_2}/L_{Nk_2} \\ \vdots & \vdots & \vdots & \vdots \\ L_{Nk_N}/L_{1k_N} & L_{Nk_N}/L_{2k_N} & \cdots & 1 \end{bmatrix} \quad \mathbf{p} = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_N \end{bmatrix}\tag{8}$$

where the $k_u, u \in (1, N)$ means that the u -th user is distributed to k -th cell.

Based on the **PerronFrobenius theorem**, there exists an eigenvector that all the elements are positive. Since $\text{trace}(\mathbf{G} - \mathbf{I}) = 0$, there must exist an eigenvalue larger than 0.