Appendix for SIR-Based Power Control Used in CDMA Systems

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1 Derivation for Eigenvalue problem

In this system, the N users are distributed in K cells randomly as shown in Fig.1. The users are grouped by calculating the minimal distance on the center of each cell.

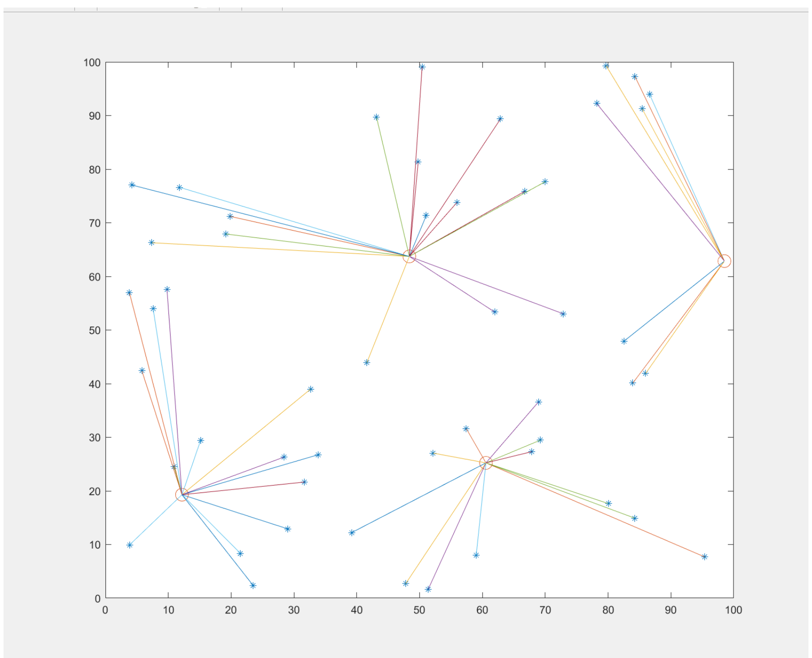

Figure 1: An example with 50 users and 4 cells.

The pathloss of each user is given by

$$L_{uk} = d_{uk}^3 \tag{1}$$

where d_{uk} is the distance from u-th user to k-th center.

The SIR γ_1 of 1st user in 1st cell, *i.e.* $k_1 = 1$ can be represented as

$$\begin{aligned}
\gamma_1 &= \frac{P_1/L_{11}}{\sum_{u\neq 1}^{N} P_u/L_{u1}} \\
&= \frac{P_1/L_{11}}{\sum_{u=1}^{N} P_u/L_{u1} - P_1/L_{11}} \\
&= \frac{P_1}{L_{11}\sum_{u=1}^{N} P_u/L_{u1} - P_1}
\end{aligned} \tag{2}$$

From Eq. (2), we have

$$\frac{\gamma+1}{\gamma}P_1 = L_{11}\sum_{u=1}^{N} P_u/L_{u1} \tag{3}$$

$$= L_{11}[1/L_{11}, 1/L_{21}, \cdots, 1/L_{N1}] \begin{bmatrix} P_1 \\ \cdots \\ P_N \end{bmatrix} \tag{4}$$

$$= [1, L_{11}/L_{21}, \cdots, L_{11}/L_{N1}] \begin{bmatrix} P_1 \\ \cdots \\ P_N \end{bmatrix} \tag{5}$$

By applying the balanced SIR algorithm on all users, the problem is formulated as

$$\frac{\gamma+1}{\gamma}\boldsymbol{p} = \boldsymbol{G}\boldsymbol{p} \tag{6}$$

$$\frac{1}{\gamma}\boldsymbol{p} = (\boldsymbol{G}-\boldsymbol{I})\boldsymbol{p} \tag{7}$$

where

$$\boldsymbol{G} = \begin{bmatrix} 1 & L_{1k_1}/L_{2k_1} & \cdots & L_{1k_1}/L_{Nk_1} \\ L_{2k_2}/L_{1k_2} & 1 & \cdots & L_{2k_2}/L_{Nk_2} \\ \vdots & \vdots & \vdots & \vdots \\ L_{Nk_N}/L_{1k_N} & L_{Nk_N}/L_{2k_N} & \cdots & 1 \end{bmatrix} \quad \boldsymbol{p} = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_N \end{bmatrix} \tag{8}$$

where the $k_u, u \in (1, N)$ means that the u-th user is distributed to k-th cell.

Based on the *PerronFrobenius theorem*, there exists an eigenvector that all the elements are positive. Since $trace(G - I) = 0$, there must exist an eigenvalue lager than 0.

2 Proof for $\boldsymbol{G}^U = (\boldsymbol{G}^D)^T$

For 1st user in 1st cell, *i.e.* $k_1 = 1$

$$\begin{aligned}
\gamma_1 &= \frac{P_1/L_{11}}{\sum_{u\neq 1}^{N} P_u/L_{1k_u}} \\
&= \frac{P_1/L_{11}}{\sum_{u=1}^{N} P_u/L_{1k_u} - P_1/L_{11}} \\
&= \frac{P_1}{L_{11}\sum_{u=1}^{N} P_u/L_{1k_u} - P_1}
\end{aligned} \tag{9}$$

The SIR for downlink is represented as

$$\frac{\gamma + 1}{\gamma}\boldsymbol{p}^D = \boldsymbol{G}^D \boldsymbol{p}^D \tag{10}$$

$$\frac{1}{\gamma}\boldsymbol{p}^D = (\boldsymbol{G}^D - \boldsymbol{I})\boldsymbol{p}^D \tag{11}$$

where

$$\boldsymbol{G}^D = \begin{bmatrix} 1 & L_{1k_1}/L_{1k_2} & \cdots & L_{1k_1}/L_{1k_N} \\ L_{2k_2}/L_{2k_1} & 1 & \cdots & L_{2k_2}/L_{2k_N} \\ \vdots & \vdots & \vdots & \vdots \\ L_{Nk_N}/L_{Nk_1} & L_{Nk_N}/L_{Nk_2} & \cdots & 1 \end{bmatrix} \quad \boldsymbol{p}^D = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_N \end{bmatrix} \tag{12}$$