Constrained Power Control in Cellular Radio Systems

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Abstract: **High system capacities can be achieved by controlling the transmitter power in multiuser radio systems. Power control with no constraint on the maximum power level has been studied extensively in earlier work([1]-[18]). Transmitter power is at a premium in radio systems such as cellular systems and PCS. There is a limit on the maximum transmitter power especially at the terminals (eg. mobile units and handsets) since the power comes from a battery. In this paper we study power control that maximizes the minimum carrier to interference ratio (CIR), with a constraint on the maximum power. The optimal power vector solution lies on the boundary of the constrained power vector set and achieves a balance in the CIR's. Results indicate that the constraints do not induce any stability problems. A distributed scheme with favourable convergence properties and close to optimum performance is presented. Simulation results show that the algorithm tries to maximize the number of terminals served with CIR greater than or equal to the target CIR, while conserving power.**

I. Introduction

Cochannel interference, due to spectrum reuse, is one of the main limiting factors in achieving a high user density in wireless communication systems. Controlling the transmitter power level has been a frequently used tool to control this interference to achieve a high capacity as well as a high link quality (carrier to interference ratio, CIR). Previous results show that for a given link quality requirement capacity can be maximized by adjusting the transmitter power such that all admitted connections operate at the same CIR. This is referred to as CIR-balancing([5]-[8],[10]). Results show that substantial improvements in capacity, similar to the results achieved by dynamic channel allocation, can be achieved. Also distributed power control algorithms (PCA's), iteratively adjusting the power in the individual links based only on measurements in that link have been shown to achieve similar performance([9],[11]-[13]). In [18] an algorithm that integrates power control and base assignment is presented.

Most of the previously reported schemes, assume that the transmitter power can be continuously adjusted without limitations in dynamic range. This is of course very optimistic since the maximum output power of a handheld device is certainly limited, mainly due to battery capacity limitations. However, in [14] it was demonstrated that introducing transmitter power range constraints does not severely affect the performance of a centralized power control scheme. However, when using a distributed PCA, these constraints affect not only the CIR-performance but also the convergence properties of the PCA are affected. This is the topic of this paper. First, we derive an upper bound on the CIR-performance when each transmitter is confined to use at most a given power level. A centralized scheme for achieving this bound is presented. Further a distributed algorithm that has close to optimum performance is proposed.

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II. System Model

In this section we describe the system model and some relevant results needed for the analysis. We discuss power control for the uplink (from terminal to base) only. For the downlink (from base to terminal) all the results in this paper are valid with appropriate changes in the notation.

We consider a cellular radio system with a finite channel set of size N (where a channel could be a frequency or time slot). The number of terminals using the same channel is denoted by M. We assume that the channels are orthogonal i.e. terminals on different channels do not interfere with each other. We denote the transmitter power of the i^{th} terminal communicating with the i^{th} base station by P_i. The gain on the radio link from terminal j to base i is denoted by G_{ij}. All the G_{ij}'s are positive and can take values in the range $(0, 1]$. ν_i denotes the receiver noise at the i^{th} base. The link quality is measured by the carrier to interference ratio(CIR). The CIR of the i^{th} terminal at its base is given by

$$\gamma_i = \frac{P_i G_{ii}}{\sum_{\substack{j=1 \\ j\neq i}}^{M} P_j G_{ij} + \nu_i}\ , \ 1 \le i \le M\ . \quad (1)$$

Equation (1) for the CIR of the i^{th} terminal at its base can be rewritten as

$$\gamma_i = \frac{P_i}{\sum_{j=1}^{M} A_{ij} P_j + \eta_i} , \qquad (2)$$

where the $M \times M$ matrix $A = \{A_{ij}\}$ is defined by

$$A_{ij} = \begin{cases} \left(\frac{G_{ij}}{G_{ii}}\right) & \text{if } i \neq j \\ 0 & \text{if } i = j \ . \end{cases} , \qquad (3)$$

and $\eta_i = \nu_i / G_{ii}$. We let $\boldsymbol{\eta}$ be the M dimensional vector whose i^{th} element is η_i. The η_i's are greater than or equal to zero. In our analysis we assume that at least one of the η_i is not zero.

We introduce the notion that γ is achievable if there exist power vectors such that $\gamma_i \geq \gamma,\ i = 1, \ldots, M$. The maximum achievable CIR γ^* is defined as

$$\gamma^* = \max_{\boldsymbol{P} \geq \mathbf{o}} \ \min_{1 \leq i \leq M} \gamma_i \ , \qquad (4)$$

where $\boldsymbol{P}$ is the M-dimensional power vector whose i^{th} element is P_i. The notation $\boldsymbol{P} \geq \mathbf{o}$ denotes that the elements of $\boldsymbol{P}$ are nonnegative. For interference limited systems(receiver noise power equal to zero) it is shown in ([8]-[11]) that an optimal power vector $\boldsymbol{P}^*$ exists that achieves γ^*. This optimal vector $\boldsymbol{P}^*$ is unique up to scaling by a constant, and results in all links having the same CIR, i.e., $\gamma_i = \gamma^*$, $1 \leq i \leq M$. The maximum eigenvalue λ^* of matrix $\boldsymbol{A}$ is related to γ^* as $\gamma^* = 1/\lambda^*$([10],[19]). The optimal power vector $\boldsymbol{P}^*$ is the eigenvector of $\boldsymbol{A}$ corresponding to λ^*. In [12] a distributed synchronous algorithm that accounts for receiver noise power is presented. Here each of the M terminals on the channel adjust their transmitter powers synchronously at discrete time instants. The power adjustment made by the i^{th} terminal at the n^{th} time instant is given by

$$P_i^{(n)} = \gamma_t \ \frac{P_i^{(n-1)}}{\gamma_i^{(n-1)}}, \ 1 \leq i \leq M \ , \ n \geq 1 \ , \qquad (5)$$

where $P_i^{(n)}$ is the power transmitted by the i^{th} terminal at the n^{th} iteration, $\gamma_i^{(n)}$ is its resulting CIR, and γ_t is the target CIR. For $\gamma_t < \gamma^*$ the above algorithm converges to a power vector solution $\boldsymbol{P} = (\boldsymbol{I} - \gamma_t \boldsymbol{A})^{-1} \boldsymbol{\eta}$ which results in all links having a CIR equal to γ_t. In [20] it is shown that the power adjustments in the above algorithm can also be made asynchronously, where the M terminals can adjust powers in a round robin fashion. When $\gamma_t \geq \gamma^*$ we have [20]

$$\lim_{n \to \infty} \boldsymbol{P}^{(n)} = \boldsymbol{P}^* \lim_{n \to \infty} K(n), \qquad (6)$$

and

$$\lim_{n \to \infty} \gamma_i^{(n)} = \gamma^*, \ 1 \leq i \leq M \ ,$$

where $K(n)$ is given by

$$K(n) = b \frac{(\gamma_t \lambda^*)^n - 1}{\gamma_t \lambda^* - 1} + a (\gamma_t \lambda^*)^n,$$

where the constants a and b are positive, and are decided by $\boldsymbol{P}^{(0)}$ and $\gamma_t \boldsymbol{\eta}$ respectively. In [16] it is shown that the scheme given by Equation (5) converges for totally asynchronous power updates (i.e each terminal has different rate of power updates), when $\gamma_t < \gamma^*$.

III. Constrained Power control

We denote the maximum transmitter power by P_{max}. Let $\boldsymbol{P}_{max}$ be an M dimensional power vector with all elements equal to P_{max}. We define the set $\boldsymbol{\mathcal{B}}\ (P_{max})$ by

$$\boldsymbol{\mathcal{B}}\ (P_{max}) \triangleq \{\boldsymbol{P} : 0 \leq \boldsymbol{P} \leq \boldsymbol{P}_{max}\} \ . \qquad (7)$$

The set $\boldsymbol{\mathcal{B}}\ (P_{max})$ is described geometrically by a "hypercube" whose side is of length P_{max} in Euclidean space.

We are interested in finding the maximum achievable CIR within $\boldsymbol{\mathcal{B}}\ (P_{max})$, $\gamma\prime$, given by

$$\gamma\prime = \max_{\boldsymbol{P} \in \boldsymbol{\mathcal{B}}\ (P_{max})} \ \min_{1 \leq i \leq M} \gamma_i \ , \qquad (8)$$

An upperbound on $\gamma\prime$ is clearly γ^*[14]. $\gamma\prime$ is therefore confined to the range $[0, \gamma^*)$. Note that γ is achievable if there exist power vectors such that $\gamma_i \geq \gamma,\ i = 1, \ldots, M$. The inequalities $\gamma_i \geq \gamma,\ i = 1, \ldots, M$ can be rewritten in matrix form using Equation (1) as

$$\boldsymbol{P} \geq \gamma (\boldsymbol{A}\ \boldsymbol{P} + \boldsymbol{\eta}\ ) \ . \qquad (9)$$

The set of all power vectors that achieve γ is therefore defined by

$$\boldsymbol{\mathcal{C}}\ (\gamma) \triangleq \{\boldsymbol{P} : (\frac{1}{\gamma} \boldsymbol{I}\ - \boldsymbol{A}\ ) \boldsymbol{P} \geq \boldsymbol{\eta}\ \} \ , \qquad (10)$$

The set $\boldsymbol{\mathcal{C}}\ (\gamma)$ is described geometrically by an unbounded polyhedral "cone" in Euclidean space. The apex of the "cone" $\boldsymbol{\mathcal{C}}\ (\gamma)$ is the power vector $\boldsymbol{P}_a(\gamma) = (\frac{1}{\gamma} \boldsymbol{I}\ - \boldsymbol{A}\ )^{-1} \boldsymbol{\eta}$. The set of feasible power vectors $\boldsymbol{\mathcal{S}}\ (\gamma)$ that achieve γ is thus defined by

$$\boldsymbol{\mathcal{S}}\ (\gamma) \triangleq \{\boldsymbol{P} : \boldsymbol{\mathcal{C}}\ (\gamma) \cap \boldsymbol{\mathcal{B}}\ (P_{max})\} \ . \qquad (11)$$

The set $\boldsymbol{\mathcal{S}}\ (\gamma)$ is the intersection of $\boldsymbol{\mathcal{C}}\ (\gamma)$ and $\boldsymbol{\mathcal{B}}\ (P_{max})$. Note that γ is achievable in $\boldsymbol{\mathcal{B}}\ (P_{max})$ if $\boldsymbol{\mathcal{S}}\ (\gamma) \neq \Phi$. We can therefore write

$$\gamma\prime = \max\{\gamma : \boldsymbol{\mathcal{S}}\ (\gamma) \neq \Phi\} \ .$$

In the following proposition we present an expression for $\gamma\prime$. For the proof of this proposition see [21]. In the proof we show that γ' is the maximum balance (i.e. all terminals with same CIR) CIR. In [5] a similar result was stated without proof for a more general case of unequal maximum power levels.

Proposition 1: The maximum achievable CIR within $\mathcal{B}\ (P_{max})$, γ', is given by

$$\gamma' = \gamma : \max_{1\leq i\leq M} [(\frac{1}{\gamma} I \ - A\ )^{-1} \eta\ ]_i = P_{max} \qquad (12)$$

and

$$\mathcal{S}\ (\gamma') = \{P_a(\gamma')\}\ \square. \qquad (13)$$

The vector $P_a(\gamma')$ achieves a balance in CIR's such that $\gamma_i = \gamma'$, $i = 1,\ldots,M$ in the system. Note also that of all the power vectors that can achieve γ' for the system $P_a(\gamma')$ has the smallest power in each component.

A centralized CPC (CCPC) scheme to compute γ' for the system follows naturally from Proposition 1. We start with a small value of γ (close to 0)and compute $\max_i\{P_i\}$ as

$$\max_i\{P_i\} = \max_{1\leq i\leq M} [(\frac{1}{\gamma} I \ - A\ )^{-1} \eta\ ]_i \qquad (14)$$

If $\max_i\{P_i\} < P_{max}$ then we increase γ by a small factor. The above process is repeated until $|\max_i\{P_i\} - P_{max}|$ is sufficiently small. The computation of $\max_i\{P_i\}$ in the CCPC scheme involves solving for P in the system of linear equations $P = \gamma_t(A\ P + \eta\ )$. This can be done iteratively using Equation (5).

Note that the above analysis for finding γ' is for non zero receiver noise power i.e. $\eta\ \neq 0$. For $\eta\ = 0$ it is easy to see that all the power vectors that are scaled versions of P^* and lie within $\mathcal{B}\ (P_{max})$ achieve γ^*, i.e. γ^* is always achievable in $\mathcal{B}\ (P_{max})$ and $\gamma' = \gamma^*$.

IV. Distributed CPC

A distributed power control scheme is derived by making a simple modification to the power control of [12]. Each of the M terminals on the channel adjust their transmitter powers synchronously at discrete time instants. The power adjustment made by the i^{th} terminal at the n^{th} time instant is given by

$$P_i^{(n)} = \min\{P_{max}\ ,\ \gamma_t\ \frac{P_i^{(n-1)}}{\gamma_i^{(n-1)}}\},\ 1 \leq i \leq M\ ,\ n \geq 1\ , \qquad (15)$$

where P_{max} is the maximum transmitter power for the system. We shall refer to the above power control scheme as the distributed CPC (DCPC) scheme. We shall now proceed to analyse the convergence properties of the DCPC scheme.

We first define a set of powers $\mathcal{C}\ ^{-}(\gamma)$. The inequalities $\gamma_i \leq \gamma$, $i = 1,\ldots,M$ can be rewritten in matrix form using Equation (1) as $P \geq \gamma(A\ P + \eta\ )$. The set of all power vectors that satisfy $\gamma_i \leq \gamma$, $i = 1,\ldots,M$ is therefore defined by

$$\mathcal{C}\ ^{-}(\gamma) \triangleq \{P : (\frac{1}{\gamma} I \ - A\ )P \leq \eta\ \}\ , \qquad (16)$$

The set $\mathcal{C}\ ^{-}(\gamma)$ is described geometrically by a polyhedral "cone" in Euclidean space with the apex vector $P_a(\gamma)$ given by $P_a(\gamma) = (\frac{1}{\gamma} I \ - A\ )^{-1}\eta$. Note that $\mathcal{C}\ ^{-}(\gamma)$ and $\mathcal{C}\ (\gamma)$ have the same apex vector. We define the set of power vectors $\mathcal{S}\ ^{-}(\gamma)$ as

$$\mathcal{S}\ ^{-}(\gamma) \triangleq \{P : \mathcal{C}\ ^{-}(\gamma) \cap \mathcal{B}\ (P_{max})\}\ . \qquad (17)$$

The set $\mathcal{S}\ ^{-}(\gamma)$ is $\mathcal{C}\ ^{-}(\gamma)$ "truncated" by $\mathcal{B}\ (P_{max})$ We define the mapping $\mathcal{T}$ from the set of all nonnegative power vectors into itself by

$$\mathcal{T}\ (P) \triangleq \min\{P_{max}, \gamma_t(A\ P + \eta\ )\}, \qquad (18)$$

where min{.,.} is component wise. We can now describe the DCPC scheme by

$$P^{(n)} = \mathcal{T}\ (P^{(n-1)}),\ n \geq 1. \qquad (19)$$

It is shown in [21], using contraction mapping arguments[22], that the DCPC scheme converges to the fixed point $P^{(*)}$ given by $P^{(*)} = \mathcal{T}\ (P^{(*)})$, starting with any nonnegative power vector $P^{(0)}$. However for $\gamma_t \geq \gamma^*$, $\mathcal{T}$ is no longer a contraction mapping. So we use other techniques to show that for any $\gamma_t \geq 0$ the DCPC scheme still converges to the fixed point. The mapping $\mathcal{T}$ has a unique fixed point $P^{(*)}$ given by $P^{(*)} = \mathcal{T}\ (P^{(*)})$ and is the "supremum" of the set $\mathcal{S}\ ^{-}(\gamma)$ [21]. The mapping $\mathcal{T}$ also has a "monotonicity" property that

$$\mathcal{T}\ (P_1) \leq \mathcal{T}\ (P_2) \text{for} P_1 \leq P_2\ .$$

Let us denote $P^{(k)}$ by $\mathcal{T}\ ^k(P^{(0)})$. Using the above property it can be shown [21] that

$$\lim_{k\to\infty} \mathcal{T}\ ^k(0) = P^{(*)} \text{and} \lim_{k\to\infty} \mathcal{T}\ ^k(P_{max}) = P^{(*)}\ .$$

Now for $0 \leq P^{(0)} \leq P_{max}$ applying the "monotonicity" property repeatedly we will have

$$P^{(*)} \leq \lim_{k\to\infty} \mathcal{T}\ ^k(P^{(0)}) \leq P^{(*)},$$

or

$$\lim_{k\to\infty} \mathcal{T}\ ^k(P^{(0)}) = P^{(*)}\ .$$

Hence we have the following proposition for the DCPC scheme (for a more detailed proof see [21])

Proposition 2: Starting with any nonnegative power vector $P^{(0)}$ the DCPC scheme converges to the fixed point $P^{(*)}$ given by $P^{(*)} = \mathcal{T}\ (P^{(*)})$ $\square$.

The only requirement on γ_t in Proposition 2 is $\gamma_t \geq 0$. So Proposition 2 guarantees convergence of the DCPC scheme for any $\gamma_t \geq 0$. In [21] it is shown that a totally asynchronous version of the DCPC scheme, that allows different power update rates for terminals, also converges to the fixed point $\boldsymbol{P}^{(*)}$ given by $\boldsymbol{P}^{(*)} = \boldsymbol{T}\ (\boldsymbol{P}^{(*)})$. Note that setting $\gamma_t \geq \frac{P_{max}}{\min_i\{\eta_i\}}$ will result in $\boldsymbol{P}^{(*)} = \boldsymbol{P}_{max}$, which is equivalent to operating with maximum transmitter power.

V. Numerical Results

In this section the behaviour of the DCPC scheme as γ_t is varied is illustrated by simulations. A one-dimensional cellular system with 20 cells in a circle is considered. The cell size is 2Km with the base station at the center. Fixed channel assignment is assumed with channels reused in every other cell. Each base station has 5 channel allocated to it. The link gains are given by $G_{ij} = d_{ij}^{-4}S$, where S is the slow fading factor that is log-normal distributed with log-variance of 10dB. The maximum transmitter power was set to 1W and the receiver noise power was assumed to be 10^{-15}W. A full load situation is assumed in all cases,i.e. all channels at each base station are in use. We consider only snapshots of the cellular system, where a snapshot is essentially a set of terminals frozen in their positions. The terminal positions are uniformly distributed in each cell. The DCPC scheme is applied to each snapshot and the powers and CIR's are recorded. In each snapshot 5 iterations of the DCPC scheme is carried out. The Powers and CIR's, rounded to the nearest integer in dB, are recorded at the end of the five iterations. Any CIR's lower than 0 dB are counted in the bin corresponding to 0 dB. The CIR's above 50 dB are accounted for in the bin corresponding to 50 dB. This is repeated for 100 snapshots.

We denote the fraction of terminals that have CIR equal to the abscissa as f_{CIR}. The fraction of terminals that have CIR (Power) less than or equal to the abscissa is denoted by $F_{CIR}(F_{POWER}$ respectively). f_{CIR}, F_{CIR}, and F_{POWER} are plotted in Fig. 1,2, and 3 respectively. The plots include FCA(with all terminals transmitting 1W, which is the maximum power) and DCPC with γ_t =15,20,25,30,35,40 dB. From Fig. 1 we see that the CIR distribution of FCA is rather evenly spread. For DCPC as γ_t is increased fewer terminals are able to attain γ_t. From Fig. 2 we see that F_{CIR} corresponding to γ_t is always lower for DCPC than for FCA. This means that the fraction of terminals that have CIR lower than or equal to γ_t is smaller for DCPC than for FCA. In Fig. 3 we observe that increasing γ_t results in increased power levels. Finally from Fig. 1,2, and 3 we see that as γ_t increases DCPC tends to FCA. Setting $\gamma_t \gg \gamma^*$ will result in $\boldsymbol{P}^{(*)} = \boldsymbol{P}_{max}$ for DCPC which is equivalent to operating as in FCA.

VI. Conclusions

The optimum power vector that maximizes the minimum CIR in the system under a constraint on the maximum power level is derived. A distributed constrained power control scheme with favorable convergence properties and close to optimum performance is presented. Simulation results show that the DCPC scheme supports more number of users ,with CIR equal to or above the target CIR, than when operating with maxmimum transmitter powers.

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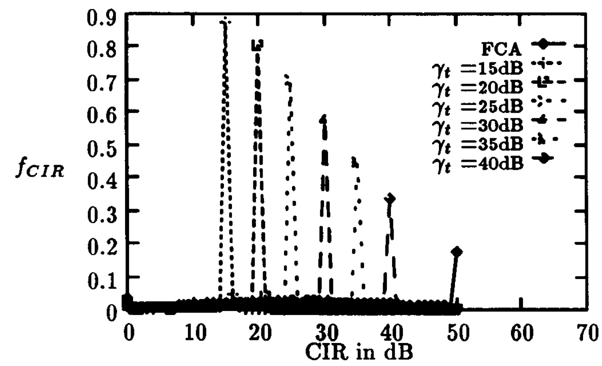

Figure 1: f_{CIR}, the fraction of terminals with CIR equal to the abscissa, for FCA and DCPC with γ_t =15,20,25,30,35,40 dB.

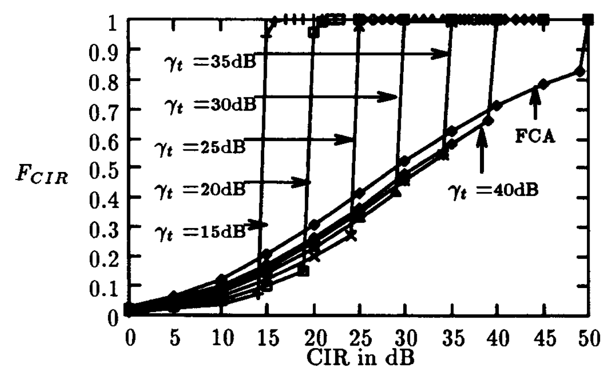

Figure 2: F_{CIR}, the fraction of terminals with CIR less than or equal to the abscissa, for FCA and DCPC with γ_t =15,20,25,30,35,40 dB.

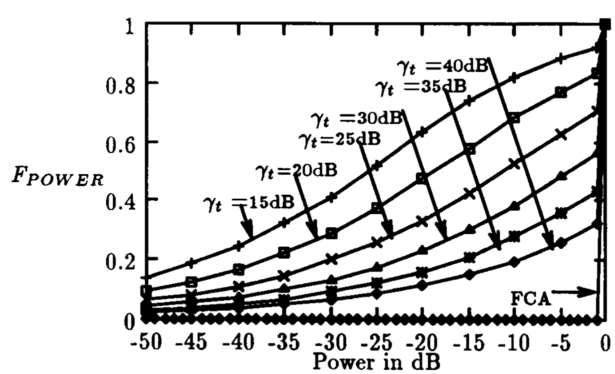

Figure 3: F_{POWER},the fraction of terminals with Power less than or equal to the abscissa, for FCA and DCPC with γ_t =15,20,25,30,35,40 dB.