Analysis of Power Control and Its Imperfections in CDMA Cellular Systems

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***Abstract*—In this paper, we develop a general model to study the forward-link capacity of a code-division multiple access (CDMA) cellular system with power control. Using this model, two forward-link power control schemes (nth-power-of-distance power control scheme and optimum power control scheme) are examined. The increase in capacity by using power control has been studied. The capacities of the forward link are also compared with those of the reverse link with perfect power control. In the case of imperfect power control, an analytical model is presented to study the effect of power control error in a CDMA cellular system. The effect of the dynamic range of transmission, moreover, has also been analyzed.**

***Index Terms*—CDMA cellular systems, power control.**

I. Introduction

CODE-DIVISION multiple access (CDMA) has attracted more attention due to its ever-increasing capacity [1], [2]. Power control is one effective way to avoid the near–far problem and to increase the capacity in CDMA cellular systems. For the forward link, the transmitted power for a mobile at different locations is adjusted according to the power control law. With knowledge on the locations of mobiles, it is possible to minimize the total transmitted power by transmitting high power level for far-end users and low power level for near-in users, and hence reduce the interference level. An nth-power-of-distance power control law has been presented and discussed using a simple power control model [1]. This model based on the distance from the home base station has been further investigated by Gejji in [2]. In [1], analysis has been carried out only for users at the cell boundary with 11 surrounding cells. In [2], the close-in, boundary, and intermediate users located at two fixed directions are studied taking into account four adjacent cells. In this paper, assuming that the locations of the mobiles can be determined by using the radiolocation techniques based on angle-of-arrival or time-of-arrival measurements [3], we develop a general model for users located at any given position within the cell. Our model is not only based on the distance from the home base station, but also depends on the direction from it. Two surrounding tiers of cells are considered and the path loss exponent ranges from two to four. The optimum threshold values are obtained for various power control factors and path loss exponents. The increase in capacity due to the use of power control will be examined. The optimum power control law proposed by Gejji [2] has also been developed as a function of the distance and the direction from the base station to provide a more accurate power control scheme. In order to achieve a uniform quality of service throughout the coverage area, power controls for the forward link and the reverse link are essential. Under perfect power control, the forward-link capacities for nth-power-of-distance power control law and optimum power control law are compared with the reverse-link capacity.

In a practical system, the control of power is not perfect. The power control error is caused by the power measurement error and the measurement delay in the power control process. Finite dynamic range of transmission is another imperfection in power control. In a practical situation, a mobile is not able to change its power to any desired value. The transmitted power is limited by a finite dynamic range. Hence, the performance of a power control algorithm will be affected by the power control error and the dynamic range of transmission.

In [4], simulation schemes have been presented for both links where power control error and finite dynamic range of transmission exist. Analysis on the effect of the power control error has been carried out for the reverse link in [5]. In [6] and [7], considering the global influence of the speed of the adaptive power control system, dynamic range of the transmitter, spatial distribution of users and propagation statistics (such as fading and shadowing), a combined scheme is also given for the reverse link. In this paper, we present a theoretical model to evaluate the forward-link capacity taking into account the power control error. In fact, the model can also be applied to the reverse link. Finally, the effects of the dynamic range of transmission on capacity will be evaluated analytically both for the forward link and reverse link.

The remainder of the paper is organized as follows. Section II presents the interference model. In Section III, we use the model to analyze the forward-link power control schemes. The effects of the power control error and dynamic range of transmission are studied by analysis in Section IV. Next, numerical results are discussed in Section V. Finally, Section VI provides conclusions of the paper.

II. Interference Model

The calculation geometry of the interference model is shown in Fig. 1. The cellular layout consists of two tiers of surrounding cells centrally located base stations. The users are assumed

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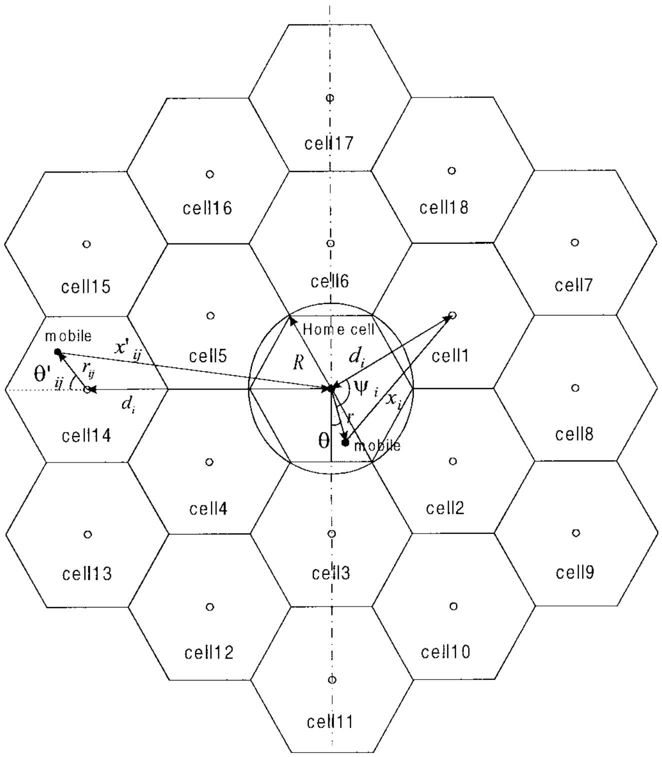


Fig. 1. Forward-link and reverse-link interference geometry.

to be uniformly distributed. By symmetry, only the C/I of users within the shaded triangle needs to be evaluated [8]. The reference axis is vertical as shown in Fig. 1. Then the location of a given mobile in the home cell is (r, θ), where $0 \le r \le y(\theta)$, $y(\theta) = (\sqrt{3}R/2)/\cos\theta$, and $0 \le \theta \le 30°$. The distance between the base stations of the tier-1 cells and the given mobile located at position (r, θ) in the home cell is

$$
\begin{aligned}
x_1 &= \sqrt{d_1^2 + r^2 + 2d_1 r \cos(\theta + 60°)} \\
x_4 &= \sqrt{d_1^2 + r^2 - 2d_1 r \cos(\theta + 60°)} \\
x_2 &= \sqrt{d_1^2 + r^2 + 2d_1 r \cos(\theta + 120°)} \\
x_5 &= \sqrt{d_1^2 + r^2 - 2d_1 r \cos(\theta + 120°)} \\
x_3 &= \sqrt{d_1^2 + r^2 - 2d_1 r \cos\theta} \\
x_6 &= \sqrt{d_1^2 + r^2 + 2d_1 r \cos\theta}
\end{aligned}
$$

where $d_1 = \sqrt{3}R$ is the distance between the home base station and tier-1 cell base stations. The distance between the base stations of the tier-2 cells and the given mobile in the home cell is

$$
\begin{aligned}
x_7 &= \sqrt{d_2^2 + r^2 + 2d_2 r \cos(\theta + 60°)} \\
x_{13} &= \sqrt{d_2^2 + r^2 - 2d_2 r \cos(\theta + 60°)} \\
x_8 &= \sqrt{d_3^2 + r^2 + 2d_3 r \cos(\theta + 90°)} \\
x_{14} &= \sqrt{d_3^2 + r^2 - 2d_3 r \cos(\theta + 90°)} \\
x_9 &= \sqrt{d_2^2 + r^2 + 2d_2 r \cos(\theta + 120°)} \\
x_{15} &= \sqrt{d_2^2 + r^2 - 2d_2 r \cos(\theta + 120°)} \\
x_{10} &= \sqrt{d_3^2 + r^2 + 2d_3 r \cos(\theta + 150°)} \\
x_{16} &= \sqrt{d_3^2 + r^2 - 2d_3 r \cos(\theta + 150°)} \\
x_{11} &= \sqrt{d_2^2 + r^2 - 2d_2 r \cos\theta} \\
x_{17} &= \sqrt{d_2^2 + r^2 + 2d_2 r \cos\theta} \\
x_{12} &= \sqrt{d_3^2 + r^2 - 2d_3 r \cos(\theta + 30°)} \\
x_{18} &= \sqrt{d_3^2 + r^2 + 2d_3 r \cos(\theta + 30°)}
\end{aligned}
$$

where $d_2 = 2\sqrt{3}R$ and $d_3 = 3R$ are the distances between the home base station and tier-2 cell base stations. If the total transmitted power of each base station is P_T, the total power received at the mobile located at (r, θ) will be given by [9]

$$
\begin{aligned}
P_r &= P_T \cdot \left(r^{-m} + \sum_{i=1}^{18} x_i^{-m} \right) \\
&= P_T \cdot r^{-m} \cdot x_I
\end{aligned} \tag{1}
$$

where $x_I = \left(\sum_{i=0}^{18} x_i^{-m}\right) r^m$ is defined as the total interference factor; $x_0 = r$ is the distance between the given mobile and its base station; m is the path loss exponent.

III. Power Control Schemes

A. Forward-Link Power Control

1) nth-Power-of-Distance Power Control Law: A distance-driven power control scheme (nth-power-of-distance power control) has been proposed in [1]. It assumes that continuous power control can be implemented. It is suggested the transmitted power depends on the distance and the power threshold. With knowledge on the location of a mobile, the transmitted power for a mobile can be adjusted according to

$$
Pt_j(r) = F_j \cdot P_R \tag{2}
$$

where

$$
F_j = \begin{cases} \left(\dfrac{r_0}{R}\right)^n, & \text{for } 0 \le r \le r_0 \\ \left(\dfrac{r}{R}\right)^n, & \text{for } r_0 < r \le R \end{cases}
$$

where r_0 is the close-in distance and n is the power control factor; P_R is the power required to reach the users at the cell corner $(R, 30°)$. Then the carrier-to-interference power ratio (C/I) for a mobile located at (r, θ) is given by [9]

$$
C/I = \frac{Pt_j(r) \cdot r^{-m}}{\sum_{i=0}^{18} \left(P_{T_i} \cdot x_i^{-m}\right) - Pt_j(r) \cdot r^{-m}} \tag{3}
$$

where P_{T_i} is the total transmitted power for the ith base station. Although $Pt_j(r)$ is not related to θ, the C/I ratio is a function of r and θ due to the different locations of the mobile. So we should compute C/I for all (r, θ) in the cell to find the minimal value. Given a required $(C/I)_{\min}$, the capacity can be obtained. For analytical convenience, hexagonal cells are approximated by circular cells. Assuming circular cells with radius R, N users are uniformly distributed in each cell with density $\rho = \frac{N}{\pi R^2}$, P_T can be calculated by [2]

$$\begin{aligned} P_T &= \frac{N}{\pi R^2} \int_0^{2\pi} \int_0^R Pt_j \cdot r\,dr\,d\theta \\ &= N P_R \left[\frac{2}{n+2} + \frac{n}{n+2} \left(\frac{r_0}{R} \right)^{n+2} \right]. \end{aligned} \tag{4}$$

Let

$$f(r_0) = \frac{2}{n+2} + \frac{n}{n+2} \left(\frac{r_0}{R} \right)^{n+2}$$

we have

$$P_T = P_R \cdot N \cdot f(r_0). \tag{5}$$

The total transmitted power relates to the power control factor n, the close-in distance r_0, and capacity N. The forward link C/I ratio of a mobile (r, θ) is given by

$$C/I = \frac{Pt_j \cdot r^{-m}}{P_T \cdot x_I \cdot r^{-m} - Pt_j \cdot r^{-m}} = \frac{F_j}{N \cdot f(r_0) \cdot x_I - F_j}. \tag{6}$$

For a required C/I ratio

$$N(r, \theta) = \frac{[(C/I)_{\text{req}} + 1] \cdot F_j}{(C/I)_{\text{req}} \cdot f(r_0) \cdot x_I}. \tag{7}$$

The carrier power is a small part of the received power in CDMA. Neglecting the carrier power in the denominator in (6), we get

$$C/I \approx \frac{F_j}{N \cdot f(r_0) \cdot x_I} \tag{8}$$

or

$$N(r, \theta) \approx \frac{F_j}{(C/I)_{\text{req}} \cdot f(r_0) \cdot x_I}. \tag{9}$$

From the equations above, for a given C/I, N has the same shape as that of the C/I in the case when N is a constant. Since N depends on r, θ, and r_0, $N(r, \theta)$ is different for different locations. To achieve at least the required C/I ratio in all locations, the minimum value of $N(r, \theta)$ will be chosen as the system capacity of the forward link.

2) Optimum Power Control Scheme: To achieve a uniform service, the transmitted power for a mobile located at (r, θ) can be adjusted to have the same shape as the total interference factor $x_I(r, \theta)$. Then Pt_j changes as interference changes with high Pt_j for large interference and low Pt_j for small interference. It is required that Pt_j should be proportional to the interference. If we designate

$$Pt_j(r, \theta) = p(r, \theta) \cdot P_R = \frac{x_I(r, \theta)}{x_I(R, 30^\circ)} \cdot P_R \tag{10}$$

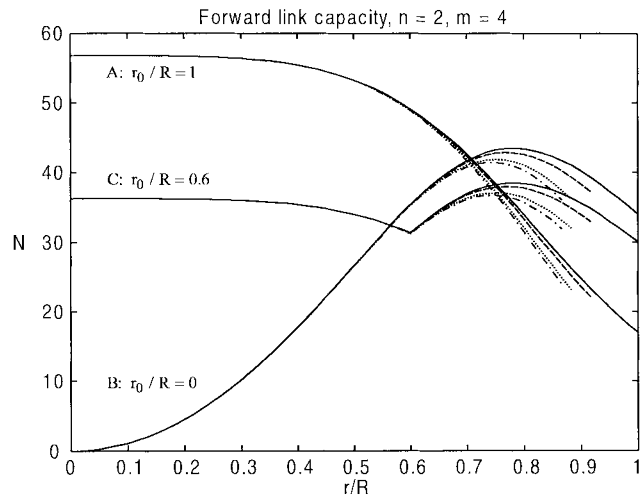


Fig. 2. Forward-link capacity N versus r/R for $n = 2$, $m = 4$, and $\theta = 0^\circ$ (-.-), $\theta = 10^\circ$ (...), $\theta = 20^\circ$ (- - -) , $\theta = 30^\circ$ (__).

where $p(r, \theta)$ is the optimum transmitted power function given by $p(r, \theta) = x_I(r, \theta)/x_I(R, 30^\circ)$, the C/I ratio becomes

$$C/I = \frac{Pt_j(r, \theta) \cdot r^{-m}}{\sum_{i=0}^{18} \left(P_{T_j} \cdot x_I^{-m} \right) - Pt_j(r, \theta) \cdot r^{-m}}. \tag{11}$$

Assuming circular cells, P_T can be evaluated analytically

$$P_T = \frac{N}{\pi R^2} \int_0^{2\pi} \int_0^R Pt_j(r, \theta) r\,dr\,d\theta \tag{12}$$

and the C/I will be

$$\begin{aligned} C/I &\approx \frac{Pt_j \cdot r^{-m}}{P_T \cdot x_I \cdot r^{-m}} \\ &= \frac{p(r, \theta)}{\left[N \cdot \frac{12}{\pi R^2} \int_0^{30^\circ} \int_0^R p(r, \theta) r\,dr\,d\theta \right] \cdot x_I(r, \theta)} \\ &= \frac{1}{N \cdot \frac{12}{\pi R^2} \int_0^{30^\circ} \int_0^R x_I(r, \theta) r\,dr\,d\theta}. \end{aligned} \tag{13}$$

For a required C/I ratio, the capacity can be written as

$$N = \frac{1}{(C/I)_{\text{req}} \cdot \frac{12}{\pi R^2} \int_0^{30^\circ} \int_0^R x_I(r, \theta) r\,dr\,d\theta} \tag{14}$$

Evaluating (14) numerically, we can obtain the approximate expressions for $m = 2$

$$N = \frac{0.3424}{(C/I)_{\text{req}}}$$

for $m = 3$

$$N = \frac{0.4921}{(C/I)_{\text{req}}}$$

for $m = 4$

$$N = \frac{0.5794}{(C/I)_{\text{req}}}.$$

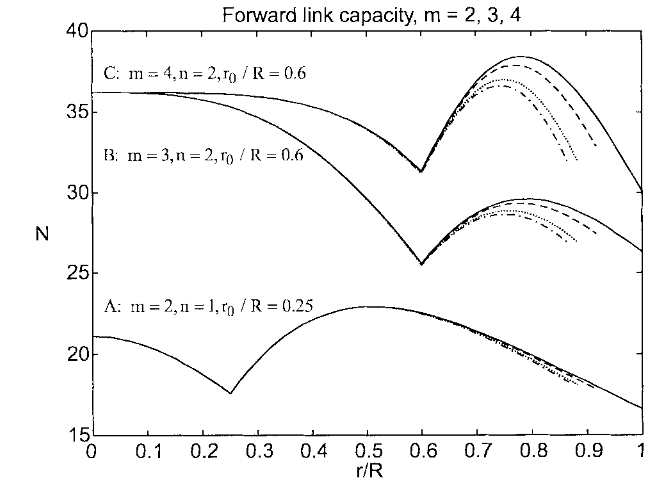


Fig. 3. Forward-link capacity N versus r/R for $\theta = 0°$ (-.-), $\theta = 10°$ (...), $\theta = 20°$ (- - -), $\theta = 30°$ (__).

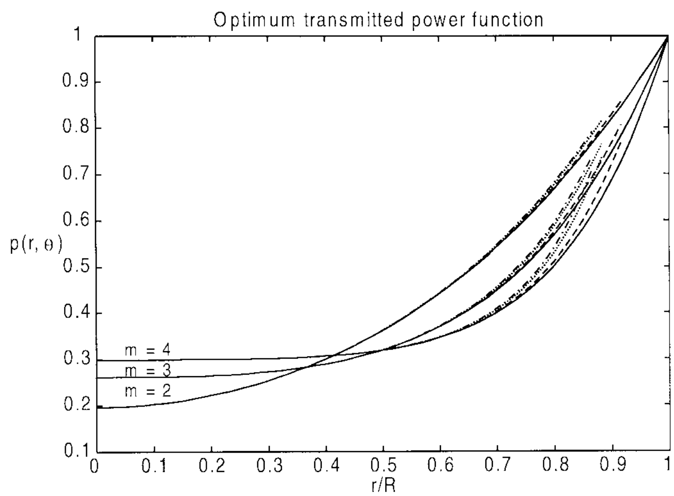


Fig. 4. Optimum transmitted power function $p(r, \theta)$ versus r/R as a function of path loss exponent m for $\theta = 0°$ (-.-), $\theta = 10°$ (...), $\theta = 20°$ (- - -), $\theta = 30°$ (__).

For a given required C/I ratio, N is a constant for users at any location. If N is fixed, C/I is a constant. The system can then provide uniform service for all users within the cell.

B. Reverse-Link Power Control

The interference model for the reverse link is similar to [10], which is also shown in Fig. 1. Assuming the received power at the base station is the same for all users with power control, the C/I ratio is

$$C/I = \frac{P_c}{(N-1)\cdot P_c + \sum_{i=1}^{18}\sum_{j=1}^{N}\left[\left(\frac{r_{ij}}{x'_{ij}}\right)^m \cdot P_c\right]} \tag{15}$$

where P_c is the power received at the base station and N is the number of users uniformly distributed in each cell. x'_{ij} represents the distance between the jth mobile in cell i and the base station of the home cell, which is given by

$$x'_{ij} = \sqrt{d_i^2 + r_{ij}^2 + 2d_i r_{ij}\cos\theta'_{ij}}.$$

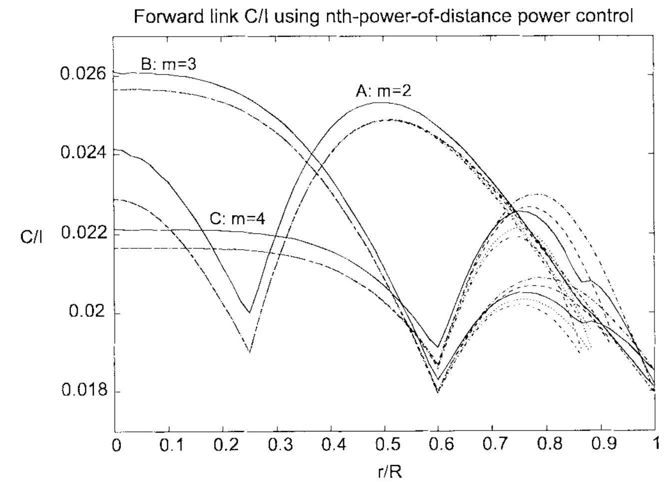


Fig. 5. The calculated C/I versus r/R for $\theta = 0°$ (-.-), $\theta = 10°$(...), $\theta = 20°$ (- - -), $\theta = 30°$ (- -.) and the mean C/I (solid line) conducted by simulations. Case *A*: $m = 2$, $n = 1$, and $r_0/R = 0.25$ (in the analysis $N = 16.63$ and in the simulation $N = 18$). Case *B*: $m = 3$, $n = 2$, and $r_0/R = 0.60$ (in the analysis $N = 25.48$ and in the simulation $N = 29$). Case *C*: $m = 4$, $n = 2$, and $r_0/R = 0.60$ (in the analysis $N = 30.08$ and in the simulation $N = 34$).

$d_i = a_i R$ is the distance between the ith base station of surrounding cells and the home base station, which is the same as those used in the forward-link analysis. r_{ij} represents the distance between the jth mobile and its base station in the ith cell. The received power at the home base station due to mobiles in the ith surrounding cell can also be calculated by analysis, which is given by

$$(Pr_c)_i = \int_0^{2\pi}\int_0^R P_c\left(\frac{r}{\sqrt{d_i^2 + r^2 + 2d_i r\cos\theta'}}\right)^m \times \frac{N}{\pi R^2} r\, dr\, d\theta'. \tag{16}$$

The C/I ratio becomes

$$C/I = \frac{P_c}{(N-1)P_c + \sum_{i=1}^{18}(Pr_c)_i} = \frac{1}{(N-1) + N\cdot I_{\text{out}}} \tag{17}$$

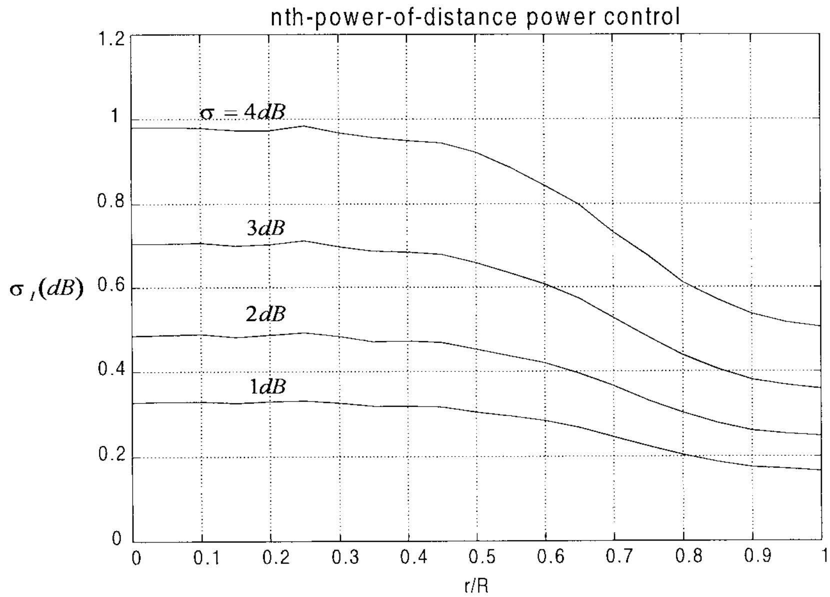
where

$$I_{\text{out}} = \frac{\sum_{i=1}^{18}(Pr_c)_i}{N\cdot P_c}.$$

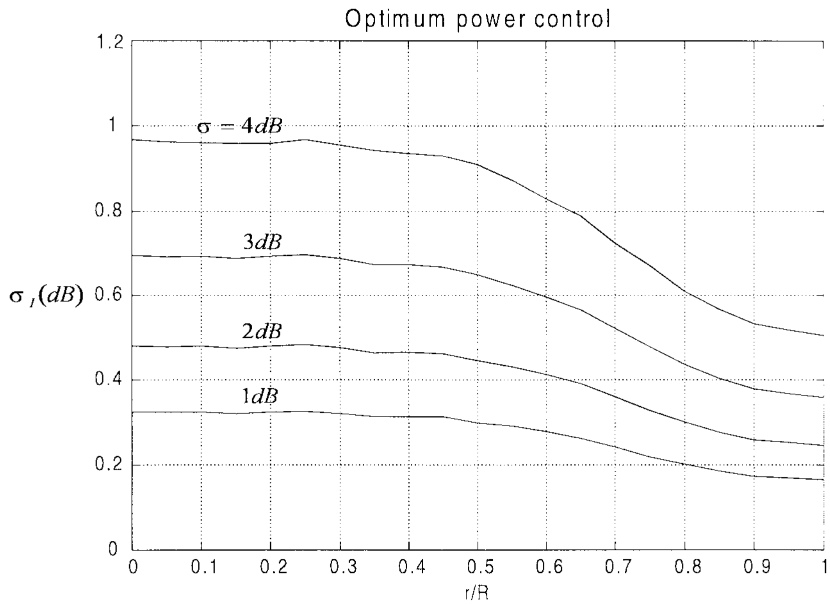
We can find that the C/I ratio is a constant for users located at different locations within the same cell. For a required C/I ratio, N has to satisfy

$$N = \frac{(C/I)_{\text{req}} + 1}{(C/I)_{\text{req}}\cdot(1 + I_{\text{out}})}. \tag{18}$$

Since I_{out} is a constant, N is also a constant for users at different locations within the cell.



(a)



(b)

Fig. 6. (a) σ_I versus r/R for the nth-power-of-distance power control scheme. (b) σ_I versus r/R for the optimum power control scheme.

IV. Imperfect Power Control

A. Effect of Power Control Error

When power control is not ideal, the error is assumed to be log-normally distributed with standard deviation of σ (in decibels). The effect of the power control error can be studied by multiplying the transmitted power by a log-normal random variable [4], [11]. The received power for a mobile is rewritten as

$$Pr = Pt \cdot r^{-m} \cdot 10^{\frac{\gamma}{10}} \tag{19}$$

where Pt is the transmitter power; r is the distance between the transmitter and receiver; γ is a zero-mean Gaussian random variable with a standard deviation of σ, which denotes the power control error. When $\sigma = 0$ dB, the case is corresponding to perfect power control. If power control is not perfect, σ is assumed to be 1–4 dB. The probability density function (pdf) of the received power is expressed as

$$f(Pr) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ \frac{-[10\log_{10}(Pr) - 10\log_{10}(\overline{Pr})]^2}{2\sigma^2} \right\} \tag{20}$$

and the mean received power for a mobile is given by

$$\overline{Pr} = Pt \cdot r^{-m}. \tag{21}$$

In the following analysis, the path loss exponent m is assumed to be 4.

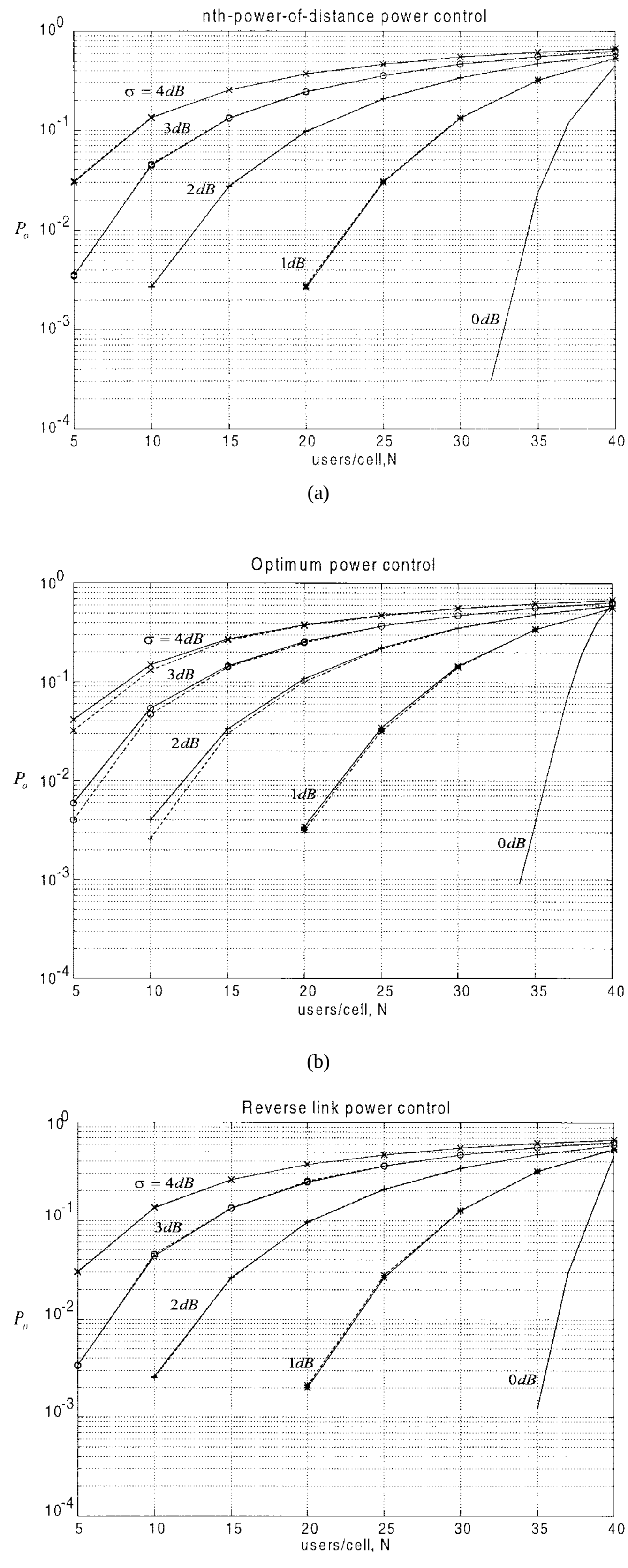


Fig. 7. (a) Outage probability versus capacity for the nth-power-of-distance power control scheme. (b) Outage probability versus capacity for the optimum power control scheme. (c) Outage probability versus capacity for the reverse-link power control scheme. (___ Analytical results. - - - - - Simulated results).

1) Forward-Link C/I Ratio: In the presence of power control error, the total interference power received for the jth mobile within the home cell is

$$I_j = \sum_{i=0}^{18} \left[\sum_{k=1}^{N} \left(Pt_{ik} \cdot 10^{\frac{\gamma_{ik}}{10}} \right) \cdot x_{ij}^{-4} \right] - Pt_{0j} \cdot x_{0j}^{-4} \cdot 10^{\frac{\gamma_{0j}}{10}} \tag{22}$$

where x_{ij} is the distance between the ith base station and the jth mobile in the home cell; Pt_{ik} is the transmitted power for the kth mobile inside the ith cell under perfect power control. γ_{ik} represents the power control error for the kth mobile inside the ith cell. The C/I ratio for the jth mobile in the home cell will be

$$C/I = \frac{Pt_{0j} \cdot x_{0j}^{-4} \cdot 10^{\frac{\gamma_{0j}}{10}}}{I_j}. \tag{23}$$

2) Reverse-Link C/I Ratio: In the reverse link, the total interference power received at the base station for the jth mobile within home cell becomes

$$I_j = \sum_{i=0}^{18} \sum_{k=1}^{N} \left[P_c \cdot \left(\frac{r_{ik}}{x'_{ik}} \right)^4 \cdot 10^{\frac{\gamma_{ik}}{10}} \right] - P_c \cdot 10^{\frac{\gamma_{0j}}{10}} \tag{24}$$

where x'_{ik} is the distance between the kth mobile in the ith cell and the home base station; r_{ik} is the distance between the kth mobile in cell i and its base station. γ_{ik} represents the power control error for the kth mobile in the ith cell. The C/I ratio for the desired mobile is given by

$$C/I = \frac{P_c \cdot 10^{\frac{\gamma_{0j}}{10}}}{I_j}. \tag{25}$$

3) Outage Probability Analysis: In mobile cellular systems, outage probability is generally used to evaluate the capacity. It is defined as the failing probability to achieve a required C/I ratio. It is well known that

$$\Pr\left[C/I < (C/I)_{\text{req}}\right] = \frac{1}{2} + \frac{1}{2}\text{erf}\left[\frac{(C/I)_{\text{req}} - m_{CI}}{\sqrt{2}\sigma_{CI}}\right] \tag{26}$$

where

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt.$$

The mean C/I (in decibels) is [12]

$$\begin{aligned} m_{CI} &= 10\log_{10}(Pr) - 10\log_{10}(I) \\ &= 10\log_{10}(\overline{Pr}/\bar{I}) - \frac{\ln 10}{20}\left(\sigma^2 - \sigma_I^2\right). \end{aligned} \tag{27}$$

$\overline{Pr}/\bar{I}$ is the mean-carrier-power to mean-interference-power ratio. It is equivalent to the C/I ratio when power control is perfect. The total interference power I is the sum of a finite number of log-normal random variables, the mean and standard deviation of which are $\overline{Pr}$ and σ, respectively. Referring to [12], the total interference power is also approximately log-normal with standard deviation of σ_I. Since Pr and I depend on the location of the mobile in the forward link, σ_I is a function of r and θ. Therefore, σ_{CI} and m_{CI} change as the

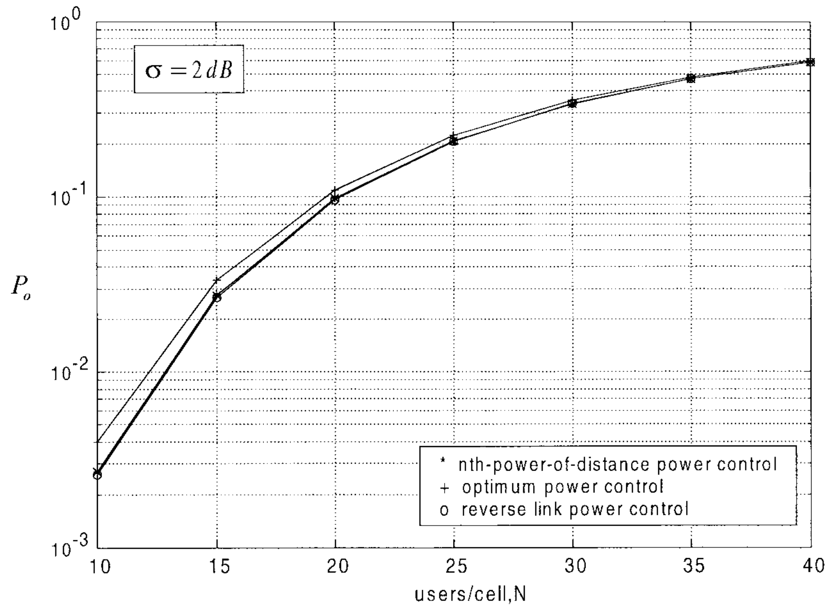


Fig. 8. Comparison of outage probability for the three power control schemes.

position of the mobile changes. The variance of C/I is derived by

$$\sigma_{CI}^2 = \sigma^2 + \sigma_I^2. \tag{28}$$

Then the outage probability is obtained by

$$P_o = \int_\theta \int_r \Pr\left[C/I < (C/I)_{\text{req}}\right] \cdot f(r,\theta) r\, dr\, d\theta \tag{29}$$

where $f(r,\theta)$ stands for the pdf for the location of a desired mobile. The double integral denotes one hexagonal cell coverage area. P_o can be simulated by first generating a large set of points uniformly distributed in the cell. For each position, simulation is repeated to check if the C/I falls below the required value. Finally, the outage probability is derived from the outage probabilities for different locations using the equation

$$P_o = \lim_{K\to\infty} \frac{1}{K} \sum_{k=1}^{K} \{\Pr\left[C/I < (C/I)_{\text{req}}\right]\}_k. \tag{30}$$

B. Effect of Dynamic Range of Transmission

According to the above power control schemes, the transmitted power in a CDMA system can be adjusted to any desired value. However, in practice, the transmitter power is limited by both lower bound and upper bound. Hence, power control may not be fully implemented. If the transmitter power is restricted by a dynamic range D, the transmitted power may be recast into

$$P_d = \begin{cases} Pt_{\max}/D, & Pt < Pt_{\max}/D \\ Pt, & Pt \geq Pt_{\max}/D \end{cases} \tag{31}$$

when Pt is the transmitted power for a mobile without power limitation. If $Pt_{\min} \geq Pt_{\max}/D$, perfect power control can be implemented. When $Pt_{\min} < Pt_{\max}/D$, the C/I ratio for the forward link and reverse link can be evaluated as follows.

1) nth-Power-of-Distance Power Control: When $Pt_{\min} < Pt_{\max}/D$, the total transmitted power at the base station can be calculated by

$$\begin{aligned} P_T &= \frac{N}{\pi R^2} \int_0^{2\pi} \int_0^R P_d \cdot r\, dr\, d\theta \\ &= \frac{2N}{R^2} \left(\int_0^{r_c} \frac{P_R}{D} \cdot r\, dr + \int_{r_c}^R Pt \cdot r\, dr \right) \\ &= NP_R \left[\frac{2}{n+2} + \frac{n}{n+2} D^{-\left(\frac{n+2}{n}\right)} \right] \end{aligned} \tag{32}$$

where r_c is the distance corresponding to the transmitted power $Pt_{\max}/D = (r_c/R)^n P_R$. $Pt_{\max} = P_R$ is the maximum transmitted power for a mobile located at distance R. The C/I ratio is

$$C/I = \frac{P_d \cdot r^{-4}}{P_T \cdot \sum_{i=0}^{18} x_i^{-4} - P_d \cdot r^{-4}}. \tag{33}$$

2) Optimum Power Control: If optimum power control scheme is adopted, the total transmitted power at the base station is

$$\begin{aligned} P_T &= \frac{N}{\pi R^2} \int_0^{2\pi} \int_0^R P_d \cdot r\, dr\, d\theta \\ &= \frac{N}{\pi R^2} \left[\int_0^{2\pi} \int_0^{r_c(\theta)} \frac{P_R}{D} \cdot r\, dr\, d\theta \right. \\ &\qquad \left. + \int_0^{2\pi} \int_{r_c(\theta)}^R Pt \cdot r\, dr\, d\theta \right] \end{aligned} \tag{34}$$

where $r_c(\theta)$ is the distance corresponding to the transmitted power

$$\frac{P_R}{D} = \frac{x_I[r_c(\theta), \theta]}{x_I(R, \pi/6)} P_R.$$

TABLE I
THE NORMALIZED TOTAL TRANSMITTED POWER $P_T/(NP_R)$ AS A FUNCTION OF DYNAMIC RANGE OF TRANSMISSION D

Dynamic range of transmission	$P_T/(NP_R)$
$D = 2$ dB	0.7020
$D = 3$ dB	0.6152
$D = 4$ dB	0.5593
$D = 5$ dB	0.5171

When θ changes, r_c changes to achieve the required transmitted power. Solving this integral numerically, we get $P_T/(NP_R)$ as shown in Table I. Then the C/I ratio for the mobile is given by

$$C/I = \frac{P_d}{P_T \cdot x_I - P_d}. \tag{35}$$

3) Reverse-Link Power Control: The total in-cell received power is

$$\begin{aligned} Pr_{\text{in}} &= \frac{N}{\pi R^2}\int_0^{2\pi}\int_0^R Pr \cdot r\,dr\,d\theta' \\ &\approx \frac{2N}{R^2}\left(\int_{r_f}^{r_c} \frac{P_c \cdot R^4}{D}\cdot r^{-4}\cdot r\,dr + \int_{r_c}^{R} P_c \cdot r\,dr\right) \\ &\approx \frac{NP_c}{D}\left[\left(\frac{R}{r_f}\right)^2 - \sqrt{D}\right] + NP_c\left(1-\frac{1}{\sqrt{D}}\right) \end{aligned} \tag{36}$$

where r_c is the distance corresponding to the transmitted power $(P_c \cdot R^4)/D = r_c^4 P_c$. Increasing the transmitted power for near-in mobiles to the tolerable minimum value leads to an increase in the total received power. This will result in excessive interference to the far-end mobiles. If mobiles can be very close to the base station, the integral from 0 to r_c will tend to infinity. Therefore, a small circular forbidden zone is assumed to be around the base station with radius r_f [13]. The received power from the ith surrounding cell is given by

$$\begin{aligned} (Pr_{\text{out}})_i &\approx \frac{N}{\pi R^2}\left[\int_0^{2\pi}\int_{r_f}^{r_c} \frac{P_c \cdot R^4}{D}\cdot x_i'^{-4}\cdot r\,dr\,d\theta' \right. \\ &\qquad \left. + \int_0^{2\pi}\int_{r_c}^{R} P_c \cdot \left(\frac{r}{x_i'}\right)^4 \cdot r\,dr\,d\theta'\right] \\ &\approx \frac{NP_c}{D}\frac{\left[\sqrt{D}\left(\frac{r_f}{R}\right)^2 - 1\right]\left[\left(\frac{r_f}{R}\right)^2 - a_i^4\sqrt{D}\right]}{\left(a_i^2\sqrt{D}-1\right)^2\left[a_i^2 - \left(\frac{r_f}{R}\right)^2\right]^2} \\ &\quad + NP_c\left[4a_i^2 \ln\left(\frac{a_i^2\sqrt{D}-1}{a_i^2\sqrt{D}-\sqrt{D}}\right) - \frac{4a_i^6 - 5a_i^4}{\left(a_i^2-1\right)^2}\right. \\ &\qquad \left. + \frac{4a_i^6 D - 5a_i^4\sqrt{D}}{\left(a_i^2\sqrt{D}-1\right)^2} - 1 + \frac{1}{\sqrt{D}}\right] \end{aligned} \tag{37}$$

where x_i' is the distance between the interfering mobile and the home base station. The C/I ratio at the base station for the desired mobile is

$$C/I = \frac{Pr(r)}{Pr_{\text{in}} + \sum_{i=1}^{18}(Pr_{\text{out}})_i - Pr(r)}. \tag{38}$$

V. RESULTS AND DISCUSSIONS

A. Effect of Perfect Power Control

In our analysis and simulations, two surrounding tiers of cells are considered and $(C/I)_{\text{req}} = 0.01792$ (-17.47 dB). Equation (7) has been plotted in Fig. 2 for $r_0/R = 0$, 0.6, 1.0, and $\theta = 0°$, 10°, 20°, 30°. Since the cell has hexagonal configuration, r must satisfy $0 \le r \le y(\theta)$. When $\theta = 0°$, r can reach $\sqrt{3}R/2$. Case *A* $(r_0/R = 1.0)$ corresponds to the case when no power control is applied. Case *B* $(r_0/R = 0)$ corresponds to the power control scheme without threshold adjustment. Case *C* $(r_0/R = 0.6)$ corresponds to the power control scheme with power threshold adjustment. In case *A*, capacity N decreases as r/R increases because equal power is transmitted to users at any location within each cell. The signal received by the far-end users becomes weak. To maintain the required C/I ratio, the number of users that can be supported will be decreased. In case *B*, capacity N increases as r/R increases due to the reduction of the transmitted power for the near-in users. Case *A* only benefits the close-in users and case *B* only benefits the far-end users. Both cases are not acceptable for system design. Adjusting the transmitted power for close-in users, we obtain curve *C*, which is the best case for path loss exponent $m = 4$. The minimum value of $N(r,\theta)$ is 30.08 users/cell at $r/R = 1.0$ and $\theta = 30°$, which will be chosen as the system capacity. When $r/R > 0.6$, the capacity increases to a maximum value and then decreases because the far-end users are more sensitive to the adjacent cell interference. From curves *A*, *B*, and *C*, capacity N is different for $\theta = 0°$, 10°, 20°, 30°, indicating that the capacity is also impacted by the direction of the users. Fig. 3 plots the capacity N for different path loss exponents $m = 2$, 3, 4, and $\theta = 0°$, 10°, 20°, 30°. Curves *A*, *B*, and *C* are the cases where the largest minimum capacity for each m is obtained. It can be observed that the capacity for $m = 4$ is obviously larger than that for $m = 2$ and $m = 3$. This is due to the smaller impact of the out-of-cell interference when the path loss becomes larger.

Table II gives values of the normalized maximum capacity (compared with no power control applied when $n = 0$) as a function of r, θ, and r_0 with power control factors $n = 0$, 1, 2, 3, 4, and 5. For a fixed r_0/R, the minimum value $N_{\min}$ of $N(r,\theta)$ is found within the area of the triangle. When r_0/R ranges from 0 to 1, the maximum value of $N_{\min}$ can be obtained for different m and n. The corresponding $(r/R, \theta)$ and r_0/R are shown in the table to achieve this maximum $N_{\min}$. Consider the case $m = 2$ and $n = 1$, $r_0/R = 0.25$ is the best choice to achieve the largest minimum 1.49 times capacity occurring at $(r/R, \theta) = (1, 30°)$. In cases $m = 3$ and 4, it is observed that the maximum capacity is obtained with a power control factor $n = 2$. The capacity N for $m = 2$, 3, and 4 without power control are 11.18,

TABLE II
THE MAXIMUM VALUES OF THE NORMALIZED CAPACITY $\frac{N_{\max}(r_0/R)|_{n=0-5}}{N_{\max}(r_0/R)|_{n=0}}$ AND THE CORRESPONDING $(r/R, \theta)$ AND r_0/R FOR DIFFERENT POWER CONTROL FACTOR n AND PATH LOSS EXPONENT m

path loss exponent		power control factor					
m		$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
m=2	Normalized N	1.0000	1.4884	1.4557	1.2761	1.1855	1.1336
	$(r/R, \theta)$	any	$(1.00, 30^\circ)$	$(0.65, 0^\circ)$	$(0.75, 0^\circ)$	$(0.80, 0^\circ)$	$(0.85, 0^\circ)$
	r_0/R	any	0.25	0.65	0.75	0.80	0.85
m=3	Normalized N	1.0000	1.4800	1.7154	1.5056	1.3551	1.2648
	$(r/R, \theta)$	any	$(1.00, 30^\circ)$	$(0.60, 0^\circ)$	$(0.75, 0^\circ)$	$(0.80, 0^\circ)$	$(0.80, 0^\circ)$
	r_0/R	any	0.30	0.60	0.75	0.80	0.80
m=4	Normalized N	1.0000	1.4739	1.7705	1.6831	1.5148	1.3913
	$(r/R, \theta)$	any	$(0.30, 0^\circ)$	$(1.00, 30^\circ)$	$(0.75, 0^\circ)$	$(0.75, 0^\circ)$	$(0.80, 0^\circ)$
	r_0/R	any	0.30	0.60	0.75	0.75	0.80

14.85, and 16.99, respectively. For $m = 4$, the maximum capacity $N_{\max} = 30.08$ users/cell is achieved with $n = 2$, and $r_0/R = 0.6$. The results reported in [1] is $N_{\max} = 31$ and in [2] is $N < 32$. So our results are quite consistent. In Fig. 4, optimum transmitted power function $p(r, \theta)$ is plotted for $\theta = 0^\circ$, 10°, 20°, 30°. When $m = 4$, $p(r, 30^\circ)$ increases from $0.30P_R$ to P_R. When $m = 2$, the transmitted power starts at $0.20P_R$. As r/R increases, the impact of θ becomes larger. For $m = 4$, at $r/R = \sqrt{3}/2$, the transmitted power is $0.71P_R$ for $\theta = 0^\circ$ and $0.61P_R$ for $\theta = 30^\circ$. The transmitted power for far-end users is sensitive to θ. So θ cannot be neglected.

Fig. 5 plots the C/I ratio calculated by analysis for $\theta = 0^\circ$, 10°, 20°, 30°, and the mean C/I conducted by simulation as a function of r/R for different path-loss exponents $m = 2$, 3, and 4. In the simulations, the mean C/I is the average value of the users at different direction for the same r. Curves *A*, *B*, and *C* are the cases where the largest capacity for each m is obtained. $n = 1$ and $r_0/R = 0.25$ is the best choice for $m = 2$ that can provide 1.49 times capacity compared with no power control applied when $n = 0$. When $n = 2$, $r_0/R = 0.6$, the maximum capacity can be achieved for $m = 3$ and $m = 4$. For a similar C/I as a function of r, in case *C*, capacity $N = 30.08$ for analysis and $N = 34$ for simulation. The calculated capacity is smaller than the simulated capacity since the service area is assumed to be covered by hexagonal cells in the simulations.

In Table III, the forward-link capacities for the two power control schemes are compared with the reverse-link capacity with power control. Adopting the optimum power control scheme, it can be found that the capacity of the forward link is slightly smaller than that of the reverse link, but they are very close. When $m = 4$, employing the nth-power-of-distance power-control scheme can achieve about 93% of the optimum capacity and the percentages for the cases when $m = 2$ and $m = 3$ are 87% and 93%, respectively.

For both links, capacity analysis using circular cell configuration has been derived. Using hexagonal cell layout, the calculation of the capacity and C/I become complicated. Therefore, for these two forward-link power control schemes, we use our model to evaluate the C/I through computer simulations. For the outage probability P_o(the probability that $C/I < 0.01792) < 0.1$, the simulated results of forward-link capacity and reverse-link capacity are shown in Table IV. The capacity in all cases is larger than the calculated capacity shown in Table III due to the different cell configuration.

B. Effect of Power Control Error

σ_I in (27) and (28) is the standard deviation of the total interference for a mobile, which is conducted through computer simulations. For a given mobile located at r/R ranging from 0 to 1, mobiles are generated in 19 cells and the sum of the total interference is calculated. This calculation is repeated many times to evaluate σ_I. For the nth-power-of-distance power control, $n = 2$, $r_0/R = 0.6$ are used. Fig. 6(a) and (b) shows σ_I against r/R for the nth-power-of-distance power control and optimum power control when $N = 30$ users/cell and $\theta = \pi/6$. It can be seen that σ_I decreases as r/R increases since the close-in users are more sensitive to in-cell interference. Any change in the total transmitted power of the home cell will affect the total interference for a close-in user significantly. When the distance from the base station becomes large, this effect becomes small. It is also noted that σ_I is higher for a larger σ.

Assuming hexagonal cell configuration, the effect of the imperfect power control is depicted in Fig. 7(a)–(c) for the forward and reverse links. Both analytical and simulated results are shown in the figure. In the situation with perfect power control, $\sigma = 0$ dB. For the same capacity, the outage probability increases with an increase in σ. When $N = 30$ users/cell and using nth-power-of-distance power control, $P_o = 0.13$ for $\sigma = 1$ dB, $P_o = 0.46$ for $\sigma = 3$ dB, and $P_o = 0.55$ for $\sigma = 4$ dB. Obviously, to achieve a required outage probability, the capacity will be reduced due to the nonideal power control. When employing nth-power-of-distance power control or reverse-link power control, good agreement has been observed between the analytical results

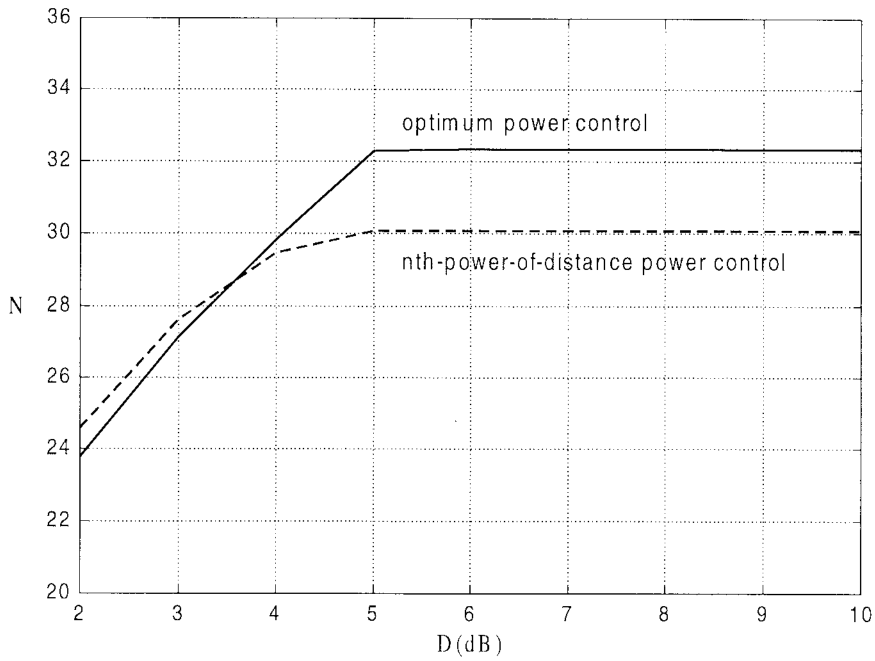


Fig. 9. Forward-link capacity N versus dynamic range of transmission D.

TABLE III
THE CALCULATED FORWARD-LINK CAPACITY AND REVERSE-LINK CAPACITY WITH PERFECT POWER CONTROL (CIRCULAR CELL CONFIGURATION)

	path loss exponent m	$m = 2$	$m = 3$	$m = 4$
Forward link capacity	nth-power-of-distance power control law	16.63	25.48	30.08
	optimum distance and direction dependent power control law	19.11	27.46	32.33
Reverse link capacity	power control	19.43	27.87	32.53

TABLE IV
THE SIMULATED FORWARD-LINK CAPACITY AND REVERSE-LINK CAPACITY WITH PERFECT POWER CONTROL (HEXAGONAL CELL CONFIGURATION)

	path loss exponent m	m = 2	m = 3	m = 4
Forward link capacity	nth-power-of-distance power control law	19	30	36
	optimum power control law	20	30	37
Reverse link capacity	power control	21	31	38

and the simulated results for each σ. In the case of optimum power control, the difference between the analytical result and the simulated result becomes larger as capacity decreases. This is because $\overline{Pr}/\bar{I}$ in (27) is evaluated by simulation using hexagonal cells. According to [9], Pt_j is adjusted to have the same shape as $x_I(r,\theta)$, not $\left[\sum_{i=0}^{18}(Pt_i \cdot x_i^{-m})\right]/r^{-m}$. So only the approximated result is given in (27). When capacity decreases, $\overline{Pr}/\bar{I}$ deviates from its ideal value. If circular cells are assumed and $\overline{Pr}/\bar{I}$ is calculated using (13), it is found that the theoretical results are quite consistent with the simulated results (not shown here). This implies that the difference in Fig. 7(b) comes from the approximate value of $\overline{Pr}/\bar{I}$, not the analytical model of the outage probability.

When $\sigma = 2$ dB, the analytical outage probability is plotted in Fig. 8 for the three power control schemes. Note that P_o for the nth-power-of-distance power control is very close to that for the reverse-link power control, and P_o for the optimum power control is slightly larger than the other two power control schemes. When N increases, the difference of these three power control schemes becomes smaller.

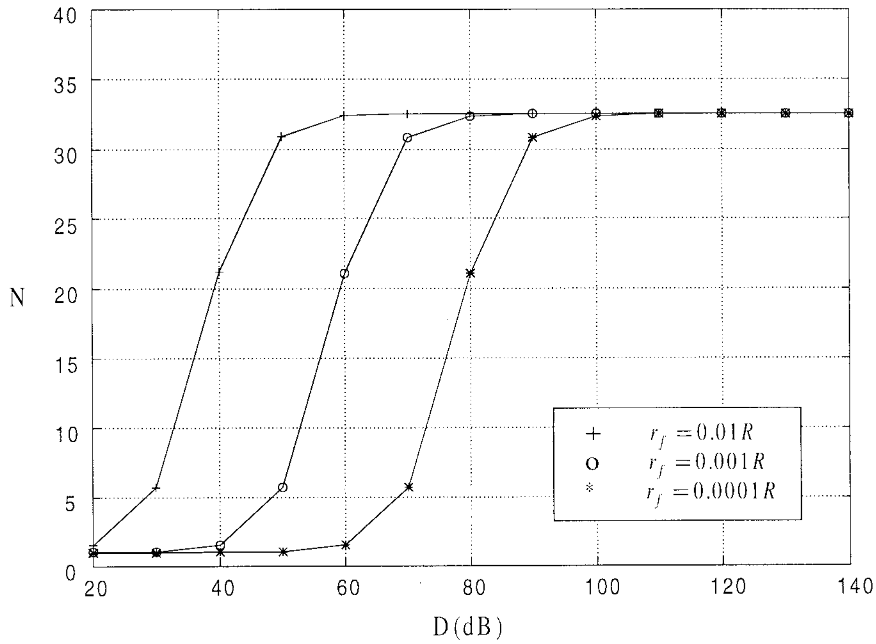


Fig. 10. Reverse-link capacity N versus dynamic range of transmission D.

C. Effect of Dynamic Range of Transmission

In our study of the effect of dynamic range of transmission, we compute the capacity by analysis using circular cell layout. Assuming the dynamic range of the transmitter power is limited to D, Fig. 9 shows the forward-link capacity against D. Results indicate that the required dynamic ranges of transmission for the nth-power-of-distance power control and optimum power control are 5 and 6 dB, respectively. When D is less than the required value, capacity decreases due to the excessive interference. Fig. 10 shows the reverse-link capacity loss due to the limited dynamic range of transmitter power. When $r_f = 0.01R$, $0.001R$, and $0.0001R$, the required D are 80, 100, and 120 dB, respectively. A decrease in D will result in dramatic reduction in capacity.

VI. Conclusion

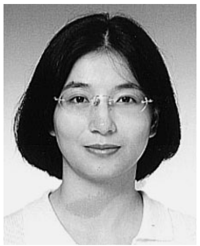
In this paper, the forward-link capacity of a CDMA cellular system has been analyzed with two power control schemes: nth-power-of-distance and optimum power control. The C/I of a user located anywhere within the cell has been derived and the capacity is evaluated given a required C/I. The maximum capacities are found for different path loss factors and the optimum power control outperforms the nth-power-of-distance scheme in all cases. The reverse-link capacity has also been investigated based on the perfect power control scheme. The reverse-link capacity is found to be slightly larger than that of the forward link with optimum power control for the same path loss exponent. The figure increases with the path loss exponent due to the smaller out-of-cell interference.

Also, a methodology has been presented to analyze the outage probability of the forward and reverse links in a CDMA cellular system with perfect and imperfect power control schemes. Simulations have been performed to verify the analytical results. In all cases, the outage probability is found to be very sensitive to the power control error. For a higher capacity, the difference in the outage probability for the three power control schemes becomes smaller. The capacity loss due to the limited dynamic range of transmission has also been examined. It is found that the required dynamic ranges of transmission for the nth-power-of-distance power control and optimum power control are 5 and 6 dB, respectively.

References

[1] W. C. Y. Lee, "Overview of cellular CDMA," *IEEE Trans. Veh. Technol.*, vol. 40, pp. 291–302, May 1991.
[2] R. R. Gejji, "Forward-link-power control in CDMA cellular systems," *IEEE Trans. Veh. Technol.*, vol. 41, pp. 532–536, Nov. 1992.
[3] J. Caffery Jr. and G. L. Stüber, "Overview of radiolocation in CDMA cellular systems," *IEEE Commun. Mag.*, vol 36, pp. 38–45, Apr. 1998.
[4] E. Kudoh, "On the capacity of DS/CDMA cellular mobile radios under imperfect transmitter power control," *IEICE Trans Commun.*, vol. E76-B, pp. 886–893. Aug. 1993.
[5] P. Newson and M. R. Heath, "The capacity of a spread spectrum CDMA system for cellular mobile radio with consideration of system imperfections," *IEEE J. Select. Areas Commun.*, vol. 12, pp. 673–684, May 1994.
[6] R. Prasad, M. G. Jansen, and A. Kegel, "Capacity analysis of a cellular direct sequence code division multiple access system with imperfect power control," *IEICE Trans. Commun.*, vol. E76-B, pp. 894–904, Aug. 1993.
[7] M. G. Jansen and R. Parasad, "Capacity, throughput, and delay analysis of a cellular DS CDMA system with imperfect power control and imperfect sectorization," *IEEE Trans. Veh. Technol.*, vol. 44, pp. 67–74, Feb. 1995.
[8] K. S. Gilhousen, I. M. Jacobs, R. Padovani, A. J. Viterbi, L. A. Weaver Jr., and E. Wheatley, "On the capacity of a cellular CDMA system," *IEEE Trans. Veh. Technol.*, vol. 40, pp. 303–312, May 1991.
[9] W. M. Tam and F. C. M. Lau, "Effect of power control on the forward link capacity of a CDMA mobile cellular system," in *Proc. IEEE ICCS'96*, 1996, pp. 1235–1238.
[10] F. C. M. Lau and W. M. Tam, "Effects of increased capacity in CDMA cellular system," in *Proc. ISSSE'95*, 1995, pp. 579–582.
[11] W. M. Tam and F. C. M. Lau, "Analysis of imperfect power control in CDMA cellular systems," in *Proc. PIMRC'97*, 1997, pp. 892–897.
[12] D. C. Cox, "Cochannel interference considerations in frequency reuse small-coverage-area radio system," *IEEE Trans. Commun.*, vol. COM-32, pp. 135–141, Jan. 1982.

[13] T. S. Rappaport and L. B. Milstein, "Effects of radio propagation path loss on DS-CDMA cellular frequency reuse efficiency for the reverse channel," *IEEE Trans Veh. Technol.*, vol. 41, pp. 231–241, Aug. 1992.



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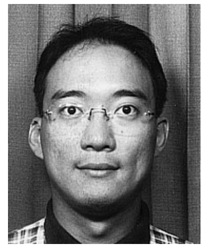

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