Performance of Optimum Transmitter Power Control in Cellular Radio Systems

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***Abstract*—Most cellular radio systems provide for the use of transmitter power control to reduce cochannel interference for a given channel allocation. Efficient interference management aims at achieving acceptable carrier-to-interference (C/I) ratios in all active communication links in the system. In this paper, such schemes for the control of cochannel interference are investigated. The effect of adjacent channel interference is neglected. As a performance measure, the interference (outage) probability is used, i.e., the probability that a randomly chosen link is subject to excessive interference. In order to derive upper performance bounds for transmitter power control schemes, algorithms are suggested that are optimum in the sense that the interference probability is minimized. Numerical results indicate that these upper bounds exceed the performance of conventional systems by an order of magnitude regarding interference suppression and by a factor of 3 to 4 regarding the system capacity. The structure of the optimum algorithm shows that efficient power control and dynamic channel assignment algorithms are closely related.**

I. Introduction

EFFICIENT channel reuse is of paramount importance in the design of high capacity cellular radio systems. Cochannel interference caused by frequency reuse is the single most restraining factor on the system capacity. Systems may be designed either to tolerate low signal-to-interference ratios (i.e., efficient modulation and coding schemes), or to reduce the interference with efficient cell planning. In addition, dynamic channel allocation may be used to reduce the interference even more by adapting the cell plan to changing traffic and unforeseen propagation conditions.

In addition, the use of transmitter power control has been proposed to control cochannel interference. The main idea is to adjust the power of each transmitter for a given channel allocation, such that the interference levels at the receiver locations are minimized. Maintaining adequate transmission quality on the actual communications links is an obvious constraint. The measure of quality usually employed in cellular systems design is the *carrier-to-interference* (C/I) *ratio.* The benefits of power control are, however, not as obvious as they may seem at first glance. It should be borne in mind that by reducing the transmitter power in a certain link, that link will also be more vulnerable to interference. That the design of power control algorithms is not trivial is illustrated by the wide range of algorithms and results found in the literature.

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Several aspects of power control for cochannel interference reduction have been investigated in the literature. Early analytical work by Bock and Ebstein [1] has developed into a well-known technique in government spectrum management [17]. The problem is here approached by means of linear programming. In [2], Aein investigates cochannel interference management in satellite systems. He introduces the concept of C/I *balancing,* which yields a "fair" distribution of the interference in the sense that all users experience the same C/I level. The problem is identified as an eigenvalue problem for positive matrices. In [3]–[5], Nettleton and Alavi apply and extend these results to spread spectrum cellular radio systems. In these systems, the adjacent channel ("adjacent code") interference also has to be taken into account. In fact, this type of interference is shown to be dominant in spread spectrum systems [18]. Nettleton and Alavi show that C/I balancing substantially improves the capacity of such systems.

Other power control algorithms implemented this far basically keep the *received power at a constant level.* This scheme is very efficient in controlling adjacent channel (intracell) interference, but has been shown to have limited effect on cochannel interference [9]. Most work done in this area involves simulation studies with quite realistic but complex system models [6]–[10]. Unfortunately, the large variety of model assumptions makes these results hard to compare.

The aim of this paper is to further investigate the performance of transmitter power control algorithms. The basic models and ideas in [2]–[5] form the point of departure for the following discussion. The approach in this paper will be to find performance bounds and conditions of stability for all types of transmitter power control algorithms. To derive these bounds, a (hypothetical) power control scheme is proposed that is optimum in the sense that it minimizes the interference (or outage) probability. The latter quantity is the probability that some randomly chosen mobile (or base station) has a "to low" C/I. It will be seen that the C/I balancing technique plays a key role in this minimization process. Finally, some implementational aspects regarding distributed C/I control algorithms will be discussed. First, however, we will define our system model and derive some useful results from the theory of positive matrices.

II. System Model

Throughout this paper we will study a large, but finite, cellular radio system consisting of N cells. For the system design we have at our disposal M *independent* channel pairs, each consisting of *independent* up- and downlink channels.

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This means that the crosstalk between channels (adjacent channel interference) is neglected. The actual implementation of the channel is not of importance to our model. Frequency slots (FDMA), times slots (TDMA), or even spreading codes (CDMA) may be used. In CDMA systems, however, the assumption of having no crosstalk between channels may not be very realistic. The channel pairs will be allocated to mobiles and base stations according to some arbitrary channel assignment strategy. Each channel pair is assumed to be used only once in each cell, which means that there will be no intracell interference. The set of cells using a certain channel pair m at some given instant, will be called the *cochannel set* of m. The size (cardinality) of this set is denoted $Q(m)$.

Let us now consider a call in progress in some cell of the cochannel set of m. Both the base station and the mobile station will experience interference from the other $Q(m) - 1$ cochannel cells of the set. We will in the sequel assume the transmission quality to be dependent only on the C/I ratio, Γ. The total interference power is modeled as the sum of the powers of all active interferers [13], [16]. We will further consider high capacity, interference limited systems only. The consequence of this is that other noise sources, like termal noise, may be neglected. Using these assumptions we may express Γ as

$$\Gamma = \frac{P_{rx,i}}{\sum_{j=1}^{Q(m)-1} I_j} \tag{1}$$

where $P_{rx,i}$ is the received power from the "desired" transmitter and I_j is the received power from interferer j.

Now, let us denote the *link gain* on the path between the mobile in cell i and the base station in cell j at some given moment by G_{ij} (Fig. 1). Note that, due to our assumptions above, there is only one mobile in cell i using this particular channel. Using this notation, we may derive the C/I *at mobile* i

$$\Gamma_i = \frac{P_{rx,i}}{\sum_{j \neq i} I_j} = \frac{G_{ii} P_i}{\sum_{j \neq i} G_{ij} P_j} \tag{2}$$

where P_i is the transmitter power used by the base station in cell i. Note that G_{ii} is the path gain of the "desired" signal path in cell i. To simplify further derivations, we introduce some matrix notation similar to that in [2]–[5]. First, we define the *link gain matrix* $\boldsymbol{G} = \{G_{ij}\}$. It should been borne in mind that, in a mobile system, the link gains will constantly change. $\boldsymbol{G}$ may, therefore, be described as a matrix valued random process. At some given instant of time, however, $\boldsymbol{G}$ will be a $\boldsymbol{Q} \times \boldsymbol{Q}$ square matrix of random variables. Further dividing the right-hand side (RHS) of (2) by G_{ii} yields

$$\Gamma_i = \frac{P_i}{\sum_{j \neq i} P_j \frac{G_{ij}}{G_{ii}}} = \frac{P_i}{\sum_{j=1}^{Q} P_j Z_{ij} - P_i} \tag{3}$$

where we have introduced the stochastic variables Z_{ij} defined as

$$Z_{ij} = \frac{G_{ij}}{G_{ii}}. \tag{4}$$

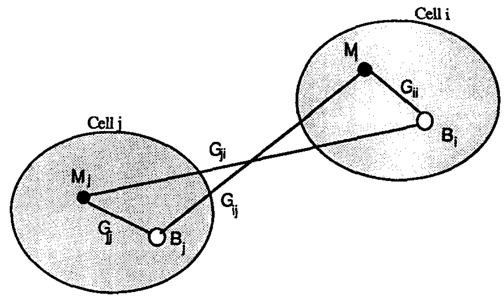

Fig. 1. Link geometry and link gain model.

Z_{ij} is the link gain from the base station in cell j to the mobile in cell i, normalized to the link gain in the desired path in cell i (i.e., from base station i to mobile i). Again using matrix notation we introduce the normalized *downlink gain matrix* $\boldsymbol{Z} = \{Z_{ij}\}$. We may in the same manner derive the C/I at the base station by letting $\boldsymbol{P}$ denote the mobile transmitter powers, and replacing $\boldsymbol{Z}$ with the *uplink gain* matrix $\boldsymbol{W} = \{W_{ij}\}$ defined as

$$W_{ij} = \frac{G_{ji}}{G_{ii}} = Z_{ji} \frac{G_{ii}}{G_{jj}}.$$

Note that in general G_{ii} is not equal to G_{jj}. Therefore, $\boldsymbol{W}$ will, in general, not be identical to the transpose of $\boldsymbol{Z}$. The properties of a given uplink matrix are thus not identical to the properties of the corresponding downlink matrix. However, it is reasonable to assume that they exhibit the same statistical properties.

Γ_i as defined by (3) will be a stochastic variable. Now, let Γ denote the C/I at some randomly chosen mobile. We define the distribution of Γ as

$$F(\gamma) = \Pr\{\Gamma \leq \gamma\} = \frac{1}{Q} \sum_{j=1}^{Q} \Pr\{\Gamma_j \leq \gamma\} = \frac{1}{Q} \sum_{j=1}^{Q} F_j(\gamma) \tag{5}$$

where $F_j(\gamma)$ is the distribution of Γ_j. In general, the distributions $F_j(\gamma)$ are not identical for all j's. In all finite networks there will be boundary effects, slightly favoring boundary cells. Let us now assume that the transmission system requires a minimum (threshold) C/I ratio γ_0. This C/I level is called the system *protection ratio.* As our performance measure, we will throughout the paper use the *interference probability*

$$F(\gamma_0) = \Pr\{\Gamma \leq \gamma_0\}. \tag{6}$$

This is the probability that some randomly chosen mobile has a C/I below the system protection ratio [13], [16].

Now, let us define a *power control algorithm* (PCA) as an algorithm using some measurement information to determine the transmitter power vector $\boldsymbol{P} = \{P_j\}$. The performance of the algorithm is measured by the resulting interference probability, $F(\gamma_0)$. We will define a *global* PCA, Ψ_G, as an algorithm that in every moment has access to the entire link gain matrix $\boldsymbol{Z}$ and may instantaneously control the entire vector $\boldsymbol{P}$. Ψ_G can be described by a vector valued function

$$\boldsymbol{P} = \Psi_G(\boldsymbol{Z}) \tag{7}$$

In a *local* PCA, however, each base station (or mobile) controls its own transmitter power based on only limited knowledge about $\boldsymbol{Z}$. It is clear that the class of local PCA's is included in the class of global PCA's. Therefore, the lowest interference probability that may be achieved by a local PCA cannot be better than the interference probability achieved by the best global PCA. The global algorithm may, therefore, be used to derive an upper bound on the performance of any local algorithm. From a practical point of view, however, local algorithms are preferred since they are simpler and require considerably less information exchange.

III. Achievable Cochannel Interference

Let us consider a system with global power control and some given instantaneous link gain matrix $\boldsymbol{Z}$.

Definition: The C/I-level γ is *achievable* if there exists a power vector $\boldsymbol{P} \geq \boldsymbol{0}$ such that $\Gamma_i \geq \gamma$ for all cells (mobiles) i.

The notation $\boldsymbol{P} \geq \boldsymbol{0}$ is used to denote that all elements of $\boldsymbol{P}$ are nonnegative. The following result is extremely useful to determine the performance of a global PCA. Given a nondegenerate stochastic link gain matrix $\boldsymbol{Z}$ we have the following proposition.

Proposition 1: With probability one, there exists a unique maximum achievable C/I level

$$\gamma^* = \max\{\gamma | \exists\, \boldsymbol{P} \geq \boldsymbol{0} : \Gamma_i \geq \gamma, \forall\, i\}.$$

The maximum is given by

$$\gamma^* = \frac{1}{\lambda^* - 1}$$

where λ^* is the largest real eigenvalue of the matrix $\boldsymbol{Z}$. The power vector $\boldsymbol{P}^*$ achieving this maximum is the eigenvector corresponding to λ^*.

Proof: The matrix $\boldsymbol{Z}$ is by definition a nonnegative matrix (all elements are nonnegative). Since all elements of $\boldsymbol{Z}$ are stochastic variables, the matrix will (for nondegenerate cases) have full rank with probability one. From the theory of nonnegative matrices we collect the following results originally due to Perron, Frobenius, and Wielandt [11].

i) $\boldsymbol{Z}$ has exactly one real positive eigenvalue λ^* for which the corresponding eigenvector is positive (i.e., all components have the same sign).

ii) The minimum real λ such that the inequality

$$\lambda \boldsymbol{P} \geq \boldsymbol{Z}\boldsymbol{P} \tag{8}$$

which has solutions for $\boldsymbol{P} \geq \boldsymbol{0}$ is $\lambda = \lambda^*$.

Now, rewriting (3), our proposition states that

$$\Gamma_i = \frac{P_i}{\sum_{j=1}^{Q} P_j Z_{ij} - P_i} \geq \gamma, \qquad \forall\, i \tag{9}$$

or

$$\frac{1+\gamma}{\gamma} P_i \geq \sum_{j=1}^{Q} P_j Z_{ij}, \quad \forall\, i.$$

Rewriting this in matrix form yields

$$\frac{1+\gamma}{\gamma} \boldsymbol{P} \geq \boldsymbol{Z}\boldsymbol{P}.$$

Combining results i) and ii) yields the required result. The simple bounds on the eigenvalues of $\boldsymbol{Z}$ derived from row sums of $\boldsymbol{Z}$ given in [2], [11], ensure that $\lambda^* > 1$ and thus $\gamma^* > 0$. □

Proposition 1 tells us that the maximum achievable C/I level is in fact reached when all inequalities (9) are satisfied with equality. This means that a C/I *balanced* system as proposed in [2]–[5] reaches the largest achievable C/I level, γ^*.

IV. Optimum Power Control

Now, let us assume that we use a global PCA, i.e., a vector valued function defined in (7). Let the achieved C/I vector given in (3) be denoted $\mathbb{\Gamma} = \{\Gamma_i\}$ where

$$\mathbb{\Gamma} = \mathbb{\Gamma}(\boldsymbol{P}, \boldsymbol{Z}) = \mathbb{\Gamma}(\Psi_G(\boldsymbol{Z}), \boldsymbol{Z}). \tag{10}$$

Now using (5) and conditioning on the link gain matrix, $\boldsymbol{Z}$, we have

$$F(\gamma_0 | \boldsymbol{Z}; \Psi_G) = \frac{1}{Q} \sum_{j=1}^{Q} F_j(\gamma_0 | \boldsymbol{Z}; \Psi_G). \tag{11}$$

Now, since Ψ_G is a deterministic function and $\boldsymbol{Z}$ is given, the distribution F_j becomes a simple step-function, i.e.

$$F_j(\gamma_0 | \boldsymbol{Z}; \Psi_G) = \begin{cases} 1, & \Gamma_j(\Psi_G, \boldsymbol{Z}) < \gamma_0 \\ 0, & \Gamma_j(\Psi_G, \boldsymbol{Z}) \geq \gamma_0. \end{cases} \tag{12}$$

We may thus express (11) as

$$F(\gamma_0 | \boldsymbol{Z}, \Psi_G) = \frac{1}{Q} B(\boldsymbol{Z}, \Psi_G) \tag{13}$$

where the integer function B is defined as

$$B(\boldsymbol{Z}, \Psi_G(\boldsymbol{Z})) = \#\ \text{elements in}\ \mathbb{\Gamma}(\Psi_G(\boldsymbol{Z})\boldsymbol{Z}) < \gamma_0. \tag{14}$$

Taking the expectation over $\boldsymbol{Z}$ we obtain

$$\begin{aligned} F(\gamma_0; \Psi_G) &= \frac{1}{Q} E[B(\boldsymbol{Z}, \Psi_G)] \\ &= \frac{1}{Q} \sum_{j=1}^{\infty} i \Pr\{B(\boldsymbol{Z}, \Psi_G) = i\}. \end{aligned} \tag{15}$$

To minimize $F(\gamma_0)$ we seek to minimize the function $B(\boldsymbol{Z}\Psi_G)$ for every outcome $\boldsymbol{Z}$. This minimum number we denote by $B^*(\boldsymbol{Z})$. $B^*(\boldsymbol{Z})$ is the minimum number of inequalities (9) that do not hold, i.e., the number of links in which the protection ratio γ_0 may not be achieved. Since B^* is an integer function, the minimum may be reached by several transmitter power vectors. In particular, we have the following proposition.

Proposition 2: At least one optimum power vector $\boldsymbol{P}$ has the form

$$\begin{aligned} P_i &= 0, & i &\in \mathfrak{R} \\ P_i &\neq 0, & i &\notin \mathfrak{R} \end{aligned}$$

where $\mathfrak{R} = \{i : \Gamma_i < \gamma_0\}, |\mathfrak{R}| = B^*$.

Proof: Assume we have an optimum power vector $\boldsymbol{P}^*$ not fulfilling the above requirement, i.e., $P_j^* \neq 0$ for all j. Let Γ^* be the corresponding C/I vector. We form a vector $\boldsymbol{P}$ such that, for some $k \in \mathfrak{R}$:

$$P_i = P_i^*, \quad i \neq k$$
$$P_k = 0.$$

Computing the C/I vector Γ for this power vector we obtain

$$\Gamma_i = \frac{P_i}{\sum_{j=1}^{Q} P_j Z_{ij} - P_i} = \frac{P_i^*}{\sum_{j=1}^{Q} P_j^* Z_{ij} - P_i^* - P_k^* Z_{ik}}$$
$$\geq \Gamma_i^* \geq \gamma_0, \qquad i \neq k$$
$$\Gamma_k = 0$$

since all quantities are nonnegative. $\boldsymbol{P}$ achieves a C/I that is not less than γ_0 whenever this is achieved by $\boldsymbol{P}^*$. Therefore, $\boldsymbol{P}$ is also an optimal power vector. Lowering the C/I for the link in cell k will be of no consequence since the link is considered to be unusable anyway. Repeating the argument stepwise for all indexes $k \in \mathfrak{R}$ proves the proposition. □

Due to Proposition 2, we may limit our search for optimum power vectors to those vectors with positive components for those cells where the required C/I is achieved. The other components are zero, corresponding to no transmission at all. By setting some $P_k = 0$, we effectively *remove* cell k from the cochannel set. Now, let us form a submatrix of $\boldsymbol{Z}$, denoted $\boldsymbol{Z}^*$ where all rows and columns $\boldsymbol{k}$ corresponding to zero components in the optimum power vector have been removed. Clearly, the maximum achievable C/I-level for $\boldsymbol{Z}^*$ must be not less than γ_0.

This observation leads us to a procedure to minimize the outage probability. An *optimum* global PCA will, by removing as few cells as possible, find the largest submatrix $\boldsymbol{Z}^*$ for which γ_0 is achievable. In this way we find an optimum power vector consisting of the eigenvector of $\boldsymbol{Z}^*$ and zero components corresponding to the removed cells. The "cell removal" procedure may appear as a strange way to achieve high system capacity. However, one should bear in mind that we consider those links where the required C/I level cannot be reached to be useless. Using a positive transmitter power in such a link will only cause interference in other cells. Note that straightforward C/I balancing, without cell removal, may be disastrous since *all* links may drop below the C/I threshold.

The question arises how one would implement such an optimum procedure. A "brute force" search would first check if γ_0 is achievable for the original matrix $\boldsymbol{Z}$. If not, we would try to remove one cell, computing the eigenvalue of each reduced system until the C/I requirement was fulfilled. If not, we would try removing all combinations of 2 cells and so on. Clearly, such a procedure would find an optimum power vector in a finite number of steps since finally, by removing combinations of $Q - 1$ cells, no interference would remain. However, in the "worst case," the total number of eigenvalue computations we have to perform is exponentially increasing with Q. To overcome this practical problem we propose the following algorithm that in most cases closely approximates the behavior of the optimum algorithm. We use a sequential procedure, where each step involves only one eigenvalue computation. The idea is to remove one cell at the time until the required C/I level is achieved in the remaining cells. An obvious criterion for cell removal is of course to maximize the $(\gamma')^*$ of the reduced system. This, however, requires $Q - k$ eigenvalue computations at the kth algorithm step (and roughly a total of $Q^2/2$ computions in the worst case). Instead, we propose an even simpler criterion that requires substantially less computational effort.

Stepwise Removal Algorithm (SRA)

1) Determine γ^* corresponding $\boldsymbol{Z}$. If $\gamma^* \geq \gamma_0$ use the eigenvector $\boldsymbol{P}^*$, else set $Q' = Q$.

2) Remove the cell k for which the maximum of the row and column sums

$$r_k = \sum_{j=1}^{Q} Z_{kj}, \qquad r_k^T = \sum_{j=1}^{Q} Z_{jk}$$

is maximized (combined sum criterion) and form the $(Q' - 1) \times (Q' - 1)$ matrix $\boldsymbol{Z}'$. Determine γ^* corresponding to $\boldsymbol{Z}'$. If $\gamma^* \geq \gamma_0$, use the eigenvector P^*, else set $Q' = Q' - 1$ and repeat step 2). □

By this procedure we, one by one, remove cells stepwise until all the C/I's in all remaining cells are larger than γ_0. The row and column sums provide bounds on the dominant eigenvalue of the matrix $\boldsymbol{Z}$ [2], [11]. The cell removal criterion, therefore, seeks to maximize the lower bound for the γ^* of the next matrix $\boldsymbol{Z}'$. Other simple criteria have been evaluated in [15].

At the present we do not know if an algorithm exists to solve the original submatrix problem in polynomial time or if this problem is actually NP-complete. Thus, we cannot determine if, and under what conditions, the problem in fact may be solved by a step-by-step procedure like the SRA. This constitutes an interesting open problem for further investigations.

V. Numerical Results

In order to derive some numerical results, more specific model assumptions have to be made. We will, for this purpose, consider a fixed and symmetric channel assignment strategy that divides the cells in K different channel groups. The cells using the same frequency are place symmetrically in a hexagonal grid [12]. Each set will consist of N/K cells forming a sparse hexagonal pattern. Base stations use omnidirectional antennas and are located at the center of the cells. The locations of the mobiles are assumed to be uniformly distributed over the cell area. To simplify calculations we will express all distances normalized to the radius of the hexagonal cells.

A simple propagation model that has found widespread use in the analysis of cellular systems is used [12]. We will assume that adequate modulation, coding, and equalization schemes are used in order to suppress the explicit dependence of the multipath fading process. Therefore, the performance of the system will only be dependent on the "local average" received signal power. The link gain G_{ij} is modeled as

$$G_{ij} = \frac{A_{ij}}{d_{ij}^{\alpha}} \tag{16}$$

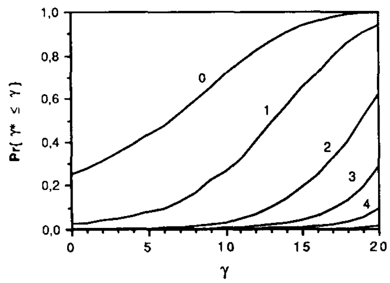

Fig. 2. Distribution of the maximum achievable C/I in all cells in the system, $\gamma^*(k)$, when k cells are removed according to the combined sum criterion. The system investigated contains 16 cochannel cells and uses a seven-cell cluster. Log–normal fading, $\sigma = 6$ dB, $\alpha = 4$.

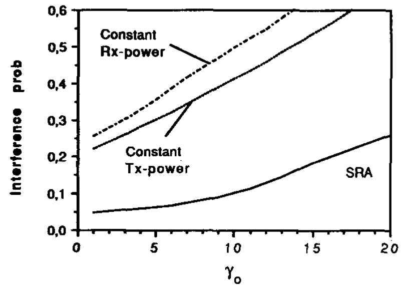

Fig. 3. Interference probability comparison. The system investigated contains 16 cochannel cells and uses a three-cell cluster. Log-normal fading, $\sigma = 6$ dB, $\alpha = 4$.

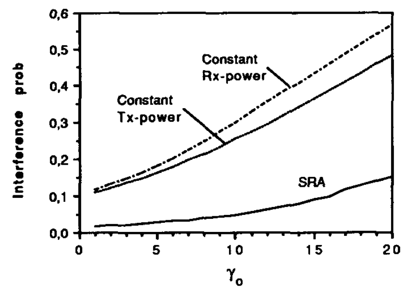

Fig. 4. Interference probability comparison. The system investigated contains 16 cochannel cells and uses a seven-cell cluster. Log-normal fading, $\sigma = 6$ dB, $\alpha = 4$.

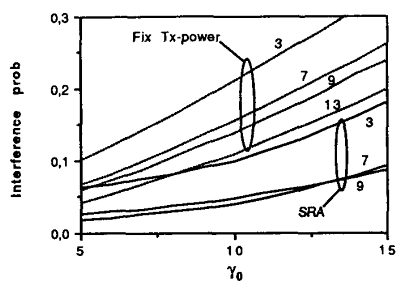

Fig. 5. Interference probability for different cluster sizes. The system investigated contains 16 cochannel cells and uses a 3, 7, 9, and 13 cell clusters. Log-normal fading, $\sigma = 6$ dB, $\alpha = 4$.

where d_{ij} is the distance between the mobile in cell i and the base station in cell j. The $1/d^\alpha$ factor models the large scale propagation loss whereas the attenuation factors A_{ij} model the power variation due to *shadowing*. We assume all A_{ij} to be indentically independent and log-normally distributed random variables with 0 dB expectation. The log-variance will be denoted by σ. Parameter values of α in the range of 3–5 and σ in the range of 4–10 dB usually provide good models for urban propagation [12].

The numerical results are for a 16 cell "square" cochannel cell pattern with "full load," i.e., all cochannel cells are assumed to be in use. The evaluation of the interference probabilities is made by means of Monte Carlo simulation. The standard deviation of the estimated probability at the 10% interference probability level is around $\pm 1\%$.

Due to the excessive computational complexity of the "brute force" optimum algorithm this paper is mainly limited to the stepwise removal algorithm. Unless otherwise noted, we use the propagation constant $\alpha = 4$ in the following graphs. Lower values of α yields somewhat more pessimistic interference probability results. However, the relative performance of the different algorithms investigated hardly changes at all.

Fig. 2 gives an insight into the operation of the SRA algorithm. It shows the achievable C/I levels, γ^*, in the system where k cells have been "removed" by the SRA. As can be seen from the graph, the expected achievable C/I level increases rapidly as a few cells are removed according to the removal criterion. Apparently very few cells cause the most interference problems. These cells are often, but not always, "interior" cells of the cochannel cell pattern. Results further indicate that, for a low number of cell removals, the performance of the SRA algorithm is very close to optimum.

Figs. 3 and 4 show the performance (interference probability) of the SRA algorithm compared with the performance of two reference algorithms, fixed transmitter power (i.e., no power control at all) and constant received power. The cluster sizes $K = 3, 7$ are covered in these graphs. Results show that the SRA algorithm outperforms the reference algorithms by an order of magnitude. At the 10% outage probability level C/I gains in excess of 10 dB may be achieved. The fixed transmitter power and the constant received power algorithm achieve roughly the same performance for interesting outage probabilities.

Fig. 5 finally, shows a comparison between-systems of different cluster sizes. It can be seen clearly that a system with cluster size 3 using the SRA algorithm outperforms a cluster size 13 system using the reference algorithms for most protection ratios. Capacity gains in excess of 4 times are, therefore, to be expected.

VI. Discussion

In this paper we have studied performance bounds for power control algorithms. Our approach has been to study the general properties of some hypothetical global power control algorithms (PCA's). These algorithms assume knowledge of the attenuation in all transmission and interference paths. A PCA is described that is optimum in the sense that it minimizes the interference probability for a given threshold C/I (i.e., protection ratio). This algorithm was, however, found to be somewhat impractical, since the computational complexity was exponentially increasing with the size of the cochannel set. However, a sequential algorithm that approximates the behavior of the optimum algorithm in "linear" time, the SRA, is presented. The numerical results presented for this algorithm indicate that there are potentially large gains to be achieved by using C/I control. Gains in excess of 10 dB in interference reduction may be achieved. This would correspond to capacity gains in the order of a factor 4 compared to system using fixed transmitter power or traditional power control. However most of the derivations are concerned with the downlink channel, results are equally valid for the uplink channel. It should be noted that the algorithms discussed in Section IV are not dependent on any of the explicit model assumptions used for deriving the numerical results.

All these algorithms apparently involve quite drastic measures to achieve a low interference probability. Usually, in one or more cells, base stations and/or mobiles are required to discontinue their transmissions. This is, however, a straightforward consequence of the performance measure. Links that end up with a C/I level below the required protection ratio are considered useless. In digital communications with extensive error control coding this is a quite realistic assumption. Wasting transmitter power in these cells will only add to the interference in the system. In fact, this is the usual remedy to poor C/I levels in modern cellular systems. A mobile experiencing poor transmission quality is handed over to another channel and/or base station which improves the performance locally. The power control algorithms discussed above tell us which of the mobiles that should be handed over to maximize *global* system performance. The intimate relation between dynamic channel assignment and efficient power control is obvious.

Attempting a practical implementation of the SRA algorithm would be difficult, since the path gain elements of the (time varying) link gain matrix $\boldsymbol{Z}$, are usually not known. Estimating these gains would require a significant measurement effort. Even if this would be possible, the amount of data that would have to be communicated and managed by the central controller would be enormous in a reasonably sized network. The main benefit of these results is, therefore, that they provide an estimate of the optimum performance of all C/I control schemes.

Further studies should, therefore, be devoted to studying algorithms that use only limited path gain information. Of special interest are local PCA's that may be operated only with path gain information that can be derived from measurements in a single cell. One interesting direction would be to investigate distributed C/I balancing algorithms, i.e., algorithms that attempt to achieve a given C/I level [14]. The result in Proposition 1 would establish for which preset C/I levels such an algorithm would be stable, i.e., that C/I balance could be reached.

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REFERENCES

[1] F. Bock, B. Ebstein, "Assignment of transmitter power control by linear programming," *IEEE Trans. Electromagn. Compat.*, vol. EMC-6, p. 36, 1964.

[2] J. M. Aein, "Power balancing in systems employing frequency reuse," *COMSAT Tech. Rev.*, vol. 3, no. 2, Fall 1973.

[3] R. W. Nettleton, "Traffic theory and interference management for a spread spectrum cellular radio system," in *Proc. ICC-80,* Seattle, WA, 1980.

[4] R. W. Nettleton and H. Alavi, "Downstream power control for spread-spectrum cellular mobile radio system," in *Proc. Globecom'82,* Miami, FL, 1982.

[5] ——, "Power control for spread-spectrum cellular mobile radio system," in *Proc. IEEE Veh. Technol. Conf.,* 1983, pp. 242–246.

[6] T. Nagatsu, T. Tsuruhara, and M. Sakamoto, "Transmitter power control for cellular land mobile radio," in *Proc. IEEE Globecom,* 1983, pp. 1430–1434.

[7] A. N. Rosenberg, "Simulation of power control and voice-channel selection in cellular systems," in *Proc. IEEE Veh. Technol. Conf.,* 1985, pp. 12–15.

[8] T. Fujii and M. Sakamoto, "Reduction of cochannel interference in cellular systems by intra-zone channel reassignment and adaptive transmitter power control," in *Proc. IEEE Veh. Technol. Conf.,* 1988, pp. 668–672.

[9] W. Tschirks, "Effect of transmission power control on the cochannel interference in cellular radio networks," *Elektrotechnik Inform.*, vol. 106, no. 5, 1989.

[10] H. Axén, "Power control in cellular mobile telephone systems," Diploma thesis, Ericsson Radio Syst., Int. Rep., BT/SU90:1708, 1990 (in Swedish).

[11] F. R. Gantmacher, *The Theory of Matrices.* New York: Chelsea, 1974, vol. 2, ch. XIII.

[12] W. C. Y. Lee, "Elements of cellular mobile radio," *IEEE Trans. Veh. Technol.,* vol. VT-35, pp. 48–56, 1986.

[13] R. Prasad, A. Kegel, J. Arnbak, "Analysis of system performance of high-capacity mobile radio," in *Proc. IEEE Veh. Technol. Conf.,* 1989, pp. 306–310.

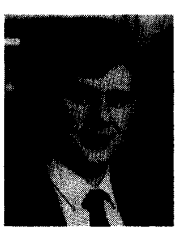
[14] H. Axén, "Uplink C/I level as control parameter for mobile transmitter power control- a preliminiary study," Ericsson Int. Rep., BT/ST 90:2377, Dec. 1990 (in Swedish).

[15] J. Zander, "Optimum power control in cellular radio systems," Int. Rep. TRITA-TTT-9101, Royal Instit. of Technol., Stockholm, Jan. 1991.

[16] R. Prasad, and A. Kegel, "Improved assessment of interference limits in cellular radio performance," *IEEE Trans. Veh. Technol.,* vol. 40, pp. 412–419, 1991.

[17] *Spectrum Management and Computer-Aided Techniques CCIR Handbook,* Geneva, Switzerland, 1983.

[18] R. W. Nettleton and G. R. Schloemer, "A high-capacity assignment method for cellular mobile telephone systems," in *Proc. IEEE Veh. Technol. Conf.,* San Francisco, CA, 1989.



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