

Coordinated Beamforming for the Multicell Multi-Antenna Wireless System

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Abstract—In a conventional wireless cellular system, signal processing is performed on a per-cell basis; out-of-cell interference is treated as background noise. This paper considers the benefit of coordinating base-stations across multiple cells in a multi-antenna beamforming system, where multiple base-stations may jointly optimize their respective beamformers to improve the overall system performance. Consider a multicell downlink scenario where base-stations are equipped with multiple transmit antennas employing either linear beamforming or nonlinear dirty-paper coding, and where remote users are equipped with a single antenna each, but where multiple remote users may be active simultaneously in each cell. This paper focuses on the design criteria of minimizing either the total weighted transmitted power or the maximum per-antenna power across the base-stations subject to signal-to-interference-and-noise-ratio (SINR) constraints at the remote users. The main contribution of the paper is an efficient algorithm for finding the joint globally optimal beamformers across all base-stations. The proposed algorithm is based on a generalization of uplink-downlink duality to the multicell setting using the Lagrangian duality theory. An important feature is that it naturally leads to a distributed implementation in time-division duplex (TDD) systems. Simulation results suggest that coordinating the beamforming vectors alone already provide appreciable performance improvements as compared to the conventional per-cell optimized network.

Index Terms—Beamforming, dirty-paper coding, uplink-downlink duality, multiple-input multiple-output (MIMO), multicell systems.

I. INTRODUCTION

CONVENTIONAL wireless systems are designed with a cellular architecture in which base-stations from different cells communicate with their respective remote terminals independently. Signal processing is performed on a per-cell basis; intercell interference is treated as background noise; intercell coordination is limited to the handling of mobile handoff. Conventional cellular networks are also typically designed to operate in the intercell-interference limited regime. Consequently, the performance of a conventional network can be significantly improved if joint signal processing is enabled

across the different base-stations to minimize or even to cancel intercell interference.

This paper evaluates the benefit of one particular type of base-station coordination for the multicell downlink system. The setup here is a scenario in which the base-stations are equipped with multiple antennas and the remote receivers are equipped with a single antenna each. Within each cell, multiple remote users may be active simultaneously and are separated via spatial multiplexing using beamforming. In a conventional system, the beamforming vectors in each cell are set independently. The main point of this paper is that it is possible to improve the overall network performance by allowing beamforming vectors from different base-stations to be coordinated.

The system setup in this paper differs from many of the coordinated joint processing systems studied in the literature (e.g. [1], [2], [3], [4]) where antennas from multiple base-stations act as a single antenna array. Such a fully coordinated case is sometime referred to as “network MIMO” (multiple-input multiple-output), where much larger performance gains are conceivable. However, network MIMO comes at a cost of signal-level coordination, i.e. data streams intended for different mobile users belonging to different cells need to be shared among the base-stations. In contrast, the system considered in this paper only requires coordination at the beamforming level, and is therefore much easier to implement.

Downlink beamforming for multi-antenna wireless systems has been studied extensively in the literature. A concept known as uplink-downlink duality has emerged as a main tool. This paper extends duality to the multicell setting by establishing that the multicell downlink beamforming problem for minimizing either the total weighted transmit power or the maximum per-antenna power subject to the received signal-to-noise-and-interference-ratio (SINR) constraints can be solved via a dual uplink problem. The duality holds for both linear transmit beamforming and for the nonlinear dirty-paper coded system. The main contribution of this paper is an efficient algorithm, which is capable of finding the globally optimal downlink beamforming vector across all base-stations. This algorithm is a multicell generalization of a similar algorithm proposed in [5] for the single-cell case. A key advantage of the proposed algorithm is that it naturally leads to a distributed implementation in a time-division duplex (TDD) system.

As an alternative to the transmit power minimization problem mentioned above, one can also formulate a rate-region maximization problem subject to power constraints at the base-stations. Both problems are of practical interest. Although it is possible to show that uplink-downlink duality continues to

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hold for the rate region maximization problem under suitable conditions (e.g. sum power constraint across all the transmitters), numerical optimization of achievable rate regions becomes much more difficult. For this reason, the remainder of this paper restricts its attention to the power minimization problem subject to SINR constraints.

A. Related Work

The benefit of base-station coordination in a cellular network has been a subject of many recent studies. Motivated by the joint detection and cooperation techniques for intracell interference mitigation, [1], [2], [3], [4], [6], [7], [8] study the capacity improvement due to the joint encoding or decoding across the base-stations for intercell interference mitigation. Base-station coordination is practically conceivable because in a cellular network base-stations are connected by high-capacity backhaul links. However, the amount of backhaul communications required to achieve joint processing is also substantial. This motivates network models with either constrained backhaul capacity [9], [10], [11] or with cooperation only among neighboring base-stations [12], [13], [14] or among cluster of base-stations [15].

The problem setting in this paper differs from the above series of works in that rather than joint processing at a signal level, we consider coordination at the beamformer level, which requires much less overhead and is more practical to implement. In particular, the data stream for each user only needs to be (pre-)processed at its own base-station (and not across all the base-stations). Further, the base-stations need not be symbol synchronized as required with signal-level coordination. With these practical considerations, this paper focuses on the joint optimization of transmit beamformers across the base-stations for minimizing the transmit power subject to quality-of-service constraints at the mobiles. This setup is suitable for constant bit-rate traffic with stringent delay constraints. As the simulation results of this paper show, appreciable performance gain can already be obtained with this limited form of coordination. Related works based on the minimization of packet loss probability [16] and the maximization of capacity [17], [18] have also been reported in the literature.

The transmit beamforming design problem goes back to the classic work of [19], where an iterative algorithm is proposed for the optimization of the beamforming vectors and power allocations to satisfy a set of target SINRs for an arbitrary set of transmission links. The main contribution of [19] is a beamformer-power update algorithm based on an uplink-downlink duality that converges to a feasible solution to the problem. In the single-cell multi-user downlink case, the optimality of this duality-based approach is proved in [20] and [21], [22]. More recently, [5] shows that the single-cell downlink beamforming problem can be formulated as a second-order cone-programming problem. This crucial insight allows an interpretation of duality via Lagrangian theory in convex optimization [23].

The single-cell uplink-downlink duality can be immediately generalized to the multicell setting if signal-level coordination between the base-stations is assumed. This is shown in [24]

in a CDMA context, where a beamformer-power iteration algorithm similar to that of [19] is proposed.

Less obvious is the question of whether uplink-downlink duality continues to hold in a multicell network with beamforming-level coordination only. This paper uses a Lagrangian duality approach to establish that duality indeed exists in this case. In addition, this paper proposes an optimization procedure based on power iteration alone. The proposed algorithm has a key advantage of being amenable to distributed implementation, which is highly desirable in a multicell network. Further, this paper takes realistic power constraint into account by solving the problem of minimizing the maximum per-antenna power constraint across all the base-stations. Finally, this paper provides a generalization to include dirty-paper coding within each cell.

The multicell uplink-downlink duality considered in this paper is related to the concept of network duality proposed in [25]. However, the network duality established in [25] is derived based on a linear programming approach, which is different from the Lagrangian approach taken in this paper. Consequently, [25] arrives at a different set of numerical algorithms for the coordinated beamforming problem, which happen to be not as easily implementable in a distributed fashion. In fact, to obtain distributed solutions, [25] has to resort to suboptimal algorithms. As a further note, [25] deals with the more general problem of joint transmit and receive beamforming optimization for a multicell network where both the base-station and the remote users are equipped with multiple antennas. Although uplink-downlink duality continues to hold, one consequence of this more general setup is that the proof of global optimality is no longer available, in contrast to the simpler single-antenna-per-remote-user case considered in this paper. Finally, the coordinated beamforming problem can also be solved using yet another different approach based on treating all the base-stations as a single transmitter, then modifying the corresponding channel matrix and determining the corresponding optimal beamforming vectors [26]. However, distributed implementation of the resulting algorithm is not yet available.

Coordination at the beamforming-level for the multicell multi-antenna channel has also been explored in [27], [28] from a viewpoint of egotistic vs. altruistic strategies. These works focus on the Pareto boundary of the achievable rate region of the multicell network under fixed power constraint, which is complementary to the problem of power minimization under fixed SINR constraints considered in this paper. The achievable rate region of multicell systems has also been explored in [29] using duality, and in [30]. In particular, [30] deals with a more general setting with different possible levels of coordination between the base-stations. The problem setting of the current paper corresponds to a particular limited coordination scenario in [30]. Further, [30] also addresses the issue of imperfect channel knowledge at the base-stations.

The problem setting of this paper assumes that the set of active mobile users with each cell and their respective SINR constraints are fixed. User scheduling (e.g. [31], [32], [33]) and congestion control strategies (e.g. [26]) are assumed to be performed separately. Throughout the paper, perfect channel side information for mobile users within each cell

is assumed to be available at each base-station. In a practical implementation, channel side information may be estimated via channel reciprocity for a TDD system or via a feedback mechanism; see [34], [35].

B. Organization

The remainder of the paper is organized as follows. Section II contains the problem formulation and the system model. In Section III, we establish uplink-downlink duality for the multicell network for problem of minimizing either the total weighted transmit power or the maximum per-antenna power subject to SINR constraints. Section IV contains distributed algorithms for multicell downlink beamforming. Section V provides simulation results. Concluding remarks are made in Section VI.

Notations: \mathcal{R} and \mathcal{C} denote the real and complex spaces. The identity matrix is denoted as I . The transpose and Hermitian transpose of a matrix are denoted as $(\cdot)^T$ and $(\cdot)^H$ respectively.

II. PROBLEM FORMULATION

A. System Model

This paper considers a multicell multi-user spatial multiplex system with N cells and K users per cell with N_t antennas at each base-station and a single antenna at each remote user. Multiuser downlink transmit beamforming is employed at each base-station. Let $x_{i,j}$ be a complex scalar denoting the information signal for the j th user in the i th cell, and $\mathbf{w}_{i,j} \in \mathcal{C}^{N_t \times 1}$ be its associated beamforming vector. The received signal at the j th remote user in the i th cell, denoted as $y_{i,j} \in \mathcal{C}$, is a summation of the intended signal, intracell interference, and intercell interference:

$$y_{i,j} = \sum_l \mathbf{h}_{l,i,j}^H \mathbf{w}_{l,i,j} x_{l,i,j} + \sum_{m \neq i,n} \mathbf{h}_{m,i,j}^H \mathbf{w}_{m,n} x_{m,n} + z_{i,j} \quad (1)$$

where $\mathbf{h}_{l,i,j} \in \mathcal{C}^{N_t \times 1}$ is the vector channel from the base-station of the l th cell to the j th user in the i th cell, and $z_{i,j}$ is the additive white circularly symmetric Gaussian complex noise with variance $\sigma^2/2$ on each of its real and imaginary components. Fig. 1 illustrates the system model for a network with seven cells and three users per cell.

B. Transmit Beamforming Problem

The beamformer design problem is that of minimizing some function of transmit power across all base-stations subject to SINR constraints at the remote users. With $\mathbf{w}_{i,j}$ as the beamforming vectors, the SINR for the j th user in the i th cell can be expressed as:

$$\Gamma_{i,j} = \frac{|\mathbf{w}_{i,j}^H \mathbf{h}_{i,i,j}|^2}{\sum_{l \neq j} |\mathbf{w}_{i,l}^H \mathbf{h}_{i,i,j}|^2 + \sum_{m \neq i,n} |\mathbf{w}_{m,n}^H \mathbf{h}_{m,i,j}|^2 + \sigma^2} \quad (2)$$

Let $\gamma_{i,j}$ be the SINR target for the j th user in the i th cell. We can then formulate, for example, a total transmit power minimization problem as follows:

$$\begin{aligned} & \text{minimize} \quad \sum_{i,j} \mathbf{w}_{i,j}^H \mathbf{w}_{i,j} \\ & \text{subject to} \quad \Gamma_{i,j} \geq \gamma_{i,j}, \quad \forall i = 1 \dots N, j = 1 \dots K \end{aligned} \quad (3)$$

where the minimization is over the $\mathbf{w}_{i,j}$'s. Throughout this paper, we assume that the set of SINR targets are feasible.

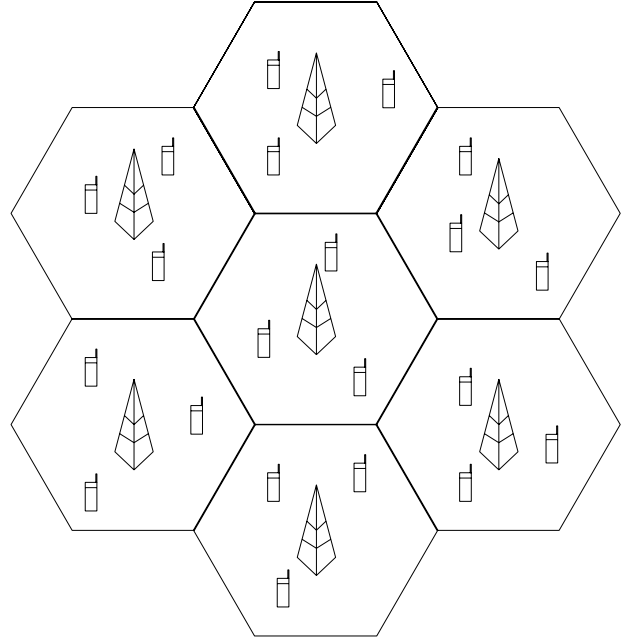


Fig. 1. Multicell wireless network.

C. Conventional Systems

In a conventional wireless cellular system, the multiuser beamforming problem is solved on a per-cell basis; out-of-cell interference is regarded as a part of background noise. In particular, for a fixed base-station \hat{i} , a conventional system finds the optimal set of $\mathbf{w}_{\hat{i},j}$, $j = 1 \dots K$, assuming that all other $(N-1)K$ beamformers are fixed:

$$\begin{aligned} & \text{minimize} \quad \sum_j \mathbf{w}_{\hat{i},j}^H \mathbf{w}_{\hat{i},j} \\ & \text{subject to} \quad \Gamma_{\hat{i},j} \geq \gamma_{\hat{i},j}, \quad \forall j = 1 \dots K \end{aligned} \quad (4)$$

where $\Gamma_{\hat{i},j}$ is given by (2). This single-cell downlink problem has a classic solution as given in [19], [20], [21], [5].

Note that in a conventional system, the choice of beamformers at each base-station affects the background noise level at neighboring cells, and hence the setting of beamformers in neighboring base-stations. Thus, the above per-cell optimization is in practice performed iteratively until the system converges to a per-cell optimal solution.

D. Motivating Example for Joint Optimization

This paper is motivated by the observation that the per-cell optimization above does not necessarily lead to a joint optimal solution. Significant performance improvement may be obtained if base-stations coordinate in jointly optimizing all of their beamformers at the same time. The following example illustrates this point.

Consider a multicell network but with only a single user per cell. The per-cell optimization reduces to the optimal transmit beamforming problem for a multi-input single-output (MISO) system with a background noise level which includes out-of-cell interference. Note that regardless of the level of the background noise, the optimal per-cell transmit beamformer is

a vector that matches the channel. Thus, in this example, per-cell optimization across the cells converges in one iteration—every base-station uses a transmit beamformer that matches the MISO channel.

This channel-matching solution is not necessarily the joint optimum. For example, when two users belonging to two different cells are near each other at the cell edge, it may be advantageous to steer the beamforming vectors for the two base-stations away from each other so as to reduce mutual interference. Such a joint optimal beamforming solution may lead to higher received SINRs at a fixed transmit power, or conversely a lower transmit power at fixed SINRs.

One of the first algorithms for solving the multicell joint beamforming optimization problem is given by Rashid-Farrokhi, Liu and Tassiulas [19]. They showed that the optimal downlink beamforming problem under SINR constraints can be solved efficiently by an iterative uplink beamformer and power update algorithm. It is well known that the uplink beamforming problem is much easier to solve [36]. Thus, by transforming the downlink problem into the uplink domain, the downlink problem may be solved efficiently as well.

The global optimality of the beamformer-power iteration algorithm has been shown for the single-cell case in [20], [21], [5]. This paper gives a rigorous derivation of duality for the multicell case, then proposes a new algorithm for solving the joint multicell downlink beamforming problem.

III. UPLINK-DOWNLINK DUALITY FOR MULTICELL SYSTEMS

A. Minimization of Weighted Transmitted Power

We begin by formulating a slightly more general version of the transmit beamforming problem (3). In a multicell system, each base-station (or sometime each antenna) typically has its own power constraint. Thus, it is useful to consider a problem of minimizing the weighted total transmit power, with the transmit power at the i th base-station weighted by a factor α_i . In this case, (3) becomes

$$\begin{aligned} & \text{minimize} \quad \sum_{i,j} \alpha_i \mathbf{w}_{i,j}^H \mathbf{w}_{i,j} \\ & \text{subject to} \quad \Gamma_{i,j} \geq \gamma_{i,j}, \quad \forall i = 1 \dots N, j = 1 \dots K \end{aligned} \quad (5)$$

The SINR target constraints in (5) may appear nonconvex. In a study of single-cell downlink beamforming problem, [5] showed that SINR constraints of this type can be transformed into a second-order-cone constraint (see also [37]). This crucial observation enables methods for solving (5) via convex optimization.

Uplink-downlink duality refers to the fact that the minimum transmit power needed to achieve a certain set of SINR constraints in a downlink channel is the same as the minimum total transmit power needed to achieve the same set of SINR targets in an uplink channel, where the uplink channel is obtained by reversing the input and the output of the downlink. The main goal of this section is to show that uplink-downlink duality, previously established for the single-cell case, carries over to the multicell setting. The following theorem is a multicell generalization of the single-cell duality result as

stated in [23]. The proof is based on a Lagrangian technique, similar to the approach used in [23].

Theorem 1: The optimal transmit beamforming problem (5) for the downlink multiuser multi-cellular network can be solved via a dual uplink channel in which the SINR constraints remain the same and the noise power is scaled by α_i . Mathematically, a Lagrangian dual of the optimization problem for the downlink (5) is the following uplink problem:

$$\begin{aligned} & \text{minimize} \quad \sum_{i,j} \lambda_{i,j} \sigma^2 \\ & \text{subject to} \quad \Lambda_{i,j} \geq \gamma_{i,j} \end{aligned} \quad (6)$$

where the minimization is over $\lambda_{i,j}$, and

$$\Lambda_{i,j} = \max_{\hat{\mathbf{w}}_{i,j}} \frac{\lambda_{i,j} |\hat{\mathbf{w}}_{i,j}^H \mathbf{h}_{i,i,j}|^2}{\sum_{(m,l) \neq (i,j)} \lambda_{m,l} |\hat{\mathbf{w}}_{i,j}^H \mathbf{h}_{i,m,l}|^2 + \alpha_i \|\hat{\mathbf{w}}_{i,j}\|^2}.$$

The optimal $\hat{\mathbf{w}}_{i,j}$ has the interpretation of being the receiver beamformer of the dual uplink channel, and is a scaled version of the optimal $\mathbf{w}_{i,j}$. The optimal $\lambda_{i,j}$ has the interpretation of being the dual uplink power, and it corresponds to the dual variable associated with the SINR constraint of (5).

Proof: The proof hinges upon the fact that the SINR constraints can be reformulated as a second-order cone-programming problem as shown in [5]. Therefore, strong duality holds for (5). This allows us to characterize the solution of (5) via its Lagrangian:

$$\begin{aligned} L(\mathbf{w}_{i,j}, \lambda_{i,j}) = & \sum_{i,j} \alpha_i \mathbf{w}_{i,j}^H \mathbf{w}_{i,j} - \sum_{i,j} \lambda_{i,j} \left[\frac{|\mathbf{w}_{i,j}^H \mathbf{h}_{i,i,j}|^2}{\gamma_{i,j}} - \right. \\ & \left. \sum_{l \neq j} |\mathbf{w}_{i,l}^H \mathbf{h}_{i,i,j}|^2 - \sum_{m \neq i,n} |\mathbf{w}_{m,n}^H \mathbf{h}_{m,i,j}|^2 - \sigma^2 \right] \end{aligned} \quad (7)$$

Rearranging (7), we get:

$$\begin{aligned} L(\mathbf{w}_{i,j}, \lambda_{i,j}) = & \sum_{i,j} \lambda_{i,j} \sigma^2 + \sum_{i,j} \mathbf{w}_{i,j}^H \left[\alpha_i \mathbf{I} - \right. \\ & \left. \left(1 + \frac{1}{\gamma_{i,j}} \right) \lambda_{i,j} \mathbf{h}_{i,i,j} \mathbf{h}_{i,i,j}^H + \sum_{m,n} \lambda_{m,n} \mathbf{h}_{i,m,n} \mathbf{h}_{i,m,n}^H \right] \mathbf{w}_{i,j} \end{aligned} \quad (8)$$

The dual objective is

$$g(\lambda_{i,j}) = \min_{\mathbf{w}_{i,j}} L(\mathbf{w}_{i,j}, \lambda_{i,j}) \quad (9)$$

It is easy to see that if $\alpha_i \mathbf{I} - \left(1 + \frac{1}{\gamma_{i,j}} \right) \lambda_{i,j} \mathbf{h}_{i,i,j} \mathbf{h}_{i,i,j}^H + \sum_{m,n} \lambda_{m,n} \mathbf{h}_{i,m,n} \mathbf{h}_{i,m,n}^H$ is not a positive definite matrix, then there exists a set of $\mathbf{w}_{i,j}$ that would make $g(\lambda_{i,j}) = -\infty$. Thus, the Lagrangian dual of (5), which is the maximum of $g(\lambda_{i,j})$, is

$$\text{maximize} \quad \sum_{i,j} \lambda_{i,j} \sigma^2 \quad (10)$$

$$\text{subject to} \quad \Sigma_i \succeq \left(1 + \frac{1}{\gamma_{i,j}} \right) \lambda_{i,j} \mathbf{h}_{i,i,j} \mathbf{h}_{i,i,j}^H$$

where

$$\Sigma_i \triangleq \alpha_i \mathbf{I} + \sum_{m,n} \lambda_{m,n} \mathbf{h}_{i,m,n} \mathbf{h}_{i,m,n}^H \quad (11)$$

Next, we show that the above dual is equivalent to (6). The problem (6) corresponds to an uplink channel with receive beamformers $\hat{\mathbf{w}}_{i,j}$, where the noise power of the dual channel is scaled by α_i . The optimal receive beamformers $\hat{\mathbf{w}}_{i,j}$ that maximize the SINR are the minimum-mean-squared-error (MMSE) receivers, which can be expressed as:

$$\hat{\mathbf{w}}_{i,j} = \left(\sum_{m,l} \lambda_{m,l} \sigma^2 \mathbf{h}_{i,m,l} \mathbf{h}_{i,m,l}^H + \alpha_i \sigma^2 \mathbf{I} \right)^{-1} \mathbf{h}_{i,j} \quad (12)$$

Plugging $\hat{\mathbf{w}}_{i,j}$ into the SINR constraint of (6), one can show that the SINR constraint is equivalent to

$$\alpha_i \mathbf{I} + \sum_{m,n} \lambda_{m,n} \mathbf{h}_{i,m,n} \mathbf{h}_{i,m,n}^H \preceq \left(1 + \frac{1}{\gamma_{i,j}} \right) \lambda_{i,j} \mathbf{h}_{i,i,j} \mathbf{h}_{i,i,j}^H$$

Thus, one can rewrite (6) as follows:

$$\begin{aligned} & \text{minimize} \quad \sum_{i,j} \lambda_{i,j} \sigma^2 \\ & \text{subject to} \quad \Sigma_i \preceq \left(1 + \frac{1}{\gamma_{i,j}} \right) \lambda_{i,j} \mathbf{h}_{i,i,j} \mathbf{h}_{i,i,j}^H \end{aligned} \quad (13)$$

Note that the problems in (10) and (13) are identical except that the maximization is replaced by minimization and the inequality constraints are reversed. It can be shown that the optimal solutions for both problems are such that the constraints are satisfied with equality. Thus, (10) and (13) give the same solutions.

In addition, it can be shown that $\mathbf{w}_{i,j}$ and $\hat{\mathbf{w}}_{i,j}$ are scaled versions of each other. Thus, one would be able to find $\mathbf{w}_{i,j}$ by first finding $\hat{\mathbf{w}}_{i,j}$, then updating it through scalar multiples $\delta_{i,j}$

$$\mathbf{w}_{i,j} = \sqrt{\delta_{i,j}} \hat{\mathbf{w}}_{i,j}. \quad (14)$$

The $\delta_{i,j}$ can be found through a matrix inversion using the fact that the SINR constraints in (5) are satisfied with equality. Plugging (14) into the SINR constraint of (5), one can rewrite the SINR constraint as:

$$\begin{aligned} & \frac{1}{\gamma_{i,j}} |\hat{\mathbf{w}}_{i,j}^H \mathbf{h}_{i,i,j}|^2 \delta_{i,j} - \sum_{n \neq j} |\hat{\mathbf{w}}_{i,n}^H \mathbf{h}_{i,i,j}|^2 \delta_{i,n} \\ & - \sum_{m \neq i,n} |\hat{\mathbf{w}}_{m,n}^H \mathbf{h}_{m,i,j}|^2 \delta_{m,n} = \sigma^2 \end{aligned} \quad (15)$$

Define $\boldsymbol{\delta}_i = [\delta_{i,1}, \delta_{i,2}, \dots, \delta_{i,K}]^T$ for $i = 1 \dots N$ and $\boldsymbol{\delta} = [\boldsymbol{\delta}_1^T, \boldsymbol{\delta}_2^T, \dots, \boldsymbol{\delta}_N^T]^T$. Based on (15), one can write

$$\mathbf{F} \boldsymbol{\delta} = \mathbf{1} \sigma^2. \quad (16)$$

Here, $\mathbf{1}$ is the $NK \times 1$ all ones-vector and \mathbf{F} is the following $NK \times NK$ matrix:

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}^{11} & \mathbf{F}^{12} & \dots & \mathbf{F}^{1N} \\ \mathbf{F}^{21} & \mathbf{F}^{22} & \dots & \mathbf{F}^{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{F}^{N1} & \mathbf{F}^{N2} & \dots & \mathbf{F}^{NN} \end{bmatrix}$$

where the (j,n) -th entry of each $K \times K$ sub-matrix \mathbf{F}^{im} is defined as follows:

$$\mathbf{F}_{jn}^{im} = \begin{cases} \frac{1}{\gamma_{i,j}} |\hat{\mathbf{w}}_{i,j}^H \mathbf{h}_{i,i,j}|^2 & \text{if } m = i \text{ and } n = j, \\ -|\hat{\mathbf{w}}_{i,n}^H \mathbf{h}_{i,i,j}|^2 & \text{if } m = i \text{ and } n \neq j, \\ -|\hat{\mathbf{w}}_{m,n}^H \mathbf{h}_{m,i,j}|^2 & \text{if } m \neq i \end{cases} \quad (17)$$

The $\delta_{i,j}$'s can be found by taking the inverse of the matrix \mathbf{F} :

$$\boldsymbol{\delta} = \mathbf{F}^{-1} \mathbf{1} \sigma^2 \quad (18)$$

B. Minimization of Maximum Antenna Power

The weighting factors α_i in Theorem 1 provide a mechanism to trade off the power consumptions at different base-stations in a multicell network. It is also straightforward to introduce additional weighting factors to account for power consumption tradeoff at the per-antenna level. However, choosing the right weights is often not easy. But if we consider a practical scenario of minimizing the maximum antenna power across all the base-stations, the weights adjustment can be done automatically using a further extension of duality. The optimization problem in this case is formulated as follows:

$$\begin{aligned} & \text{minimize} \quad \tau \\ & \text{subject to} \quad \Gamma_{i,j} \geq \gamma_{i,j}, \quad \forall i, j \\ & \quad \left[\sum_j \mathbf{w}_{i,j} \mathbf{w}_{i,j}^H \right]_{m,m} \leq \tau, \quad \forall i, m \end{aligned} \quad (19)$$

where $[\cdot]_{m,n}$ denotes the (m,n) -th entry of a matrix. The following theorem is a multicell generalization of the single-cell per-antenna power minimization problem treated in [23].

Theorem 2: The optimal maximum antenna power transmit beamforming problem (19) for the downlink multiuser multicellular network can be solved via a dual uplink channel in which the SINR constraints remain the same and the noise is uncertain. Mathematically, a Lagrangian dual of the optimization problem (19) is the following max-min problem:

$$\begin{aligned} & \max_{\mathbf{Q}_i} \min_{\lambda_{i,j}} \quad \sum_{i,j} \lambda_{i,j} \sigma^2 \\ & \text{subject to} \quad \Lambda_{i,j} \geq \gamma_{i,j} \quad \forall i, j \\ & \quad \text{tr}(\mathbf{Q}_i) \leq N_t, \quad \mathbf{Q}_i \text{ diagonal}, \\ & \quad \mathbf{Q}_i \succeq 0 \quad \forall i \end{aligned} \quad (20)$$

where

$$\Lambda_{i,j} = \max_{\hat{\mathbf{w}}_{i,j}} \frac{\lambda_{i,j} |\hat{\mathbf{w}}_{i,j}^H \mathbf{h}_{i,i,j}|^2}{\sum_{(m,l) \neq (i,j)} \lambda_{m,l} |\hat{\mathbf{w}}_{i,j}^H \mathbf{h}_{i,m,l}|^2 + \hat{\mathbf{w}}_{i,j}^H \mathbf{Q}_i \hat{\mathbf{w}}_{i,j}}.$$

The optimal $\hat{\mathbf{w}}_{i,j}$ has the interpretation of being the receiver beamformer of the dual uplink channel, and is a scaled version of the optimal $\mathbf{w}_{i,j}$. The optimal $\lambda_{i,j}$ has the interpretation of being the dual uplink power, and it corresponds to the dual variable associated with the SINR constraint of (19). The optimal uplink noise covariance matrix $\mathbf{Q}_i = \text{diag}(q_{i,1}, \dots, q_{i,N_t})$ is a diagonal matrix of dual variables associated with the downlink per-antenna power constraint in (19).

Proof: The proof mirrors that of Theorem 1. It again hinges upon the fact that the SINR constraints can be reformulated as a second-order cone-programming problem. Therefore, strong duality holds for (19). This allows us to characterize the solution of (19) via its Lagrangian. First, one can rewrite (19) as follows:

$$\begin{aligned} & \text{minimize} && NN_t\tau \\ & \text{subject to} && \Gamma_{i,j} \geq \gamma_{i,j}, \quad \forall i, j \\ & && \left[\sum_j \mathbf{w}_{i,j} \mathbf{w}_{i,j}^H \right]_{m,m} \leq \tau, \quad \forall i, m \end{aligned} \quad (21)$$

The Lagrangian of (21) is

$$\begin{aligned} L(\mathbf{w}_{i,j}, \lambda_{i,j}, \mathbf{Q}_i, \tau) = & \sum_{i,j} \lambda_{i,j} \sigma^2 + \sum_{i,j} \mathbf{w}_{i,j}^H \left[\mathbf{Q}_i - \right. \\ & \left. \left(1 + \frac{1}{\gamma_{i,j}} \right) \lambda_{i,j} \mathbf{h}_{i,i,j} \mathbf{h}_{i,i,j}^H + \sum_{m,n} \lambda_{m,n} \mathbf{h}_{i,m,n} \mathbf{h}_{i,m,n}^H \right] \mathbf{w}_{i,j} \\ & - \tau \sum_i [\text{tr}(\mathbf{Q}_i) - N_t] \end{aligned} \quad (22)$$

Using the same argument as in the proof of Theorem 1, we can write the Lagrangian dual of (19) as

$$\begin{aligned} & \max_{\mathbf{Q}_i} \max_{\lambda_{i,j}} \sum_{i,j} \lambda_{i,j} \sigma^2 \\ & \text{subject to} \quad \mathbf{\Theta}_i \succeq \left(1 + \frac{1}{\gamma_{i,j}} \right) \lambda_{i,j} \mathbf{h}_{i,i,j} \mathbf{h}_{i,i,j}^H \\ & \quad \text{tr}(\mathbf{Q}_i) \leq N_t, \quad \mathbf{Q}_i \text{ diagonal}, \\ & \quad \mathbf{Q}_i \succeq 0 \quad \forall i \end{aligned} \quad (23)$$

where

$$\mathbf{\Theta}_i \triangleq \mathbf{Q}_i + \sum_{m,n} \lambda_{m,n} \mathbf{h}_{i,m,n} \mathbf{h}_{i,m,n}^H \quad (24)$$

Also, following the same steps as in the proof of Theorem 1, one can rewrite (20) as follows

$$\begin{aligned} & \max_{\mathbf{Q}_i} \min_{\lambda_{i,j}} \sum_{i,j} \lambda_{i,j} \sigma^2 \\ & \text{subject to} \quad \mathbf{\Theta}_i \preceq \left(1 + \frac{1}{\gamma_{i,j}} \right) \lambda_{i,j} \mathbf{h}_{i,i,j} \mathbf{h}_{i,i,j}^H \\ & \quad \text{tr}(\mathbf{Q}_i) \leq N_t, \quad \mathbf{Q}_i \text{ diagonal}, \\ & \quad \mathbf{Q}_i \succeq 0 \quad \forall i \end{aligned} \quad (25)$$

The problems in (25) and (23) are identical except that the maximization is replaced by minimization and the inequality constraints are reversed. Thus, as in the previous proof, (25) and (23) give the same solutions. ■

Comparing (6) and (20), it is now clear that the weighted power minimization problem with α_i as weights corresponds to the setting of $\mathbf{Q}_i = \alpha_i \mathbf{I}$ in (20). An outer maximization over \mathbf{Q}_i provides a way to set the weights optimally to minimize the maximum per-antenna power.

C. Beamforming With Dirty-Paper Coding

Uplink-downlink duality can be extended to include the possibility of implementing dirty-paper coding (DPC) within each cell. DPC refers to an information theoretical operation where the downlink intracell interference can be pre-subtracted at the base-station. Dirty-paper coding may be implemented in practice using Tomlinson-Harashima precoding-like techniques. It can be thought of as the dual operation of receiver-based interference cancellation for the uplink.

Assuming a particular pre-subtraction order $1, 2, \dots, K$ in each cell, i.e. the j th user of the i th cell, is encoded at base-station i by subtracting the intracell interference caused by the first $(j-1)$ users of the same cell. The SINR of the downlink now becomes

$$\Gamma'_{i,j} = \frac{|\mathbf{w}_{i,j}^H \mathbf{h}_{i,i,j}|^2}{\sum_{l>j} |\mathbf{w}_{i,l}^H \mathbf{h}_{i,i,j}|^2 + \sum_{m \neq i,n} |\mathbf{w}_{m,n}^H \mathbf{h}_{m,i,j}|^2 + \sigma^2}$$

It is not difficult to see that the dual uplink problem is exactly the same as in the linear beamforming case, except the uplink SINRs are modified as

$$\Lambda'_{i,j} = \max_{\hat{\mathbf{w}}_{i,j}} \frac{\lambda_{i,j} |\hat{\mathbf{w}}_{i,j}^H \mathbf{h}_{i,i,j}|^2}{\sum_{(m,l) \succ (i,j)} \lambda_{m,l} |\hat{\mathbf{w}}_{i,j}^H \mathbf{h}_{i,m,l}|^2 + \alpha_i \|\hat{\mathbf{w}}_{i,j}\|^2}$$

where the notation $(m, l) \succ (i, j)$ denotes that either $m > i$ or $m = i$ and $l > m$. A similar modification applies to the maximum per-antenna power minimization problem.

IV. DISTRIBUTED DOWNLINK BEAMFORMING

The derivation of uplink-downlink duality via Lagrangian theory forms the basis for numerical algorithms for computing the optimal coordinated beamformers for the downlink multicell system. Our algorithms are based on the idea of iterative function evaluation, first proposed for the single-cell case in [5]. This paper generalizes the algorithm to the multicell system.

An important consideration for algorithms design in a multicell system is the issue of distributed implementation. In the second half of this section, we show that for TDD systems, the proposed algorithms naturally lead to a distributed per-cell implementation.

A. Iterative Function Evaluation Algorithm

We first present numerical algorithm for finding the optimal beamformer for the weighted sum power minimization problem (5). The main idea is to solve the downlink beamforming problem in the dual uplink domain by first finding the optimal $\lambda_{i,j}$, then the corresponding $\hat{\mathbf{w}}_{i,j}$. To find the optimal $\lambda_{i,j}$, we first take the gradient of the Lagrangian (8) with respect to $\mathbf{w}_{i,j}$ and set it to zero:

$$\left[\alpha_i \mathbf{I} - \left(1 + \frac{1}{\gamma_{i,j}} \right) \lambda_{i,j} \mathbf{h}_{i,i,j} \mathbf{h}_{i,i,j}^H + \sum_{m,n} \lambda_{m,n} \mathbf{h}_{i,m,n} \mathbf{h}_{i,m,n}^H \right] \mathbf{w}_{i,j} = 0 \quad (26)$$

Thus

$$\Sigma_i \mathbf{w}_{i,j} = \left(1 + \frac{1}{\gamma_{i,j}} \right) \lambda_{i,j} \mathbf{h}_{i,i,j} \mathbf{h}_{i,i,j}^H \mathbf{w}_{i,j} \quad (27)$$

where Σ_i is as defined in (11).

Now, multiplying both sides by $\mathbf{h}_{i,i,j}^H \Sigma_i^{-1}$, we get:

$$\mathbf{h}_{i,i,j}^H \mathbf{w}_{i,j} = \left(1 + \frac{1}{\gamma_{i,j}}\right) \lambda_{i,j} \mathbf{h}_{i,i,j}^H \Sigma_i^{-1} \mathbf{h}_{i,i,j} \mathbf{h}_{i,i,j}^H \mathbf{w}_{i,j} \quad (28)$$

Finally, cancelling out the $\mathbf{h}_{i,i,j}^H \mathbf{w}_{i,j}$ factor on both sides of the equation, we obtain a necessary condition for optimal $\lambda_{i,j}$:

$$\lambda_{i,j} = \frac{1}{\left(1 + \frac{1}{\gamma_{i,j}}\right) \mathbf{h}_{i,i,j}^H \Sigma_i^{-1} \mathbf{h}_{i,i,j}} \quad (29)$$

which can be used iteratively to obtain the optimal $\lambda_{i,j}$.

The algorithm is summarized as follows:

- 1) Find the optimal uplink power allocation $\lambda_{i,j}$ using the iterative function evaluation:

$$\lambda_{i,j} = \frac{1}{\left(1 + \frac{1}{\gamma_{i,j}}\right) \mathbf{h}_{i,i,j}^H \Sigma_i^{-1} \mathbf{h}_{i,i,j}} \quad (30)$$

where

$$\Sigma_i = \alpha_i \mathbf{I} + \sum_{m,n} \lambda_{m,n} \mathbf{h}_{i,m,n} \mathbf{h}_{i,m,n}^H \quad (31)$$

- 2) Find the optimal uplink receive beamformers based on the optimal uplink power allocation $\lambda_{i,j}$:

$$\hat{\mathbf{w}}_{i,j} = \left(\sum_{m,l} \lambda_{m,l} \sigma^2 \mathbf{h}_{i,m,l} \mathbf{h}_{i,m,l}^H + \sigma^2 \alpha_i \mathbf{I} \right)^{-1} \mathbf{h}_{i,i,j} \quad (32)$$

- 3) Find the optimal transmit downlink beamformers by scaling $\hat{\mathbf{w}}_{i,j}$:

$$\mathbf{w}_{i,j} = \sqrt{\delta_{i,j}} \hat{\mathbf{w}}_{i,j} \quad (33)$$

The global convergence of this algorithm is guaranteed by the duality result discussed in the previous section together with the convergence of the iterative function evaluation which can be justified by a line of reasoning similar to that in [5]. The proof is based on the property of standard functions [38]. In particular, one can stack the dual variables $\lambda_{i,j}$ into one vector Υ . Then (30) can be rewritten as

$$\lambda_{i,j}^{(t+1)} = f_{i,j}(\Upsilon^{(t)}), \quad i = 1 \dots N, \quad j = 1 \dots K \quad (34)$$

The function $f_{i,j}$ satisfies the following properties:

- 1) If $\lambda_{i,j} \geq 0 \forall i, j$, then $f_{i,j}(\Upsilon) > 0$.
- 2) If $\lambda_{i,j} \geq \lambda'_{i,j} \forall i, j$, then $f_{i,j}(\Upsilon) \geq f_{i,j}(\Upsilon')$
- 3) For $\rho > 1$, we have $\rho f_{i,j}(\Upsilon) > f_{i,j}(\rho \Upsilon) \forall i, j$.

as shown in the Appendix. These properties guarantee that $f_{i,j}$ is a standard function. Thus, starting with some initial $\Upsilon^{(0)}$, the iterative function evaluation algorithm converges to a unique fixed point, which must be the optimal downlink power by duality.

B. Minimizing the Maximum Antenna Power

To solve the problem of minimizing the maximum antenna power (19), we use Theorem 2 to solve the dual uplink beamforming problem with uncertain noise (20). Unlike the total weighted transmitted power minimization problem, solving the dual of the maximum antenna power minimization problem requires finding both the uncertain noise covariance matrices

\mathbf{Q}_i and the transmit uplink powers $\lambda_{i,j}$. The idea is to solve the dual problem by iteratively computing the inner minimization on $(\lambda_{i,j}, \hat{\mathbf{w}}_{i,j})$ and the outer maximization on \mathbf{Q}_i .

The inner minimization can be solved using the iterative function evaluation approach presented in the previous section. As to the outer maximization, we use a subgradient projection approach similar to the one presented in [23]. Consider the function $\phi(\mathbf{Q}_1, \dots, \mathbf{Q}_N)$ which is a subproblem of (20) with fixed \mathbf{Q}_i :

$$\begin{aligned} \phi(\mathbf{Q}_1, \dots, \mathbf{Q}_N) = & \underset{\lambda_{i,j}}{\text{minimize}} \quad \sum_{i,j} \lambda_{i,j} \sigma^2 \\ & \text{subject to} \quad \Lambda_{i,j} \geq \gamma_{i,j} \quad \forall i, j \end{aligned} \quad (35)$$

where

$$\Lambda_{i,j} = \max_{\hat{\mathbf{w}}_{i,j}} \frac{\lambda_{i,j} |\hat{\mathbf{w}}_{i,j}^H \mathbf{h}_{i,i,j}|^2}{\sum_{(m,l) \neq (i,j)} \lambda_{m,l} |\hat{\mathbf{w}}_{i,j}^H \mathbf{h}_{i,m,l}|^2 + \hat{\mathbf{w}}_{i,j}^H \mathbf{Q}_i \hat{\mathbf{w}}_{i,j}}$$

It is shown in [23] that ϕ is concave in $(\mathbf{Q}_1, \dots, \mathbf{Q}_N)$. Further, let $\mathbf{w}_{i,j}$ be the optimal downlink beamforming vector, then $\text{diag}[\sum_j \mathbf{w}_{i,j} \mathbf{w}_{i,j}^H]$ is a subgradient for ϕ with respect to \mathbf{Q}_i . Therefore, the outer maximization can be done based on a subgradient projection approach, with projection onto the constraint set $S_{\mathbf{Q}_i} = \{\mathbf{Q}_i : \text{tr}(\mathbf{Q}_i) \leq N_t, \mathbf{Q}_i \succeq 0\}$.

The algorithm is summarized as follows:

- 1) Initialize $\mathbf{Q}_i^{(0)}$ for $i = 1, 2, \dots, N$
- 2) Fix $\mathbf{Q}_i^{(n)}$. Find the optimal uplink power allocation $\lambda_{i,j}$ using the iterative function evaluation:

$$\lambda_{i,j} = \frac{1}{\left(1 + \frac{1}{\gamma_{i,j}}\right) \mathbf{h}_{i,i,j}^H [\Theta_i^{(n)}]^{-1} \mathbf{h}_{i,i,j}} \quad (36)$$

where

$$\Theta_i^{(n)} \triangleq \mathbf{Q}_i^{(n)} + \sum_{m,n} \lambda_{m,n} \mathbf{h}_{i,m,n} \mathbf{h}_{i,m,n}^H \quad (37)$$

- 3) Find the optimal uplink receive beamformers based on the optimal uplink power allocation $\lambda_{i,j}$:

$$\hat{\mathbf{w}}_{i,j} = \left(\sum_{m,l} \lambda_{m,l} \sigma^2 \mathbf{h}_{i,m,l} \mathbf{h}_{i,m,l}^H + \sigma^2 \mathbf{Q}_i^{(n)} \right)^{-1} \mathbf{h}_{i,i,j}$$

- 4) Find the optimal transmit downlink beamformers by scaling $\hat{\mathbf{w}}_{i,j}$:

$$\mathbf{w}_{i,j} = \sqrt{\delta_{i,j}} \hat{\mathbf{w}}_{i,j} \quad (38)$$

- 5) Update $\mathbf{Q}_i^{(n)}$ using the subgradient projection method with step size t_n :

$$\mathbf{Q}_i^{(n+1)} = \mathcal{P}_{S_{\mathbf{Q}_i}} \left\{ \mathbf{Q}_i^{(n)} + t_n \text{diag} \left[\sum_j \mathbf{w}_{i,j} \mathbf{w}_{i,j}^H \right] \right\} \quad (39)$$

The projection operation is a simple renormalization, i.e. multiplication by a constant so that $\text{tr}(\mathbf{Q}_i^{(n+1)}) = N_t$.

- 6) Increment n . Return to step 2) until convergence.

The global convergence of this algorithm is guaranteed by the duality result discussed earlier, the convergence of the iterative function evaluation and the convergence of the subgradient projection method due to the concavity of $\phi(\mathbf{Q}_1, \dots, \mathbf{Q}_N)$ [39]. Note that the algorithm can be extended in a straightforward fashion when dirty-paper coding is used.

C. Distributed Implementation

An important feature of the iterative function evaluation algorithms proposed in subsection IV-A and IV-B is that they can be implemented in a distributed fashion in a TDD system, where uplink and downlink channels are reciprocals of each other. In this case the virtual dual uplink is the real uplink.

Consider first the iterative function evaluation step of the algorithm. The function iteration (30) on the uplink power $\lambda_{i,j}$ involves channel vectors $\mathbf{h}_{i,j}$ within each cell only, which the base-station typically has the knowledge of, and the matrix $\mathbf{\Sigma}_i$. Observe that $\mathbf{\Sigma}_i$ is essentially the covariance matrix of the received signal in the uplink direction at the base-station i , which includes the intended signal, the interference, and a scaled version of the background noise. In a TDD system, this covariance matrix may be estimated locally at each base-station in the uplink direction. Channel reciprocity implies that the signal and interference covariance matrices at the uplink receiver are exactly as required. In addition, the background noise level is typically known, so the scaling factor α_i can be easily compensated for. Thus, we can perform the iterative updates of $\lambda_{i,j}$ and the estimation of $\mathbf{\Sigma}_i$ on a per-cell basis, i.e. the iterative function evaluation process can be done locally without the need of explicit inter-base-station coordination. This uplink per-cell iteration process always converges. It converges to the optimal uplink power. Note that base-station coordination is achieved implicitly via uplink power control (i.e. the update of $\lambda_{i,j}$'s, which affect all other $\mathbf{\Sigma}_i$'s). The channel side information is needed only for users within each cell, and not for out-of-cell interferers.

Secondly, the beamforming vectors can also be easily obtained in the dual uplink. This is essentially receiver MMSE beamforming at the base-station.

Finally, to use the uplink beamformer for downlink transmission, one needs to scale it by the right $\delta_{i,j}$. This involves a matrix inversion (18). But, this process is equivalent to a downlink power control problem on the effective matrix channel for achieving a desirable set of SINRs, which, by the classic result of Foschini and Miljanic [40], has a distributed implementation using a per-user power update algorithm. Each step of the algorithm sets a $\delta_{i,j}$ to satisfy its corresponding SINR constraint with equality assuming all other $\delta_{i,j}$'s are fixed. The convergence of the algorithm can be proved either using the method of [40] or by a standard function argument [38].

In fact, both the uplink per-cell iterative function evaluation and the downlink power control part of the algorithm can even be implemented asynchronously at each base-station using possibly outdated power information. The convergence of such asynchronous update is still guaranteed by the standard function argument as shown in Theorem 4 of [38]. The only necessary synchronization is that the base-stations must all be in the uplink phase or the downlink phase together, so that the three steps of the algorithm can be executed consecutively.

The above remarks are also applicable to the maximum per-antenna power minimization problem. The only modifications are the projection operation on \mathbf{Q}_i , i.e. (39), and the modification of the received covariance matrix (37) based on \mathbf{Q}_i , both of which can be done on a per-cell basis.

D. Beamformer-Power Iteration Algorithm

An alternative approach for solving the downlink beamforming problem is to iteratively update the beamformers and the power, as proposed in [19], [36]. For the minimization of the total weighted transmitted power, the algorithm goes as follows:

- 1) Initialize $\hat{\mathbf{w}}_{i,j}$;
- 2) Find the $\lambda_{i,j}$ to satisfy the SINR constraints of (6) with equality;
- 3) Find the uplink receive beamformers based on the uplink power allocation $\lambda_{i,j}$:

$$\hat{\mathbf{w}}_{i,j} = \left(\sum_{m,l} \lambda_{m,l} \sigma^2 \mathbf{h}_{i,m,l} \mathbf{h}_{i,m,l}^H + \sigma^2 \alpha_i \mathbf{I} \right)^{-1} \mathbf{h}_{i,j} \quad (40)$$

- 4) Go to step 2 until convergence;
- 5) Update the transmit downlink beamformers

$$\mathbf{w}_{i,j} = \sqrt{\delta_{i,j}} \hat{\mathbf{w}}_{i,j} \quad (41)$$

The convergence of the iterations involving Steps 2 and 3 is shown in [36]. By our multicell duality result, the algorithm must also converge to the global optimal solution for the downlink.

Both the iterative function evaluation algorithm and the beamformer-power iteration algorithm give the optimal solution for the multicell downlink beamforming problem. Both can be implemented directly in a TDD system. However, an implementation of the beamformer-power iteration algorithm requires repeated updates of uplink powers in Step 2 above, which would itself require either centralized processing (e.g. by performing a matrix inversion) or a separate iterative process (e.g. by fixed-point iteration). As both Steps 2 and 3 above need to be repeatedly executed by all users at the same time, the beamforming-power iteration algorithm would require a higher level of synchronization among all the users and all the base-stations than the iterative function evaluation algorithm proposed in the previous section.

V. SIMULATIONS

We begin by investigating the benefit of coordinated beamforming in a two-cell configuration shown in Fig. 2. Standard cellular network parameters are used in simulation: the noise power spectral density is set to -162 dBm/Hz; the channel vectors are chosen according to the distance-dependent path loss $L = 128.1 + 37.6 \log_{10}(d)$, where d is the distance in kilometers, with 8dB log-normal shadowing, and a Rayleigh component. The distance between neighboring base-stations is set to be 2.8km. An antenna gain of 15dBi is assumed. For illustration purposes, the weighting factors corresponding to the base-station antenna power constraints are set to be $\alpha_i = 1$.

Fig. 3 shows the benefit of coordinated beamforming when there are two users per cell. Fig. 4 shows a similar case with three users per cell. In both cases, one of the users in each cell is located at distance d away from its own base-station on the straight line connecting the two base-stations. The other users are located randomly elsewhere in the cell.

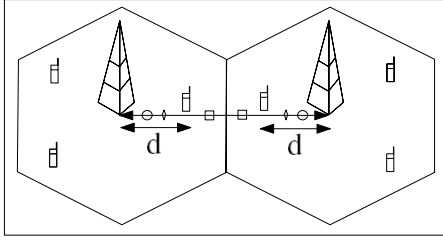


Fig. 2. A two-cell configuration with two users located between two base-stations at distance d .

The base-stations are equipped with four antennas. It is clear from the figures that the coordinated beamforming system significantly outperforms the conventional per-cell optimized system, especially at the high SINR target range. The benefit is the largest when the users are close to the cell edge as expected.

Fig. 3 and Fig. 4 also illustrate that the performance gain due to coordinated beamforming is larger in the two-user-per-cell case as compared to the three-user-per-cell case. Intuitively, when the number of users per cell is small as compared to the number of base-station antennas, there are spare dimensions available for interference suppression. This is when coordinated beamforming shows the most benefit.

It is interesting to note that in the two-cell two-user-per-cell case, the conventional system eventually becomes infeasible as the SNR target increases. Yet, the coordinated beamforming system is always feasible. This is because the base-stations are equipped with four antennas. With a total of four users between the two cells, the coordinated system has the capability of zero-forcing the out-of-cell users, thus completely eliminating the out-of-cell interference. In contrast, out-of-cell interference is always present in the conventional system. Note that complete zero-forcing is no longer possible in the three-users-per-cell configuration. Nevertheless, coordinated beamforming still produces significant power saving.

Fig. 5 and Fig. 6 show the performance of coordinated beamforming in a 7-cell network with 3 randomly located users per cell (as in Fig. 1) under the minimum total transmit power and the minimum maximum antenna power criteria, respectively. Again, each base-station is equipped with 4 antennas. It is again observed that while the joint optimization algorithm outperforms the conventional per-cell update in the range of high SINR targets. This is due to the fact that at high SINRs, the multicell network becomes predominantly interference limited. The figures also show the dirty paper coding gain for both the joint optimization and the per-cell update algorithms.

Fig. 7 illustrates the power saving in the maximum antenna power by running the per-antenna optimization algorithm. For this 7-cell 3-user-per-cell case, the power saving is in the 1-2dB range.

To illustrate the convergence behavior of the proposed iterative evaluation algorithm, we compare it with the beamformer-power iterative algorithm of [19]. Fig. 8 shows the norm residue of the uplink transmitted power (in mW) versus the

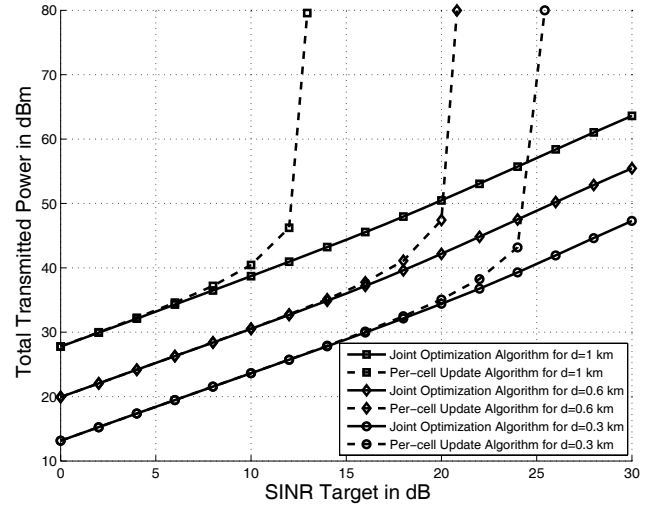


Fig. 3. Total transmitted power versus the SINR targets for the joint optimization of a coordinated beamforming system and the per-cell update of a conventional system for two-cell network with two users per cell. One of the users are at various distances d away from its own base-stations. The base-stations are equipped with four antennas.

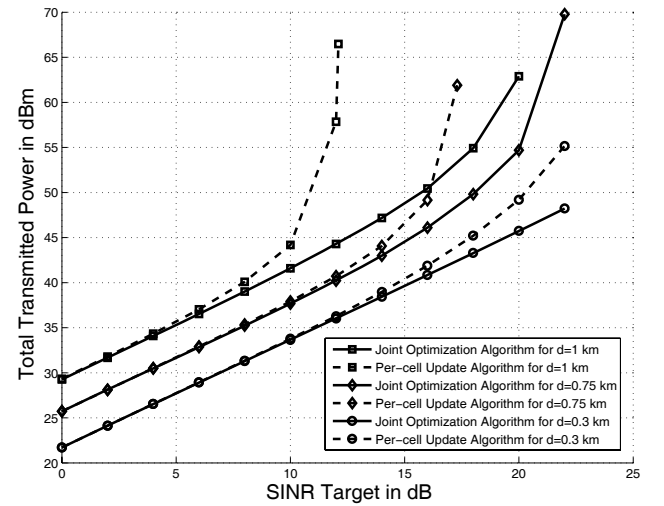


Fig. 4. Total transmitted power versus the SINR targets for the joint optimization of a coordinated beamforming system and the per-cell update of a conventional system for two-cell network with three users per cell. One of the users are at various distances d away from its own base-stations. The base-stations are equipped with four antennas.

number of iterations. The norm residue is defined as:

$$R^{(n)} = \sigma^2 \|\Upsilon^{(n)} - \Upsilon^*\|_2 \quad (42)$$

where Υ^* represents the optimal power vector. It is observed that while the beamformer-power iterative algorithm converges more rapidly at the beginning, the distributed iterative function evaluation algorithm in fact provides faster convergence asymptotically. Note that for the distributed iterative function evaluation algorithm, each iteration step requires a covariance matrix estimation of the received signal at the base-stations. The algorithm convergence speed will therefore typically be affected by the accuracy of the estimation process, especially when channels change over time.

Finally, we note that the convergence speed of the iterative

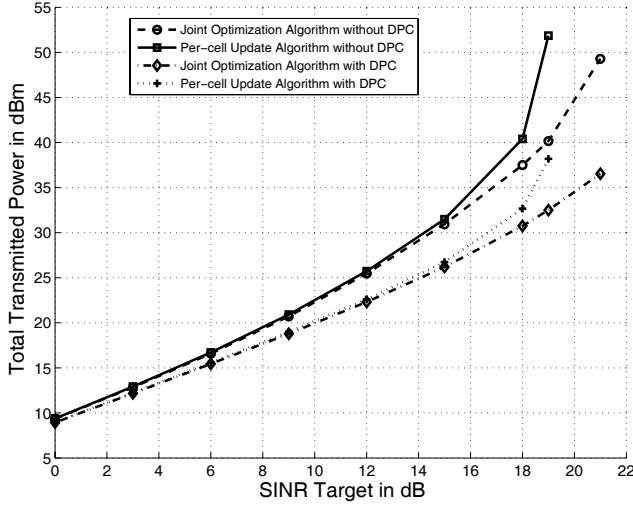


Fig. 5. Total transmitted power versus the SINR targets for the joint optimization of a coordinated beamforming system and the per-cell update of a conventional system for a 7-cell network with 3 users per cell with and without dirty-paper coding. The base-stations are equipped with four antennas.

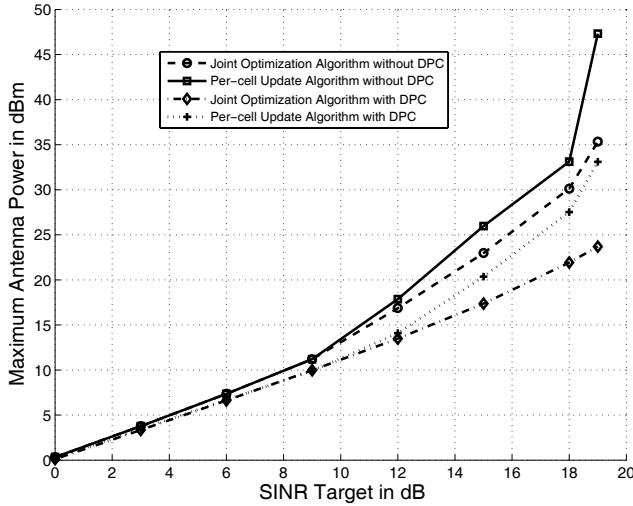


Fig. 6. The maximum per-antenna power versus the SINR targets for the joint optimization of a coordinated beamforming system and the per-cell update of a conventional system for a 7-cell network with 3 users per cell with and without dirty-paper coding. The base-stations are equipped with four antennas.

function evaluation algorithm depends not only on the size of the problem (e.g. the number of antennas, the number of cells and the number of users per cell), but also on the SINR targets. Fig. 9 shows the norm residue versus the number of iterations for different values of the SINR targets. It is observed that convergence becomes slower when the SINR target increases.

VI. CONCLUSION

This paper provides a solution to the optimal coordinated downlink beamforming design problem for a multicell network with multiple users per cell. The uplink-downlink duality is generalized to the multicell case using the Lagrangian theory for two different design criteria: minimizing the total weighted transmitted power and minimizing the maximum antenna power subject to SINR constraints. An iterative function

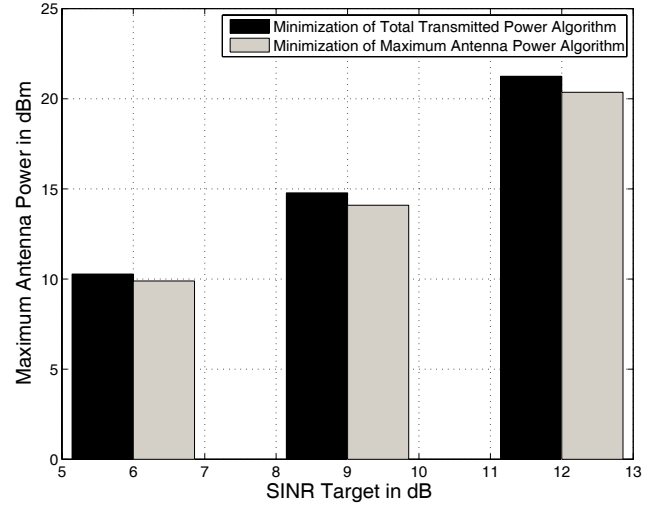


Fig. 7. Comparing the maximum antenna power for the total power minimization algorithm and the maximum antenna power minimization algorithm.

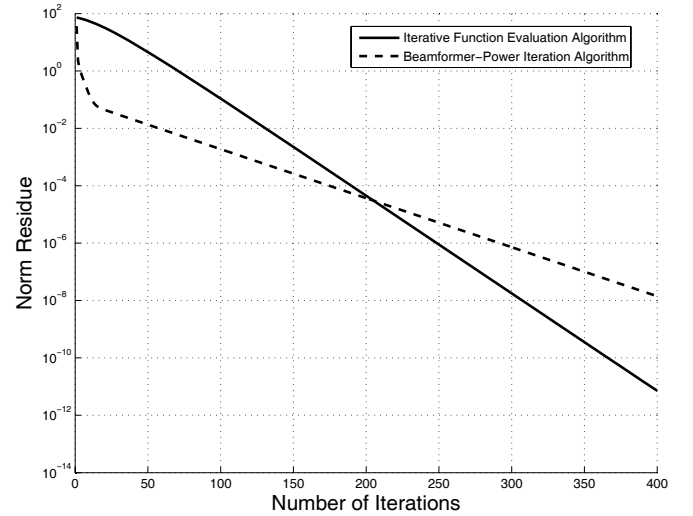


Fig. 8. The norm residue versus the number of iterations for the two optimization algorithms.

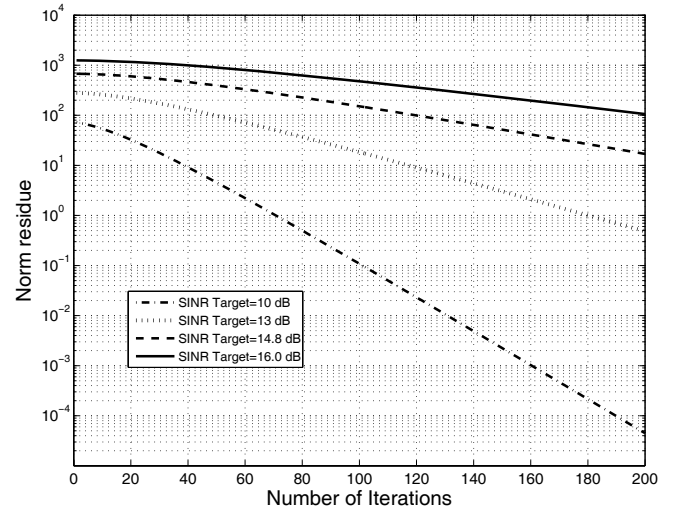


Fig. 9. The norm residue versus the number of iterations for different SINR targets.

evaluation based algorithm which is capable of finding the globally optimal coordinated beamformers is presented. A key feature of the algorithm is that it can be implemented in a distributed fashion for the TDD system. The proposed algorithm is efficient, and it outperforms conventional wireless systems with per-cell signal processing.

APPENDIX

This appendix presents proofs of standard function properties satisfied by f_{ij} in (34). The proofs are similar to the ones presented in [5], and are included here for completeness.

- 1) If $\lambda_{i,j} \geq 0 \forall i, j$, then $f_{i,j}(\Upsilon) > 0$.

Proof: This property holds because if $\lambda_{i,j} \geq 0$ then $\Sigma_i \succ 0$ and consequently $\Sigma_i^{-1} \succ 0$. Thus $\mathbf{h}_{i,i,j}^H \Sigma_i^{-1} \mathbf{h}_{i,i,j} > 0$ and consequently $f_{i,j}(\Upsilon) > 0$. ■

- 2) If $\lambda_{i,j} \geq \lambda'_{i,j} \forall i, j$, then $f_{i,j}(\Upsilon) \geq f_{i,j}(\Upsilon')$.

Proof: Assume $\lambda_{i,j} \geq \lambda'_{i,j}$. Then,

$$f_{i,j}(\Upsilon) = \frac{1}{\left(1 + \frac{1}{\gamma_{i,j}}\right) \mathbf{h}_{i,i,j}^H \Sigma_i^{-1} \mathbf{h}_{i,i,j}} \quad (43)$$

where

$$\begin{aligned} \Sigma_i &= \alpha_i \mathbf{I} + \sum_{m,n} \lambda_{m,n} \mathbf{h}_{i,m,n} \mathbf{h}_{i,m,n}^H \\ &= \alpha_i \mathbf{I} + \sum_{m,n} \lambda'_{m,n} \mathbf{h}_{i,m,n} \mathbf{h}_{i,m,n}^H \\ &\quad + \sum_{m,n} (\lambda_{m,n} - \lambda'_{m,n}) \mathbf{h}_{i,m,n} \mathbf{h}_{i,m,n}^H \end{aligned} \quad (44)$$

Now, since $\lambda_{i,j} \geq \lambda'_{i,j}$, we have $\sum_{m,n} (\lambda_{m,n} - \lambda'_{m,n}) \mathbf{h}_{i,m,n} \mathbf{h}_{i,m,n}^H \succeq 0$. But as shown in [5], for positive semidefinite matrices \mathbf{C} and \mathbf{D} and vector \mathbf{x} in the range of \mathbf{C} :

$$\frac{1}{\mathbf{x}^T (\mathbf{C} + \mathbf{D})^{-1} \mathbf{x}} \geq \frac{1}{\mathbf{x}^T \mathbf{C}^{-1} \mathbf{x}} \quad (45)$$

with equality if and only if $\mathbf{D}(\mathbf{C} + \mathbf{D})^{-1} \mathbf{x} = 0$. Thus

$$\frac{1}{\mathbf{h}_{i,i,j}^H \Sigma_i^{-1} \mathbf{h}_{i,i,j}} \geq \frac{1}{\mathbf{h}_{i,i,j}^H \Sigma_i'^{-1} \mathbf{h}_{i,i,j}} \quad (46)$$

where

$$\Sigma_i' = \left(\sum_{m,n} \lambda'_{m,n} \mathbf{h}_{i,m,n} \mathbf{h}_{i,m,n}^H + \alpha_i \mathbf{I} \right) \quad (47)$$

Hence, $f_{i,j}(\Upsilon) \geq f_{i,j}(\Upsilon')$. ■

- 3) For $\rho > 1$, $\rho f_{i,j}(\Upsilon) > f_{i,j}(\rho \Upsilon) \forall i, j$.

Proof: Let $\rho > 1$,

$$\rho f_{i,j}(\Upsilon) = \frac{1}{\left(1 + \frac{1}{\gamma_{i,j}}\right) \mathbf{h}_{i,i,j}^H (\rho \Sigma_i)^{-1} \mathbf{h}_{i,i,j}} \quad (48)$$

where

$$\begin{aligned} \rho \Sigma_i &= \rho \alpha_i \mathbf{I} + \rho \sum_{m,n} \lambda_{m,n} \mathbf{h}_{i,m,n} \mathbf{h}_{i,m,n}^H \\ &= (\rho - 1) \alpha_i \mathbf{I} + \alpha_i \mathbf{I} + \rho \sum_{m,n} \lambda_{m,n} \mathbf{h}_{i,m,n} \mathbf{h}_{i,m,n}^H \end{aligned}$$

Since $\rho > 1$, we have $(\rho - 1) \alpha_i \mathbf{I} \succeq 0$. Based on (45), we get

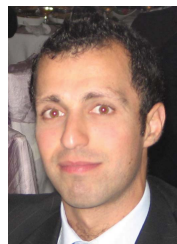
$$\frac{1}{\mathbf{h}_{i,i,j}^H (\rho \Sigma_i)^{-1} \mathbf{h}_{i,i,j}} \geq \frac{1}{\mathbf{h}_{i,i,j}^H \left(\alpha_i \mathbf{I} + \rho \sum_{m,n} \lambda_{m,n} \mathbf{h}_{i,m,n} \mathbf{h}_{i,m,n}^H \right)^{-1} \mathbf{h}_{i,i,j}} \quad (49)$$

Thus, $\rho f_{i,j}(\Upsilon) \geq f_{i,j}(\rho \Upsilon)$. Finally, it is easy to check that the equality condition is not satisfied. Thus, $\rho f_{i,j}(\Upsilon) > f_{i,j}(\rho \Upsilon)$ strictly. ■

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