SIR-Based Power Control Used in CDMA Systems

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***Abstract*—This passage summarizes some results on signal to interference (SIR) based power control algorithms The classic works of Nettleton, and Alavi attracted considerable attention in the nineties. The modern approach to the power balancing control problem in wireless networks, formulated by Zander in 1992, and made a progress in 1997. It converts the real problems into a matrix-associated questiones.**

I. Introduction

CDMA has attracted more attention to its increasing capacity. Power control is an useful way to the near-far effect in uplink and the corner effect in the downlink, so as to increase the capacity of or has an efficiency on the CDMA system.In cellular wireless communication system, an uplink specific is based on the mobile battery power.Also, downlink codes are synchronous and can be made orthogonal; but uplink codes arrive at the base station asynchronously, resulting in cross correlation, and causing high in-cell interference potential unless power is controlled. For this reason we concentrate on the uplink.

Unlike the conditions in FDMA/TDMA cellular radio systems where in any cochannel set only one mobile in each cell, in CDMA cellular systems all the mobiles in each cell use the same frequency, which results in three dimensional link gain matrices. SIR balancing is not an eigenvalue problem any more unless intra-cell SIR balancing is assumed and the link gain matrices are simplified to two dimensions [1]. An n th-power-of-distance power control law has been presented and discussed using a simple power control model. This model based on the distance from the base station has been investigated by [1]. He introduces the concept of SIR balancing, which yields a "fair" distribution of the interference in the sense that all users experience the same SIR level. The problem is identified as an eigenvalue problem for positive matrices. Besides, Zander further apply and extend these results to spread spectrum cellular radio systems. In these systems, the adjacent channel interference also has to be taken into account. Also, Zander made the progress on the Centralized Power Control(CPC). He do the calculation in the sense that it minimizes the interference probability. It will be seen that the SIR balancing technique plays a key role in this minimization process [2]. In [3], a centralized constrained power control scheme to compute SIR for the system follows naturally from a proposition so as to get a high quality of communication, which can be seemed as an opposite algorithm. The matrix above is based on the link gain consists of BS to BS, seemed not to be available for CDMA cellular systems directly. It would caused deviations when the system reaches the highest capacity. So Qiang Wu converts the matrix into a user $\times$ user-based matrix in order to get the lowest effect when a user is canceled by the algorithm [**?**] [4].

The remainder of the paper is organized as follows. Section II presents the interference model. In Section III, we use the model to analyze the initial power control schemes. The constrained CPC algorithm will be discussed in Section IV. Next, numerical results are discussed in Section V. Finally, Section VI provides conclusions of the paper.

II. Initial System Model

This section will show that if the effect of noise power on the interfering signal experienced is sufficiently small to be neglected, formulating the power control problem mathematically leads to an eigenvalue problem involving positive matrices. If we let γ_d and γ_u denote the desired downlink and uplink SIR's, respectively, for all users, Nettleton and Alavi [1] showed that the balanced power vectors, $\boldsymbol{p}^D$ and $\boldsymbol{p}^U$ must satisfy the eigenvalue problems:

$$\boldsymbol{G}^U\boldsymbol{p}^U = \frac{1+\gamma^U}{\gamma^U}\boldsymbol{p}^U \tag{1}$$

$$\boldsymbol{G}^D\boldsymbol{p}^D = \frac{1+\gamma^D}{\gamma^D}\boldsymbol{p}^D \tag{2}$$

where the matrix $\boldsymbol{G}^U = (\boldsymbol{G}^D)^T \in \mathbb{C}^{N\times N}$, that we prove in the appendix, is a nonnegative matrix of known parameters whose size depends on the number of mobiles of each cell and whose entries depend on the distances from each user to each base station. N is the total number of users under considered cells. The derivation will be given later.

The users in CDMA share same bandwidth compared with TD/FDMA, so the data have three dimension. SIR balancing is not an eigenvalue problem any more unless intra-cell SIR balancing is assumed and matrices are simplified to two dimensions [1]. So we set up the assumptions as following:

1) The service area consists of K cells. The analysis here is general and has less influences due to the shape of the cell.
2) The load of the i-th cell is L_i. Although the general traffic model permits variations about this mean from time to time, we simplify the model which is L_i users are linked to the i-th base station at the given time. The total number of users is

$$\sum_{i=1}^{K} L_i = N \tag{3}$$

3) The i-th base station receives P_i^U of power from one user in the i-th cell' BS. The power transmitted by the k-th user in the i-th cell is P_{ik}^U, k = 1, 2, ...,L_i.
4) Base station i transmits an average power P_i^D per user at user's position, and the actual transmitter power from i-th BS to k-th user at the BS,P_{ik}^D. Thus,

$$\sum_{k=1}^{L_i} P_{ik}^D = L_i P_i^D \tag{4}$$

5) The control information state between each base station and each user is required. Assumed that the users have a random distribution and has uniform propagation characteristics with an inverse-$alpha$ law. The distance between the k-th user and the i-th cell and the base station of the j-th cell is denoted by d_{ikj}. Thus the totoal power received by user k in cell i is given by

$$P_{rx,k}^D = \sum_{j=1}^{N} \frac{L_j P_j^D}{d_{ikj}^{\alpha}} \tag{5}$$

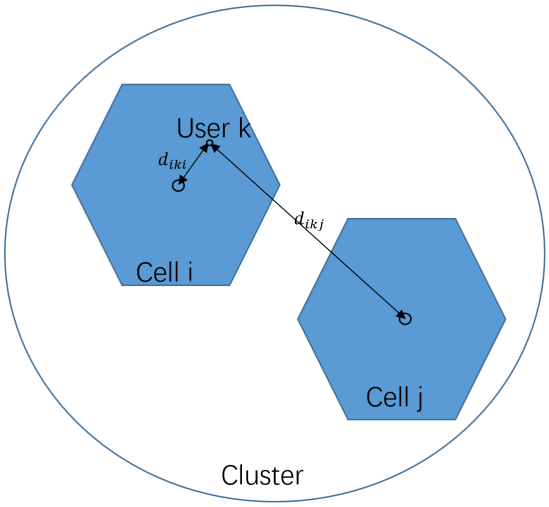

Fig. 1. Uplink and downlink interference geometry.

Abviously, it's easy to know that

$$P_{ik}^D = d_{iki}^{\alpha} P_i^D \tag{6}$$

In an actual implementation, it would probably be more difficult to acquire the control information state for each user due to their motions. However, the model is convenient for the initial simulation.

We have neglected the effects of shadow fading in this work, which leads to an optimistic results. We will add the fading at next step.

Proposition 1: There exists a unique maximum achievable SIR level:

$$\gamma^* = \max\{\gamma | \exists P \geq 0 : \Gamma_i \geq \gamma, \forall i\} \tag{7}$$

For the uplink, the total power received by the base station express as:

$$P_{sum} = \sum_{j=1}^{N} P_j^U \sum_{k=1}^{L_j} \left[\frac{d_{jkj}}{d_{jki}}\right]^{\alpha} \tag{8}$$

The signal from the users thus suffer the signal-to-interference ration at the base station:

$$\Gamma_i^U = \frac{P_i^U}{P_{sum} - P_i^U}$$

Hence,

$$\frac{1+\gamma_i^U}{\gamma_i^U} P_i^U = \sum_{j=1}^{N} P_j^U \sum_{k=1}^{L_j} \left[\frac{d_j kj}{d_j ki}\right]^{\alpha} \tag{9}$$

To balance the SIR for each user in cell i, and set all the SIR in cell i to be equal, and solve the P_i^U for all i, then we can get the desired signal in the uplink by(4). This needs the solution of linear algebraic equation. We define this operation as in-cell balancing because different cells may have different SIR^U values.

To get the SIR^U for all cells, we should know that:

$$\begin{aligned} &\boldsymbol{P}^U = \left[P_1^U, P_2^U, \ldots, P_N^U\right]^T; \\ &\boldsymbol{G}^U = \left[g_{ij}^U \times a_{ij}^U\right], where\ g_{ij}^U = \sum_{k=1}^{L_j} \left[\frac{d_{jkj}}{d_{jki}}\right]^{\alpha} \end{aligned} \tag{10}$$

Since we neglected the shadow fading, the factor a_{ij^U} can be set as 1. And now we have:

$$\boldsymbol{G}^U \boldsymbol{P}^U = \frac{1+\gamma^U}{\gamma^U} \boldsymbol{P}^U \tag{11}$$

as showed in (1). Thus, we have the relationship between the SIR and the eigenvalue:

$$\lambda^U = \frac{1+\gamma^U}{\gamma^U} \tag{12}$$

Then we convert the three-dimension control information to the two-dimension matrix problems, which is a classic eigenvalue problem with a eigenvector of P^U.

The downlink's situation seems like it is in the uplink. We can talk about it later. The downlink's gain link matrix is a transpose of the uplink matrix, so that they have the same eigenvalue.

III. The Optimal Centralized Power Control Algorithm

Let us consider a power control algorithm of the uplink, which has the uplinlk gain matrix G and may control the power vector P instantaneously. And assume that the transmission system requires a minimum (threshold) SIR γ_0^U. There are several methods in order to minimizes the interference probability [2].

Up to now, it caused a problem because of the matrix' composition. Each row or column consists of a whole cell's gain link information, while the removal of whole cell results in a worse case by the algorithm. We transform the link gain matrix from BS×BS to user×user. We set a CDMA system consisting of N cells with L_k users in the k-th cell,and the

total number is n. If the i-th user is m-th user in the k-th cell, we can get that:

$$i = \sum_{num=1}^{k} L_{num-1} + m \qquad 1 \le m \le L_{num} \tag{13}$$

where L_0 = 0, L_k(n = 1,2,...,N) is the number of active users in cell k at one moment. By using the equations, we can derive the uplink SIR matrix for mobile i in cell k by (2) - (9):

$$\gamma_i^U = \frac{g_{ik}P_i}{\sum\limits_{j\neq i} g_{jk}P_j} \tag{14}$$

We defind G_{ij} = $\frac{g_{jk}}{g_{ik}}$, so that we can get the G matrix as:

$$\gamma_i^U = \frac{P_i}{\sum\limits_{j\neq i} G_{ij}P_j} \tag{15}$$

Thus, the G geometry of uplink can be express as:

$$\boldsymbol{G}^U = \begin{bmatrix} \frac{g_{11}}{g_{11}} & \frac{g_{21}}{g_{11}} & \dots & \frac{g_{n1}}{g_{11}} \\ \frac{g_{11}}{g_{21}} & \frac{g_{21}}{g_{21}} & \dots & \frac{g_{n1}}{g_{21}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{g_{12}}{g_{L_1,1}} & \frac{g_{21}}{g_{L_1,1}} & \dots & \frac{g_{n1}}{g_{L_1,1}} \\ \frac{g_{12}}{g_{L_1+1,2}} & \frac{g_{22}}{g_{L_1+1,2}} & \dots & \frac{g_{n2}}{g_{L_1+1,2}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{g_{12}}{g_{L_1+L_2,2}} & \frac{g_{22}}{g_{L_1+L_2,2}} & \dots & \frac{g_{n2}}{g_{L_1+L_2,2}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{g_{1N}}{g_{nN}} & \frac{g_{2N}}{g_{nN}} & \dots & \frac{g_{nN}}{g_{nN}} \end{bmatrix} \tag{16}$$

Define: Compared with the initial power vector P^*, there is at least one optimum power vector P, of which the u-th power is zero, satisfy the demend of the PCA and have a better SIR value, then it has the form:

$$P_i^U = P_i^{*U}, i \neq u;$$
$$P_u^U = 0, \quad i = u.$$

proof: We can obtain the SIR value by equation(15):

$$\begin{aligned}\gamma_i^U &= \frac{P_i^U}{\sum\limits_{j=1}^{n} P_j^U G_{ij}^U - P_i^U} \\ &= \frac{P_I^{*U}}{\sum\limits_{j=1}^{n} P_j^{*U} G_{ij}^U - P_i^{*U} - P_k^{*U} G_{ik}^U} \\ &\ge \gamma_i^{*U} \ge \gamma_0, (i \neq k).\end{aligned}$$

P achieves a SIR that is not less than γ_0 while this is absolutely achieved by P^*. Therefore, P is also an optimal vector. So the removal will leads to a better SIR rather than make it decreased.

Besides, there is a new algorithm, which is called SRA, will need n eigenvalue computations in the worst case, while it will cost n^2 times roughly by removing the user freely. This action means the BS cannot maintain the communication among the exist users, so we need to canceled the user or set up a new BS for the demands.

Stepwise Removal Algorithm(**SRA**)

1) Step 1: Determine γ^* corresponding to G. if $\gamma^* \ge \gamma_0^U$, then use the eigenvector P^* and stop; else set $G'(\in \mathbb{C}^{n\times n}) = G(\in \mathbb{C}^{n\times n})$ and follow step 2.
2) Step 2: Remove user u for which the maximum of th row and colunm sums

$$r_u = \sum_{j=1}^{n} G_{uj}^{U'}$$
$$r_u^T = \sum_{j=1}^{n} G_{ju}^{U'}$$

is maximized and form the $G'(\in \mathbb{C}^{n\times n})$ matrix. Determine the γ^* correspoding to $G'(\in \mathbb{C}^{n\times n})$ by(9) ,where γ^* is the minimum of the SIR we can get, Furthermore, if the γ^* is still bigger than the limitation, of which can maintain the communication, use the eigenvector P^*, set $G'(\in \mathbb{C}^{n\times n}) = G\left(\in \mathbb{C}^{n-1\times n-1}\right)$, and repeat step 2.

Stepwise Maximum-Inteference Removal Algorithm (**SMIRA**)

The SMIRA is the same like SRA except the step 2, user k is removed if the maximum fo the sums

$$r_u = \sum_{j=1}^{n} G_{uj}^{U'} P_j, \qquad r_u^T = \sum_{j=1}^{n} G_{ju}^{U'} P_u \tag{17}$$

is maximized.

r_k represents the total interference for user k in the uplink and r_u^T of it represents the total interference to other users caused by user u [5]. Sometimes, the link gain cannot represent the interference clearly, so as to times the power vector, it can be more accuracy, although they still cannot represent these two concept exactly.

IV. Numerical Result

To derive the numerical result of the system model, more assumptions for simulation are given as following. The system includes a total 4 square cells, and a cluster of 16 cell at the final simulation, at the center there is the base station with omni-directional antennas. The mobiles are located randomly with a uniform density, which means every cell has the same number of users, and transport the signal at a constant rate. The link gain G_{ij} is describe as: G_{ij} = $\frac{A_{ij}}{d_{ij}^{\alpha}}$, where **A** is the fading component with 0dB in this case. d_{ij} is the propagation distance and α is the path loss exponent, which we set α = 3 here. Furthermore, there is a minimization of distance d_0, when the propagation distance is less than d_0, we should set the distance equal to d_0. Besides, the system model will be improved by the UAV channels. The situation becomes more complicated while they have a same properties.

Fig2 describes the relationship between SIR and the number of users per cell with the initial algorithm. The value of SIR has a threshold, of which we set as -20dB. So that we can make sure the range of the capacity and do the simulation more accuracy.

In Fig3, we change the algorithm by the SRA. Then the figure shows the SIR value, for the uniform distribution case, with cell to cell balancing, removing the users whose interference have the worst effects. Uplink and downlink results are distinguishable distribution. For this case, we just list the uplink performance in Fig3. The latter cancels the PCA algorithm and set the users' transmit power all to be equal, and choose the worst-performance user's SIR. All the values get from an average of 500-time iterations. The results of the two types of SIR are barely distinguishable, particularly when the number of users reaches 50, that the worst SIR is extremely lower than the balancing one.

Fig4 noted the performance achieved by the SRA, SMIRA and original algorithm, with global SIR balancing and perfect average power control of N=16, the spreading channel rate are constant, and also have a omni-directional antennas. We set the SIR threshold for -10dB for the removing algorithm. Thus, the system has at least a -10dB SIR performance, and cannot support the users be removed. The moving algorithm is just the same with removing algorithm, except change the user, of which cannot achieve the standard SIR, belong to another BS instead of canceling the users. It behaves better than the original one but not much. However, it solves the problem of assignments of the users, so as to be more flexible than the original one.

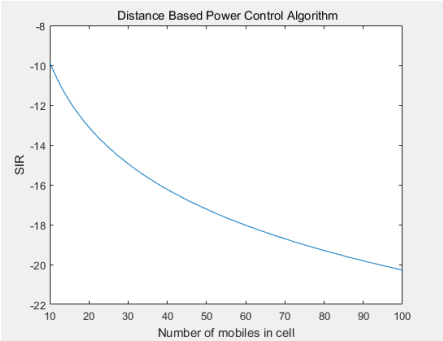

Fig. 2. Initial Global PCA

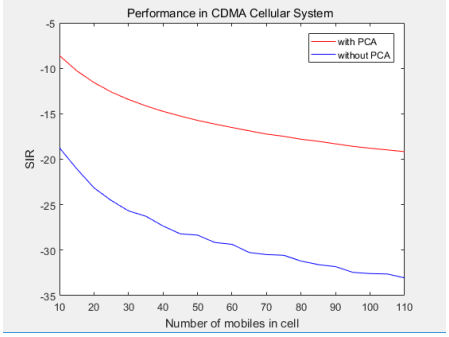

Fig. 3. SIR with Fully Balanced Systems vs. without Balanced Systems

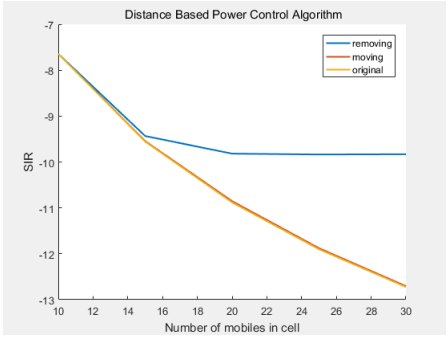

Fig. 4. SIR Comparison

V. Conclusion

The conclusion goes here.

Acknowledgment

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