

Fitness Fatigue Model

Systems Model that takes an input (training load $w(t)$) and predicts performance $p(t)$ that is described as the sum of a positive fitness effect $g(t)$ and a negative fatigue effect $h(t)$, such that $p(t) = p_0 + g(t) - h(t)$. Convolution and discretisation with the most common models in practice to identify performance on day n gives

$$p(n) = p_0 + k_1 \sum_{s=1}^n w(s) e^{-(n-s)/\tau_1} - k_2 \sum_{s=1}^n w(s) e^{-(n-s)/\tau_2}$$

With the four parameters k_1, k_2, τ_1, τ_2 .

$$p(n) = p_0 + k_1 \sum_{s=1}^n w(s) e^{-(n-s)/\tau_1} - k_2^i \sum_{s=1}^n w(s) e^{-(n-s)/\tau_2}, \quad k_2^i = k_3 \sum_{j=1}^i w(j) e^{-(i-j)/\tau_3}$$

With the five parameters $k_1, k_3, \tau_1, \tau_2, \tau_3$

$$p(n) = p_0 + k_A \sum_{s=1}^n w(s) (1 - e^{-(n-s)/\tau_{A1}})U + k_A \sum_{s=1}^n w(t) (e^{-(n-s-TD)/\tau_{A2}})|U - 1| - k_B \sum_{s=1}^n w(s) e^{-(n-s)/\tau_B}$$

Where $U = 1$, when $n - s \leq TD$ (time delay) and $U = 0$ when $n - s > TD$. We can set TD to an integer k and $\frac{k-0.5}{2} < \tau_{A1} < \frac{k+0.5}{2}$. We could also estimate TD and put the constraint around τ_{A1} to give five/six parameters $k_A, k_B, \tau_{A1}, \tau_{A2}, \tau_B, (TD)$

Presumably this would also lead to the following

$$p(n) = p_0 + k_A \sum_{s=1}^n w(s) (1 - e^{-(n-s)/\tau_{A1}})U + k_A \sum_{s=1}^n w(t) (e^{-(n-s-TD)/\tau_{A2}})|U - 1| - k_B^i \sum_{s=1}^n w(s) e^{-(n-s)/\tau_B}, \quad k_B^i = k_{B2} \sum_{j=1}^i w(j) e^{-(i-j)/\tau_{B2}}$$

With the six/seven parameters $k_A, k_{B2}, \tau_{A1}, \tau_{A2}, \tau_{B1}, \tau_{B2}, (TD)$

Note, index notation above runs from 1 to n such that performance on any day where there is training is assumed to occur immediately after training such that $\exp(-(n-s)/\tau) = \exp(0) = 1$.

For sequential models the $1 - e^{-(n-s)/\tau_{A1}}$ means that there are immediate fatigue effects from a days training, but a 1 day delay for fitness effects.

We can change the notation to 1 to $n - 1$. This just means that performance is measured prior to any training on that day so that some of the effect has decayed by the time performance is predicted the next day.