PFPL Syntax Package*

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Overview

Write \usepackage{pfpl-syntax} to define syntax macros for PFPL. The package provides an integrated, systematic treatment of abstract and concrete syntax for a wide range of languages.

Abstract Binding Trees

To typeset an abstract binding tree yourself, provide the package with the [abt] option, and use the following commands:

- \Opn{kwd}: Format an operator as a keyword; use starred form for an operator whose display is to be set in math mode. For example, \Opn{fun} produces fun and \Opn*{\lambda} produces λ .
- \Abt<parameters>[optional] (arguments): produce abstract syntax, with all arguments typeset; the starred form omits the optional arguments. If the parnethesized arguments are empty, then no parentheses are typeset. For example, the command

produces $get\langle a\rangle[\tau]$, and the command

\Abt{\Opn{set}}[\tau]<a>(e)

produces $\operatorname{\mathtt{set}}\langle a\rangle[\tau](e)$. The starred forms produce $\operatorname{\mathtt{get}}\langle a\rangle$, and $\operatorname{\mathtt{set}}\langle a\rangle(e)$, respectively.

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• \Abs<symbols>(variables){abt}: produce an abstractor over symbols and/or variables within an ABT. For example, the command

```
\label{lambda} $$ \Lambda (x) = (\Lambda (x) {e}) $$ (Abs(x) {e}) $$
```

produces $\lambda[\tau](x \cdot e)$ and the starred form produces $\lambda(x \cdot e)$. Similarly, the command

```
\  \hf{\operatorname{dcl}}[\tau,\tau] (e;\Lambda s<a>\{m\})
```

produces $dcl[\tau; \rho](e; a.m)$ and the starred form produces dcl(e; a.m).

The package option [sf] indicates that operations are to be set in sans serif font, and the package option [sc] indicates that they are to be set in small capitals. These forms are not ordinarily used directly, they are rather for authors to define their own syntax macros, as illustrated for PFPL in the next section.

Language-Specific Definitions

The commands for formatting the many languages considered in PFPL are formulated in a series of figures organized around type structure. The tables display the abstract and concrete syntax, and the literal command used to create them (with starred forms for the concrete syntax).

When I is a finite set of indices, the notation τ_I stands for the finite map $i \hookrightarrow \tau_i \mid i \in I$ and e_I stands for $i \hookrightarrow e_i \mid i \in I$.

```
\begin{array}{lll} & & & & \\ & \text{fun}(\tau_1\,;\tau_2) & & \tau_1 \rightarrow \tau_2 & & \\ & & \text{lam}[\tau_1\,;\tau_2](x\,.\,e) & & \lambda(x\,.\,e) & & \\ & & \text{ap}[\tau_1\,;\tau_2](e_1\,;e_2) & & \text{ap}(e_1\,;e_2) & & \\ & & \text{appEx\{e\_1\}\{e\_2\}} \end{array}
```

Figure 1: Function Types

```
unit
                                                                            \unitTy
                                         1
                                                                            \unitEx
null
                                          \langle \rangle
\mathtt{prod}(\tau_1; \tau_2)
                                                                            \displaystyle \prodTy{\tilde 1}_{\tilde 2}
                                          \tau_1 \times \tau_2
                                                                            \texttt{\projEx<i>\{e\}} \quad (i=1,2)
\operatorname{proj}\langle i\rangle[	au_1\,;	au_2](e)
                                          e \cdot i
pair[\tau_1; \tau_2](e_1; e_2)
                                                                            \pairEx[\tau_1][\tau_2]{e_1}{e_2}
                                          \langle e_1, e_2 \rangle
                                                                            \vprodTy<I><i>{\tau}
\operatorname{\mathtt{vprod}}\langle I 
angle (	au_I)
                                          \times_{i\in I}(i\hookrightarrow \tau_i)
\mathtt{vtuple}\langle I\rangle[\tau_I](e_I)
                                          \langle i \hookrightarrow e_i \mid i \in I \rangle
                                                                            \vtupleEx<I>[\tau]{e}
\operatorname{vproj}\langle I;i\rangle[\tau_I](e)
                                                                            \vert = (i \in I)
```

Figure 2: Product Types

```
void
                                                                                              \voidTy
absurd[\rho](e)
                                                 absurd(e)
                                                                                              \absurdEx[\rho]{e}
                                                                                              \t 1){\begin{array}{c} \\ \\ \\ \end{array}}
sum(\tau_1; \tau_2)
                                                 \tau_1 + \tau_2
                                                                                              \injEx<i>[\tau_1][\tau_2]{e} (i = 1, i)
\operatorname{inj}\langle i\rangle[\tau_1;\tau_2](e)
                                                 i \cdot e
case[\tau_1 ; \tau_2 ; \rho](e ; x . e_1 ; x . e_2)
                                                 case e\{x.e_1 | x.e_2\}
                                                                                              \caseEx[	][\rho]{e}{x}{e}
bool
                                                                                              \boolTy
true
                                                 true
                                                                                              \trueEx
false
                                                 false
                                                                                              \falseEx
                                                 \mathtt{if}(e \mathbin{;} e_1 \mathbin{;} e_2)
if[\rho](e; e_1; e_2)
                                                                                              \left[ \right] {e}_{e_1}{e_2}
                                                 +_{i\in I}(i\hookrightarrow \tau_i)
	exttt{vsum}\langle I
angle(	au_I)
                                                                                              \vsumTy<I><i>{\tau}
\operatorname{vinj}\langle I;i\rangle[	au_I](e)
                                                                                              \ \injEx<I><i>[\tau]{e} \ (i \in I)
\operatorname{vcase}\langle I\rangle[\tau_I\,;\rho](e\,;x\,.\,e_I')
                                                 vcase e\{i \hookrightarrow x \cdot e'_i \mid i \in I\}
```

Figure 3: Sum Types

```
\operatorname{ind}(t.\tau)
                                        ind(t.\tau)
                                                                           \int Ty{t}{\tau}
\inf[t \cdot \tau](e)
                                                                           \in Ex[t][\tau]{e}
                                        in(e)
rec[t.\tau;\rho](e;x.e')
                                                                            rec(e; x.e')
                                        nat
                                                                           \n
                                                                           \zeroEx
zero
                                        zero
                                                                           \succEx{e}
succ(e)
                                        succ(e)
                                                                           \natitEx[\rho]{e}{e_0}{x}{e_1}
\mathtt{natit}[\rho](e \; ; e_0 \; ; x \; . \; e_1)
                                        natit e \{ e_0 \mid x . e_1 \}
ifz[\rho](e; e_0; x.e_1)
                                        ifz e \{ e_0 \mid x . e_1 \}
                                                                           \ifzEx[\rho]{e}{e_0}{x}{e_1}
list(\tau)
                                        \tau list
                                                                           \left\langle \right\}
{\tt nil}[\tau]
                                                                           \nilEx[\tau]
                                        nil
                                                                            \consEx[	au]{e_1}{e_2}
\mathsf{cons}[\tau](e_1;e_2)
                                        cons(e_1;e_2)
                                        listrec e \left\{ \left. e_1 \mid x, y \right. . e_2 \right\}
                                                                           \line Ex[\rho][\tau]{e}{e_1}{x}{y}{e_1}{e_2}
listrec[\rho;\tau](e;e_1;x,y.e_2)
listcase[\rho;\tau](e;e_1;x,y.e_2)
                                        listcase e \{ e_1 \mid x, y \cdot e_2 \}
                                                                           \line Ex[\rho][\tau]{e}{e_1}{x}{y}{d}
coi(t.\tau)
                                        coi(t.\tau)
                                                                           \coiTy{t}{\tau}
\mathtt{out}[t \ . \ \tau](e)
                                        out(e)
                                                                           \outEx[t][\tau]{e}
gen[t.\tau;\sigma](e;x.e')
                                        gen(e; x.e')
                                                                            \genEx[t][\tau][\sigma]{e}{x}{e'}
conat
                                        conat
                                                                           \conatTy
\mathtt{stream}(\tau)
                                                                           \streamTy{\tau}
                                        	au stream
```

Figure 4: Inductive and Coinductive Types

```
\mathtt{all}(t.\tau)
                                 \forall (t . \tau)
                                                          \Lambda Ty{t}{\tau}
tlam(t.e)
                                 \Lambda(t \cdot e)
                                                          \LamEx{t}[\tau]{e}
tap[t.\tau](e;\sigma)
                                 tap(e;\sigma)
                                                          \AppEx[t][\tau]{e}{\sigma}
some(t.\tau)
                                 \exists (t . \tau)
                                                          \SomeTy{t}{\lambda}
pack[t.\tau](\rho;e)
                                                          \PackEx[t][\tau]{\rho}{e}
                                 pack(\rho; e)
open[t.\tau;\rho](e;t,x.e')
                                                         \OpenEx[t][\tau]{e}[\rho]{x}{e'}
                                 open(e; t, x . e')
```

Figure 5: Polymorphic Types

```
\ansTy
ans
                          ans
                                              \yesEx
yes
                         yes
                                              \noEx
no
                          no
                                              \contTy{\tau}
cont(\tau)
                          \tau \, \mathtt{cont}
\mathtt{cont}[\tau](k)
                          cont(k)
                                              \contEx{k}
letcc[\tau](x.e)
                         letcc(x.e)
                                              \label{letccEx[tau]{x}{e}}
\mathtt{throw}[\tau\,;\rho](e_1\,;e_2)
                          throw(e_1; e_2)
                                              \t \sum [\tau] {e_1}{e_2}
{\rm emp}[\tau]
                                              \empStk[\tau]
\operatorname{ext}[\tau_1;\tau_2](k;f)
                          k ; f)
```

Figure 6: Continuation Types

```
parr(\tau_1; \tau_2)
                                                                                                                 \tau_1 \rightharpoonup \tau_2
                                                                                                                                                                                                \parrTy{\tau_1}{\tau_2}
fun[\tau_1;\tau_2](f,x\cdot e)
                                                                                                                fun(f, x \cdot e)
                                                                                                                                                                                                \int funEx\{f\}\{x\}\{e\}
ap[\tau_1; \tau_2](e_1; e_2)
                                                                                                                ap(e_1;e_2)
                                                                                                                                                                                               \appEx{e_1}{e_2}
fix[\tau](x.e)
                                                                                                                fix(x.e)
                                                                                                                                                                                               fixEx[\tau]{x}{e}
\operatorname{rec} t \cdot (\tau)
                                                                                                                \operatorname{rec} t \cdot (\tau)
                                                                                                                                                                                               \rcTy{t}{\lambda u}
\operatorname{fold}[t\,.\,\tau](e)
                                                                                                                 fold(e)
                                                                                                                                                                                               \foldEx[t][\tau]{e}
\mathtt{unfold}[t \, . \, \tau](e)
                                                                                                                                                                                                \unfoldEx[t][\tau]{e}
                                                                                                                 unfold(e)
self(\tau)
                                                                                                                 self(\tau)
                                                                                                                                                                                               \left\{ \right\}
\mathtt{roll}[\tau](e)
                                                                                                                roll(e)
                                                                                                                                                                                               \rollEx[\tau]{e}
self[\tau](x.e)
                                                                                                                 self(x.e)
                                                                                                                                                                                               \left[ \left( x\right) \right] 
lam(x.M)
                                                                                                                 \lambda(x . M)
                                                                                                                                                                                               \ullet \ullet
app(M_1; M_2)
                                                                                                                 M_1(M_2))
                                                                                                                                                                                               \upEx{M_1}{M_2}
Ι
                                                                                                                 Ι
                                                                                                                                                                                                \uIEx
K
                                                                                                               K
                                                                                                                                                                                               \uKEx
S
                                                                                                                S
                                                                                                                                                                                                \uSEx
В
                                                                                                                В
                                                                                                                                                                                                \uBEx
```

Figure 7: Recursive Types

```
\mathtt{cmd}(\tau)
                                                                     \cmdTy{\tau}
                                      	au cmd
{\rm cmd}[\tau](m)
                                      {\tt cmd}(m)
                                                                     \cmdEx[\tau]{m}
\mathtt{ret}[\tau](e)
                                     \mathtt{ret}(e)
                                                                     \verb|\retCmd[\tau]{e}|
\mathtt{bnd}[\tau\,;\rho](e\,;x\,.\,m)
                                      bnd x \leftarrow e ; m
                                                                     \bndCmd{e}{x}{m}
dcl[\tau;\rho](e;a.m)
                                                                     \label{local_e} $$\clcmd{e}{a}{m}$
                                      dcl \ a := e ; m
\operatorname{ref}(\tau)
                                                                     \rghty{\tau u}
                                      \tau \,\, \mathtt{ref}
\operatorname{ref}\langle a \rangle
                                      \&\,a
                                                                     \refEx{a}
\mathtt{get}\langle a \rangle
                                                                     \getCmd{a}
                                      ! a
                                                                     \getrefCmd[\tau]{e}
\mathtt{getref}[\tau](e)
                                      *e
\operatorname{set}\langle a \rangle(e)
                                                                     \setCmd{a}{e}
                                      a := e
\mathtt{setref}[	au](e_1\,;e_2)
                                                                     \label{lem:cond} $$\left[ \tau_1 \right] = 1 . $$ e_1 = 2. $$
                                      e<sub>1</sub> *= e<sub>2</sub>
```

Figure 8: Command Types

$\begin{array}{c} \mathtt{st} \\ \mathtt{type} \\ \mathtt{val}(\tau) \end{array}$	$\begin{array}{c} \mathtt{st} \\ \mathtt{type} \\ \mathtt{val}(\tau) \end{array}$	\staticLock \univSg \valSg{\tau}
$\operatorname{Ext}(S; M)$ $\operatorname{in}[S; M](M')$ $\operatorname{out}[S; M](M')$	$\{S \mid M\}$ $\operatorname{in}(M')$ $\operatorname{out}(M')$	\extSg{S}{M} \inMd[S][M]{M'} \outMd[S][M]{M'}
$\mathtt{Comp}(S)$	$S \operatorname{Comp}$	\compSg{S}
$ ext{Pi}(S_1;X.S_2) \ ext{fun}[S_1](X.M_2) \ ext{inst}[S_1;X.S_2](M_1;M_2)$	$X{:}S_1 o S_2 \ ext{fun} X{:}S_1 ext{in} M_2 \ M_1(M_2)$	\piSg{S_1}{X}{S_2} \funMd{S_1}{X}{M_2} \instMd[S_1][X][S_2]{M_1}{M_2}
$egin{aligned} \mathtt{Sig}(S_1;X.S_2) \ \mathtt{str}[S_1;X.S_2](M_1;M_2) \ \mathtt{proj}\langle i angle[S_1;X.S_2](M) \end{aligned}$	$X:S_1 \times S_2$ $\operatorname{str} M_1 \text{ in } M_2$ $M \cdot \text{i}$	$\label{eq:sigSg} $$ \strMd[S_1]_{X}_{S_2} \strMd[S_1]_{X}_{S_2}_{M_1}_{M_2} \projMd[S_1]_{X}_{S_2}_{M} (i=1,2)$

Figure 9: Module Types

```
one
                                                                                                                                                                                   \unitPr
par(p_1; p_2)
                                                                                               p_1 \otimes p_2
                                                                                                                                                                                   \prec{p_1}{p_2}
null
                                                                                                                                                                                   \nullPr
                                                                                                                                                                                   \choosePr{p_1}{p_2}
or(p_1; p_2)
                                                                                                p_1 \oplus p_2
que\langle a\rangle(p)
                                                                                                 ?\langle a\rangle(p)
                                                                                                                                                                                   \quePr{a}{p}
\operatorname{sig}\langle a \rangle(p)
                                                                                                !\langle a\rangle(p)
rcv\langle a\rangle(x.p)
                                                                                                 ?\langle a\rangle(x\cdot p)
                                                                                                                                                                                   \sigPr{a}{p}
\operatorname{snd}\langle a\rangle(e\,;p)
                                                                                                                                                                                   \space{2} \spa
                                                                                                !\langle a\rangle(e;p)
\operatorname{asnd}\langle a\rangle(e)
                                                                                                !\langle a\rangle(e)
                                                                                                                                                                                   \asndPr{a}{e}
\operatorname{\mathsf{sync}}(\varepsilon)
                                                                                                 \operatorname{sync}(\varepsilon)
                                                                                                                                                                                   \syncPr{\varepsilon}
new[\tau](a.p)
                                                                                                 \nu(a \cdot p)
                                                                                                                                                                                   \newPr{a}{p}
sil
                                                                                                                                                                                   \silAc
\operatorname{sig}\langle a\rangle
                                                                                                 a!
                                                                                                                                                                                   \sigAc{a}
que\langle a \rangle
                                                                                                 a?
                                                                                                                                                                                   \queAc{a}
\operatorname{snd}\langle a \rangle(e)
                                                                                                 a!e
                                                                                                                                                                                   \sndAc{a}{e}
rcv\langle a\rangle(e)
                                                                                                 a?e
                                                                                                                                                                                   \rcvAc{a}{e}
any(e)
                                                                                                 ?e
                                                                                                                                                                                   \anyAc{e}
emit(e)
                                                                                                !e
                                                                                                                                                                                   \emitAc{e}
```

Figure 10: Proceses

```
F(\tau^{+})
                            \uparrow(\tau^+)
                                                 \freeTy{\posTy{\tau}}
\mathsf{ret}[\tau^+](v)
                             ret(v)
                                                 \freeEx[\posTy{\tau}]{v}
let[\tau^{-}; \rho^{-}](e; x . e')
                            let(e; x.e')
                                                 \fletEx[\negTy{\tau}][\negTy{\rho}]{e}{x}{e'}
\mathtt{U}(\tau^{-})
                             \downarrow(\tau^-)
                                                 \operatorname{thunk}[\tau^{\text{-}}](e)
                             thunk(e)
                                                 \thunkEx[\negTy{\tau}]{e}
force(v)
                             force(v)
                                                 \forceEx{v}
```

Figure 11: Polarized Types

```
\tensorTy{\tau_1}{\tau_2}
tens(\tau_1; \tau_2)
                                                            \tau_1 \otimes \tau_2
with [\tau_1; \tau_2](v_1; v_2)
                                                                                                       \tensorEx{v_1}{v_2}
                                                            v_1 \otimes v_2
                                                            \operatorname{split} v\left\{x_1, x_2 \cdot e\right\}
split[\tau_1 ; \tau_2 ; \rho](v ; x_1, x_2 . e)
                                                                                                       \splitEx{v}{x_1}{x_2}{e}
and(\tau_1; \tau_2)
                                                            \tau_1 \& \tau_2
                                                                                                       \begin{array}{ll} \begin{array}{ll} \begin{array}{ll} \begin{array}{ll} \begin{array}{ll} \\ \end{array} \end{array} \end{array}
both(e_1; e_2)
                                                            e_1 \& e_2
                                                                                                       \begin{array}{l} \begin{array}{l} \begin{array}{l} \\ \\ \end{array} \end{array}
par e; x, y.(e')
                                                            par e; x, y.(e')
                                                                                                       \parEx{e}{x}{y}{e'}
```

Figure 12: Parallel Types

```
truth
                                                                                                                                                     Т
                                                                                                                                                                                                                                                          \trueProp
                                                                                                                                                     \top I
                                                                                                                                                                                                                                                          \trueIPf
truthI
and(\phi_1;\phi_2)
                                                                                                                                                     \phi_1 \wedge \phi_2
                                                                                                                                                                                                                                                          \andProp{\phii_1}{\phii_2}
andI[\phi_1; \phi_2](\pi_1; \pi_2)
                                                                                                                                                     \wedge I(\pi_1; \pi_2)
                                                                                                                                                                                                                                                          \\ \\ and IPf{\pi_1}{\pi_2}
andE\langle i\rangle[\phi_1;\phi_2](\pi)
                                                                                                                                                     \wedge \mathbf{E} \langle i \rangle (\pi)
                                                                                                                                                                                                                                                          \falseProp
falsity
\mathtt{falseI}[\phi](\pi)
                                                                                                                                                    \perp \mathrm{I}(\pi)
                                                                                                                                                                                                                                                          \falseIPf[\phi]{\pi}
falseE[\phi](\pi)
                                                                                                                                                     \perp E(\pi)
                                                                                                                                                                                                                                                          \falseEPf[\phi]{\pi}
\operatorname{or}(\phi_1;\phi_2)
                                                                                                                                                     \phi_1 \lor \phi_2
                                                                                                                                                                                                                                                          \orProp{\phi_1}{\phi_2}
\mathtt{orI}\langle i\rangle[\phi_1\ ;\phi_2](\pi)
                                                                                                                                                    \forall \mathtt{I}\langle i\rangle(\pi)
                                                                                                                                                                                                                                                          \orIPf[\phi_1][\phi_2]<i>{\pi}
orE[\phi_1; \phi_2; \rho](\pi; x.\pi_1; x.\pi_2)
                                                                                                                                                     \vee E(\pi; x.\pi_1; x.\pi_2)
                                                                                                                                                                                                                                                          imp(\phi_1;\phi_2)
                                                                                                                                                     \phi_1 \supset \phi_2
                                                                                                                                                                                                                                                          \impProp{\phi_1}{\phi_2}
\mathtt{impI}[\phi_1;\phi_2](x.\pi_2)
                                                                                                                                                     \supset I(x.\pi_2)
                                                                                                                                                                                                                                                          \label{limpipf} $$ \displaystyle \prod_1 [\phi_2]_{x}_{\phi_2}$
\mathtt{impE}[\phi_1\,;\phi_2](\pi\,;\pi_1)
                                                                                                                                                     \supset E(\pi; \pi_2)
                                                                                                                                                                                                                                                          \label{limpepf} $$ \displaystyle \lim_1 [\phi_2]_{\phi_1} $$ in EPf[\phi_1] $$ in EPf[\phi_
not(\phi)
                                                                                                                                                     \neg \phi
                                                                                                                                                                                                                                                           \notProp{\phi}
\mathtt{notI}[\phi](x \cdot \pi)
                                                                                                                                                     \neg \mathtt{I}(x \cdot \pi)
                                                                                                                                                                                                                                                           \notIPf[\phi]{x}{\pi}
\mathtt{notE}[\phi](\pi \; ; \pi_2)
                                                                                                                                                     \neg \mathtt{E}(\pi ; \pi_2)
                                                                                                                                                                                                                                                          \notEPf[\phi]{\pi_2}
```

Figure 13: Propositions and Proofs