# PFPL Syntax Package\*

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December 24, 2022

### Overview

Write \usepackage{pfpl-syntax} to define syntax macros for PFPL. The package provides an integrated, systematic treatment of abstract and concrete syntax for a wide range of languages.

## **Abstract Binding Trees**

To typeset an abstract binding tree yourself, provide the package with the [abt] option, and use the following commands:

- \Opn{kwd}: Format an operator as a keyword, or \Opn\*{symbol} for an operator whose display is in math mode. For example, \Opn{fun} produces fun and \Opn\*{\lambda} produces  $\lambda$ .
- \Abt<parameters>[optional] (arguments): produce abstract syntax, with all arguments typeset, and the starred form to omit the optional arguments. Omitted arguments are omitted. For example, the command

produces  $get\langle a\rangle[\tau]$ , and the command

produces  $\operatorname{\mathtt{set}}\langle a\rangle[\tau](e)$ . The starred forms produce  $\operatorname{\mathtt{get}}\langle a\rangle$ , and  $\operatorname{\mathtt{set}}\langle a\rangle(e)$ , respectively.

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• \Abs<symbols>(variables){abt}: produce an abstractor over symbols and/or variables within an ABT. For example, the command

produces  $\lambda[\tau](x \cdot e)$  and the starred form produces  $\lambda(x \cdot e)$ . Similarly, the command

produces  $dcl[\tau; \rho](e; a.m)$  and the starred form produces dcl(e; a.m).

These general forms are not ordinarily used directly, they are rather for authors to define their own syntax macros, as illustrated for PFPL in the next section.

## Language-Specific Definitions

The commands for formatting the many languages considered in PFPL are formulated in a series of figures organized around type structure. The tables display the abstract and concrete syntax, and the literal command used to create them (with starred forms for the concrete syntax).

```
\begin{array}{lll} & & & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & &
```

Figure 1: Function Types

```
unit
                                       1
                                                                       \unitTy
null
                                                                       \unitEx
prod(\tau_1; \tau_2)
                                       \tau_1 \times \tau_2
                                                                       \prodTy{\tau_1}{\tau_2}
\operatorname{proj}\langle i\rangle[	au_1\,;	au_2](e)
                                       e \cdot i
                                                                       \verb|projEx<i>|e| (i=1,2)|
                                                                       \price{1} [\au_2] {e_1} {e_2}
pair[\tau_1; \tau_2](e_1; e_2)
                                       \langle e_1, e_2 \rangle
\operatorname{vprod}\langle I \rangle(\tau_I)
                                       \times_{i\in I}(i\hookrightarrow \tau_i)
                                                                       \vprodTy<I><i>{\tau}
\mathtt{vtuple}\langle I \rangle [\tau_I](e_I)
                                       \langle i \hookrightarrow e_i \mid i \in I \rangle
                                                                      \vtupleEx<I>[\tau]{e}
\operatorname{vproj}\langle I;i\rangle[\tau_I](e)
                                                                       \vert vprojEx<I><i>[\tau]{e} (i \in I)
```

The notation  $\tau_I$  stands for the finite map  $i \hookrightarrow \tau_i \mid i \in I$  and  $e_I$  stands for  $i \hookrightarrow e_i \mid i \in I$ .

Figure 2: Product Types

```
void
                                                 0
                                                                                             \voidTy
absurd[\rho](e)
                                                 absurd(e)
                                                                                             \absurdEx[\rho]{e}
sum(\tau_1; \tau_2)
                                                                                             \sum_{1}{\sum_{1}{\sum_{2}}}
                                                 \tau_1 + \tau_2
\operatorname{inj}\langle i\rangle[\tau_1;\tau_2](e)
                                                 i \cdot e
                                                                                             (i = 1, 2)
case[\tau_1 ; \tau_2 ; \rho](e ; x . e_1 ; x . e_2)
                                                 case e\{x.e_1 | x.e_2\}
                                                                                             \caseEx[\tau][\rho]{e}{x}{e}
bool
                                                                                             \boolTv
true
                                                                                             \trueEx
                                                 true
false
                                                 false
                                                                                             \falseEx
if[\rho](e; e_1; e_2)
                                                 if(e; e_1; e_2)
                                                                                             \ifEx[\rho]{e}{e_1}{e_2}
	exttt{vsum} \langle I 
angle (	au_I)
                                                 +_{i\in I}(i\hookrightarrow \tau_i)
                                                                                             \vsumTy<I><i>{\tau}
\operatorname{vinj}\langle I;i\rangle[	au_I](e)
                                                                                             \verb|\vinjEx<I><i>[\tau]{e} \quad (i \in I)
\operatorname{vcase}\langle I \rangle [\tau_I \, ; \rho](e \, ; x \, . \, e_I')
                                                 vcase e\{i \hookrightarrow x \cdot e'_i \mid i \in I\}
```

Figure 3: Sum Types

```
\operatorname{ind}(t.\tau)
                                        ind(t.\tau)
                                                                          \int Ty{t}{\tau}
\inf[t \cdot \tau](e)
                                                                          \in Ex[t][\tau]{e}
                                        in(e)
rec[t.\tau;\rho](e;x.e')
                                                                          rec(e; x.e')
                                        nat
                                                                          \n
                                                                          \zeroEx
zero
                                        zero
                                                                          \succEx{e}
succ(e)
                                        succ(e)
\mathtt{natit}[\rho](e;e_0;x.e_1)
                                       natit e \{ e_0 \mid x . e_1 \}
                                                                          \natitEx[\rho]{e}{e_0}{x}{e_1}
ifz[\rho](e; e_0; x.e_1)
                                        ifz e \{ e_0 \mid x . e_1 \}
                                                                          \ifzEx[\rho]{e}{e_0}{x}{e_1}
list(\tau)
                                        \tau list
                                                                          \left\langle \right\}
{\tt nil}[\tau]
                                                                          \nilEx[\tau]
                                        nil
                                                                          \consEx[	au]{e_1}{e_2}
\mathsf{cons}[\tau](e_1;e_2)
                                        cons(e_1;e_2)
                                        listrec e \left\{ \left. e_1 \mid x, y \right. . e_2 \right\}
                                                                          \line Ex[\rho][\tau]{e}{e_1}{x}{y}{e_1}{e_2}
listrec[\rho;\tau](e;e_1;x,y.e_2)
listcase[\rho;\tau](e;e_1;x,y.e_2)
                                        listcase e \{ e_1 \mid x, y \cdot e_2 \}
                                                                          \line Ex[\rho][\tau]{e}{e_1}{x}{y}{d}
coi(t.\tau)
                                        coi(t.\tau)
                                                                          \coiTy{t}{\tau}
\mathtt{out}[t \ . \ \tau](e)
                                        out(e)
                                                                          \outEx[t][\tau]{e}
gen[t.\tau;\sigma](e;x.e')
                                        gen(e; x.e')
                                                                          \genEx[t][\tau][\sigma]{e}{x}{e'}
conat
                                        conat
                                                                          \conatTy
\mathtt{stream}(\tau)
                                                                          \streamTy{\tau}
                                        	au stream
```

Figure 4: Inductive and Coinductive Types

```
\mathtt{All}(t \, . \, \tau)
                                      \forall (t . \tau)
                                                                 \Lambda 11Ty{t}{\tau}
Lam(t.e)
                                      \Lambda(t \cdot e)
                                                                 \LamEx{t}[\tau]{e}
Ap[t . \tau](e ; \sigma)
                                      Ap(e;\sigma)
                                                                  \AppEx[t][\tau]{e}{\sigma}
Some(t.\tau)
                                      \exists (t . \tau)
                                                                  \Sigma Ty{t}{\tau}
Pack[t.\tau](\rho;e)
                                     Pack(\rho; e)
                                                                  \PackEx[t][\tau]{\rho}{e}
\mathtt{Open}[t \, . \, \tau \, ; \rho](e \, ; t, x \, . \, e')
                                                                 \OpenEx[t][\tau]{e}[\rho]{x}{e'}
                                      Open(e; t, x . e')
```

Figure 5: Polymorphic Types

```
\ansTy
ans
                         ans
                                              \yesEx
yes
                         yes
                                              \noEx
no
                         no
                                             \contTy{\tau}
cont(\tau)
                         \tau \, \mathtt{cont}
\mathtt{cont}[\tau](k)
                         cont(k)
                                             \contEx{k}
letcc[\tau](x.e)
                         letcc(x.e)
                                             \label{letccEx[tau]{x}{e}}
\mathtt{throw}[\tau\,;\rho](e_1\,;e_2)
                         throw(e_1; e_2)
                                             \t \sum [\tau] {e_1}{e_2}
{\rm emp}[\tau]
                                             \empStk[\tau]
\operatorname{ext}[\tau_1;\tau_2](k;f)
                         k; f
```

Figure 6: Continuation Types

```
parr(\tau_1; \tau_2)
                                                                                                                 \tau_1 \rightharpoonup \tau_2
                                                                                                                                                                                                \parrTy{\tau_1}{\tau_2}
fun[\tau_1;\tau_2](f,x\cdot e)
                                                                                                                fun(f, x \cdot e)
                                                                                                                                                                                                \int funEx\{f\}\{x\}\{e\}
ap[\tau_1; \tau_2](e_1; e_2)
                                                                                                                ap(e_1;e_2)
                                                                                                                                                                                               \appEx{e_1}{e_2}
fix[\tau](x.e)
                                                                                                                fix(x.e)
                                                                                                                                                                                               fixEx[\tau]{x}{e}
\operatorname{rec} t \cdot (\tau)
                                                                                                                \operatorname{rec} t \cdot (\tau)
                                                                                                                                                                                               \rcTy{t}{\lambda u}
\operatorname{fold}[t\,.\,\tau](e)
                                                                                                                 fold(e)
                                                                                                                                                                                               \foldEx[t][\tau]{e}
\mathtt{unfold}[t \, . \, \tau](e)
                                                                                                                                                                                                \unfoldEx[t][\tau]{e}
                                                                                                                 unfold(e)
self(\tau)
                                                                                                                 self(\tau)
                                                                                                                                                                                               \left\{ \right\}
\mathtt{roll}[\tau](e)
                                                                                                                roll(e)
                                                                                                                                                                                               \rollEx[\tau]{e}
self[\tau](x.e)
                                                                                                                 self(x.e)
                                                                                                                                                                                               \left[ \left( x\right) \right] 
lam(x.M)
                                                                                                                 \lambda(x.M)
                                                                                                                                                                                               \ullet \ullet
app(M_1; M_2)
                                                                                                                 M_1(M_2))
                                                                                                                                                                                               \upEx{M_1}{M_2}
Ι
                                                                                                                 Ι
                                                                                                                                                                                                \uIEx
K
                                                                                                               K
                                                                                                                                                                                               \uKEx
S
                                                                                                                S
                                                                                                                                                                                                \uSEx
В
                                                                                                                В
                                                                                                                                                                                                \uBEx
```

Figure 7: Recursive Types

```
\mathtt{cmd}(\tau)
                                                                                                                                                   \tau \, \operatorname{cmd}
                                                                                                                                                                                                                                                                           \cmdTy{\tau}
 {\rm cmd}[\tau](m)
                                                                                                                                                   \operatorname{cmd}(m)
                                                                                                                                                                                                                                                                           \cmdEx[\tau]{m}
\mathtt{ret}[\tau](e)
                                                                                                                                                                                                                                                                            \retCmd[\tau]{e}
                                                                                                                                                  ret(e)
\mathtt{bnd}[\tau\,;\rho](e\,;x\,.\,m)
                                                                                                                                                                                                                                                                           \bndCmd{e}{x}{m}
                                                                                                                                                  bnd x \leftarrow e ; m
dcl[\tau;\rho](e;a.m)
                                                                                                                                                  \operatorname{dcl} a := e ; m
                                                                                                                                                                                                                                                                           \cline{dclCmd{e}{a}{m}}
ref(\tau)
                                                                                                                                                   	au ref
                                                                                                                                                                                                                                                                           \rghty{\tau u}
\operatorname{ref}\langle a \rangle
                                                                                                                                                                                                                                                                           \refEx{a}
                                                                                                                                                   & a
\gcd\langle a \rangle
                                                                                                                                                                                                                                                                            \getCmd{a}
                                                                                                                                                   ! a
\mathtt{getref}[\tau](e)
                                                                                                                                                                                                                                                                            \getrefCmd[\tau]{e}
                                                                                                                                                   *e
\operatorname{\mathfrak{set}}\langle a \rangle(e)
                                                                                                                                                   a := e
                                                                                                                                                                                                                                                                            \scalebox{ } \sc
\mathtt{setref}[\tau](e_1\,;e_2)
                                                                                                                                                   e_1 *= e_2
                                                                                                                                                                                                                                                                           \verb|\setrefCmd[\tau]{e_1}{e_2}|
```

Symbols a are constants of sort loc.

Figure 8: Command Types

st	st	\staticLock
type	type	\univSg
$\mathtt{val}(\tau)$	$\mathtt{val}(\tau)$	\valSg{\tau}
$\operatorname{Ext}(S; M)$	$\{S \mid M\}$	\extSg{S}{M}
$\mathtt{in}[\hat{S}\ ; M](M')$	$\operatorname{in}(M')$	\inMd[S][M]{M'}
$\mathtt{out}[S;M](M')$	$\check{\operatorname{\mathtt{out}}}(M')$	\outMd[S][M]{M'}
${\tt Comp}(S)$	$S \operatorname{Comp}$	\compSg{S}
$\mathtt{Pi}(S_1;X.S_2)$	$X:S_1 \to S_2$	\piSg{S_1}{X}{S_2}
$\operatorname{fun}[S_1](X.M_2)$	$\operatorname{fun} X:S_1\operatorname{in} M_2$	\funMd{S_1}{X}{M_2}
$\mathtt{inst}[S_1;X.S_2](M_1;M_2)$	$M_1(M_2)$	$\label{limited} $$ \left[ S_1 \right] [X] [S_2] \{M_1\} \{M_2\} $$$
$\mathtt{Sig}(S_1;X.S_2)$	$X:S_1 \times S_2$	$\sigSg{S_1}{X}{S_2}$
$\mathtt{str}[S_1\ ; X\ .\ S_2](M_1\ ; M_2)$	$\mathtt{str} M_1  \mathtt{in} M_2$	$\t \mathbb{S}_{1}[X][S_{2}]\{M_{1}\}\{M_{2}\}$
$\mathtt{proj}\langle i angle[S_1;X.S_2](M)$	$M \cdot \mathtt{i}$	$\label{eq:special_special} $$  \projMd[S_1][X][S_2]\{M\}  (i=1,2) $$

Figure 9: Module Types

```
one
                                                    \unitPr
par(p_1; p_2)
                           p_1 \otimes p_2
                                                    \prec{p_1}{p_2}
null
                                                    \nullPr
                                                    \choosePr{p_1}{p_2}
or(p_1; p_2)
                            p_1 \oplus p_2
que\langle a\rangle(p)
                            ?\langle a\rangle(p)
                                                    \quePr{a}{p}
\operatorname{sig}\langle a \rangle(p)
                            !\langle a\rangle(p)
rcv\langle a\rangle(x.p)
                            ?\langle a\rangle(x\cdot p)
                                                    \sigPr{a}{p}
\operatorname{snd}\langle a\rangle(e\,;p)
                                                    \sl Pr{a}{e}{p}
                            !\langle a\rangle(e;p)
\operatorname{asnd}\langle a\rangle(e)
                            !\langle a\rangle(e)
                                                    \asndPr{a}{e}
\operatorname{\mathsf{sync}}(\varepsilon)
                            \operatorname{sync}(\varepsilon)
                                                    \syncPr{\varepsilon}
new[\tau](a.p)
                            \nu(a \cdot p)
                                                    \newPr{a}{p}
sil
                                                    \silAc
\operatorname{sig}\langle a \rangle
                            a!
                                                    \sigAc{a}
que\langle a \rangle
                            a?
                                                    \queAc{a}
\operatorname{snd}\langle a \rangle(e)
                            a!e
                                                    \sndAc{a}{e}
rcv\langle a\rangle(e)
                            a?e
any(e)
                            ?e
                                                    \rcvAc{a}{e}
emit(e)
                            !e
                                                    \emitAc{e}
```

Figure 10: Proceses

```
F(\tau^{+})
                             \uparrow(\tau^+)
                                                 \freeTy{\posTy{\tau}}
\mathsf{ret}[\tau^+](v)
                             ret(v)
                                                 \freeEx[\posTy{\tau}]{v}
let[\tau^{-}; \rho^{-}](e; x . e')
                             let(e; x.e')
                                                 \fletEx[\negTy{\tau}][\negTy{\rho}]{e}{x}{e'}
\mathtt{U}(\tau^{-})
                             \downarrow(\tau^-)
                                                 \operatorname{thunk}[\tau^{\text{-}}](e)
                             thunk(e)
                                                 \thunkEx[\negTy{\tau}]{e}
force(v)
                             force(v)
                                                 force(v)
```

Figure 11: Polarized Types

```
\tensorTy{\tau_1}{\tau_2}
tens(\tau_1; \tau_2)
                                                         \tau_1 \otimes \tau_2
with [\tau_1; \tau_2](v_1; v_2)
                                                                                                  \tensorEx{v_1}{v_2}
                                                         v_1 \otimes v_2
                                                         \operatorname{split} v\left\{x_1, x_2 \cdot e\right\}
split[\tau_1 ; \tau_2 ; \rho](v ; x_1, x_2 . e)
                                                                                                  \splitEx{v}{x_1}{x_2}{e}
and(\tau_1; \tau_2)
                                                         \tau_1 \& \tau_2
                                                                                                  \begin{array}{ll} \begin{array}{ll} \begin{array}{ll} \begin{array}{ll} \\ \end{array} \end{array}
both(e_1; e_2)
                                                         e_1 \& e_2
                                                                                                  \begin{array}{l} \begin{array}{l} \begin{array}{l} \\ \\ \end{array} \end{array}
par e; x, y.(e')
                                                         par e; x, y.(e')
                                                                                                  \parEx{e}{x}{y}{e'}
```

Figure 12: Parallel Types

```
truth
                                                                                                                                                                                                                                                       Т
                                                                                                                                                                                                                                                                                                                                                                                                                               \trueProp
                                                                                                                                                                                                                                                       \top I
                                                                                                                                                                                                                                                                                                                                                                                                                               \trueIPf
truthI
 and(\phi_1;\phi_2)
                                                                                                                                                                                                                                                       \phi_1 \wedge \phi_2
                                                                                                                                                                                                                                                                                                                                                                                                                               \andProp{\phii_1}{\phii_2}
 andI[\phi_1; \phi_2](\pi_1; \pi_2)
                                                                                                                                                                                                                                                       \wedge I(\pi_1; \pi_2)
                                                                                                                                                                                                                                                                                                                                                                                                                               \label{pi_1}_{\pi_1}_{\pi_2}
 andE\langle i\rangle[\phi_1;\phi_2](\pi)
                                                                                                                                                                                                                                                       \wedge \mathbf{E} \langle i \rangle (\pi)
                                                                                                                                                                                                                                                                                                                                                                                                                               \falseProp
falsity
\mathtt{falseI}[\phi](\pi)
                                                                                                                                                                                                                                                      \perp \mathrm{I}(\pi)
                                                                                                                                                                                                                                                                                                                                                                                                                               \falseIPf[\phi]{\pi}
falseE[\phi](\pi)
                                                                                                                                                                                                                                                       \perp E(\pi)
                                                                                                                                                                                                                                                                                                                                                                                                                               \falseEPf[\phi]{\pi}
\operatorname{or}(\phi_1;\phi_2)
                                                                                                                                                                                                                                                       \phi_1 \lor \phi_2
                                                                                                                                                                                                                                                                                                                                                                                                                               \orProp{\phi_1}{\phi_2}
\mathtt{orI}\langle i\rangle[\phi_1\ ;\phi_2](\pi)
                                                                                                                                                                                                                                                      \forall \mathtt{I}\langle i\rangle(\pi)
                                                                                                                                                                                                                                                                                                                                                                                                                               \orIPf[\phi_1][\phi_2]<i>{\pi}
 orE[\phi_1; \phi_2; \rho](\pi; x.\pi_1; x.\pi_2)
                                                                                                                                                                                                                                                       \vee E(\pi; x.\pi_1; x.\pi_2)
                                                                                                                                                                                                                                                                                                                                                                                                                               imp(\phi_1;\phi_2)
                                                                                                                                                                                                                                                       \phi_1 \supset \phi_2
                                                                                                                                                                                                                                                                                                                                                                                                                               \impProp{\phi_1}{\phi_2}
 \mathtt{impI}[\phi_1;\phi_2](x.\pi_2)
                                                                                                                                                                                                                                                       \supset I(x.\pi_2)
                                                                                                                                                                                                                                                                                                                                                                                                                               \label{limpipf} $$ \displaystyle \prod_1 [\phi_2]_{x}_{\phi_2}$
 \mathtt{impE}[\phi_1\,;\phi_2](\pi\,;\pi_1)
                                                                                                                                                                                                                                                       \supset E(\pi; \pi_2)
                                                                                                                                                                                                                                                                                                                                                                                                                               \label{limpepf} $$ \displaystyle \lim_1 [\phi_2]_{\phi_1} $$ in EPf[\phi_1] $$ in EPf[\phi_
not(\phi)
                                                                                                                                                                                                                                                       \neg \phi
                                                                                                                                                                                                                                                                                                                                                                                                                                \notProp{\phi}
\mathtt{notI}[\phi](x \cdot \pi)
                                                                                                                                                                                                                                                       \neg \mathtt{I}(x \cdot \pi)
                                                                                                                                                                                                                                                                                                                                                                                                                                \begin{array}{l} \begin{array}{l} & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & \\ & \\ & & \\ & \\ & \\ & & \\ & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & 
\mathtt{notE}[\phi](\pi \; ; \pi_2)
                                                                                                                                                                                                                                                       \neg \mathtt{E}(\pi ; \pi_2)
                                                                                                                                                                                                                                                                                                                                                                                                                               \notEPf[\phi]{\pi_2}
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Figure 13: Propositions and Proofs