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Static Hedging Strategies

Static hedging strategies are used in derivatives trading, which primarily uses various types of financial products (options) to enter into forward transactions. Besides static hedging strategies, semi-static and dynamic hedging strategies can be distinguished.

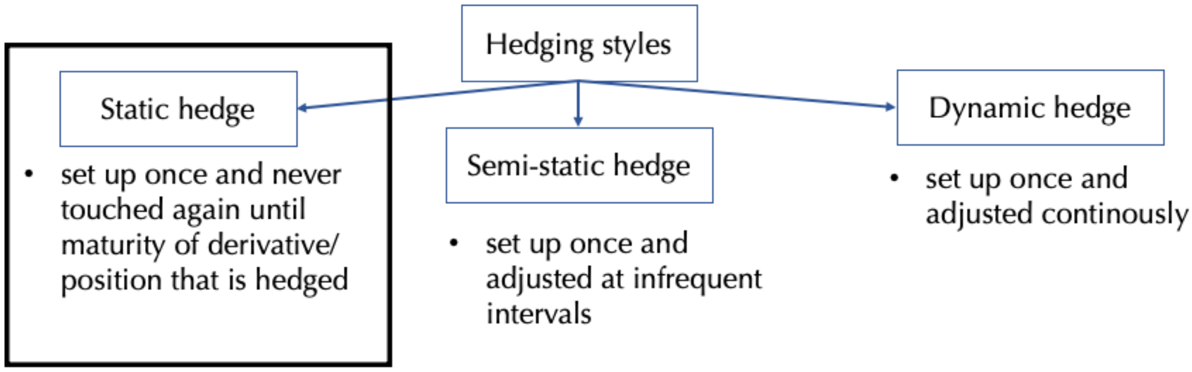

Illustration No.1) Overview of different Hedging Approaches

In the present case, reference is made only to static hedging in the context of delta hedging, as well as delta-gamma hedging.

Delta Hedging

The goal is to set up a portfolio that has a constant value regardless of the price fluctuation of the underlying or can even generate a minimum profit in the event of price movements. For this purpose, the delta of the position held in the option in the portfolio must first be determined. This then results in the amount of underlying that must be additionally built up either long (positive sign) or short (negative sign) in order to obtain, in combination with the delta of the option held, a portfolio delta value of zero as a solution to a simple linear system of equations. The input parameters needed for this are: Value of the total position, risk-free interest rate, current underlying price, strike price of the option(s), maturity of the option, implied volatility.

The procedure is as follows:

1) Using the BSM formula, determine the fair price of the option.
2) Calculate the number of options included from the total value of the position.
3) Calculate delta of one option, then of the whole position
4) Calculate the amount of underlying (short/long) needed to obtain a portfolio delta of zero.

If there is a price movement in the underlying, the effect can be easily observed by determining the P/L of the portfolio. Depending on the long/short position and the movement of the underlying price (up/down), the underlying position will have a positive or negative value. Thereupon, the fair option price must be calculated again at the new underlying price and transferred to the overall position. Then P/L from the underlying position and the P/L from the option position can be compared and an overall P/L can be established.

Delta-Gamma-Hedging

In addition to the elimination of the delta, the gamma can also be eliminated from the portfolio. For this, however, another option is needed in addition to the underlying, since the underlying has a gamma of zero. First the portfolio gamma is made neutral by investing in another option, then the entire portfolio delta is made neutral by investing in the underlying.

The procedure is as follows:

1) Calculate gamma for the held option as well as for the additional available one, then apply to the total position.
2) Calculate total gamma for the position, resolve by quantity of option available to get a total gamma value of zero.
3) Calculate delta for both options, then calculate for the total portfolio
4) Resolve by quantity of underlying to make the total portfolio also delta neutralAuch hier kann der Effekt wieder verdeutlicht werden, indem die errechneten Werte für Portfolio und Option bei einem Preissprung/ -verfall des Underlyings erneut betrachtet werden.

Literature:

Hull, John C. (2014): Options, Futures, and Other Derivatives, 9. Edition, P.402-409, P.411-415, P.418f.

Used Equations:

<u>Black-Scholes-Merton Equation:</u>

$$C(S,t) = S\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2)$$
$$P(S,t) = Ke^{-r(T-t)}\Phi(-d_2) - S\Phi(-d_1)$$

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$$
$$d_2 = d_1 - \sigma\sqrt{T-t}$$
$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp\frac{-z^2}{2} dz$$

<u>Greeks for Call and Put Options:</u>

Calculating the Greeks for calls

We find the following Greeks for the price of a European call option $C(S,t)$:

- Delta: $\Delta_C := \frac{\partial C}{\partial S} = \Phi(d_1) > 0$
- Vega: $\mathcal{V}_C := \frac{\partial C}{\partial \sigma} = S\sqrt{\tau}\varphi(d_1) > 0$
- Theta: $\Theta_C := \frac{\partial C}{\partial \tau} = \frac{S\sigma}{2\sqrt{\tau}}\varphi(d_1) + Kre^{-r\tau}\Phi(d_2) > 0$
- Rho: $\rho_C := \frac{\partial C}{\partial r} = \tau Ke^{-r\tau}\Phi(d_2) > 0$
- Gamma: $\Gamma_C := \frac{\partial^2 C}{\partial S^2} = \frac{\partial \Delta}{\partial S} = \frac{\varphi(d_1)}{S\sigma\sqrt{\tau}} > 0$

Calculating the Greeks for puts

Similar Greeks are found for the price of a European put option $P(S,t)$:

- Delta: $\Delta_P := \frac{\partial P}{\partial S} = \Phi(d_1) - 1 = -\Phi(-d_1) < 0$
- Vega: $\mathcal{V}_P := \frac{\partial P}{\partial \sigma} = \frac{\partial C}{\partial \sigma} = S\sqrt{\tau}\varphi(d_1) > 0$
- Theta: $\Theta_P := \frac{\partial P}{\partial \tau} = \frac{S\sigma}{2\sqrt{\tau}}\varphi(d_1) - Kre^{-r\tau}\Phi(-d_2)$
- Rho: $\rho_P := \frac{\partial P}{\partial r} = -\tau Ke^{-r\tau}\Phi(-d_2) < 0$
- Gamma: $\Gamma_P := \frac{\partial^2 P}{\partial S^2} = \frac{\partial^2 C}{\partial S^2} = \frac{\partial \Delta}{\partial S} = \frac{\varphi(d_1)}{S\sigma\sqrt{\tau}} > 0$