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Regime switching in foreign exchange rates: Evidence from currency option prices

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Abstract

This paper examines the ability of regime-switching models to capture the dynamics of foreign exchange rates. First we test the ability of the models to fit foreign exchange rate data in-sample and forecast variance out-of-sample. A regime-switching model with independent shifts in mean and variance exhibits a closer fit and more accurate variance forecasts than a range of other models. Next we use exchange-traded currency options to determine whether market prices reflect regime-switching information. We find that observed option prices are significantly different from their theoretical levels determined by a regime-switching option valuation model and that a simulated trading strategy based on regime-switching option valuation generates higher profits than standard single-regime alternatives. Overall, the results indicate that observed option prices do not fully reflect regime-switching information.

JEL classification: G13

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Regime-switching models are well-suited for capturing the time series behavior of many financial variables. The US short-term interest rate, for example, can be modeled as switching between a low and a high volatility regime in response to changes in various macroeconomic and political factors. For much of the last

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25 years, the volatility of the short rate has been relatively low. Occasionally, however, there have been sudden switches to periods of extremely high volatility. The causes of these regime switches have varied. During the period 1979–1982, the Federal Reserve (Fed) experimented with a new target instrument for monetary policy, non-borrowed reserves, deviating from its usual practice of targeting interest rates. The result was high volatility in interest rates for the duration of the experiment. Other periods of high volatility in US interest rates have coincided with changes in the economic and political environments due to wars involving the US, the OPEC oil crisis, and the October 1987 stock market crash (see Hamilton, 1990; Gray, 1996).

Regime-switching models are designed to capture discrete changes in the economic mechanism that generates the data. Hamilton (1988), Cai (1994), Hamilton and Susmel (1994), and Gray (1996) use variations of the standard Markov regime-switching model to describe the time series behavior of US short-term interest rates. Dahlquist and Gray (1995) show that various foreign short-term interest rates are also well-described by regime-switching models. Engel and Hamilton (1990), Bekaert and Hodrick (1993) and Engel and Hakkio (1996) all document regime shifts in major foreign exchange rates.

In this paper, we take the level of investigation one step further by examining whether regime-switching models provide more accurate security valuation. More specifically, we analyze exchange-traded currency option prices to determine whether regime-switching option valuation models are better than standard methods at identifying mispriced options. The study has two major findings. First, we show that option values generated from a regime-switching model are significantly different than market prices. Second, we show that a regime-switching option valuation model generates higher profits than standard option valuation methods.

The remainder of the paper is organized as follows. The following section outlines the economic framework of regime-switching models, focusing on the economic causes and effects of regime switching. Section 2 develops a regime-switching model for exchange rates and discusses estimation issues. A number of GARCH models are also discussed. Empirical estimates of the competing models for three major foreign exchange rates are reported in Section 3. Section 4 presents evidence from the currency options market that option values generated from a regime-switching model are significantly different than market prices. The results of a trading simulation that pits the regime-switching model against variations on the Black–Scholes (1973) model are discussed in Section 5. The final section contains a summary of the major findings.

1. The economics of regime switching

The literature on switching of interest rate regimes has, to date, focused almost exclusively on short-term US interest rates. The economic motivation for modeling the conditional distribution of US interest rates as a regime-switching process is usually couched in terms of the Fed's changes in monetary policy rules. Under different policy regimes, different interest rate behaviors arise. Witness the Fed's decision to target non-borrowed reserves rather than interest rates during the period 1979–1982. This experiment results in a period of unprecedented interest rate volatility, changing fundamentally the structure of the dependence of interest rates on other macroeconomic variables.

Dahlquist and Gray (1995) develop a similar economic motivation for regime switching in interest rates in the European Monetary System (EMS). They argue that, under a system of exchange rate target zones, the economic motivation for regime switching in interest rates is also based on central bank policy regimes. In this setting, the stochastic process of interest rates may be different when there is pressure on the weak currency and it is being defended against speculative attacks, compared with periods when the exchange rate is credible and behaving approximately as a free float. Just as US interest rates have tended to behave quite differently during periods of interest rate targeting than during periods of monetary aggregate targeting, EMS interest rates may behave differently during periods of 'currency defense' and more 'normal' times. In both cases, regime shifts are driven, at least in part, by changes in central bank policy.

The key differences between the interest and exchange rate examples, are the expected length and the timing of a particular policy regime. In the US, policy regimes have tended to be long lived. The Fed experiment, for example, lasted three years. In contrast, the very nature of the EMS suggests that policy regimes in that setting will be relatively short lived. Episodes of the high volatility, speculative attack regime where the weak currency is being defended by the central bank are not likely to last long. In this regime, a speculative attack, or even a change in the fundamentals, drives the exchange rate towards the weak edge of the target zone. The central bank of the depreciating currency may intervene in foreign exchange markets or raise interest rates in an attempt to drive the bilateral rate back towards the center of the target zone. Sometimes central banks are successful in doing this and the current crisis is averted; sometimes they are not (or it is too costly), in which case a realignment may occur. In either case, the uncertainty is generally resolved quickly, within a few days or weeks, and the process returns to normal.

Timing is the other key difference. Unlike the situation where the Federal Reserve has control over the timing of policy changes such as the Fed experiment, the EMS central banks have virtually no control over the instigation and liquidation of speculative attacks. Yet, every speculative attack requires a change of operating policy — from a focus on domestic monetary policy to a focus on exchange rate management. Since speculative attacks and currency crises tend to be sudden and short lived, we expect to see frequent switching between regimes.

Regime switching in foreign exchange rates has been documented by Engel and Hamilton (1990), Bekaert and Hodrick (1993), and Engel and Hakkio (1996), among others. One common finding is that US dollar denominated exchange rates tend to exhibit 'long swings' in the mean. That is, the exchange rate (in units of foreign currency per US dollar) tends to trend upward with the US dollar appreciating for long periods of time (three to four years), and then trends downward with the US dollar depreciating for a similar length of time. Engel and Hamilton (1990) note the inability of standard single-regime models of exchange rate determination to explain this behavior. Although they do not focus on the economic causes of regime switching, they note that (a) regime-switching models provide better in-sample fit and forecasts than the random walk model, and, (b) differences between domestic and foreign monetary and fiscal policies may be relevant in explaining the long swings in the dollar.

The dynamics of the conditional mean are not critical to the application of regime-switching models to option valuation. Under risk-neutral valuation the conditional mean is simply the difference between the domestic and foreign interest rates. The dynamics of the conditional variance, however, are crucial. Section 2 presents the competing statistical models used in this paper to describe the dynamics of exchange rate volatility. These models are then compared empirically in Section 3 using exchange rate and currency option price data.

2. Foreign exchange rate dynamics

This section describes the competing statistical models of exchange rate dynamics studied in this paper. First, a regime-switching model with independent mean and variance shifts is presented. Second, a family of nested GARCH models is reviewed.

2.1. Markov regime-switching models

In this paper, we focus on the popular Markov regime-switching model of Hamilton (1988–1990) applied to log changes in foreign exchange rates, $y_t = \ln [E_t/E_{t-1}]$ where E_t represents a foreign exchange rate in US dollars per unit of foreign currency. We model y_t as being conditionally normal where the mean and variance depend on which regime is operative. We augment the standard model to allow two regimes for the mean log exchange rate change and two regimes for the variance of log exchange rate changes.[1]

[1] The next section presents empirical evidence supporting the extended model in place of the standard two-regime model.

Our notation is as follows. We define $S_{\mu t}$ as the mean regime. The mean exchange rate change at time t is μ_i when $S_{ut} = i, i = 1, 2$. Similarly, we define $S_{\sigma t}$ as the variance regime, with the volatility of log exchanges at time t being σ_i when $S_{\sigma t} = i, i = 1, 2$. Both $S_{\mu t}$ and $S_{\sigma t}$ evolve according to a first-order Markov scheme with transition probability matrix

$$\Pi_\mu = \begin{bmatrix} P_\mu & 1 - P_\mu \\ 1 - Q_\mu & Q_\mu \end{bmatrix} \tag{1}$$

for the mean regime, and

$$\Pi_\sigma = \begin{bmatrix} P_\sigma & 1 - P_\sigma \\ 1 - Q_\sigma & Q_\sigma \end{bmatrix} \tag{2}$$

for the variance regime. For the mean regime, $P_\mu = \Pr(S_{\mu t+1} = 1 | S_{\mu t} = 1)$ and $Q_\mu = \Pr(S_{\mu t+1} = 2 | S_{\mu t} = 2)$. A similar interpretation applies to P_σ and Q_σ.[2]

Next, we define a regime indicator variable that spans the regime space for both the mean and variance regimes as

$$S_t = \begin{cases} 1 & \text{if} \quad S_{\mu t} = 1 \quad \text{and} \quad S_{\sigma t} = 1\,, \\ 2 & \text{if} \quad S_{\mu t} = 2 \quad \text{and} \quad S_{\sigma t} = 1\,, \\ 3 & \text{if} \quad S_{\mu t} = 1 \quad \text{and} \quad S_{\sigma t} = 2\,, \\ 4 & \text{if} \quad S_{\mu t} = 2 \quad \text{and} \quad S_{\sigma t} = 2\,, \end{cases} \tag{3}$$

where S_t evolves according to a first-order Markov process with transition probability matrix

$$\Pi = \begin{bmatrix} P_\mu P_\sigma & (1-P_\mu)P_\sigma & P_\mu(1-P_\sigma) & (1-P_\mu)(1-P_\sigma) \\ (1-Q_\mu)P_\sigma & Q_\mu P_\sigma & (1-Q_\mu)(1-P_\sigma) & Q_\mu(1-P_\sigma) \\ P_\mu(1-Q_\sigma) & (1-P_\mu)(1-Q_\sigma) & P_\mu Q_\sigma & (1-P_\mu)Q_\sigma \\ (1-Q_\mu)(1-Q_\sigma) & Q_\mu(1-Q_\sigma) & (1-Q_\mu)Q_\sigma & Q_\mu Q_\sigma \end{bmatrix} \tag{4}$$

under the assumption that switches in mean and variance regimes are independent. We assume independence in order to limit the number of parameters in Π to estimate. Furthermore, in the context of option valuation, we are concerned primarily with shifts in variance.

[2] The extension of Hamilton's model to time-varying transition probabilities, which has been pursued by Diebold et al. (1994), Filardo (1993,1994) and Gray (1996), is not explored in this paper.

The model can also be described in terms of the conditional distribution of log exchange rate changes as

$$y_t|\Phi_{t-1} \sim \begin{cases} \mathrm{N}(\mu_1, \sigma_1) & \text{if} \quad S_t = 1\,, \\ \mathrm{N}(\mu_2, \sigma_1) & \text{if} \quad S_t = 2\,, \\ \mathrm{N}(\mu_1, \sigma_2) & \text{if} \quad S_t = 3\,, \\ \mathrm{N}(\mu_2, \sigma_2) & \text{if} \quad S_t = 4\,, \end{cases} \tag{5}$$

where S_t evolves according to Π. Whereas the Hamilton model is usually written in terms of the switching probabilities, $\Pr(S_t|S_{t-1})$, Hamilton (1994) and Gray (1996) show that estimation can be simplified by rewriting the model in terms of the regime probabilities, $\Pr(S_t|\Phi_{t-1})$. To do this, we define a vector of regime probabilities $P_{t,t-1}$ as

$$P_{t,t-1} = \begin{bmatrix} p_{1t,t-1} \\ p_{2t,t-1} \\ p_{3t,t-1} \\ p_{4t,t-1} \end{bmatrix} \tag{6}$$

where $p_{it,t-1} = \Pr(S_t = i|\Phi_{t-1})$ for $i = 1, \dots, 4$. Similarly, we define $p_{it,t} = \Pr(S_t = i|\Phi_t)$. Then, by the Markov property of the regime indicator variable S_t, $P_{t,t-1} = \Pi' P_{t-1,t-1}$.

Next, we define a vector of conditional likelihood values as

$$f_t = \begin{bmatrix} f(y_t|S_t = 1, \Phi_{t-1}) \\ f(y_t|S_t = 2, \Phi_{t-1}) \\ f(y_t|S_t = 3, \Phi_{t-1}) \\ f(y_t|S_t = 4, \Phi_{t-1}) \end{bmatrix} \tag{7}$$

where

$$f(y_t|S_t = 1, \Phi_{t-1}) = \frac{1}{\sqrt{2\pi\sigma_1}} \exp\left\{ -\frac{1}{2}\frac{(y_t - \mu_1)^2}{\sigma_1^2} \right\} \tag{8}$$

and so on, under the conditional normality assumption of the model.

Hamilton (1994) and Gray (1996) demonstrate that

$$p_t = \Pi' \left[\frac{f_{t-1} \otimes p_{t-1}}{\iota'(f_{t-1} \otimes p_{t-1})} \right], \tag{9}$$

where p_t is shorthand notation for $\Pr(S_t|\Phi_{t-1})$ and $\otimes$ denotes element-by-element multiplication. This Bayesian updating allows the likelihood function to be constructed recursively so that standard hill climbing algorithms can be used for estimation.

Regime-switching models are most often estimated by maximum likelihood given the relatively simple form of the distribution of the data. In the model described above, the density of the data has four components (one for each regime) and the long-likelihood function is simply constructed as a probability-weighted sum of these four components. In particular, the log-likelihood function to be maximized (up to an initial condition) is

$$\sum_{t=1}^{T} \ln[f(y_t|\tilde{y}_{t-1})], \tag{10}$$

where $\tilde{y}_{t-1} = \{y_{t-1}, y_{t-2}, \dots\}$, which is equivalent to

$$\sum_{t=1}^{T} \ln\left[\sum_{i=1}^{4} f(y_t|\tilde{y}_{t-1}, S_t = i)\Pr(S_t = i|\tilde{y}_{t-1})\right]. \tag{11}$$

This likelihood function can be constructed recursively from Eqs. (7) and (9).

In summary, the extended regime-switching model presented here allows for shifts in mean and variance to occur independently, that is, for periods of stable and unstable appreciation and periods of stable and unstable depreciation. In Section 3, we show that this added flexibility improves performance in capturing the time series dynamics of exchange rate data.

2.2. GARCH models

The regime-switching model described above provides one explanation for volatility clustering present in the data: switches in regime correspond to a change in the variance of the underlying data generating process. An alternative explanation is provided by the GARCH class of models of Engle (1982) and Bollerslev (1986). The GARCH(1, 1) model, for example, specifies a variance that is a function of the prior observation of variance and the square of the prior disturbance:

$$\sigma_t^2 = \sigma + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \text{ where } \varepsilon_t = y_t - \mu \tag{12}$$

and μ is the constant mean of the random variable y. For foreign exchange rates, persistent periods of appreciation or depreciation suggest that a time-varying mean is appropriate. To model this behavior, we define the disturbance in (12) to have the structure,

$$\varepsilon_t = y_t - \mu_t \text{ where } \mu_t = \mu + \rho y_{t-1}. \tag{13}$$

We then denote the model as ARGARCH(1, 1) to highlight this autoregressive feature.

In symmetric GARCH models such as these, only the magnitude and not the sign of the lagged disturbance affects volatility. The presence of leverage effects in common stock returns suggests a possible asymmetry, in which a negative value of ε_t has a greater effect on stock return volatility than a positive value. Foreign exchange rates may also exhibit asymmetric dependence on prior innovations, perhaps as the result of asymmetric policy decisions. Nelson (1990) introduces an exponential GARCH or EGARCH model to accommodate such an asymmetry. Yet another specification is introduced by Glosten et al. (1993) and involves using an indicator variable as follows:

$$\sigma_t^2 = \sigma + (\alpha_0 + \alpha_1 I_{t-1})\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2\,, \tag{14}$$

where I_t equals 1 if ε_t is negative and 0 otherwise.

Under the assumption of normality, the parameters of the GARCH models can be estimated in a maximum likelihood framework. Further, since the models are nested, they can be compared using standard *likelihood ratio* tests.

3. Parameter estimates for competing models of foreign exchange rates

This section compares the ability of several regime switching and GARCH models to capture the time series properties of foreign exchange rates. In-sample performance is measured by log-likelihood values and Ljung–Box statistics in a maximum likelihood framework. Out-of-sample performance is gauged by variance forecast error.

3.1. Data

The data consist of weekly spot exchange rates for the British Pound (GBP), Japanese Yen (JPY), and Deutsche Mark (DM) all in terms of US Dollars (USD) per unit of foreign currency. For the JPY, the data are scaled by multiplying by 100. These rates are observed at 11 : 00AM EST every Wednesday and were obtained from *Datastream*. The sample period starts in January 1973 (with the breakdown of the Bretton-Woods agreement) and ends in December 1996, and has 1252 weekly observations in total. The data are displayed in Fig. 1. Both the GBP and DM feature periods of relatively steady appreciation and depreciation versus the dollar over the sample period, while the JPY has generally appreciated.

3.2. Parameter estimates of regime-switching models

Estimates of the parameters of three alternative regime-switching models are reported in Table 1. For the single-regime model that uses a normal distribution

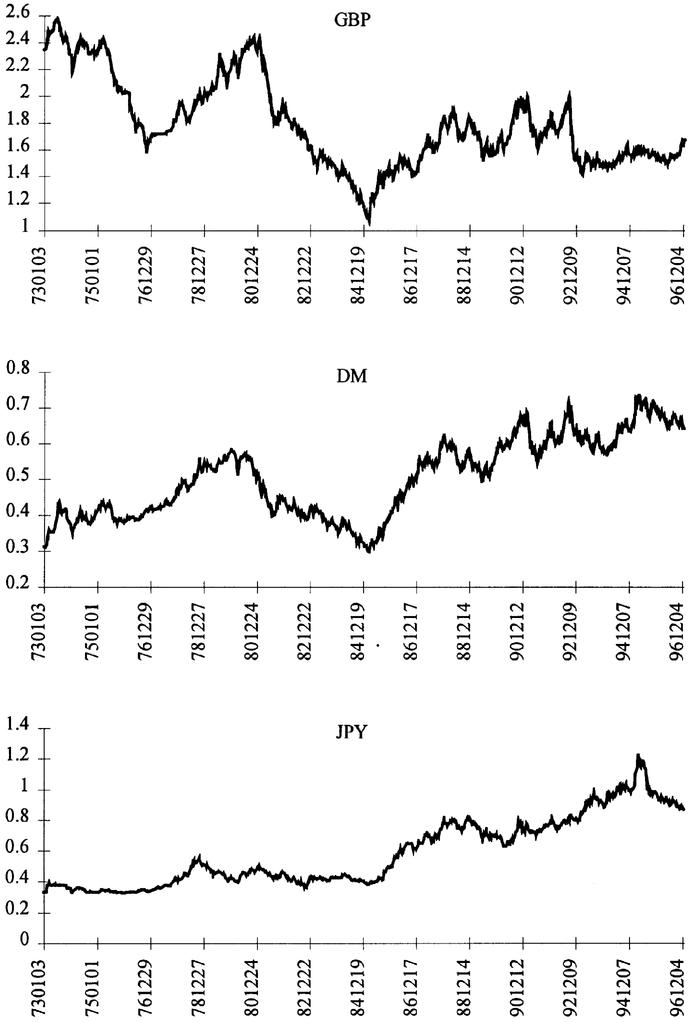


Fig. 1. Exchange rate data. Data series consist of 1252 weekly observations from January 1973 to December 1996 reported in *Datastream*. GBP and DM are in US dollars per foreign currency. JPY are in US cents per foreign currency.

to describe the data, all three currencies' volatilities are significant at any usual level. None of the currencies' mean parameters are significant, however. Apparently, when the model is restricted to one mean, the distinct periods of appreciation and depreciation of the dollar negate each other.

The standard two-regime model produces substantially increased log-likelihood values. The statistical significance of the second regime cannot be tested

Table 1
Regime-switching models. Parameters are estimated using maximum likelihood assuming normally distributed returns in each regime. White's heteroskedasticity consistent standard errors are in parentheses below each parameter estimate. Data are 100 times the weekly changes in log exchange rates from January 1973 to December 1996, a total of 1252 observations. Exchange rates are in USD per unit of foreign currency for the DM and GBP and USD per 100 units of JPY. LR test for the two (four) regime model is versus the one (two) regime model

Parameter	JPY	DM	GBP
One-regime			
μ_1	0.077	0.058	− 0.027
	(0.040)	(0.042)	(0.041)
σ_1	1.420[a]	1.495[a]	1.440[a]
	(0.054)	(0.043)	(0.048)
Log likelihood	− 2213.360	− 2278.458	− 2231.316
Two-regime			
μ_1	− 0.071	0.049	0.059
	(0.041)	(0.070)	(0.044)
σ_1	0.716	0.992	0.932
	(0.072)	(0.217)	(0.062)
μ_2	0.244	0.069	− 0.149
	(0.097)	(0.119)	(0.099)
σ_2	1.911	1.955	1.937
	(0.129)	(0.424)	(0.153)
P	0.912	0.944	0.972
	(0.031)	(0.022)	(0.013)
Q	0.900	0.929	0.959
	(0.029)	(0.080)	(0.022)
Log likelihood	− 2077.417	− 2215.651	− 2129.523
LR test statistic	271.886	125.614	203.585
Four-regime			
μ_1	− 0.152	− 0.373	− 0.273
	(0.041)	(0.196)	(0.125)
σ_1	0.608	0.960	0.860
	(0.076)	(0.090)	(0.066)
μ_2	0.987	0.454	0.333
	(0.168)	(0.158)	(0.150)
σ_2	1.852	2.031	1.906
	(0.153)	(0.216)	(0.137)
P_μ	0.941	0.882	0.948
	(0.018)	(0.066)	(0.027)
Q_μ	0.734	0.888	0.940
	(0.085)	(0.063)	(0.046)
P_σ	0.899	0.944	0.967
	(0.034)	(0.021)	(0.018)
Q_σ	0.885	0.898	0.956
	(0.034)	(0.039)	(0.023)
Log likelihood	− 2065.801	− 2207.729	− 2123.287
LR test statistic	23.232	15.844	12.473

[a]Significant at the 5% level.

using a standard Likelihood Ratio (LR) test, however, because the parameters of the second regime are not identified under the null of a single regime.[3] In this case the LR statistic is no longer distributed χ^2. Garcia (1997) derives the asymptotic distribution of the LR statistic for this two-state Markov model. He reports both the critical values of the asymptotic distribution as well as the values of a simulated empirical distribution. In both cases, the critical value for the 99% confidence level is a little less than 18. In the case at hand, the LR statistic ranges from 125.614 for the DM to 271.886 for the JPY. These large values are well in excess of the critical values and indicate the significance of the second regime. Moreover, the t-statistics on the variance parameters are large, especially for the JPY and GBP, indicating that the variance is significantly different in each regime. The same cannot be said for the mean parameters.

The four-regime model allows the mean and variance regimes to switch independently. This additional flexibility increases the log-likelihood value for all three currencies. Again, the LR test is not formal because of the identification issue. In this case, the asymptotic distribution of the LR statistic is unknown, so no conclusions about the statistical significance of the four-regime model relative to the more restrictive two-regime model can be made. We posit that the four-regime model is more intuitively appealing since it allows for bursts of higher volatility during periods of appreciation *and* depreciation of the exchange rate. The two-regime model allows volatility to be high *either* during periods of appreciation *or* during periods of depreciation. Moreover, the four-regime model performs better than the two-regime model in terms of a battery of diagnostic tests performed below. Finally, the t-statistics on the variance parameters are large, which indicates that the variance is significantly different in each regime. The t-statistics on the mean parameters are much larger than for the two-regime case, and the parameters have the appealing feature of corresponding to an appreciation and a depreciation regime for all three currencies.

The features of the four-regime model are similar for all three exchange rates that are examined. The two mean regimes correspond to periods of appreciation and depreciation of the dollar. The transition probabilities of the mean regimes, P_μ and Q_μ, are generally close to one, indicating persistence in the mean regimes. This is consistent with the 'long swings' in the dollar reported by Engel and Hamilton (1990) and Bekaert and Hodrick (1993). For the JPY, Q_μ is only 0.734, indicating less persistence of the regime in which in the dollar depreciates. Recall the sharp depreciation of the dollar in the latter half of the sample period documented in Fig. 1. Similarly, the two variance regimes are distinct, with the standard deviation of log exchange rate changes being two to three times higher in one regime than the other. The persistence of the variance regimes is also high, with P_σ and Q_σ at least 0.885 in all cases. Since volatility forecasts are the key

[3] See Hansen (1992) for a LR procedure that overcomes this difficulty.

Table 2
Ljung–Box statistics for regime-switching models. Ljung–Box statistics are reported for the residuals and squared residuals from three regime-switching models. The statistics measure serial correlation and are reported at lags of 1, 2, 3, 10 and 20 observations

Residuals	JPY		DM		GBP	
	Statistic	p-value	Statistic	p-value	Statistic	p-value
One-regime						
LB-1	5.060	0.025	3.650	0.056	1.871	0.171
LB-2	20.439	0.000	11.178	0.004	2.086	0.354
LB-3	23.561	0.000	11.214	0.011	2.174	0.537
LB-10	33.270	0.000	18.196	0.052	17.664	0.061
LB-20	51.139	0.000	23.578	0.261	35.422	0.018
Squared residuals						
LB-1	4.508	0.034	2.001	0.157	50.371	0.000
LB-2	5.819	0.055	40.865	0.000	62.671	0.000
LB-3	6.254	0.100	56.880	0.000	87.586	0.000
LB-10	18.675	0.045	69.800	0.000	151.533	0.000
LB-20	22.677	0.305	95.042	0.000	259.426	0.000
Two-regime						
LB-1	7.945	0.005	6.663	0.010	3.778	0.052
LB-2	18.104	0.000	18.281	0.000	6.318	0.043
LB-3	21.755	0.000	18.354	0.000	6.641	0.084
LB-10	29.931	0.001	24.958	0.005	20.073	0.029
LB-20	45.856	0.001	28.742	0.093	33.291	0.031
Squared residuals						
LB-1	0.042	0.839	1.548	0.214	3.497	0.062
LB-2	0.162	0.922	7.842	0.020	3.680	0.159
LB-3	0.236	0.972	10.037	0.018	7.353	0.062
LB-10	1.010	0.999	11.267	0.337	12.173	0.274
LB-20	2.085	1.000	29.853	0.072	27.112	0.132
Four-regime						
LB-1	0.339	0.561	0.125	0.724	0.157	0.692
LB-2	4.443	0.109	3.572	0.168	0.257	0.879
LB-3	5.537	0.136	4.929	0.177	0.599	0.897
LB-10	12.793	0.236	10.008	0.440	8.364	0.593
LB-20	29.557	0.077	13.915	0.835	20.851	0.406
Squared residuals						
LB-1	0.098	0.754	2.629	0.105	1.852	0.174
LB-2	0.246	0.884	5.494	0.064	1.852	0.396
LB-3	0.327	0.955	7.153	0.067	3.430	0.330
LB-10	1.607	0.996	8.984	0.534	7.347	0.692
LB-20	2.961	1.000	31.107	0.054	19.011	0.521

input to option valuation, the distinct volatility regimes imply that valuation methods that explicitly model regime-switching information may be more accurate than those that do not.

Table 2 lists the results of additional diagnostic tests of the regime-switching models. Ljung–Box statistics are listed for the residuals and squared residuals from the three models. In general, the residuals and squared residuals exhibit significant serial correlation in the single-regime model except for low lags of the GBP. The two-regime model eliminates much of the serial correlation in the squared residuals but not in the residuals. This indicates that the two-regime model captures variance dynamics well, but not shifts in mean, probably because the shifts in mean are restricted to occur simultaneously with shifts in variance. The four-regime model eliminates correlation in the residuals by allowing means to shift independently from shifts in variances. These results indicate that the four regime model captures shifts in both mean and variance better than the single and two-regime models.

3.3. Parameter estimates of GARCH models

Parameter estimates from three nested GARCH models are presented in Table 3. The three models are a normal distribution, referred to here as a constant variance model, and the GARCH(1, 1) and ARGARCH(1, 1) models of Section 2. For the JPY, most of the parameters of all three models are highly significant using White's (1982) heteroskedasticity consistent standard errors. In addition, both of the constant mean models are rejected in favor of the AR-GARCH(1, 1) model using the LR statistic based on the parameter restrictions imposed by the two simpler models. The LR statistic for the ARGARCH(1, 1) versus the constant variance model is distributed χ^2 with 3 degrees of freedom. Its value is 89.313, which is highly significant at all usual levels. For the ARGARCH(1, 1) versus the GARCH(1, 1), the LR statistic has 1 degree of freedom. Its value is 9.752, which is significant at the 5% level. Thus, for the JPY, the ARGARCH(1, 1) model appears to describe the data better than either of the simpler models. The ARGARCH(1, 1) model could not be rejected in favor of an asymmetric GARCH model, so this extension was not pursued.[4]

For both the GBP and the DM, as with the JPY, the simpler models can be rejected in favor of the ARGARCH(1, 1) model using the LR test. For the GBP, the LR statistic versus GARCH(1, 1) has 1 degree of freedom and a value of 4.698, significant at the 5% level. Compared with the constant variance model, the LR statistic has 3 degrees of freedom and a value of 168.810, significant at any usual level. For the DM, the LR statistic versus GARCH(1, 1) is 8.675 and 111.574 versus the constant variance model, both highly significant.

Table 4 lists the Ljung–Box statistics of the GARCH(1, 1) and ARGARCH(1, 1) models. In general, both models exhibit serial correlation in the

[4] The results of these tests are available from the authors upon request.

Table 3
GARCH models. Parameters are estimated using maximum likelihood assuming normally distributed innovations. White's heteroskedasticity consistent standard errors are in parentheses below each parameter estimate. Data are 100 times the weekly changes in log exchange rates from January 1973 to December 1996, a total of 1252 observations. Exchange rates are in USD per unit of foreign currency for the DM and GBP and USD per 100 units of JPY. The LR test for the GARCH(1, 1) model is versus the constant variance model. For the ARGARCH(1, 1) model, the LR tests are versus both the constant variance and the GARCH(1, 1) models

Parameter	JPY	DM	GBP
Constant variance			
μ	0.077	0.058	− 0.027
	(0.040)	(0.042)	(0.041)
σ	1.420[b]	1.495[b]	1.440[b]
	(0.054)	(0.043)	(0.048)
Log likelihood	− 2213.360	− 2278.458	− 2231.316
GARCH(1, 1)			
μ	0.076[a]	0.060	− 0.024
	(0.036)	(0.039)	(0.039)
σ	0.013[a]	0.026	0.077[a]
	(0.008)	(0.019)	(0.034)
β	0.041[b]	0.063[b]	0.081[b]
	(0.011)	(0.021)	(0.025)
α	0.953[b]	0.926[b]	0.882[b]
	(0.011)	(0.025)	(0.028)
Log-likelihood	− 2173.580	− 2227.009	− 2149.260
LR test statistic			
vs. constant variance	79.561	102.898	164.112
ARGARCH(1, 1)			
μ	0.072[a]	0.056	− 0.024
	(0.036)	(0.038)	(0.038)
ρ	0.065	0.066	0.0467[a]
	(0.040)	(0.048)	(0.021)
σ	0.013[b]	0.026[b]	0.079[b]
	(0.004)	(0.009)	(0.025)
β	0.042[b]	0.065[b]	0.083[b]
	(0.001)	(0.002)	(0.003)
α	0.952	0.925[a]	0.879
	(0.511)	(0.423)	(0.566)
Log-likelihood	− 2168.704	− 2222.671	− 2146.911
LR test statistic			
vs. constant variance	89.313	111.574	168.810
vs. GARCH(1, 1)	9.752	8.675	4.698

[a] Significant at 10% level.
[b] Significant at 5% level.

residuals for all three currencies and in the squared residuals for the DM, suggesting that the four-regime model does a better job of capturing the time-varying moments of the data.

Table 4
Ljung–Box statistics for GARCH models. Ljung–Box statistics are reported for the residual and squared residuals from two GARCH models. The statistics measure serial correlation and are reported at lags of 1, 2, 3, 10, and 20 observations

Residuals	JPY		DM		GBP	
	Statistic	*p*-value	Statistic	*p*-value	Statistic	*p*-value
GARCH(1, 1)						
LB-1	6.786	0.009	14.904	0.000	5.553	0.019
LB-2	20.184	0.000	36.146	0.000	12.867	0.002
LB-3	28.312	0.000	36.537	0.000	14.767	0.002
LB-10	37.510	0.000	41.688	0.000	30.156	0.000
LB-20	47.938	0.000	45.274	0.001	38.440	0.008
Squared residulas						
LB-1	0.281	0.596	1.344	0.246	0.547	0.460
LB-2	0.575	0.750	18.081	0.000	0.944	0.624
LB-3	0.597	0.897	19.455	0.000	0.944	0.815
LB-10	2.248	0.994	21.137	0.020	2.762	0.987
LB-20	4.404	0.999	31.192	0.053	4.903	1.000
ARGARCH(1, 1)						
LB-1	0.000	0.988	0.805	0.370	0.059	0.809
LB-2	10.817	0.005	20.510	0.000	6.462	0.040
LB-3	17.227	0.001	20.535	0.000	7.785	0.051
LB-10	26.084	0.004	27.202	0.002	22.261	0.014
LB-20	36.387	0.014	31.479	0.049	30.215	0.066
Squared residuals						
LB-1	0.199	0.655	0.342	0.558	0.544	0.461
LB-2	0.497	0.780	14.425	0.001	0.991	0.609
LB-3	0.527	0.913	16.024	0.001	0.993	0.803
LB-10	2.004	0.996	17.572	0.063	2.773	0.986
LB-20	4.467	1.000	27.901	0.112	4.788	1.000

3.4. Comparison of regime-switching and GARCH models

The regime switching and GARCH models described above differ in their representations of time-varying volatility. The regime-switching models specify constant within-regime volatility. Any heteroskedasticity in the data must therefore be described by jumps between the two regimes' volatilities. In contrast, the GARCH models incorporate innovations directly, so that the volatility can take on any positive value. Unfortunately, besides the Ljung–Box statistics, standard econometric assessments of the relative performance of these two models in capturing the features of the data are invalid since the models are non-nested.

Another way of comparing the regime switching and GARCH models is through out-of-sample forecast errors. An out-of-sample test controls for the possibility of overfitting and provides a useful framework for evaluating the merits of competing models. The parameters of the ARGARCH(1, 1) and the three regime-switching models (one, two, and four regimes) are estimated with exchange rate data through 1992, and are then held fixed for the remainder of the sample period. Variance forecasts of horizons 1, 4, and 8 weeks were constructed for each model. Each forecast is then compared with the realized variance over the forecast period, where realized variance is given by the sum of the models' squared residuals.

For a j week horizon, the forecasts are defined as

$$j\text{-week: var}\left[\sum_{i=1}^{j} y_{t+1}|\tilde{y}_t\right]. \tag{15}$$

where $\tilde{y}_t = \{y_t, y_{t-1}, \dots\}$. For the single regime model, a j-week variance forecast is simply $j\sigma^2$. For the two-regime and four-regime models, the variance forecast is a function of the regime probabilities, which are updated prior to each forecast. Consider the two-regime model. Define the variance forecast at time t of a single observation at time $t+j$ as

$$\begin{aligned} s_{t+j}^2 \equiv \text{var}[y_{t+j}|\tilde{y}_t] = \mathrm{E}_t[y_{t+j}^2] - \mathrm{E}_t[y_{t+j}]^2 \\ = p_{1t,t+j}(\sigma_1^2 + \mu_1^2) + (1 - p_{1t,t+j})(\sigma_2^2 + \mu_2^2) \\ - [p_{1t,t+j}\mu_1 + (1 - p_{1t,t+j})\mu_2], \end{aligned} \tag{16}$$

where

$$p_{1t,t+j} = \Pr[S_{t+j} = 1|\tilde{y}_t], \tag{17}$$

which is the first element in a two-element vector of regime probabilities for time $t+j$ given by

$$p_{t+j} = p_t'\Pi^j. \tag{18}$$

In the two-regime model, a j-week variance forecast is then

$$j\text{-week: } \sum_{i=1}^{j} s_{t+i}^2. \tag{19}$$

The four-regime forecasts are constructed similarly. A one-week forecast, for example, is given by

$$\begin{aligned} \mathrm{E}_t[\sigma_{t+1}^2|\tilde{y}_t] = \mathrm{E}_t[y_{t+1}^2] - (\mathrm{E}_t[y_{t+1}]^2) \\ = p_{1t,t+1}[\sigma_1^2 + \mu_1^2] + p_{2t,t+1}[\sigma_1^2 + \mu_2^2] + p_{3t,t+1}[\sigma_2^2 + \mu_1^2] \\ + p_{4t,t+1}[\sigma_2^2 + \mu_2^2] - [(p_{1t,t+1} + p_{3t,t+1})\mu_1 \\ + (p_{2t,t+1} + p_{4t,t+1})\mu_2]^2. \end{aligned} \tag{20}$$

Table 5
Variance forecasts. Variance forecasts are computed using model parameters estimated using data from January 1973 to December 1992. Forecast lengths are overlapping horizons of 1, 4, and 8 weeks over the period January 1993 to December 1996. Regime probabilities and ARGARCH volatilities are updated throughout the forecast period. 'Constant-variance' denotes a stationary normal distribution. ARGARCH denotes a GARCH(1, 1) model with a first-order auto-regressive mean

	JPY	DM	GBP		JPY	DM	GBP
Constant variance				*Two-regime*			
RMSE 1	4.842	3.397	2.749	RMSE 1	4.850	3.413	2.602
RMSE 4	10.408	7.108	6.599	RMSE 4	10.438	7.207	5.356
RMSE 8	16.916	11.116	10.905	RMSE 8	17.805	11.271	7.354
MAD 1	2.346	2.167	1.928	MAD 1	2.439	2.059	1.560
MAD 4	6.468	5.690	5.575	MAD 4	6.202	5.284	3.980
MAD 8	11.216	9.432	9.850	MAD 8	10.462	8.384	5.710
Four-regime				*ARGARCH(1, 1)*			
RMSE 1	4.906	3.404	2.591	RMSE 1	4.845	3.409	2.574
RMSE 4	10.873	6.914	5.259	RMSE 4	10.486	7.135	5.076
RMSE 8	19.315	11.060	6.758	RMSE 8	17.612	11.630	7.790
MAD 1	2.296	1.855	1.480	MAD 1	2.426	2.035	1.548
MAD 4	6.122	4.632	3.800	MAD 4	6.632	5.187	3.919
MAD 8	10.923	7.102	5.043	MAD 8	12.266	8.599	6.637

For the ARGARCH(1, 1) model, the one week forecast is actually given by the model

$$\mathrm{E}_t[\sigma_{t+1}^2|\tilde{y}_t] \equiv h_{t+1} = \sigma^2 + \alpha\varepsilon_t^2 + \beta h_t\,. \tag{21}$$

More generally a j-week forecast is given by

$$\mathrm{E}_t[\sigma_{t+j}^2|\tilde{y}_t] = j\,\frac{\sigma^2}{1-\alpha-\beta} + \left(h_{t+1} - \frac{\sigma^2}{1-\alpha-\beta}\right)\frac{1-(\alpha+\beta)^j}{1-(\alpha+\beta)}\,. \tag{22}$$

Results of the forecast experiment are listed in Table 5. Listed are the root mean squared forecast error (RMSE) and the mean absolute deviation (MAD) of weekly variance for each exchange rate and each forecast length. The four-regime model outperforms the other models for the GBP and DM using the two measures of forecast accuracy. At an eight four-week horizon, for example, the four-regime model reduces the GBP RMSE by 13% and the GBP MAD by 24% versus the ARGARCH(1, 1) model. For the DM, the four-regime model reduces the RMSE by 5% and the MAD by 17% versus the ARGARCH(1, 1) model. The four-regime model also outperforms the others for the JPY MAD, but is outperformed for the JPY RMSE. On the whole, the results indicate that the four-regime model reduces variance forecast error relative to the other models.

This section shows that the four-regime model captures the dynamics of exchange rates better than other models. Residuals and squared residuals of the four-regime model display no serial correlation, whereas the other models considered here do exhibit some serial correlation. Furthermore, the four-regime model generally outperforms the other models in variance forecast accuracy. The next two sections explore the practical implications of regime-switching models for investments in currency options.

4. Regime-switching information in currency option prices

The results in the previous section show that exchange rates exhibit shifts in their distributions as specified in a regime-switching model. If these shifts are meaningful economically, they should be incorporated within the structure of foreign currency option prices; option prices should be higher, all else equal, when the probability of the higher volatility regime exceeds the probability of the lower volatility regime. This section determines the extent to which the cross section and time series of exchange-traded option prices are consistent with a regime-switching model. There are three parts to the analysis. First, regime-switching parameters are inferred from option prices in the same manner as Black–Scholes (1973) implied volatilities. These parameter estimates are compared with those from the time series analysis of the previous section. Second, Black–Scholes implied volatilities are compared with the time series of volatility forecasts generated from the regime-switching model. Third, confidence band around model values are generated from the model parameters' variance–covariance matrix. We then determine what percentage of observed option prices fall outside the model's confidence bands.

4.1. Data

Daily option prices from February 1983 through May 1996 were obtained from the Philadelphia Stock Exchange (PHLX). On each date, open, high, low, and closing option prices, along with contemporaneous spot prices, are recorded for each contract traded that day. The total number of trades and contracts traded are also included with each record. Certain data, especially for the JPY, were unusable due to truncation of prices. The PHLX has markets for both American- and European-style foreign currency options. The trading volume in American-style options dwarfs that of their European-style counterparts. For the GBP and DM, the number of contracts of American-style options are on average about 20 times the number of contracts of European-style options. For the JPY, the difference is a factor of about 40. Since the American-style options are much more liquid than the European-style options, we use only the American-style options in our analysis.

Valuation of currency options requires observations of appropriate domestic and foreign interest rates. To proxy for these rates, we Eurodeposit rates of one, three, and six months to maturity. These data are obtained daily from *Datastream*. The rate corresponding to each option's time to expiration between one and six months is obtained by interpolating the two interest rates whose maturities straddle the option expiration date. For options with times to expiration less than one month (greater than six months), the one-month (six-month) rate is used.

4.2. Standard currency option valuation

The standard method for valuing foreign currency options is a variation of the Black–Scholes (1973) model.[5] The model assumes that the underlying foreign exchange rate S follows a geometric Brownian motion with instantaneous volatility σS and that the domestic interest rate r_d and the foreign interest rate r_f are constant continuous rates. Under these assumptions, the value of a European-style call option with exercise price X and time to maturity T is given by

$$c = Se^{-r_f T}N(d_1) - Xe^{-r_d T}N(d_2) \tag{23}$$

where

$$d_1 = \frac{\ln(S/X) + (r_d - r_f + 1/2\sigma^2)T}{\sigma\sqrt{T}},$$

$$d_2 = d_1 - \sigma\sqrt{T},$$

and $N(z)$ is the cumulative distribution function for a standard normal random variable with upper integral limit z. We dub Eq. (23) the 'modified Black–Scholes formula'. American-style options on foreign currencies can be valued straightforwardly using the Cox–Ross–Rubinstein (1979) binomial method or the Barone-Adesi and Whaley (1987) quadratic approximation.

4.3. Option valuation in regime-switching models

In Section 3, we argued that a four-regime model with independent switches in the mean and variance captures the dynamics of foreign exchange rates better

[5] The variation arises from the fact that the option's underlying asset produces income during the option's life. Merton (1973) derived the valuation equations for European-style options written on stocks with constant, proportional dividend yield. The foreign currency option valuation model is the same, except that the foreign interest rate replaces the dividend yield parameter. Developments of the European-style foreign currency option valuation equation may be found in Garman and Kohlhagen (1983), Grabbe (1983), and Biger and Hull (1983).

than a single-regime model. The four-regime model specifies four conditionally normal distributions four log exchange rate changes at any point in time. Since the governing regime can switch randomly, the volatility of log exchange rate changes can also switch randomly. Option valuation methods in regime switching models must, then, contend with stochastic volatility.

Stochastic volatility can pose significant problems in option valuation. Since volatility is not a traded asset, volatility risk cannot be hedged and therefore risk neutrality cannot be obtained using the usual Black–Scholes riskless hedge argument. In our model, stochastic volatility is driven by regime risk, and we assume that regime risk is diversifiable. If such is the case, the price of regime risk in equilibrium is zero, and option valuation may be conducted in a risk-neutral framework. Under risk neutrality, the volatility dynamics are equivalent to the volatility dynamics in the real world. In addition, the four-regime model of fitted in Section 3 collapses to a two-regime model in which the regimes differ only in volatility and persistence. The risk-neutral mean rate of appreciation of the foreign currency in each regime equals the difference between domestic and foreign interest rates.

Naik (1993) presents an analytical solution for the value of European-style options in a regime-switching model. His approach uses the regime persistence parameters to compute the expected duration for each regime over the option's life, similar to Hull and White's (1987) use of expected average volatility in their stochastic volatility option model. Naik's methodology is limited to European-style options. Since trading volume in American-style currency options dwarfs trading volume in European-style currency options, we will be primarily interested in valuing American-style options. For this reason, we use a numerical valuation method introduced in Bollen (1997).

The valuation technique uses a discrete-time approximation to the two-regime risk-neutral process. Since each regime is characterized by a conditionally normal distribution, we can represent the two-regime model by a pair of binomial distributions. In a lattice framework, this approximation would translate to four branches stemming from each node. One pair of branches corresponds to one regime and the other pair of branches corresponds to the other regime. Bollen (1997) shows, however, that this approach results in the number of nodes growing exponentially through time. An extra branch is added to one of the regimes to allow for more efficient recombining in the lattice, resulting in the number of nodes growing linearly through time.[6] The five-branch or pentanomial lattice uses a binomial distribution to represent one regime and a trinomial distribution to represent the other. The branch probabilities, conditional on regime, are calculated to match the mean and the variance of both distributions.

[6] See Boyle (1986) for a discussion of the computational advantages of increasing the number of branches in lattice-based option valuation procedures.

In the lattice, a path-dependency problem arises when regime probabilities are used in intermediate computations. The reason for this is that regime probabilities are dependent on the particular series of observed changes in the underlying variable, as described in Section 2. The path-dependency of regime probabilities is avoided by computing two conditional option values at each node, where the conditioning information is the prior regime. Options are valued in the standard way, iterating backward from the terminal array of nodes. For the terminal array, the two conditional option values are the same at each node. They are simply the maximum of zero and the option's exercise proceeds. For earlier nodes, conditional option values will depend on regime persistence since the persistence parameters are equivalent to future regime probabilities in a conditional setting. For example, suppose the lattice is used to value an American-style call option. The call option at time t, conditional on volatility regime 1, is related to conditional option values at time $t+1$ as follows:

$$c(t,1) = \mathrm{Max}[S_t - X, \mathrm{e}^{-r_{\mathrm{d}}\Delta t}\{P_\sigma \mathrm{E}[c(t+1,1)] + (1-P_\sigma)\mathrm{E}[c(t+1,2)]\}]\,, \tag{24}$$

where S_t is the exchange rate at time t, X is the exercise price of the option, and r_{d} is the domestic riskless rate of interest. The early exercise proceeds are compared with the discounted expected option value on period later. Regime 1's persistence affects the expectation of future option values by weighting the expectations of conditional option values. Expectations of the conditional option value at time $t+1$ are taken over the appropriate regime's branches using the corresponding conditional branch probabilities. Similarly, the call option at time t, conditional on volatility regime 2, is affected by volatility regime 2's persistence:

$$c(t,2) = \mathrm{Max}[S_t - X, \mathrm{e}^{-r_{\mathrm{d}}\Delta t}\{(1-Q_\sigma)\mathrm{E}[c(t+1,1)] + Q_\sigma \mathrm{E}[c(t+1,2)]\}]\,. \tag{25}$$

When the current regime is always known, one of the two conditional option values at the seed node in the lattice is correct. Since the current regime is known, traders know which option value is appropriate. The lattice can be used in this fashion to accurately value both European- and American-style options.

When the current regime is not known with certainty, the European-style option value is the probability weighted average of the two conditional option values at the seed node, where initial regime probabilities are the weights. The backward iteration technique is simply a way of computing probabilities of terminal option payoffs consistent with initial regime probabilities and the probability of switching regimes at each intermediate node. For American-style options, averaging the initial conditional option values introduces some approximation error, since the value of the early exercise feature is estimated by an average over the two extremes of regime probability, rather than a continuous integration over the entire range of regime probabilities.

In our empirical analysis, we assume that the current regime is not known with certainty, consistent with standard regime-switching models. While this means that valuation of American-style options is subject to approximation error, the size of the error will likely be small when the current regime is known with some degree of certainty. There are two reasons why traders should have reasonably precise knowledge of the current regime. First, Bollen (1997) presents simulation evidence that shows that regime probabilities close to zero or one are obtained over 97% of the time when data simulated from a regime-switching process are sampled daily. Second, traders will likely use other information besides the time series of observations to determine regime. If government policy affects regime, for example, then knowledge of the political environment may help traders determine the current regime.

4.4. Evidence of regime-switching information in option prices

Our analysis of exchange-traded currency options seeks to determine whether traders make full use of the information contained in past data. There are three components of our in-sample analysis. First, regime-switching parameters inferred from option prices are compared with the parameter estimates from the time series analysis of the previous section. Though parameters inferred from options correspond to the risk-neutral process and parameters estimated from the time series correspond to the true data-generating process, the volatility dynamics of the risk-neutral and data generating processes are equivalent.[7] We can therefore meaningfully compare the volatility and persistence of volatility parameters from the options and the time series. A close similarity between the two-parameter sets would indicate that market prices do incorporate some regime-switching information. While this analysis assumes that option prices are generated by the regime-switching model, traders may be incorporating regime-switching information simply by adjusting the volatility that they use in the modified Black–Scholes model. The second part of the in-sample analysis investigates the correspondence between the standard volatility implied by option prices and the time series of volatility forecasts generated from the regime-switching model. Finally, we determine whether observed option prices fall within confidence bands around the regime-switching model option values, where the bands are generated from the standard errors of the model's parameter estimates.

4.4.1. Implied regime-switching parameters

In this section we compare regime-switching parameters inferred from American-style option prices to those from a time series analysis of exchange rates.

[7] See related discussions in Grundy (1991), Lo and Wang (1995), and Bates (1996).

The time series of option prices includes weekly observations of the two options closest to the money. The four parameters inferred from option prices are the variance and persistence of the two volatility regimes. Consistent with the regime-switching model, the option inference procedure restricts these parameters to be constant for the entire sample of options. Note that means of the regimes cannot be inferred from options since risk-neutral valuation provides no information regarding the mean of the underlying asset's return distribution.

For this experiment, the historical exchange rate data used in the time series analysis corresponds to only the time period spanned by the options data. Since this sampling period is a subset of the period analyzed in the previous section, the time series parameter estimates differ. As before, parameters are estimated from the time series using maximum likelihood.

The parameters implied by the option prices are estimated using nonlinear least squares. To illustrate the estimation, let θ denote a vector of the two sets of regime variance and persistence parameters, let P denote a vector of regime probabilities, one for each date on which option prices are observed in the sample, let p_t denote the probability of regime 1 on date t, let $V_{i,t}$ denote the ith observed option price on date t, and let $O_{i,t}(\theta|S_t = j)$ denote the corresponding model price conditioned on regime j and parameter vector θ. The regime probabilities are constructed from θ using the Bayesian procedure outlined in Section 2. The mean of the exchange rate returns is set equal to the difference between the domestic and foreign one-month interest rates recorded each week, consistent with risk-neutral valuation. These are, in a sense, risk-neutral regime probabilities but are shown later to be consistent with standard regime probabilities. The unconditional model price for the ith observed option price on date t, denoted by $g_{i,t}(\theta)$, is given by

$$g_{i,t}(\theta) = p_t O_{i,t}(\theta|S_t = 1) + (1 - p_t) O_{i,t}(\theta|S_t = 2)\,. \tag{26}$$

Let T denote the number of dates on which option prices are observed in the sample, and suppose that two options prices are observed on each date. The non-linear least squares estimator of the parameter vector θ minimizes S, the sum of squared deviations between the model option prices and the observed option prices:

$$S = \sum_{t=1}^{T} \sum_{i=1}^{2} [V_{i,t} - g_{i,t}(\theta)]^2\,. \tag{27}$$

To estimate the variance–covariance matrix of the parameter estimates, let M equal the number of options in the sample, let S^* denote the minimum S, and let s denote the estimated variance of the deviations between model prices and observed prices. We set s equal to $S^*/(M - 4)$ to account for the four parameters estimated in the procedure. Let X denote an $M \times 4$ matrix of partial derivatives of model option prices with respect to the variance and persistence parameters evaluated at the optimal θ, denoted by θ^*. We make the necessary assumptions for asymptotic normality of the parameter estimates (see Amemiya, 1985, pp. 129–134), so that

$$\sqrt{M}(\theta^* - \theta_0) \to \mathrm{N}(0, s(X'X)^{-1}) . \tag{28}$$

The computer program that minimizes S in Eq. (27) uses the IMSL routine DBCPOL, which performs a direct search over θ using a geometric complex. Engle and Mustafa (1992) conduct a similar estimation procedure to infer the parameters of a GARCH model from option prices. They use the same IMSL routine we do to identify the parameter vector that minimizes the sum of squared deviations between model and observed prices. They use simulation methods to compute option prices consistent with candidate GARCH parameter vectors, however, whereas we use the lattice method described in Bollen (1997).

Table 6 compares the parameter values inferred from the time series of option prices with those estimated from the time series of exchange rates. Differences appear. In all cases, the volatility of the low volatility regime is lower and the volatility of the high volatility regime is higher in the time series analysis. Further, the persistence parameters are higher for the options than the time series. To gauge the importance of these differences, we use the standard errors for both sets of parameter estimates to formally test whether the two sets of parameter estimates are significantly different. The test is as follows. Let $\tilde{\theta}$ and θ^* denote the estimates from the time series and the options, respectively. Under the assumption that the estimates are independent,

$$\sigma^2(\tilde{\theta} - \theta^*) = \sigma^2(\tilde{\theta}) + \sigma^2(\theta^*) . \tag{29}$$

The test for each parameter is then a test for a significant difference between them:

$$\frac{\tilde{\theta} - \theta^*}{\sqrt{\sigma^2(\tilde{\theta} - \theta^*)}} \sim \mathrm{N}(0, 1) . \tag{30}$$

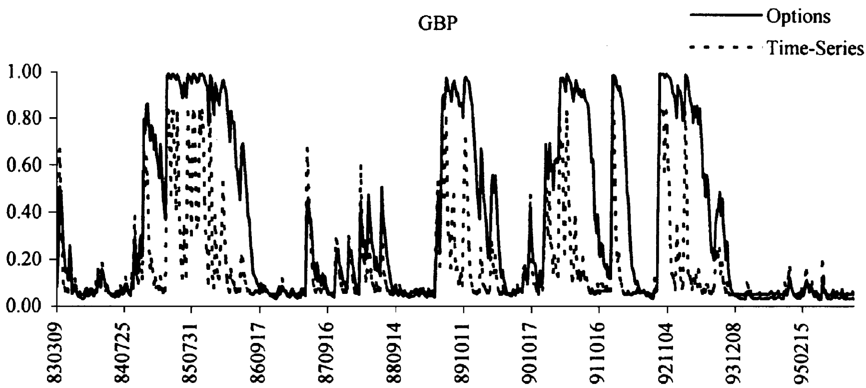
A joint test for each currency's parameters simply adds the squared standard normal deviated to form a χ^2_4 test statistic.

As seen in the last column of Table 6, for all three currencies, all four parameters show deviations that fall within standard levels of significance with the exception of the low volatility for the JPY. Furthermore, two of the three joint tests fail to reject the null hypothesis that the true parameter values are in fact the same. Thus, although the parameter estimates differ, the difference can largely be attributed to estimation error. This result can be interpreted two ways. Either the true parameter vectors are the same, or the test is too weak to distinguish a difference between them. To address this ambiguity, we conduct additional tests in Sections 4.4.2–5.

Table 6
Parameter comparisons. Time series parameter estimates are estimated using MLE assuming a four-regime model with normally distributed returns in each regime. Option parameter estimates are inferred from American-style option prices using an IMSL optimization routine and Bollen's (1997) lattice for valuing options in regime-switching models. Listed in each row are the time series parameter estimate, its standard error, the options-based parameter estimate, its standard error, and the probability of a more extreme difference between the two parameter estimates under the null hypothesis that the population parameters are equal. The test for a significant difference assumes that the measurement errors are independent

Parameter	Time series estimates	Std. error	Option-based estimates	Std. error	p-value
GBP					
σ_1	8.717%	0.800%	9.591%	3.000%	0.778
σ_2	20.856%	8.150%	15.923%	4.960%	0.605
P_σ	0.969	0.028	0.989	0.035	0.658
Q_0	0.834	0.305	0.991	0.030	0.609
Joint test					0.938
DM					
σ_1	8.761%	0.510%	8.985%	4.390%	0.960
σ_2	16.807%	1.663%	15.689%	4.642%	0.821
P_σ	0.965	0.014	0.971	0.073	0.938
Q_σ	0.893	0.037	0.973	0.069	0.311
Joint test					0.896
JPY					
σ_1	5.355%	0.766%	8.949%	2.313%	0.001
σ_2	14.256%	4.550%	13.526%	3.990%	0.381
P_σ	0.862	0.110	0.994	0.032	0.278
Q_σ	0.785	0.157	0.986	0.044	0.200
Joint test					0.005

Fig. 2 compares the time series of regime probabilities inferred from the options to those implied by the regime-switching model. As mentioned previously, the options-based regime probabilities are constructed consistent with risk-neutrality whereas the actual regime probabilities incorporate the observed regime drifts. Since the two sets of probabilities are constructed from the same set of exchange rate returns, however, and both use volatility and persistence parameters that are independent of risk preferences, the two sets of probabilities should be comparable. Indeed, the two series appear to track each other quite well for all three currencies. The relation between the regime probabilities from the time series estimation and from the options is tested more formally in OLS regressions. Table 7 lists the results from regressing the regime probabilities and weekly changes in regime probabilities inferred from options on the associated variables from the time series estimation. The regressions in all cases show a significant relation between the variables, as measured by the standard F-test.


GBP
Options
Time-Series
1.00
0.80
0.60
0.40
0.20
0.00
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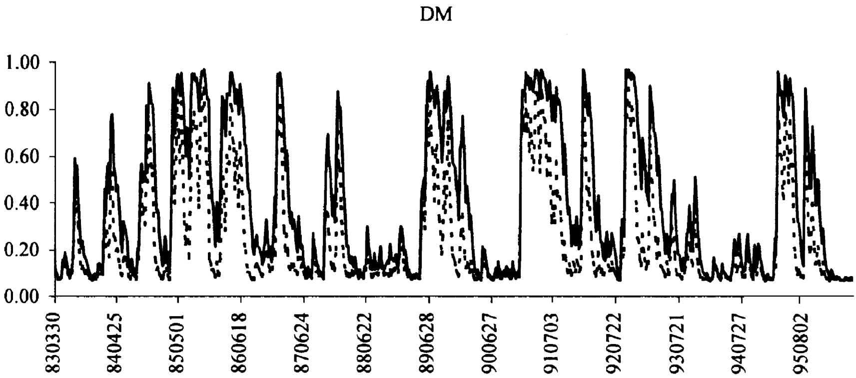
850731
860917
870916
880914
891011
901017
911016
921104
931208
950215


DM
1.00
0.80
0.60
0.40
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0.00
830330
840425
850501
860618
870624
880622
890628
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930721
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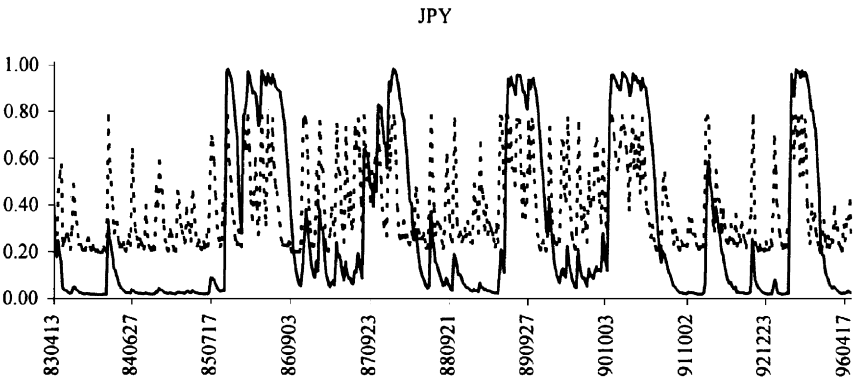
JPY
1.00
0.80
0.60
0.40
0.20
0.00
830413
840627
850717
860903
870923
880921
890927
901003
911002
921223
960417

Table 7
Regression tests of regime probabilities. Results from the OLS time series regression of the options-based regime probabilities and changes in regime probabilities on the associated time series-based counterparts. The time series-based probabilities are constructed from parameter estimates of a four-regime model using MLE on weekly exchange rate data from *Datastream*. Option-based probabilities are constructed from parameter estimates inferred from American-style option prices

	# of obs.	α	β	F-test	R^2
Regime probability					
DM	667	0.093	1.230	3320.814	0.833
		0.007	0.021	0.000	
		0.000	0.000		
GBP	612	0.151	1.233	509.363	0.454
		0.014	0.055	0.000	
		0.000	0.000		
JPY	525	− 0.111	1.063	238.400	0.312
		0.030	0.069	0.000	
		0.000	0.000		
Change in regime probability					
DM	666	0.000	0.841	2712.838	0.803
		0.002	0.016	0.000	
		0.981	0.000		
GBP	611	0.000	0.530	832.673	0.577
		0.003	0.018	0.000	
		0.962	0.000		
JPY	524	− 0.001	0.400	322.034	0.380
		0.003	0.022		
		0.827	0.000		

4.4.2. Volatility comparison

Another way to judge the information content of option prices relative to the regime-switching analysis of the underlying exchange rates is to compare standard implied volatilities of the options to the volatility forecasts of the regime-switching model. The following experiment is performed. Each week, a Black–Scholes implied volatility is computed for each currency using all options with an exercise price within 5% of the underlying exchange rate and maturity less than 40 days. This is called our 'Black–Scholes volatility forecast.' For the regime-switching model, the variance forecast is constructed using the regime

Fig. 2. Regime probabilities. Time series regime probabilities for the high volatility regime are estimated using foreign exchange rate data and MLE assuming a four-regime model with normally distributed returns in each regime. Option regime probabilities are constructed from regime parameters inferred from American-style option prices using an IMSL optimization routine and Bollen's (1997) lattice for valuing options in regime-switching models.

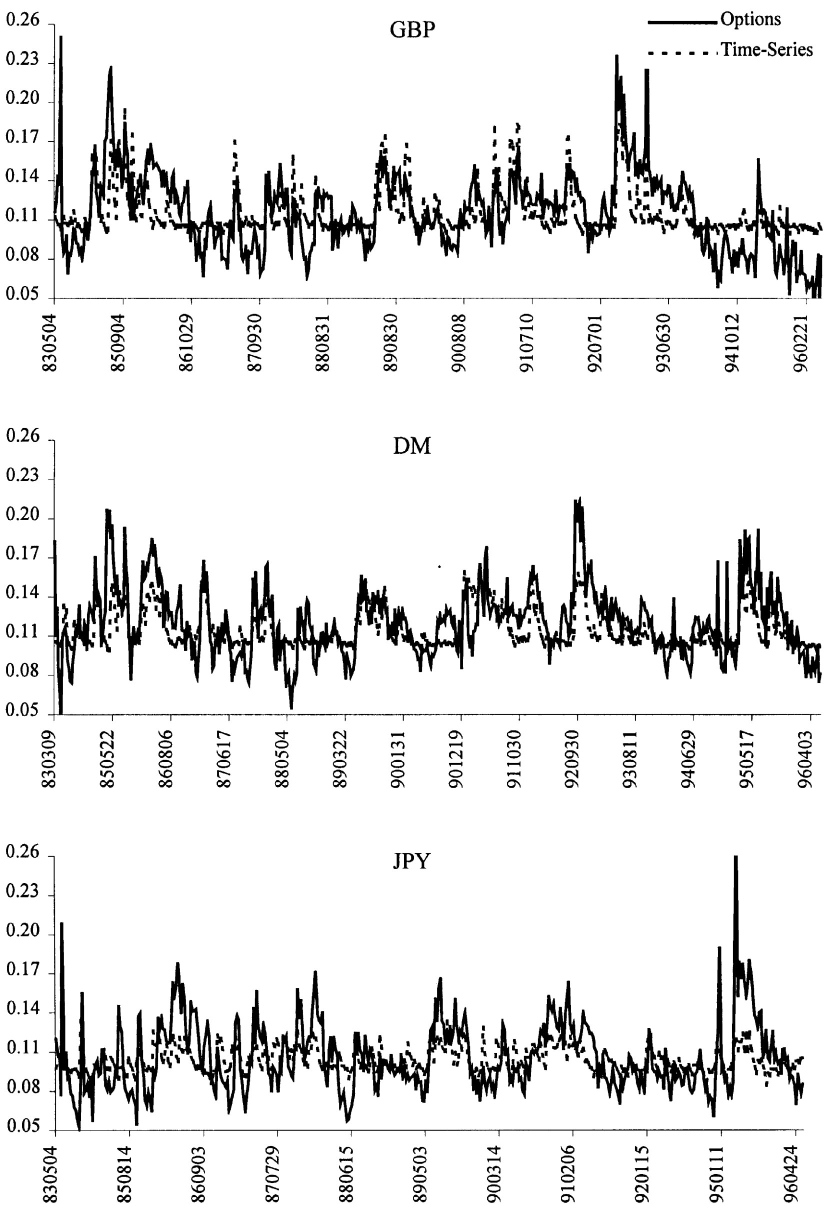

Fig. 3. Exchange rate volatility. Time series volatilities are estimated using exchange rate data and maximum likelihood assuming a four-regime model with normally distributed returns in each regime. Option volatilities are Black–Scholes (1973) volatilities implied from American option prices using a binomial lattice.

probabilities each week and the regime volatilities and persistence parameters, all estimated from a time series analysis of the underlying exchange rates. The variance forecast account for possible regime switches, as in Section 3.4, so that the horizon of the regime-switching forecast matches the average maturity of the options used each week.

Fig. 3 compares the time series of implied volatilities to the regime-switching volatility forecasts for each currency. A prominent difference between the two is that the Black–Scholes implied volatilities have spikes that exceed the maximum volatility and fall below the minimum volatility of the regime-switching forecasts. The regime-switching model implies that the volatility can never fall below the lower volatility regime's volatility nor exceed the higher volatility regime's volatility. This may be an indication that the regime-switching model is misspecified. One possible avenue for further investigation is to use of three volatility regimes, as in the Dahlquist and Gray's (1995) study of EMS interest rates. Another is to allow for conditional heteroskedasticity within regimes, as in Gray (1996).

The relation between the volatilities from the time series estimation and from the options is tested more formally in OLS regressions. Table 8 lists the results from regressing the volatilities and weekly changes in volatilities inferred from options on the associated variables from the time series estimation. As with the OLS analysis of regime probabilities, these regressions in all cases document a significant relation between the variables.

4.4.3. Option valuation bounds

Our final test of the information contained in currency options seeks to determine whether observed option prices are statistically different from model values. For this experiment, we use the regime-switching parameters estimated from the time series data. Since the model's parameter estimates have associated standard errors, we can formally test whether the observed option prices fall within confidence bands around the model values. These tests are in the spirit of Lo (1986). For a given option, valuation inputs can be separated into two groups, observable parameters (underlying currency level, time to maturity, strike price, and interest rates) and unobservable parameters (regime-switching model parameters). Since the option value is a complicated function of the underlying model parameters, we use simulation methods to generate a distribution of option values consistent with the distribution of the model's parameter estimates. We generate 1000 alternative parameter vectors from a multivariate normal distribution with mean vector equal to the vector of parameter estimates and variance–covariance matrix equal to White's (1982) heteroskedasticity consistent matrix from the maximum likelihood estimation of the parameters. We then compute 1000 option values, each consistent with one of the simulated vectors, and sort the values from low to high. The 50th value represents a 5% lower bound on the model value, and the 950th value is a 5% upper bound on

Table 8
Regression tests of volatility. Results from the OLS time series regression of the options-based volatilities and changes in volatilities on the associated time series-based parameters. The time series estimates of the parameters of four-regime model are obtained using MLE on weekly exchange rate data from *Datastream*. Parameters are inferred from options using an IMSL minimization of sum of squared errors between model values and observed option prices

	# of obs.	α	β	F-test	R^2
Volatility					
DM	593	− 0.012	1.155	441.323	0.427
		0.006	0.055	0.000	
		0.062	0.000		
GBP	539	0.004	0.965	267.610	0.331
		0.007	0.059	0.000	
		0.517	0.000		
JPY	455	− 0.048	1.488	173.215	0.275
		0.012	0.113	0.000	
		0.000	0.000		
Change in volatility					
DM	592	0.000	0.810	97.717	0.141
		0.001	0.082	0.000	
		0.838	0.000		
GBP	538	0.000	0.452	59.671	0.098
		0.001	0.059	0.000	
		0.943	0.000		
JPY	454	0.000	0.714	41.851	0.083
		0.001	0.110		
		0.925	0.000		

the model value. If the observed option price falls outside of this band, then we can conclude that the observed price is different from the model value at a 10% significance level.

Since this procedure is computationally burdensome, we limit analysis to the closest-to-maturity, closest-to-the-money option once per week for each currency from January 1993 to May 1996, a total of 347 options. Of these, 177 of the observed option prices fell outside the model's confidence bands. Furthermore, 149 fell in the upper tail. Thus, it appears that observed option prices are often 'overvalued' relative to the regime-switching model.

In summary, there are two main results from our tests of option prices in this section. First, regime-switching parameter estimates inferred from market prices are consistent with estimates from a time series analysis of the underlying exchange rates. Second, option prices generated from the time series estimates are significantly different than market prices. These seemingly inconsistent findings are likely due to differences in the power of the two tests. We interpret the results as evidence that, although market prices reflect some regime-switching information, they do not capture the information fully. This conclusion is confirmed in the trading strategy experiment that follows.

5. Trading simulation

The results of Section 3 indicate that the regime-switching model works better than competing models at describing the time series of exchange rates. Nonetheless, we are left with the distinct possibility that the improved descriptive power explains nothing more than perturbations within an option's bid–ask spread. Complicating matters further is the documented differences between the average volatilities implied by the regime-switching option valuation model and those implied by Black–Scholes model. What if the market is using the wrong option valuation model? Therein lies the danger of evaluating models using a cross section of option prices. As Whaley (1982) notes, the proper way to evaluate competing option valuation technologies is through simulating the model's abilities to generate abnormal risk-adjusted profits. The profits of the alternative trading strategies are a practical measure of the accuracy of the models' expectations of option payoffs. The out-of-sample nature of this test controls for differing levels of model complexity and should allay concerns regarding overfitting.

The three competing valuation models are: (a) a regime-switching model with constant within-regime volatility, (b) the Black–Scholes model using lagged implied Black–Scholes volatility, and (c) the Black–Scholes model using a time series estimate of volatility using the past year's data. When valuing options using the regime-switching model, parameters are estimated directly from the time series of foreign exchange rates. The regime-switching model has two regimes. Both regimes have means equal to the difference between the domestic and foreign interest rates, consistent with risk-neutral valuation. The regime variances are different, however, and are estimated using the four-regime model of Section 2. The parameters of the regime-switching model are estimated annually on the last day of trading in each year, from 1989 to 1996, a total of eight estimations. The exchange rate data used to estimate the model include weekly observations beginning in January 1973. When valuing options, the most recent regime-switching parameter estimates are used, that is, regime probabilities are updated using exchange rate data every week.

American-style options are valued once per week using all three models. Underpriced options are purchased and overpriced options are sold. The positions are then held to option expiration to ensure model convergence, for as Whaley (1986) notes, systematic biases in option valuation models will likely lead to persistent pricing errors and convergence of the observed prices to equilibrium values is only ensured at expiration. Option positions are hedged using offsetting (i.e., delta-neutral) positions in the underlying currencies. The hedge is updated weekly until maturity to respond to changes in the underlying exchange rate level and volatility. Transactions costs associated with updating the hedge are ignored.

Table 9 summarizes the profits of the three strategies trading in options on the DM, GBP, and JPY currencies and assuming no transactions costs. Profits are computed assuming one option contract is traded each time a strategy identifies an over or underpriced option.[8] Three filters (i.e. $\delta = 0$, 5 and 10%) are used. Options with prices greater than $100 + \delta\%$ of model values are sold, and options with prices lower than $100 - \delta\%$ of model values are purchased. The first column lists the results of the regime-switching strategy. The second column corresponds to the strategy that uses Black–Scholes with implied volatilities recovered from option prices of the previous day, and the third column corresponds to the strategy that uses Black–Scholes with a time series estimate of volatility using the past year's data. In all cases, the regime-switching profits are greater than Black–Scholes with lagged implied volatility, which in turn outperforms the strategy based on historical volatility. All three strategies, however, generate more profitable trades than is consistent with a profitability rate of 50%, as determined by the DeMoivre–Laplace normal approximation to the binomial. Note also that the regime-switching strategy tends to short options more than the others. This result is consistent with the earlier finding that Black–Scholes implied volatilities are higher than regime-switching volatilities. In addition, the profit per trade for the regime-switching strategy is higher than the other two.

Positive profits should not be interpreted as a rejection of market efficiency. Market inefficiency requires positive abnormal trading profits after trading costs and risk adjustment. Table 9 ignores trading costs, and the hedge is only updated weekly. Lastly, the average return (profit per option cost) on the three strategies are all positive, ranging from 3.82% for the historical volatility strategy with a filter of 0% to 28.16% for the Black–Scholes volatility strategy with a filter of 10%. Again, since trading costs are ignored, these results should be interpreted with caution.

Table 10 lists the results of the trading strategy experiment accounting for transaction costs in the form of the bid–ask spread and commissions. The bid–ask spread is assumed to raise (lower) the closing price of an option purchased (sold) by one-half the maximum allowed by the exchange.[9] In other words, the closing price is assumed to be at the midpoint between the bid and

[8] Contract sizes are 31,250 GBP, 62,500 DM, and 6,250,000 JPY.

[9] Maximum spreads are a function of the level of the option price and the units of the underlying currency. (See Rule 3014.a.ii of the *PHLX Options Rules*). The maximum spreads of the options used in our analyses are summarized in Appendix A.

Table 9

Trading profits. Profits listed for three trading strategies using PHLX American-style currency options on the DM, GBP, and JPY. Trades are made on Wednesdays from January 1990 through May 1996 at closing prices. Transaction costs are ignored. Positions are delta-hedged using foreign exchange and held to maturity. Hedges are rebalanced weekly. Interest rates used are linearly interpolated from Eurodeposit rates from *Datastream*. The three strategies differ in their valuation model: RS uses the four-regime-switching model with parameters estimated from weekly observations of underlying exchange rates from January 1973 through the December of the prior year, BS uses Black–Scholes with implied volatility from the prior day's options, TS uses Black–Scholes with implied volatility equal to the historical volatility of exchange rates from the past 50 weeks. Options with more than 100 d to maturity of more than 10% in our out-of-the-money are excluded. ITM denotes calls with $X < 0.975S$ or puts with $X > 1.025S$, OTM denotes call with $X > 1.025S$ or puts with $X < 0.975S$, and ATM denotes all other options. The filters determine whether a trade is made, with purchases made whenever option price is less than $(1-\delta)$ times model value, and sales made whenever option price is greater than $(1+\delta)$ times model value. The p-value is the probability of a greater number of profitable trades under the null hypothesis of 50% profitability

	0% filter			5% filter			10% filter		
	RS	BS	TS	RS	BS	TS	RS	BS	TS
# Trades	12,222	12,222	12,222	9890	6913	9508	8484	4651	7769
# Long	2246	5006	5509	1209	2248	4065	747	1163	3086
# Profitable	8474	7452	7124	6971	4433	5606	6074	3159	4690
% Profitable	69.33%	60.97%	58.29%	70.49%	64.13%	58.96%	71.59%	67.92%	60.37%
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
# ITM	1195	1195	1195	282	138	268	119	51	84
# ATM	7337	7337	7337	6088	3867	5832	5058	2284	4571
# OTM	3690	3690	3690	3520	2908	3408	3307	2316	3114
Profits	7011,133.23	433,113.52	− 10,741.46	622,569.59	294,961.58	− 1,277.04	547,418.58	220,253.04	17,922.53
Profit/Trade	57.37	35.44	− 0.88	62.95	42.67	− 0.13	64.52	47.36	2.31
Profit/Cost	14.69%	13.60%	3.82%	17.33%	20.76%	4.87%	19.57%	28.16%	6.09%
Profit-ITM	105,134.33	85,464.27	45,781.42	61,498.43	39,108.25	39,417.88	37,659.99	23,001.34	24,653.61
Profit-ATM	416,781.95	182,372.39	− 8360.10	385,282.50	124,611.55	8025.67	333,490.82	84,010.07	33,696.15
Profit-OTM	179,216.95	165,276.86	− 48,162.78	175,788.66	131,241.78	− 48,720.59	176,267.77	113,241.63	− 40,427.23

Table 10

Trading profits after transaction costs. Profits listed for three trading strategies using PHLX American-style currency options on the DM, GBP, and JPY. Trades are made on Wednesdays from January 1990 through May 1996 at closing prices. A $2 per contract commission and one-half the maximum spread are incorporated in the trading decision and in the cash-flow computations. Positions are delta-hedged using foreign exchange and held to maturity. Hedges are rebalanced weekly. Interest rates used are linearly interpolated from Eurodeposit rates from *Datastream*. The three strategies differ in their valuation model: RS uses the four-regime-switching model with parameters estimated from weekly observations of underlying exchange rates from January 1973 through the December of the prior year, BS uses Black–Scholes with implied volatility from the prior day's options, TS uses Black–Scholes with implied volatility equal to the historical volatility of exchange rates from the past 50 weeks. Options with more than 100 d to maturity of more than 10% in our out-of-the-money are excluded. ITM denotes calls with $X < 0.975S$ or puts with $X > 1.025S$, OTM denotes calls with $X > 1.025S$ or puts with $X < 0.975S$, and ATM denotes all other options. The filters determine whether a trade is made, with purchases made whenever option price after transaction costs is less than $(1-\delta)$ times model value, and sales made whenever option price after transaction costs is greater than $(1+\delta)$ times model value. The p-value is the probability of a greater number of profitable trades under the null hypothesis of 50% profitability

	0% filter			5% filter			10% filter		
	RS	BS	TS	RS	BS	TS	RS	BS	TS
# Trades	9690	6428	9447	7874	3508	7223	6700	2304	5683
# Long	1246	2366	4262	587	966	3039	347	473	2161
# Profitable	6487	3803	5103	5362	2180	4000	4626	1521	3284
% Profitable	66.95%	59.16%	54.02%	68.10%	62.14%	55.38%	69.04%	66.02%	57.79%
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
# ITM	681	544	716	204	99	165	88	37	58
# ATM	6064	3973	5880	4904	1979	4514	4010	1160	3386
# OTM	2945	1911	2851	2766	1430	2544	2602	1107	2239
Profits	407,548.34	178,097.22	– 237,317.74	360,881.52	122,781.53	– 148,981.98	308,290.30	89,317.98	– 91,504.25
Profit/Trade	42.06	27.71	– 25.12	45.83	35.00	– 20.63	46.01	38.77	– 16.10
Profit/Cost	6.09%	5.75%	– 7.41%	6.98%	8.44%	– 7.49%	7.87%	10.48%	– 7.03%
Profit-ITM	72,053.86	57,127.25	19,785.90	45,469.36	32,098.19	31,403.91	31,753.18	18,910.24	20,008.05
Profit-ATM	232,851.26	46,854.67	– 147,110.39	215,371.89	33,559.68	– 90,939.45	174,968.69	28,089.75	– 33,543.94
Profit-OTM	102,643.22	74,115.30	– 109,993.25	100,040.27	57,123.66	– 89,446.44	101,568.43	42,317.99	– 77,968.36

the ask, and we assume the maximum spread in an effort to bias our findings against significant trading profits. In addition, a commission of $2 is incorporated in each transaction. Note that the number of trades decreases when transaction costs are levied because the transaction costs are considered in the strategy before a trade is made. Again, all three strategies generate more profitable trades than is consistent with a profitability rate of 50%. More importantly, the strategy incorporating regime-switching dominates the others in terms of total profits generated and profits per trade.

These results indicate that the regime-switching model may have significant implications from a trading perspective. In addition, to the extent that better option prices indicate more information, the results suggest that the regime-switching model captures the dynamics of exchange rates better than the other models.

6. Conclusions

This paper investigates the ability of regime-switching models to capture the time series properties of foreign exchange rates. A model with independent mean and variance shifts is shown to provide a tighter in-sample fit and more accurate variance forecasts than competing GARCH models. The time series, cross-sectional structure of foreign currency option prices is then examined to determine whether an option valuation framework that incorporates regime-switching performs 'better' than standard models such as Black–Scholes. Exchange-traded option prices are used to infer regime-switching parameters, and these implied parameters are shown to be consistent with those of a time series analysis of the underlying exchange rates. Significant differences between observed market prices and theoretical option values are found, however, and a trading strategy that uses regime-switching option valuation is shown to generate higher profits than alternatives that do not. Overall the results indicate that regime-switching models may have practical implications for investors. Furthermore, to the extent that better option prices indicate more information, the overall results of our analyses suggest that the regime-switching model captures the dynamics of exchange rates better than alternative time series models.

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Appendix A

Maximum bid/ask spread for foreign currency options traded on the Philadelphia stock exchange (S = bid price)
Bristish Pound

$$\text{Maximum spread} = \begin{cases} \$.0015 & \text{if } S \leqslant \$.0250\,, \\ \$.0025 & \text{if } \$.0250 < S \leqslant \$.0750\,, \\ \$.0035 & \text{if } S > \$.0750\,. \end{cases}$$

German Mark

$$\text{Maximum spread} = \begin{cases} \$.0004 & \text{if } S \leqslant \$.0040\,, \\ \$.0006 & \text{if } \$.0040 < S \leqslant \$.0160\,, \\ \$.0008 & \text{if } S > \$.0160\,. \end{cases}$$

Japanese Yen

$$\text{Maximum spread} = \begin{cases} \$.000006 & \text{if } S \leqslant \$.000040\,, \\ \$.000009 & \text{if } \$.000040 < S \leqslant \$.000160\,, \\ \$.000012 & \text{if } S > \$.000160\,. \end{cases}$$

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