Exercise Sheet 02

Problems 01 - 04

Robert Hennings

Christian Albrechts University of Kiel

robert.hennings@stu.uni-kiel.de

GitHub: https://github.com/RobertHennings

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Problem Setup

Problem 1

The following fact will be used later to define the "diversification ratio".

(a) Consider random variables $Y_1, Y_2, \ldots, Y_n$ with covariance matrix $\boldsymbol{\Sigma}$ with elements $\mathrm{Var}(Y_i) = \sigma_i^2$, and $\mathrm{Cov}(Y_i, Y_j) = \sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$, $i, j = 1, \ldots, n$. Show that the standard deviation of the sum $Y_1 + Y_2 + \cdots + Y_n$ is smaller than the sum of the standard deviations $\sigma_1 + \sigma_2 + \cdots + \sigma_n$, that is,

$$\mathrm{Std}(Y_1 + Y_2 + \cdots + Y_n) \leq \sigma_1 + \sigma_2 + \cdots + \sigma_n. \tag{1}$$

Hint: Recall that

$$\begin{aligned} \mathrm{Var}(Y_1 + Y_2 + \cdots + Y_n) &= \sum_{i=1}^{n} \sigma_i^2 + \sum\sum_{i \neq j} \sigma_{ij} &(2)\\ &= \sum_{i=1}^{n} \sigma_i^2 + \sum\sum_{i \neq j} \rho_{ij}\sigma_i\sigma_j, &(3) \end{aligned}$$

and use the fact that $|\rho_{ij}| \leq 1$, $i, j = 1, \ldots, n$.

(b) Now consider a long–only portfolio with N assets. Denote the return of Asset i by R_i, and its portfolio weight by x_i, so that the portfolio return is

$$R_p = \sum_{i=1}^{N} x_i R_i. \tag{4}$$

Using elementary properties of the standard deviation, explain why the result in (a) implies that

$$\sigma_p = \mathrm{Std}(R_p) \leq \sum_{i=1}^{N} x_i \sigma_i, \tag{5}$$

where σ_i is the standard deviation of the return of Asset i. That is, the portfolio standard deviation is smaller than the portfolio–weighted average of the individual standard deviations.

Figure: Problem - 01.

Proposed Solution

$$\cos^3\theta = \frac{1}{4}\cos\theta + \frac{3}{4}\cos 3\theta \tag{1}$$

Problem Setup

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Figure: Problem - 01.

Proposed Solution

In order to solve the problem.....

$$\cos^3 \theta = \frac{1}{4} \cos \theta + \frac{3}{4} \cos 3\theta \qquad (2)$$

Paragraphs of Text

Sed iaculis dapibus gravida. Morbi sed tortor erat, nec interdum arcu. Sed id lorem lectus. Quisque viverra augue id sem ornare non aliquam nibh tristique. Aenean in ligula nisl. Nulla sed tellus ipsum. Donec vestibulum ligula non lorem vulputate fermentum accumsan neque mollis.

> *Sed diam enim, sagittis nec condimentum sit amet, ullamcorper sit amet libero. Aliquam vel dui orci, a porta odio.*
> *— Someone, somewhere…*

Nullam id suscipit ipsum. Aenean lobortis commodo sem, ut commodo leo gravida vitae. Pellentesque vehicula ante iaculis arcu pretium rutrum eget sit amet purus. Integer ornare nulla quis neque ultrices lobortis.

Lists

Bullet Points and Numbered Lists

- Lorem ipsum dolor sit amet, consectetur adipiscing elit
- Aliquam blandit faucibus nisi, sit amet dapibus enim tempus
  - Lorem ipsum dolor sit amet, consectetur adipiscing elit
  - Nam cursus est eget velit posuere pellentesque
- Nulla commodo, erat quis gravida posuere, elit lacus lobortis est, quis porttitor odio mauris at libero

1. Nam cursus est eget velit posuere pellentesque
2. Vestibulum faucibus velit a augue condimentum quis convallis nulla gravida

Blocks of Highlighted Text

Block Title

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Integer lectus nisl, ultricies in feugiat rutrum, porttitor sit amet augue.

Example Block Title

Aliquam ut tortor mauris. Sed volutpat ante purus, quis accumsan.

Alert Block Title

Pellentesque sed tellus purus. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos himenaeos.

Suspendisse tincidunt sagittis gravida. Curabitur condimentum, enim sed venenatis rutrum, ipsum neque consectetur orci.

Multiple Columns

Subtitle

Heading

1. Statement
2. Explanation
3. Example

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Integer lectus nisl, ultricies in feugiat rutrum, porttitor sit amet augue. Aliquam ut tortor mauris. Sed volutpat ante purus, quis accumsan dolor.

Table

Subtitle

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table: Table caption

Figure

Problem 1

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Figure: Problem - 01.

Definitions & Examples

Definition

A prime number is a number that has exactly two divisors.

Example

- 2 is prime (two divisors: 1 and 2).
- 3 is prime (two divisors: 1 and 3).
- 4 is not prime (three divisors: 1, 2, and 4).

You can also use the `theorem`, `lemma`, `proof` and `corollary` environments.

Theorem, Corollary & Proof

Theorem (Mass–energy equivalence)

$E = mc^2$

Corollary

$x + y = y + x$

Proof.

$\omega + \phi = \epsilon$ $\square$

Equation

$$\cos^3\theta = \frac{1}{4}\cos\theta + \frac{3}{4}\cos 3\theta \tag{3}$$

Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
```

Slide without title.

Citing References

An example of the `\cite` command to cite within the presentation:

This statement requires citation [Smith, 2022, Kennedy, 2023].

References

John Smith (2022)
Publication title
Journal Name 12(3), 45 – 678.

Annabelle Kennedy (2023)
Publication title
Journal Name 12(3), 45 – 678.

Acknowledgements

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- Devon Brown

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- Jennifer
- Yuan

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- British Royal Navy
- Norwegian Government

The End

Questions? Comments?