Non-linear effects of energy commodities on the EUR/USD spot exchange rate: An Oil and Gas Perspective on Parity Conditions.

Seminar in Applied Financial Economics: Applied Econometrics of FX Markets - Professor Reitz

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Outline

Intro: Energy Commodities and Exchange Rates

This has led some to suggest that an unidentified real factor may be causing persistent shifts in real equilibrium exchange rates.
— R.A. Amano, S. van Norden[1]

This may in fact be the case or it is also possible that the relationship between exchange rates and oil shocks is non-linear and not being detected by a linear regression framework.
— S. A. Basher, A. A. Haug, P. Sadorsky[2]

The long-run real exchange rate of these 'commodity currencies' is not constant but is time varying, being dependent on movements in the real price of commodity exports.
— P. Cashin, L. F. Cespedes, R. Sahay[3]

[1] [**amano˙1998˙oil˙prices˙exchange˙rate**], p.301

[2] [**basher˙2016˙oil˙shocks˙exchange˙rates**], p.17

[3] [**cashin˙2004˙commodity˙currencies**], p.239

Intro: The PPP puzzle[5] and Commodity Currencies

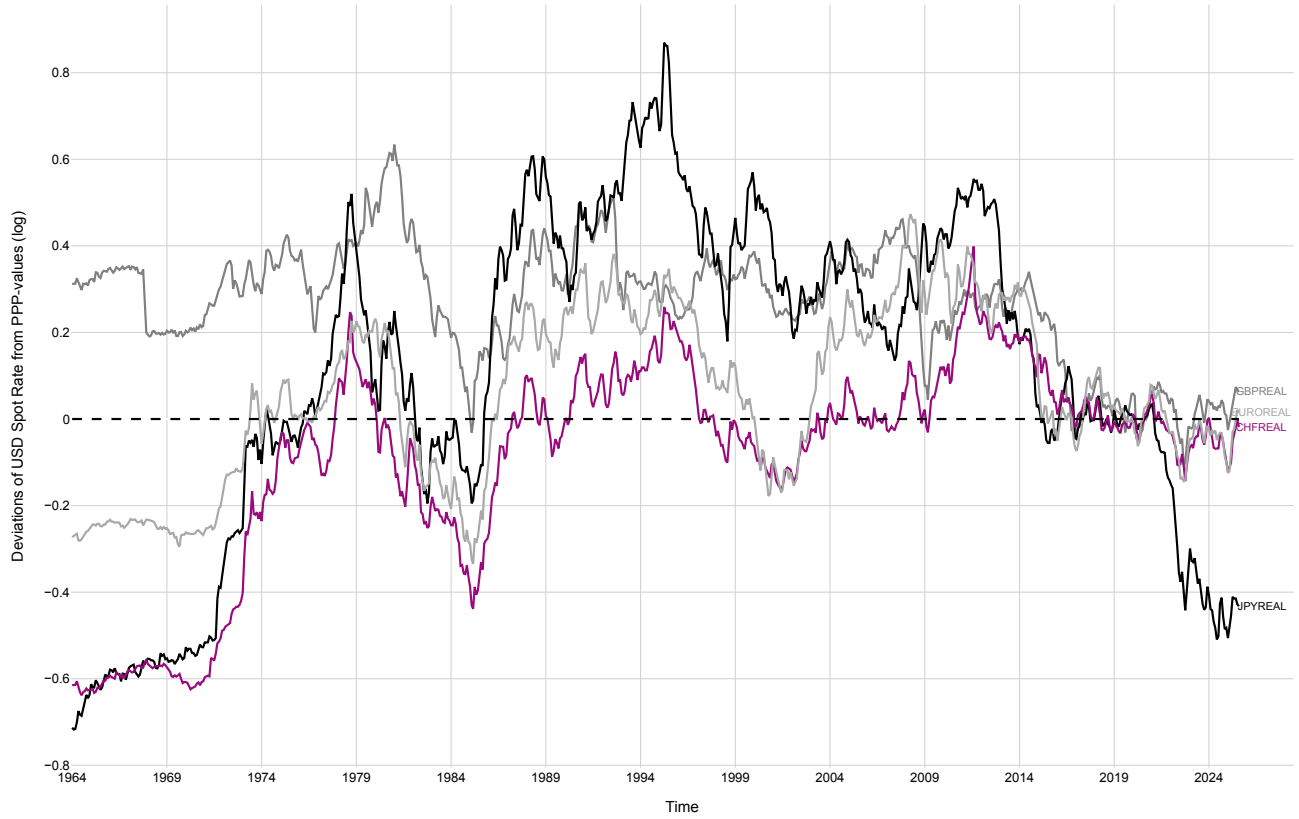


Figure: Monthly deviations of USD Spot Rate from PPP-values (in log terms) over the time: 1964 - 2025.[4]

[4] Own Illustration based on [**reitz˙2025˙applied˙econometrics˙fx**], section "Modeling Trends: Unit Roots in Time Series", page 18/18 and data taken from [**bis˙2025˙effective˙exchange˙rates**], last accessed 24.10.25, own calculations.

[5] This puzzle concerns the finding of many researchers that the speed of mean reversion of real exchange rates is too slow to be consistent with PPP, which is the proposition that exchange rates are determined by movements in relative prices.

Research Hypothesis

Energy Price Contributions to Inflation - USA

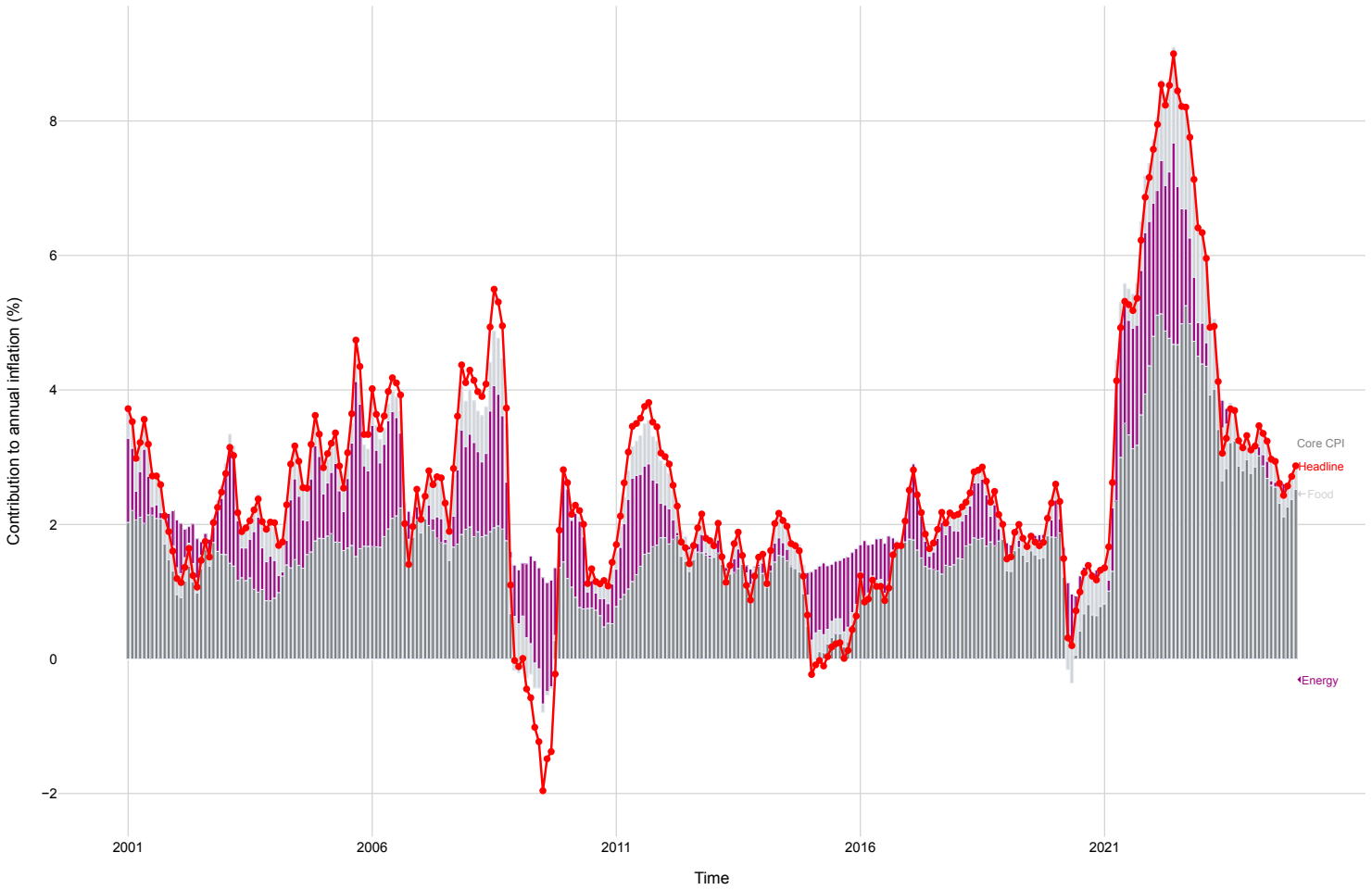


Figure: Monthly US CPI: Headline and component contributions over the time: 2001 - 2024.[6]

[6] Own Illustration based on [**ecb˙2022˙inflation˙developments˙ea˙us**], Chart A and data taken from [**fred˙2025˙federal˙reserve˙data**], last accessed 24.10.25, own calculations.

Energy Price Contributions to Inflation - EU area

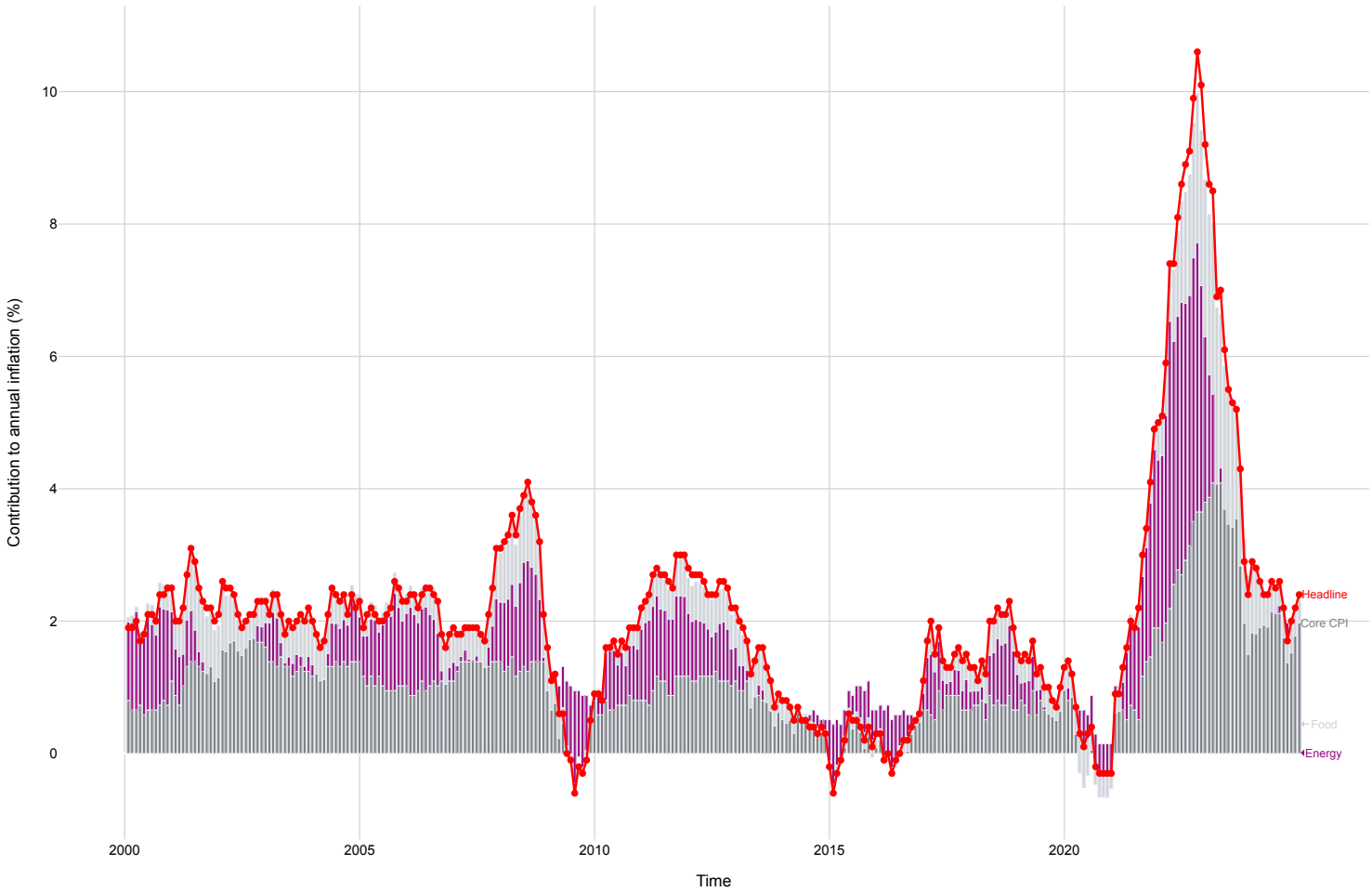


Figure: Monthly EU Area CPI: Headline and component contributions over the time: 2000 - 2024.[7]

[7] Own Illustration based on [**ecb˙2022˙inflation˙developments˙ea˙us**], Chart A and data taken from [**ecb˙2025˙data˙portal**], last accessed 24.10.25, own calculations.

Rolling Volatility of Exchange Rates and Energy Commodity Prices

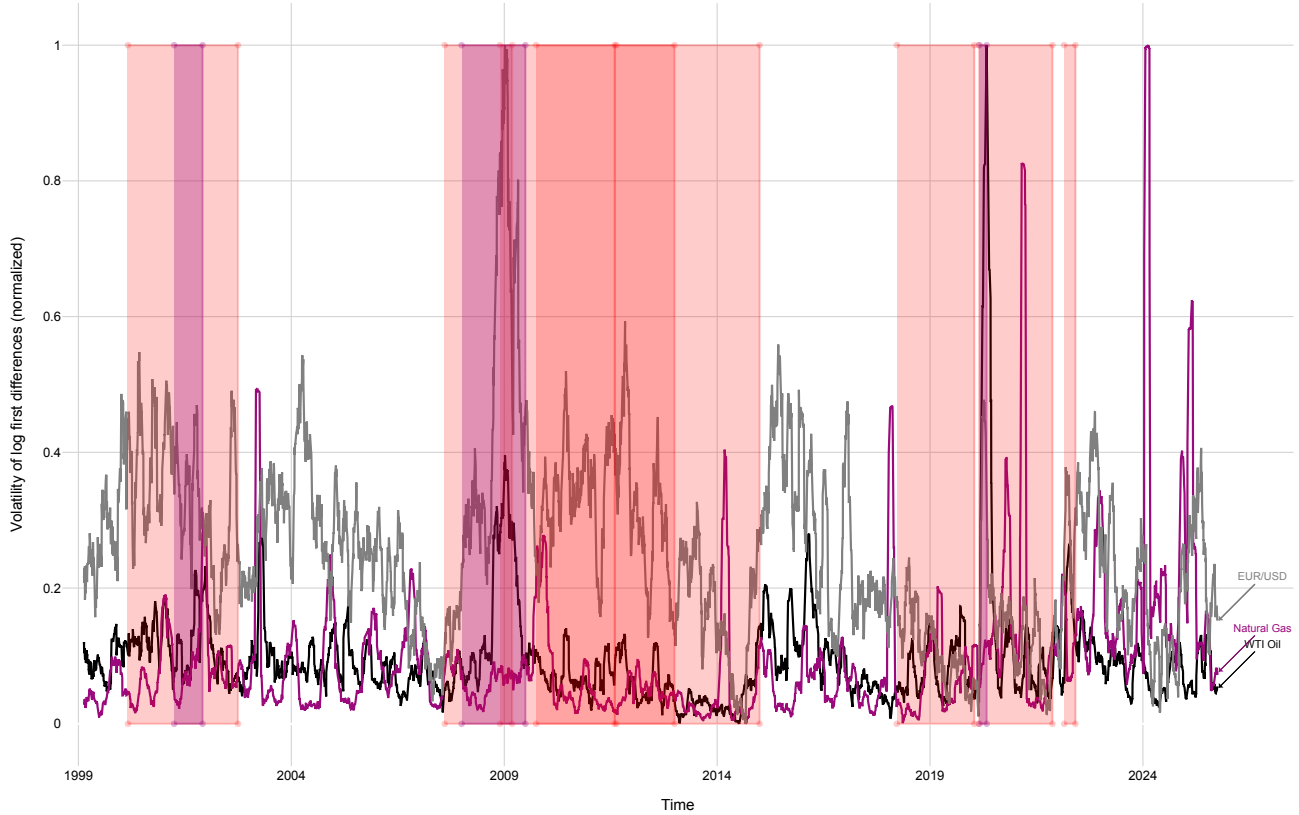


Figure: Daily normalized EUR/USD spot exchange rate, oil and gas log first differences volatility with highlighted crisis periods over the time: 2000 - 2024.[8]

[8] Own Illustration based on [**umar˙2021˙dynamic˙volatility˙spillovers**], page 7 and data taken from [**fred˙2025˙federal˙reserve˙data**], last accessed 24.10.25, own calculations.

Formulated Research Hypothesis

Main Research Hypothesis

"The standard UIP-equilibrium condition is time-dependent and primarily controlled by two main regimes, characterised by either high or low (market-) volatility."

Additional Research Hypothesis I

"Monetary policy, i.e. interest rates are, partly driven by energy commodity prices that induce volatility through the inflationary pass-through channel, especially during phases of market distress in economies heavily relying on import/export of energy commodities."

Additional Research Hypothesis II

"Factoring in variables related to energy commodity prices in combination with using alternative clustering techniques improves the identification of the regimes to better pinpoint the time-dependent testing of the standard UIP relation, compared to Markov-Switching benchmark models."

Literature Review

Systematic Literature Overview: Main Approaches

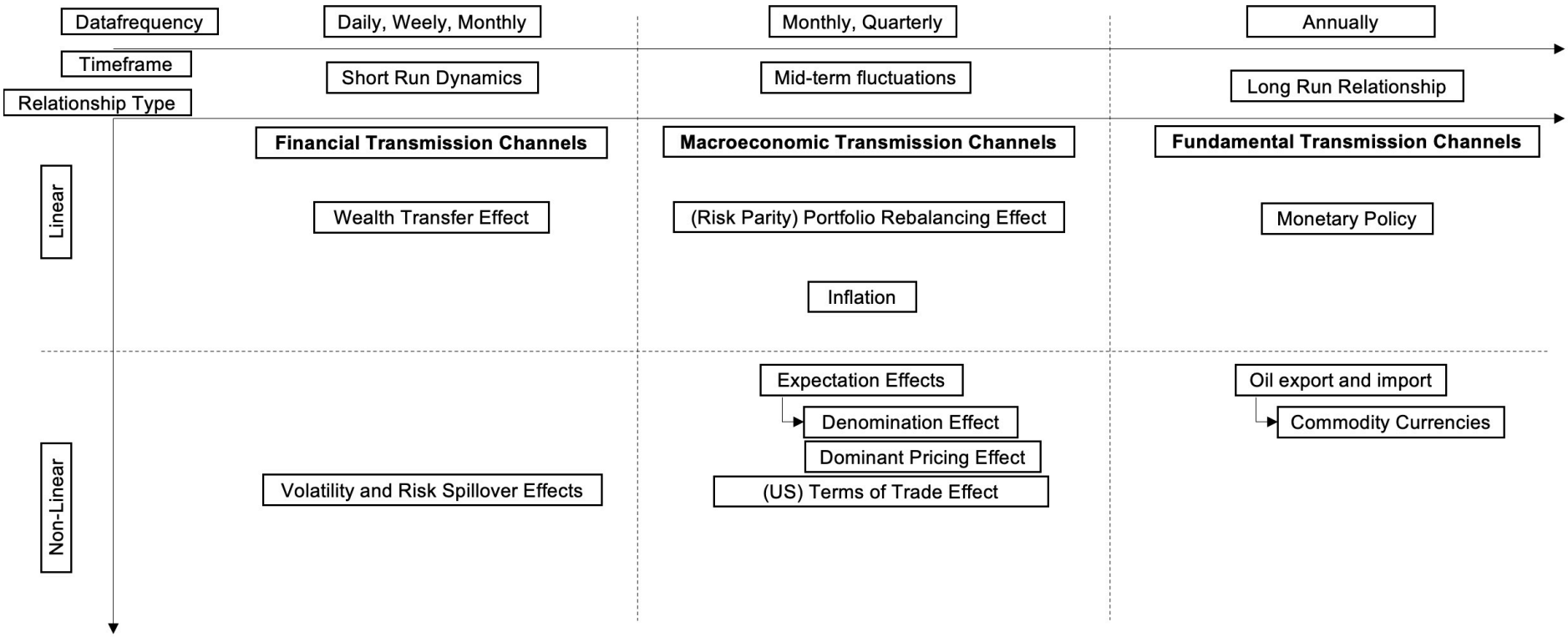


Figure: Systematic Overview about main theoretical approaches.[9]

[9] Own Illustration based on [**oberndorfer˙2020˙oil˙prices˙exchange˙rates**], Figure 5, page 5.

Theoretical Framework

Definitions - prices and measurements

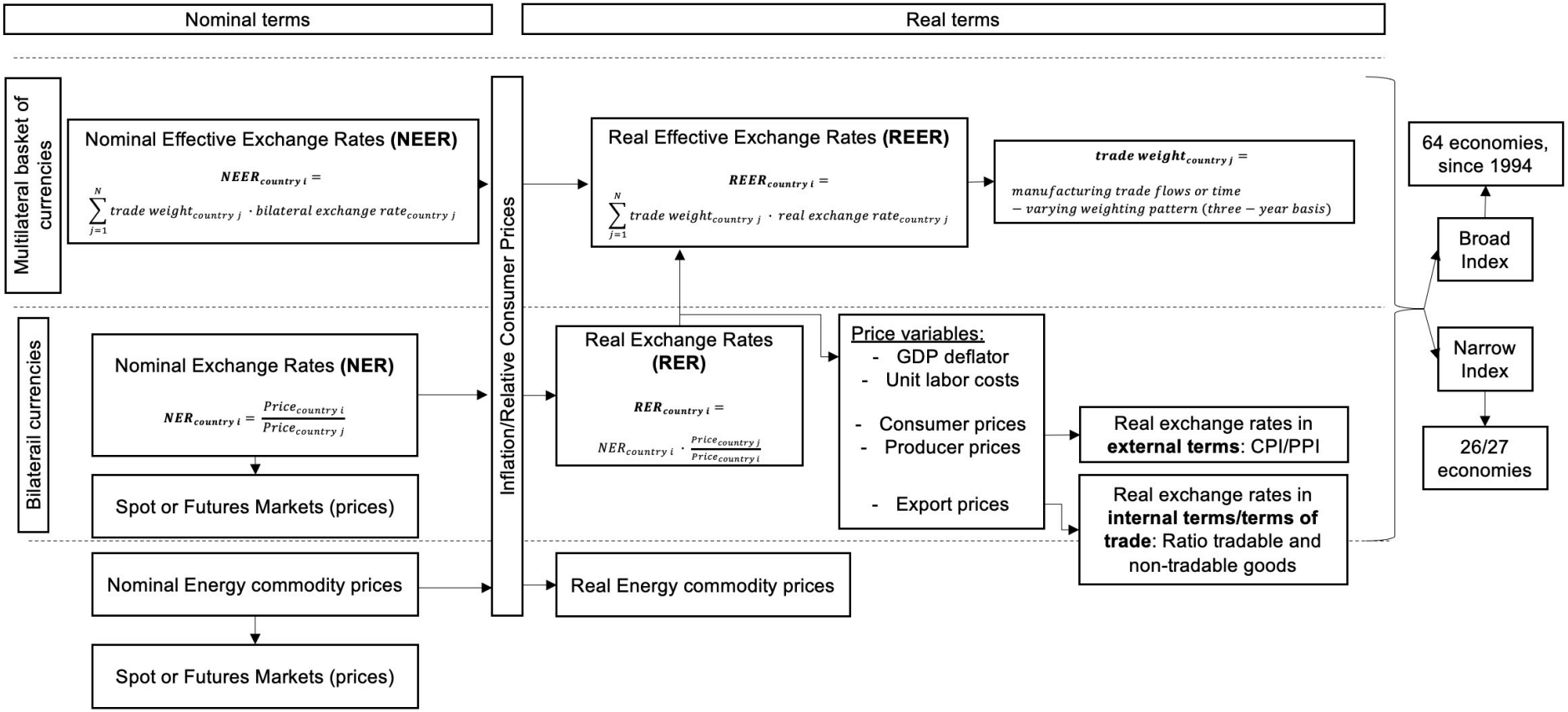


Figure: Schematic overview of prices and measurements of various exchange rate types.[10]

[10] Own Illustration based on [**rosenberg˙2003˙exchange˙rate˙determination**], page 8, exchange rates in natural logarithm (geometric averages) and data definitions from [**bis˙2025˙effective˙exchange˙rates**].

A simple model of exchange rates and commodity prices

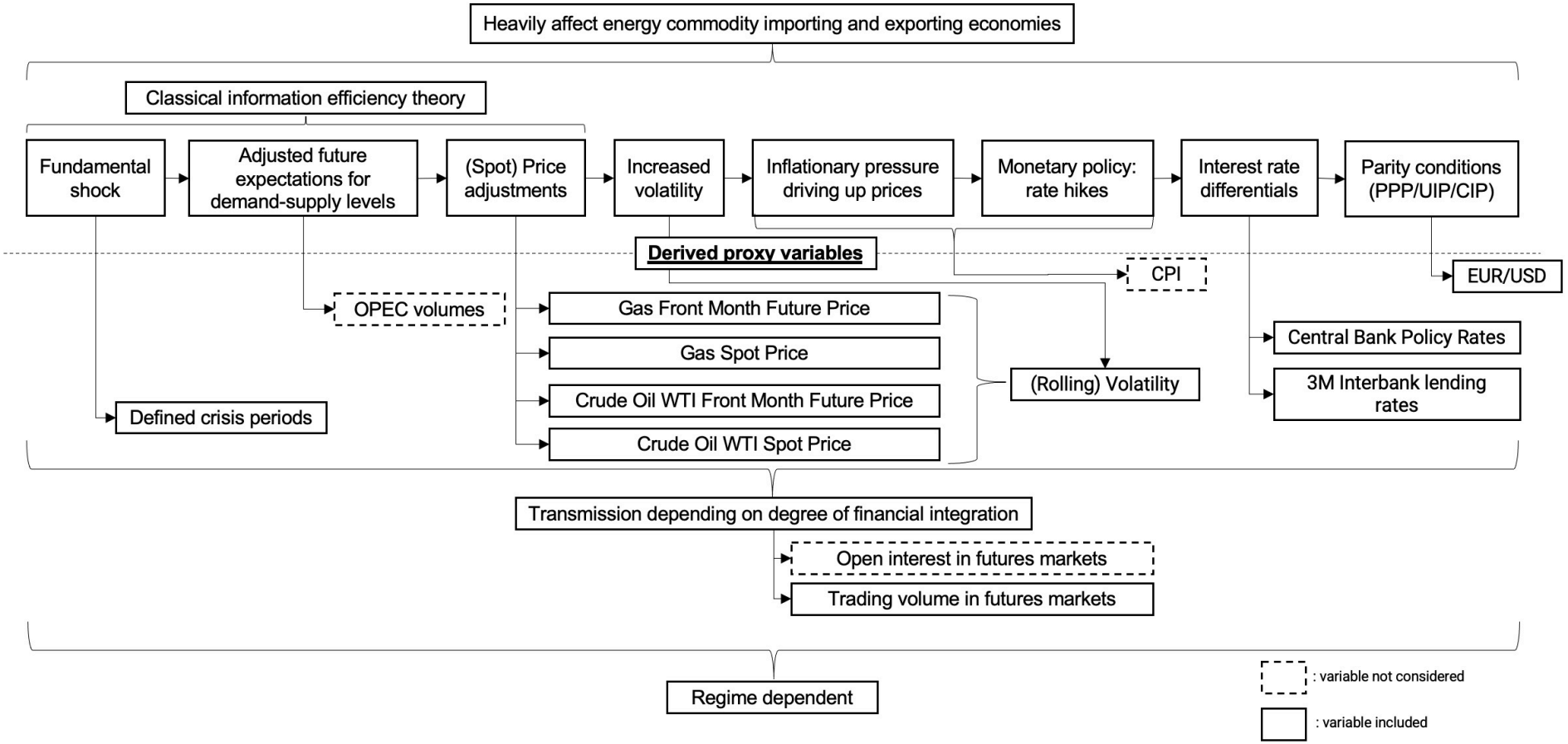
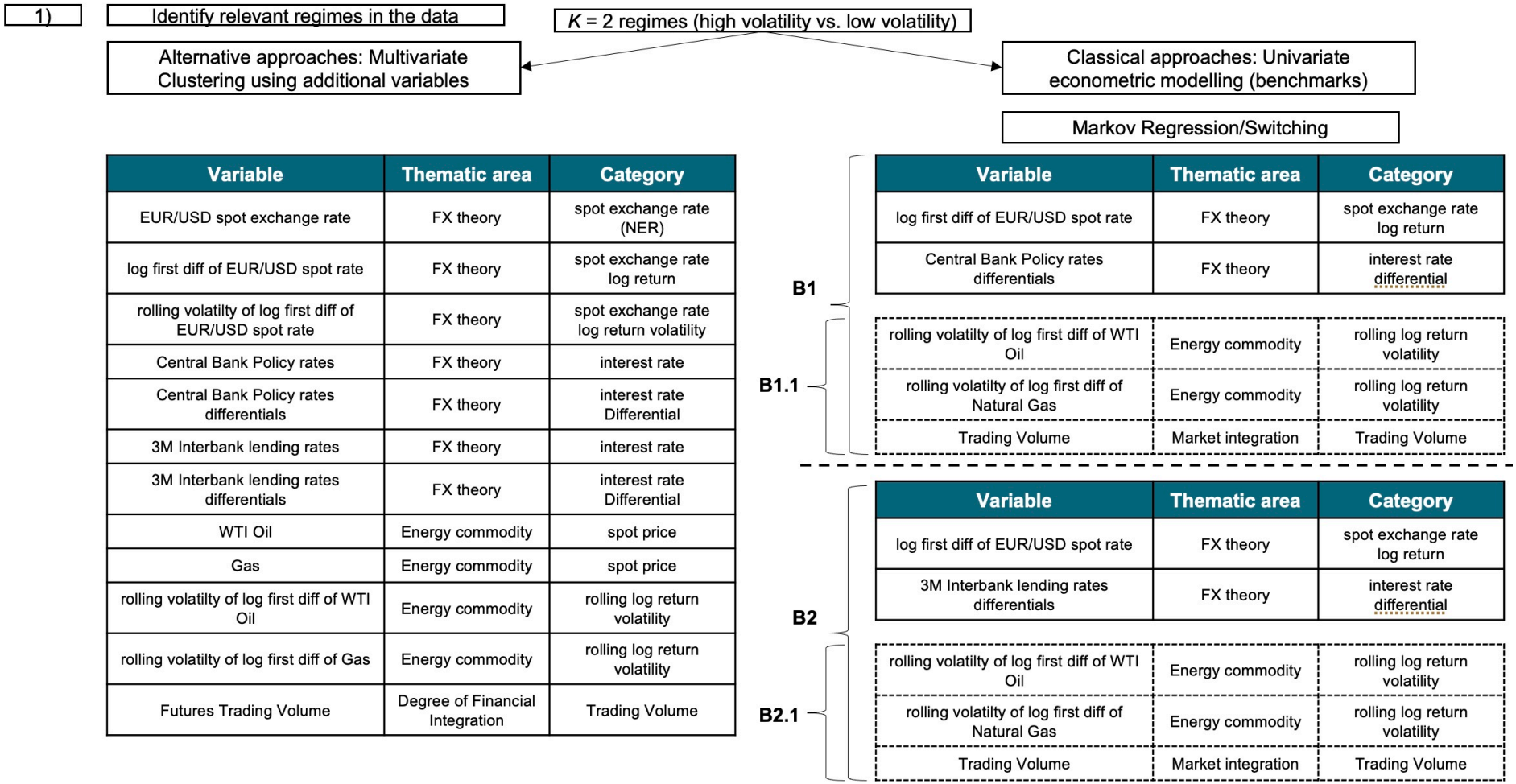


Figure: Main theoretical framework for the seminar project analysis, inflation pass-through effect of energy commodity prices.[11]

[11] Own Illustration based on [**oberndorfer˙2020˙oil˙prices˙exchange˙rates**], Figure 3, page 3.

Theoretical Framework (I)



Variable	Thematic area	Category
EUR/USD spot exchange rate	FX theory	spot exchange rate (NER)
log first diff of EUR/USD spot rate	FX theory	spot exchange rate log return
rolling volatilty of log first diff of EUR/USD spot rate	FX theory	spot exchange rate log return volatility
Central Bank Policy rates	FX theory	interest rate
Central Bank Policy rates differentials	FX theory	interest rate Differential
3M Interbank lending rates	FX theory	interest rate
3M Interbank lending rates differentials	FX theory	interest rate Differential
WTI Oil	Energy commodity	spot price
Gas	Energy commodity	spot price
rolling volatilty of log first diff of WTI Oil	Energy commodity	rolling log return volatility
rolling volatilty of log first diff of Gas	Energy commodity	rolling log return volatility
Futures Trading Volume	Degree of Financial Integration	Trading Volume

B1

Variable	Thematic area	Category
log first diff of EUR/USD spot rate	FX theory	spot exchange rate log return
Central Bank Policy rates differentials	FX theory	interest rate differential

B1.1

rolling volatilty of log first diff of WTI Oil	Energy commodity	rolling log return volatility
rolling volatilty of log first diff of Natural Gas	Energy commodity	rolling log return volatility
Trading Volume	Market integration	Trading Volume

B2

Variable	Thematic area	Category
log first diff of EUR/USD spot rate	FX theory	spot exchange rate log return
3M Interbank lending rates differentials	FX theory	interest rate differential

B2.1

rolling volatilty of log first diff of WTI Oil	Energy commodity	rolling log return volatility
rolling volatilty of log first diff of Natural Gas	Energy commodity	rolling log return volatility
Trading Volume	Market integration	Trading Volume

Figure: Main theoretical framework for the seminar project analysis, main variables and benchmark models used.[12]

[12] Own Illustration.

Theoretical Framework (II)

1) Identify relevant regimes in the data

$K = 2$ regimes (high volatility vs. low volatility)

Alternative approaches: Multivariate Clustering using additional variables

Algorithm	Source package (version)	Category
KMeans	scikit-learn (1.7.2)	Clustering
AgglomerativeClustering	scikit-learn (1.7.2)	Clustering
DBSCAN	scikit-learn (1.7.2)	Clustering
SpectralClustering	scikit-learn (1.7.2)	Clustering
MeanShift	scikit-learn (1.7.2)	Clustering
GaussianMixture	scikit-learn (1.7.2)	Clustering
Birch	scikit-learn (1.7.2)	Clustering
AffinityPropagation	scikit-learn (1.7.2)	Clustering
OPTICS	scikit-learn (1.7.2)	Clustering
MiniBatchKMeans	scikit-learn (1.7.2)	Clustering

Hyperparameter Optimisation

Technique	Source package (version)	Category
GridSearchCV	scikit-learn (1.7.2)	Parameter Optimization

TimeSeriesSplit

CombinatorialPurgedCV

Classical approaches: Univariate econometric modelling (benchmarks)

Markov Regression/Switching

Algorithm	Source package (version)	Category
MarkovRegression	statsmodels (0.14.5)	Regime Switching Regression

$$Pr(I_t = 1|I_{t-1} = 1) = P$$
$$Pr(I_t = 2|I_{t-1} = 1) = 1 - P$$
$$Pr(I_t = 2|I_{t-1} = 2) = Q$$
$$Pr(I_t = 1|I_{t-1} = 2) = 1 - Q$$

Hamilton suggests that Bayesian inference is an unbiased and efficient way to infer regime probabilities

$$Pr(I_{t-1} = 1|\Delta s_{t-1}) = \frac{f(\Delta s_{t-1}|I_{t-1} = 1)p_{1t-1}}{f(\Delta s_{t-1}|I_{t-1} = 1)p_{1t-1} + f(\Delta s_{t-1}|I_{t-1} = 2)(1 - p_{1t-1})}$$

and

$$Pr(I_{t-1} = 2|\Delta s_{t-1}) = \frac{f(\Delta s_{t-1}|I_{t-1} = 2)(1 - p_{1t-1})}{f(\Delta s_{t-1}|I_{t-1} = 1)p_{1t-1} + f(\Delta s_{t-1}|I_{t-1} = 2)(1 - p_{1t-1})}$$

where p_{1t-1} and $p_{2t-1} = 1 - p_{1t-1}$ are called prior probabilities

Using the posteriors we can calculate an expectation of the next periods regime probability as

$$p_{1t} = P\ Pr(I_{t-1} = 1|\Delta s_{t-1}) + (1 - Q)\ Pr(I_{t-1} = 2|\Delta s_{t-1})$$

or

$$p_{1t} = P\left[\frac{f_{1t-1}p_{1t-1}}{f_{1t-1}p_{1t-1} + f_{2t-1}(1 - p_{1t-1})}\right] + (1 - Q)\left[\frac{f_{2t-1}(1 - p_{1t-1})}{f_{1t-1}p_{1t-1} + f_{2t-1}(1 - p_{1t-1})}\right]$$

and

$$p_{2t} = 1 - p_{1t}$$

Estimation of the model by maximizing the log-likelihood:

$$L = \sum_{t=1}^{T} log\left[p_{1t}\frac{1}{\sqrt{2\pi h_{1t}}}exp(\Theta_1) + (1 - p_{1t})\frac{1}{\sqrt{2\pi h_{2t}}}exp(\Theta_2)\right]$$

and

$$\Theta_1 = \frac{-(\Delta s_t - \mu_{1t})^2}{2h_{1t}}, \Theta_2 = \frac{-(\Delta s_t - \mu_{2t})^2}{2h_{2t}}$$

$$Regime1 : \mu_{1t} = \alpha_1 + \beta_1(i_{t-1} - i^*_{t-1});\ h_{1t} = \sigma^2_1$$
$$Regime2 : \mu_{2t} = \alpha_2 + \beta_2(i_{t-1} - i^*_{t-1});\ h_{2t} = \sigma^2_2$$

Figure: Main theoretical framework for the seminar project analysis, main algorithms used.[13]

[13] Own Illustration based on formulas taken from [**reitz˙2025˙applied˙econometrics˙fx**], section "Modeling Nonlinearities I: Markov-Switching", page 7-15.

Theoretical Framework (III)

2) Analyze and compare the regimes

Model evaluation and selection

Evaluation metric	Source package (version)	Category	Value range	Interpretation
Silhouette Score	scikit-learn (1.7.2)	Clustering Scores	[-1, 1]	1 being best, -1 being worst, values near 0 indicate overlapping clusters.
RCM	[AB98]	MSM-Score	[0, 100]	0 (perfect regime classification), 100 (failure to detect any regime classification)
Crisis overlap percentage	Own calculations	Combined	[0, 100]	Percentage overlap ranging from 0% - 100%

Theoretically affecting crisis periods - major global shocks and US recessions

No.	Start-date	End-date	Event-Type	Event	Source
1	1970-01-01	1970-12-01	US-Recession	US Recession 1970-1970	[St 25b]
2	1971-08-15	1973-03-19	Major Global Crisis	Bretton Woods Breakdown	[Cor+25]
3	1971-08-15	1973-03-19	Major Global Crisis	Nixon Shock	[EF04]
4	1973-10-17	1974-03-01	Major Global Crisis	Oil Crisis I	[T T23]
5	1973-12-01	1975-04-01	US-Recession	US Recession 1973-1975	[St 25b]
6	1979-01-01	1981-03-01	Major Global Crisis	Oil Crisis II	[T T23]
7	1980-02-01	1980-08-01	US-Recession	US Recession 1980-1980	[St 25b]
8	1981-08-01	1982-12-01	US-Recession	US Recession 1981-1982	[St 25b]
9	1987-10-19	1987-10-19	Major Global Crisis	Black Monday Crash	[EF04]
10	1990-08-01	1991-04-01	US-Recession	US Recession 1990-1991	[St 25b]
11	1997-07-02	1998-12-31	Major Global Crisis	Asian Financial Crisis	[ML23]
12	1998-08-17	1998-09-01	Major Global Crisis	Russian Crisis	[ML23]
13	2000-03-01	2002-10-01	Major Global Crisis	Dot-com Bubble	[MNR25]
14	2001-04-01	2001-12-01	US-Recession	US Recession 2001-2001	[St 25b]
15	2007-08-09	2009-03-09	Major Global Crisis	Global Financial Crisis	[MNR25]
16	2008-01-01	2009-07-01	US-Recession	US Recession 2008-2009	[St 25b]
17	2008-11-25	2014-12-31	Major Global Crisis	US QE	[MNR25]
18	2009-10-01	2012-12-31	Major Global Crisis	European Debt Crisis	[MNR25]
19	2011-08-20	2011-08-05	Major Global Crisis	US Debt Ceiling Crisis	[MNR25]
20	2018-03-22	2020-01-15	Major Global Crisis	US-China Trade War	Various
21	2020-02-20	2021-11-16	Major Global Crisis	COVID-19 Pandemic	[ML23]
22	2020-03-01	2020-05-01	US-Recession	US Recession 2020-2020	[St 25b]
23	2022-02-24	2022-06-01	Major Global Crisis	Russia-Ukraine War	[Cha25]

Figure: Main theoretical framework for the seminar project analysis, main evaluation metrics and crisis periods used.[14]

[14] Own Illustration based on data taken from [**fred˙2025˙federal˙reserve˙data**], last accessed 24.10.25

Methodology Overview

Systematic Methodology Overview: Linear vs. Non-linear approaches

Model Results

Model comparison and selection

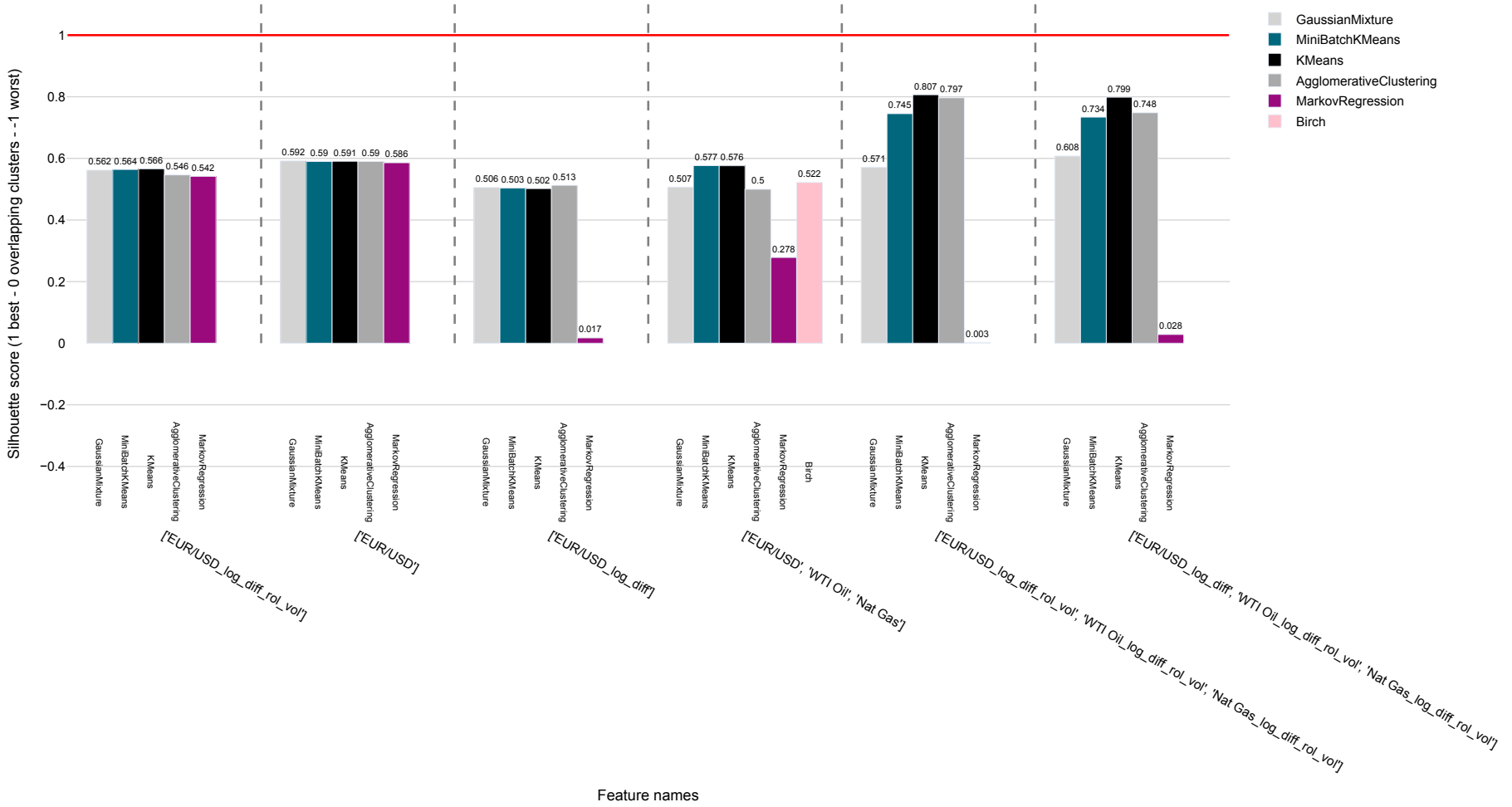


Figure: Model comparison using the silhouette score for various regime identification model configurations.[15]

[15] Own Illustration based on XYZ (2000), page 7 and data taken from [**fred˙2025˙federal˙reserve˙data**], last accessed

Predicted Regimes - Evolution over time with highlighted Crisis Periods

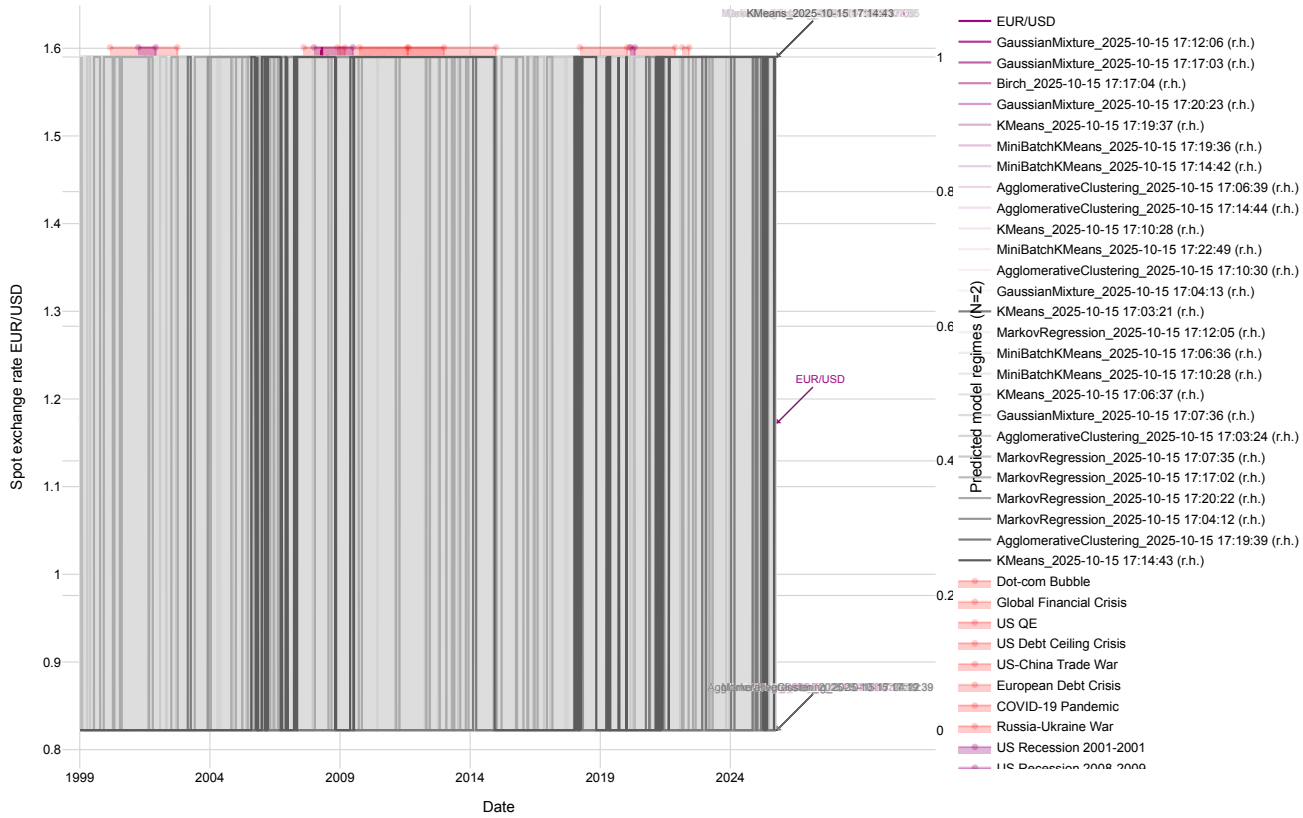


Figure: Predicted regimes over time with highlighted crisis periods.[16]

[16] Own Illustration based on XYZ (2000), page 7 and data taken from [**fred˙2025˙federal˙reserve˙data**], last accessed 24.10.25, own calculations.

Conclusion and Discussion

Seminar Project Summary

Seminar Project Limitations

Evaluation Metrics

Only used one metric for clustering (Silhouette Score).

Future Research - Possible Extensions

Other Spot rate pairs

Extend the Analysis to other Spot Exchange Rate Pairs (focus on the Commodity Currencies with a strong Commodity Linkage)

Other commodities

Extend the Analysis to also include other Commodities (e.g. Natural Gas, Brent Oil, Gold, etc.)

Other macroeconomic/external variables

Extend the Analysis by incorporating other Macroeconomic Variables (e.g. Interest Rates, Inflation Rates, etc.)

Other data frequencies

Extend the Analysis by using intraday data to capture more granular dynamics

Appendix

Appendix - Figures and Tables

Appendix - Figures and Tables

Modern Energy Commodity Markets
The current state

Oil: Global Production and Consumption over time

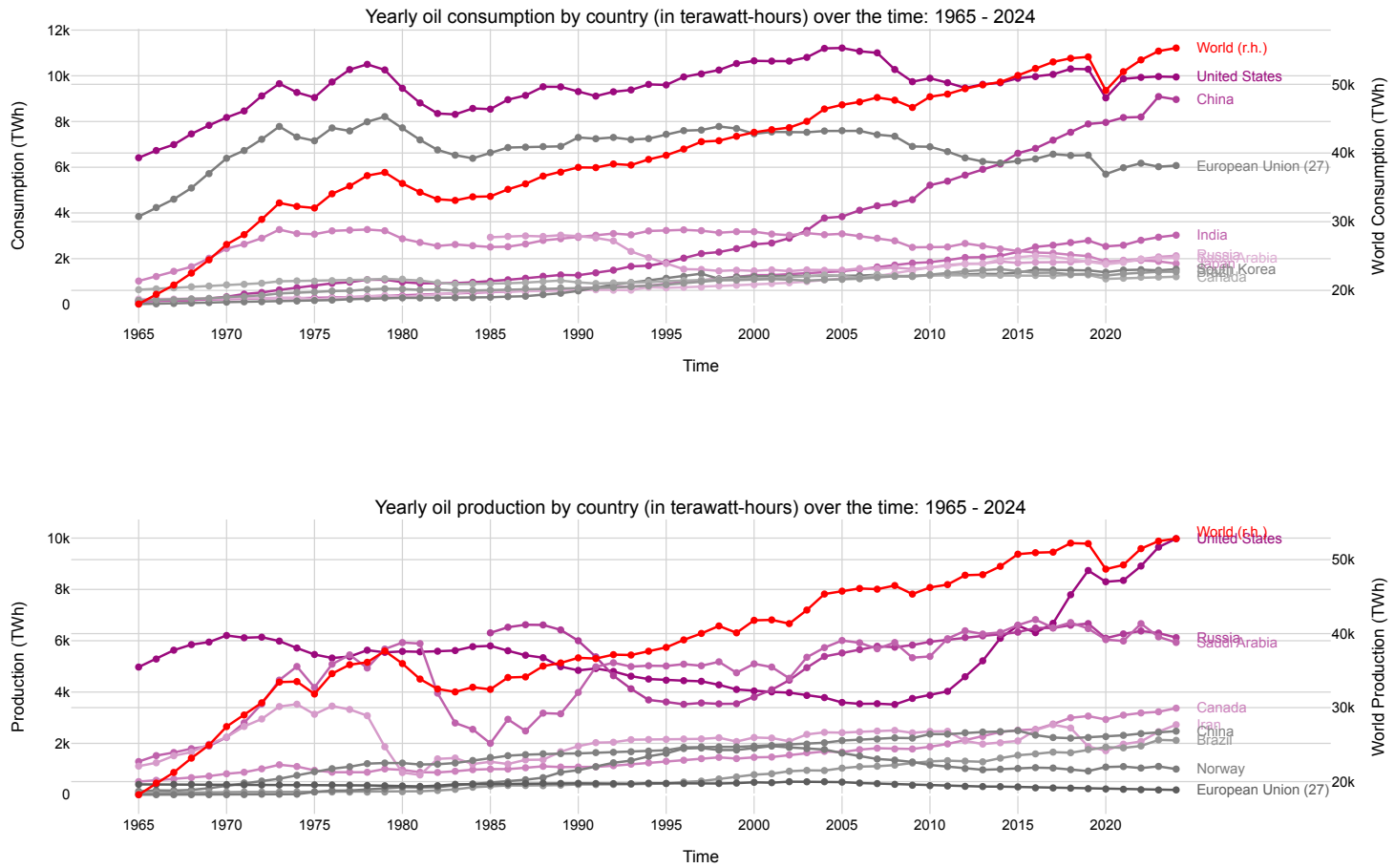


Figure: Yearly oil consumption and production by country over the time: 1965 - 2024 (in terawatt-hours).[17]

[17] Own Illustration based on [**Amir2018**], page 128 and data taken from [**energy˙institute˙2025˙oil˙consumption**], last accessed 24.10.25.

Gas: Global Production and Consumption over time

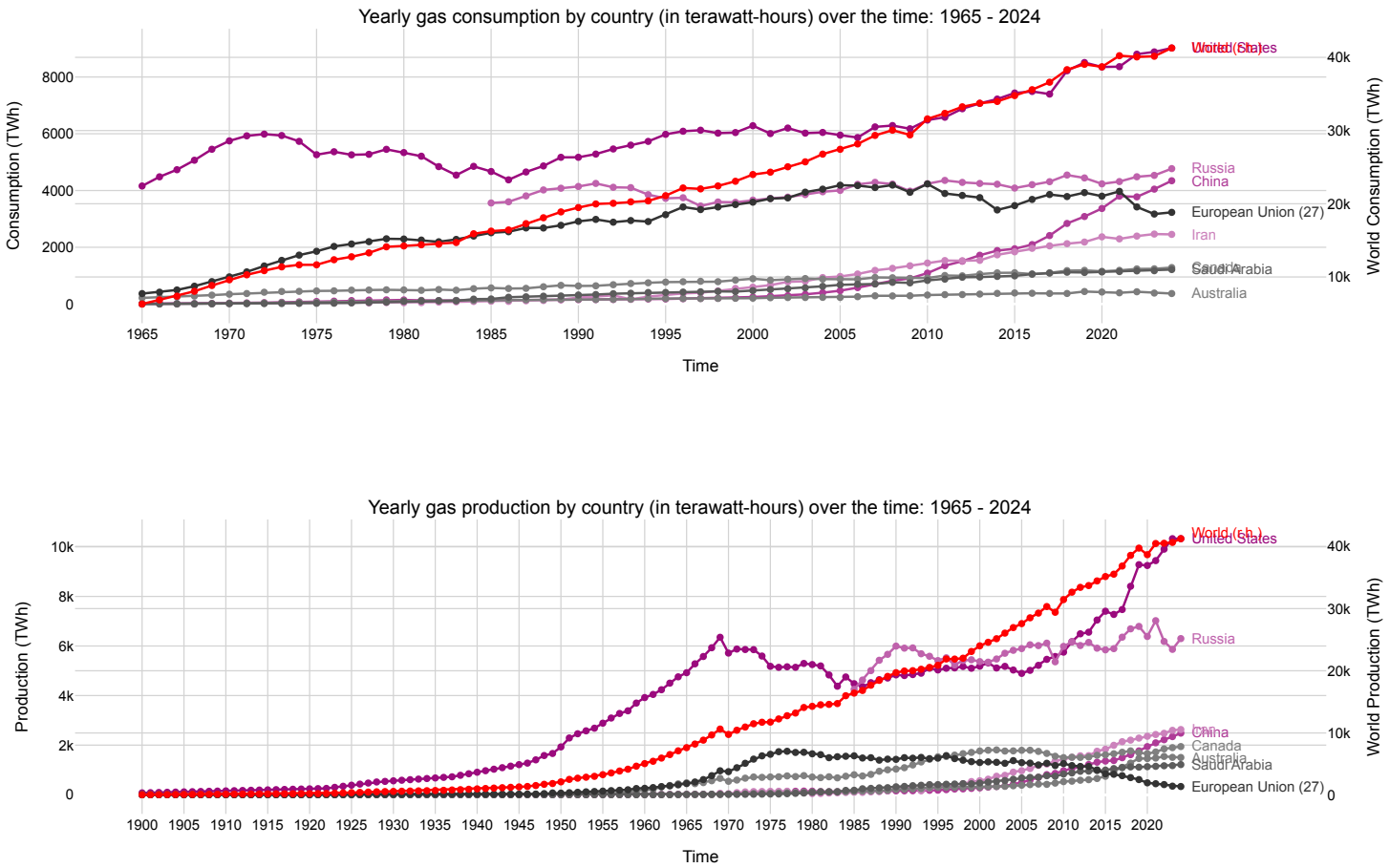


Figure: Yearly gas consumption and production by country over the time: 1965 - 2024 (in terawatt-hours).[18]

[18] Own Illustration based on [**Amir2018**], page 128 and data taken from [**energy˙institute˙2025˙gas˙consumption**], last accessed 24.10.25.

Financial Markets: Oil and Gas OI over time

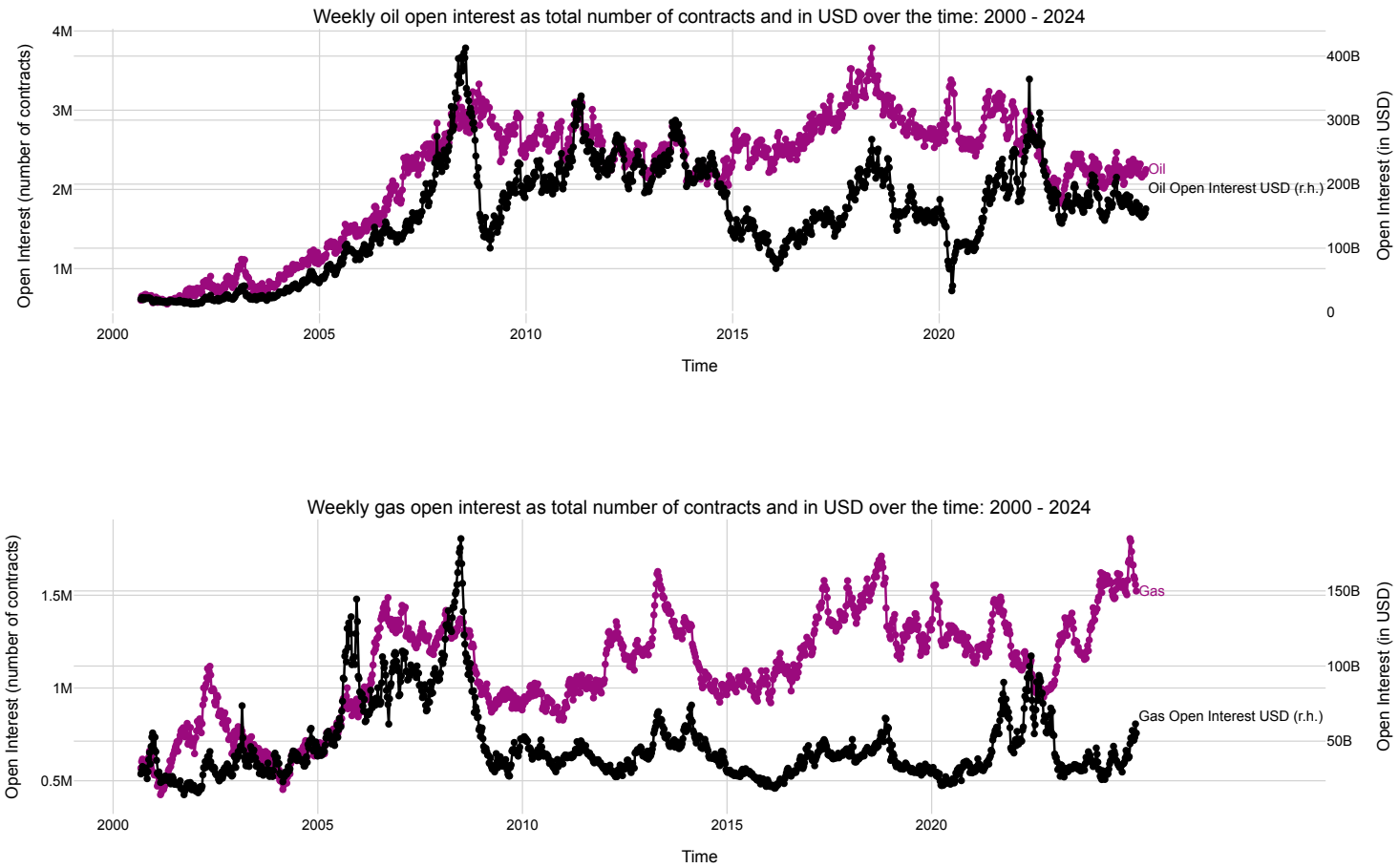


Figure: Weekly open interest of oil and gas products over the time: 2000 - 2024[19]

[19] Own Illustration based on [**Carmona2015**], page 11 and data taken from [**cftc˙2025˙commitments˙of˙traders**], last accessed 24.10.25.

Data Characteristics and Stylized Facts

Interest Rate Benchmarks - Absolute Levels

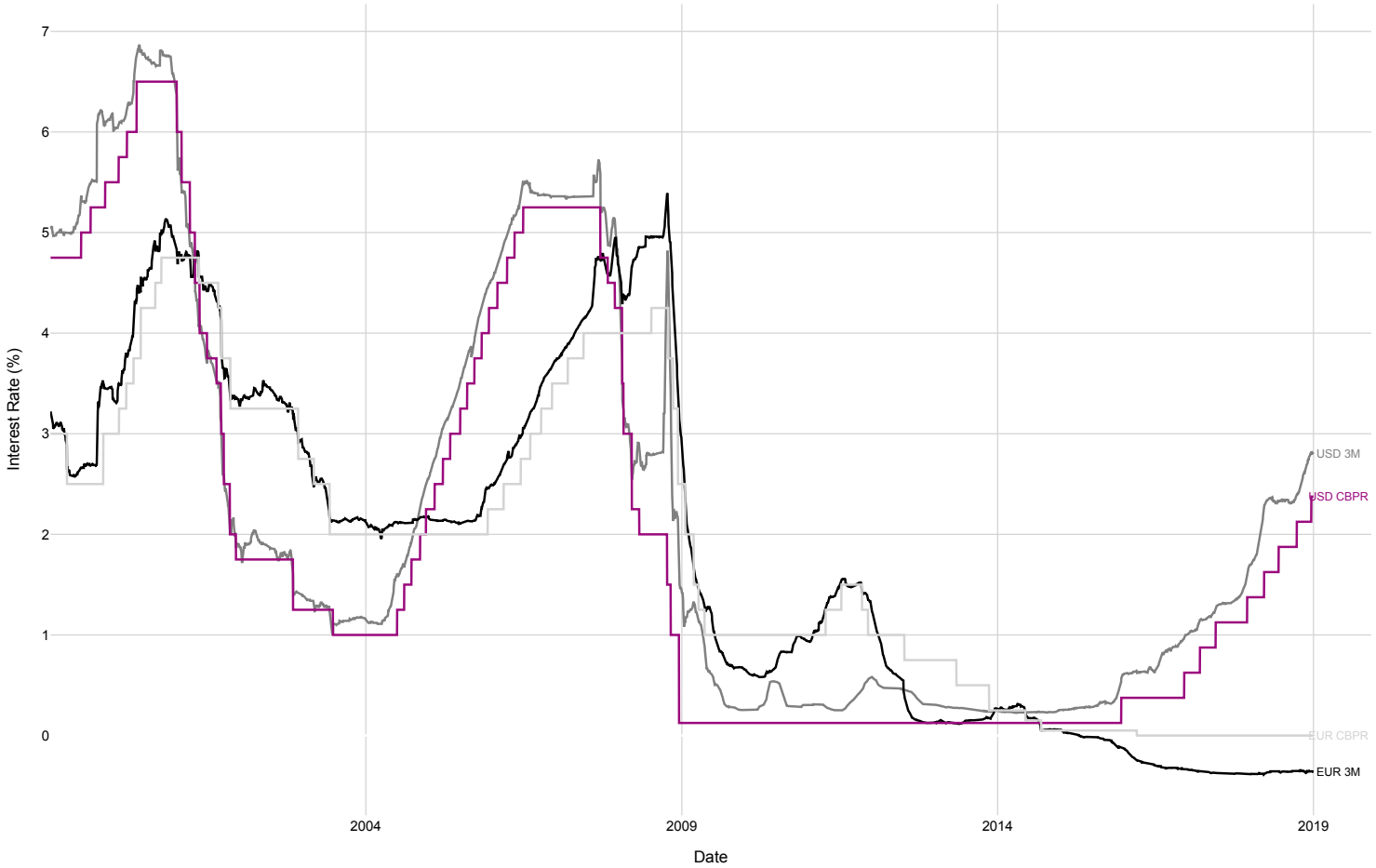


Figure: Daily BIS Central Bank Policy Rate and 3M Interbank Rates over the time: 1999 - 2019.[20]

[20] Own Illustration based [**Schnabel2023**], Figure 4, page 5 and data taken from [**bis˙2025˙effective˙exchange˙rates**] and [**reitz˙2025˙applied˙econometrics˙fx**], last accessed 24.10.25.

Interest Rate Benchmarks - Relative Levels

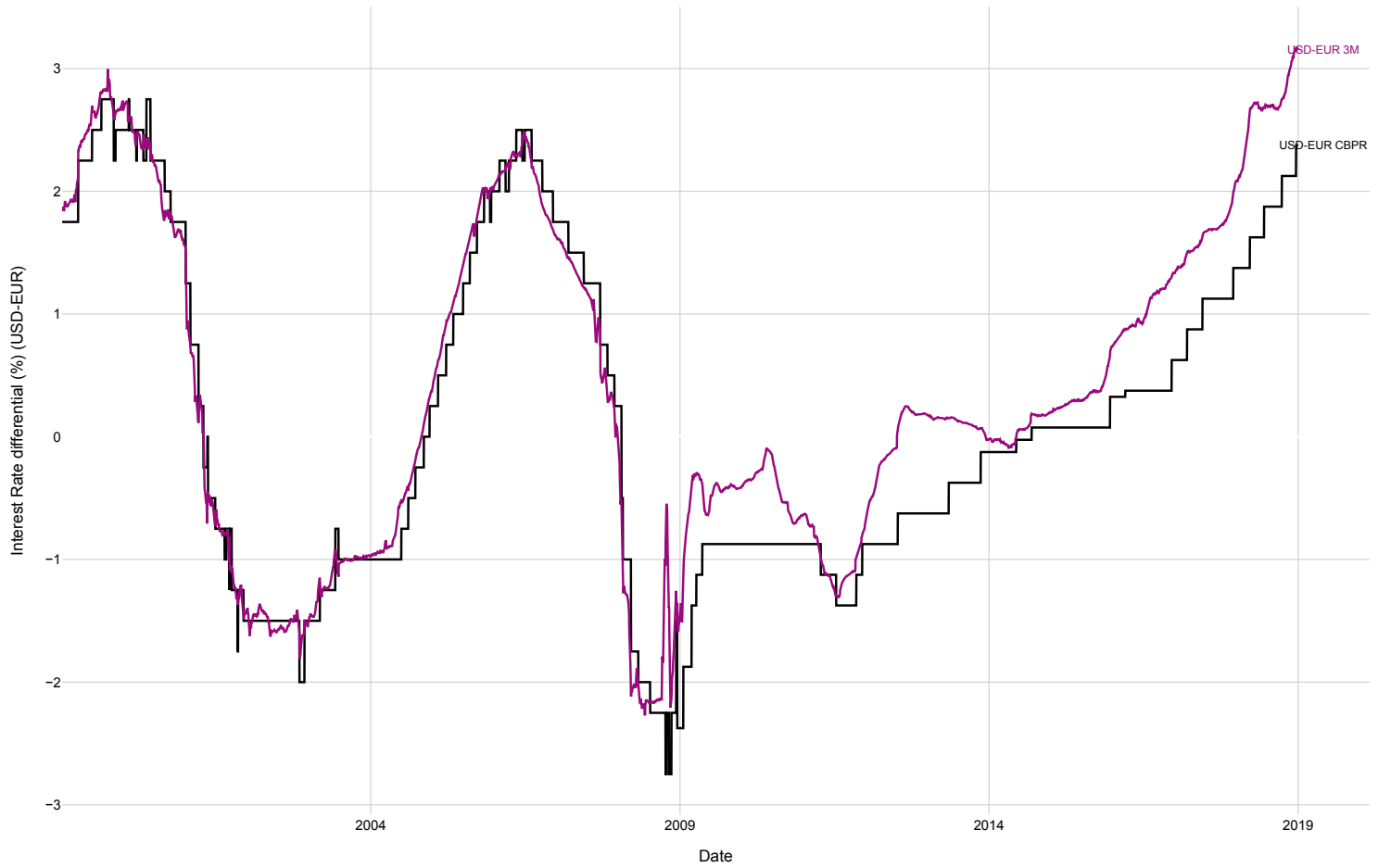


Figure: Daily BIS Central Bank Policy Rate and 3M Interbank Rates differentials (USD-EUR) over the time: 1999 - 2019.[21]

[21] Own Illustration based [**Schnabel2023**], Figure 4, page 5 and data taken from [**bis˙2025˙effective˙exchange˙rates**] and [**reitz˙2025˙applied˙econometrics˙fx**], last accessed 24.10.25.

Main variables distributions (raw data - normalized)

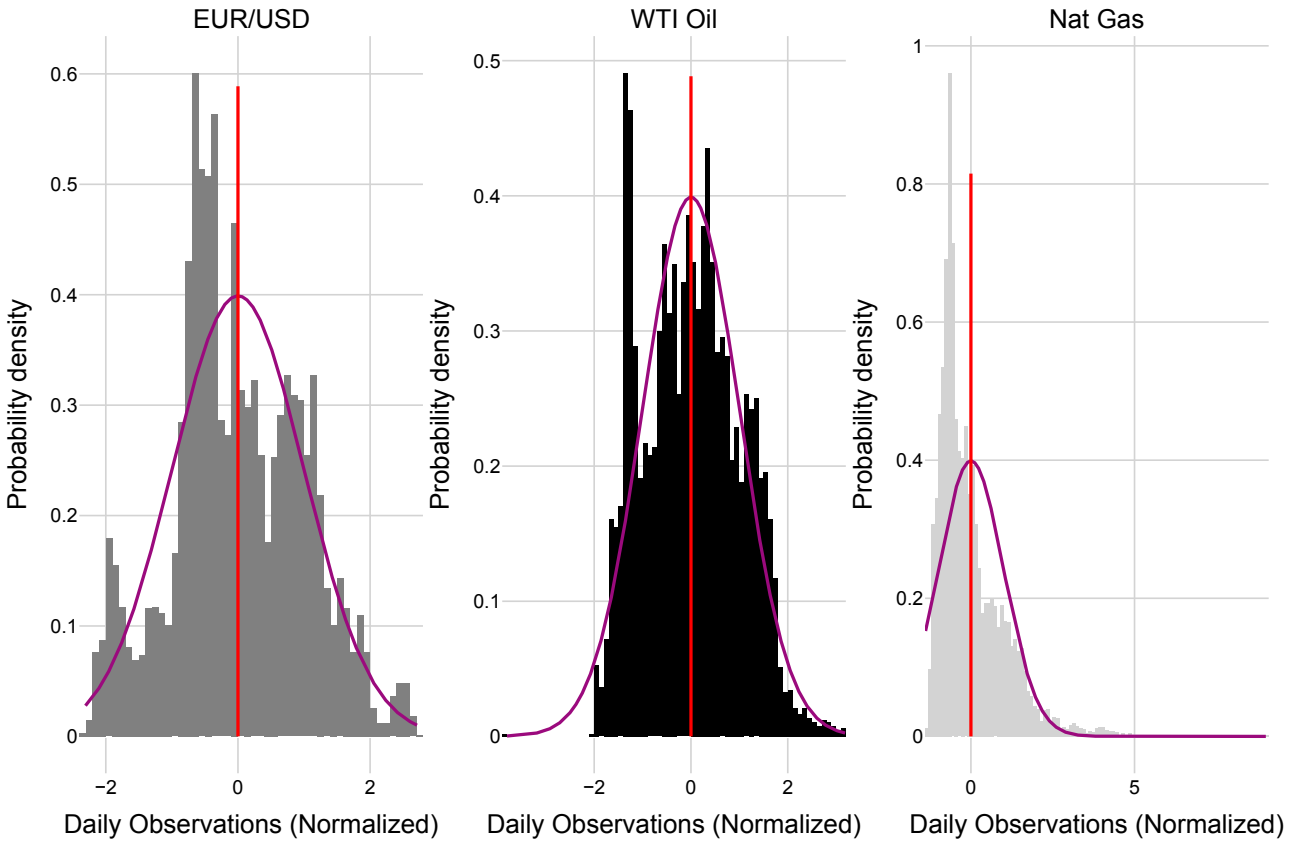


Figure: Normalized daily EUR/USD spot exchange rate, oil and gas over the time range: 1999 - 2025.[22]

[22] Own Illustration based on [**campisi˙2024˙discontinuous˙fx˙model**], page X and data taken from [**fred˙2025˙federal˙reserve˙data**], last accessed 24.10.25.

Main variables distributions (log first differences)

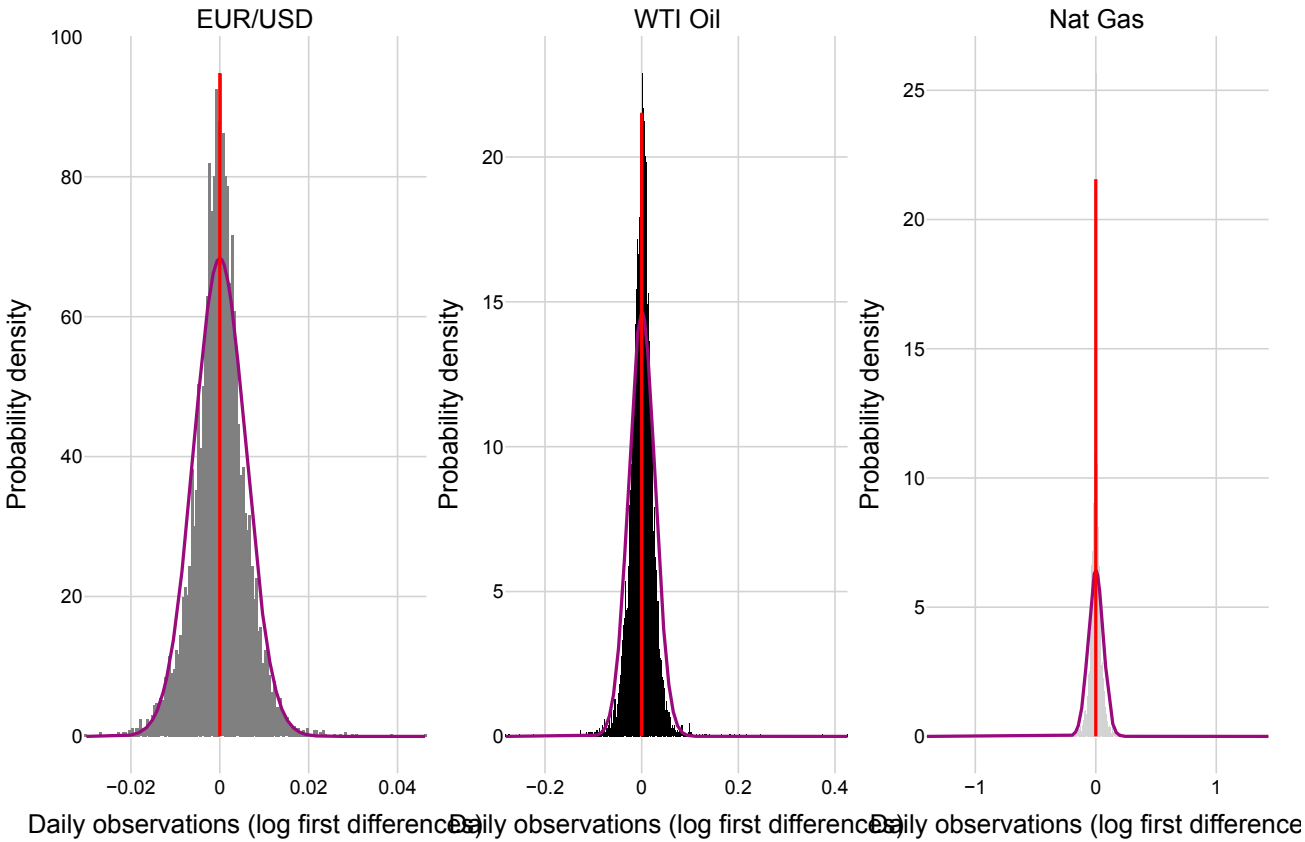


Figure: Log first differences of daily EUR/USD spot exchange rate, oil and gas over the time range: 1999 - 2025.[23]

[23] Own Illustration based on [**campisi˙2024˙discontinuous˙fx˙model**], page X and data taken from [**fred˙2025˙federal˙reserve˙data**], last accessed 24.10.25.

Tests for Normality (raw data)

Variable	Test	Statistic	p-value	Significance-level	p-value $<$ 0.05	Result
EUR/USD	Shapiro-Wilk	0.989	0.000	0.050	True	Not-Normal
EUR/USD	Kolmogorov-Smirnov	0.799	0.000	0.050	True	Not-Normal
EUR/USD	D'Agostino's K^2	55.280	0.000	0.050	True	Not-Normal
WTI Oil	Shapiro-Wilk	0.981	0.000	0.050	True	Not-Normal
WTI Oil	Kolmogorov-Smirnov	1.000	0.000	0.050	True	Not-Normal
WTI Oil	D'Agostino's K^2	340.085	0.000	0.050	True	Not-Normal
Nat Gas	Shapiro-Wilk	0.8640	0.000	0.050	True	Not-Normal
Nat Gas	Kolmogorov-Smirnov	0.938	0.000	0.050	True	Not-Normal
Nat Gas	D'Agostino's K^2	2130.008	0.000	0.050	True	Not-Normal

Table: Shapiro-Wilks, Kolmogorov-Smirnov, D'Agostino's K^2 test for normality for the EUR/USD spot exchange rate, WTI Oil and Natural Gas daily observations over the time 1999 - 2025.[24]

[24] Own Illustration based on XX, page X, tests used from: [**scipy˙2025**] and data taken from [**fred˙2025˙federal˙reserve˙data**], last accessed 24.10.25.

Tests for Stationarity - ADF Tests (raw data)

ADF Statistic	p-value	Start Time	End Time	Regression Type	Observations	Variable	Result
-1.847	0.357	04-01-1999	04-01-1999	c	6640	EUR/USD	Non-Stationary
-1.846	0.682	04-01-1999	04-01-1999	ct	6640	EUR/USD	Non-Stationary
-2.655	0.480	04-01-1999	04-01-1999	ctt	6640	EUR/USD	Non-Stationary
-0.254	0.594	04-01-1999	04-01-1999	n	6640	EUR/USD	Non-Stationary
-2.789	0.059	04-01-1999	04-01-1999	c	6624	WTI Oil	Non-Stationary
-2.770	0.208	04-01-1999	04-01-1999	ct	6624	WTI Oil	Non-Stationary
-3.060	0.267	04-01-1999	04-01-1999	ctt	6624	WTI Oil	Non-Stationary
-0.697	0.413	04-01-1999	04-01-1999	n	6624	WTI Oil	Non-Stationary
-4.341	0.000	04-01-1999	04-01-1999	c	6633	Nat Gas	Stationary
-4.743	0.001	04-01-1999	04-01-1999	ct	6633	Nat Gas	Stationary
-4.742	0.003	04-01-1999	04-01-1999	ctt	6633	Nat Gas	Stationary
-1.897	0.055	04-01-1999	04-01-1999	n	6633	Nat Gas	Non-Stationary

Table: Augmented-Dickey-Fuller (ADF) test for stationarity in various variants for the EUR/USD spot exchange rate, WTI Oil and Natural Gas daily observations over the time 1999 - 2025.[25]

[25] Own Illustration based on XX, page X, tests used from: [**seabold2010statsmodels**] and data taken from [**fred˙2025˙federal˙reserve˙data**], last accessed 24.10.25.

Tests for Stationarity - ADF Tests (log first differences)

ADF Statistic	p-value	Start Time	End Time	Regression Type	Observations	Variable	Result
-80.613	0.000	05-01-1999	05-01-1999	c	6637	EUR/USD	Stationary
-80.607	0.000	05-01-1999	05-01-1999	ct	6637	EUR/USD	Stationary
-80.601	0.000	05-01-1999	05-01-1999	ctt	6637	EUR/USD	Stationary
-80.619	0.000	05-01-1999	05-01-1999	n	6637	EUR/USD	Stationary
-14.504	0.000	05-01-1999	05-01-1999	c	6603	WTI Oil	Stationary
-14.525	0.000	05-01-1999	05-01-1999	ct	6603	WTI Oil	Stationary
-14.547	0.000	05-01-1999	05-01-1999	ctt	6603	WTI Oil	Stationary
-14.463	0.000	05-01-1999	05-01-1999	n	6603	WTI Oil	Stationary
-20.094	0.000	05-01-1999	05-01-1999	c	6616	Nat Gas	Stationary
-20.102	0.000	05-01-1999	05-01-1999	ct	6616	Nat Gas	Stationary
-20.121	0.000	05-01-1999	05-01-1999	ctt	6616	Nat Gas	Stationary
-20.095	0.000	05-01-1999	05-01-1999	n	6616	Nat Gas	Stationary

Table: Augmented-Dickey-Fuller (ADF) test for stationarity in various variants for the EUR/USD spot exchange rate, WTI Oil and Natural Gas daily observations (log first differences) over the time 1999 - 2025.[26]

[26] Own Illustration based on XX, page X, tests used from: [**seabold2010statsmodels**] and data taken from [**fred˙2025˙federal˙reserve˙data**], last accessed 24.10.25.

Tests for Cointegration (raw data)

Cointegration Score	p-value	Start Time	End Time	Observations	Trend	Variable X	Variable Y	Result
-2.967000	0.118000	04-01-1999	01-10-2025	6641	c	EUR/USD	WTI Oil	Not Cointegrated
-3.364000	0.134000	04-01-1999	01-10-2025	6641	ct	EUR/USD	WTI Oil	Not Cointegrated
-3.635000	0.167000	04-01-1999	01-10-2025	6641	ctt	EUR/USD	WTI Oil	Not Cointegrated
-3.268000	0.013000	04-01-1999	01-10-2025	6641	n	EUR/USD	WTI Oil	Cointegrated
-2.416000	0.317000	04-01-1999	01-10-2025	6641	c	EUR/USD	Nat Gas	Not Cointegrated
-2.634000	0.446000	04-01-1999	01-10-2025	6641	ct	EUR/USD	Nat Gas	Not Cointegrated
-3.530000	0.204000	04-01-1999	01-10-2025	6641	ctt	EUR/USD	Nat Gas	Not Cointegrated
-4.182000	0.001000	04-01-1999	01-10-2025	6641	n	EUR/USD	Nat Gas	Cointegrated

Table: Engle and Granger Cointegration test for the EUR/USD spot exchange rate, WTI Oil and Natural Gas daily observations over the time 1999 - 2025.[27]

[27] Own Illustration based on XX, page X, tests used from: [**seabold2010statsmodels**] and data taken from [**fred˙2025˙federal˙reserve˙data**], last accessed 24.10.25.

Tests for Cointegration (log differences)

Cointegration Score	p-value	Start Time	End Time	Observations	Trend	Variable X	Variable Y	Result
-4.293000	0.003000	17-02-1999	01-10-2025	6609	c	EUR/USD	WTI Oil	Cointegrated
-4.805000	0.002000	17-02-1999	01-10-2025	6609	ct	EUR/USD	WTI Oil	Cointegrated
-4.830000	0.006000	17-02-1999	01-10-2025	6609	ctt	EUR/USD	WTI Oil	Cointegrated
-5.724000	0.000000	17-02-1999	01-10-2025	6609	n	EUR/USD	WTI Oil	Cointegrated
-4.001000	0.007000	17-02-1999	01-10-2025	6609	c	EUR/USD	Nat Gas	Cointegrated
-4.348000	0.009000	17-02-1999	01-10-2025	6609	ct	EUR/USD	Nat Gas	Cointegrated
-4.341000	0.030000	17-02-1999	01-10-2025	6609	ctt	EUR/USD	Nat Gas	Cointegrated
-4.246000	0.000000	17-02-1999	01-10-2025	6609	n	EUR/USD	Nat Gas	Cointegrated

Table: Engle and Granger Cointegration test for the EUR/USD spot exchange rate, WTI Oil and Natural Gas daily observations (log first differences) over the time 1999 - 2025.[28]

[28] Own Illustration based on XX, page X, tests used from: [**seabold2010statsmodels**] and data taken from [**fred˙2025˙federal˙reserve˙data**], last accessed 24.10.25.

Tests for Autocorrelation (raw data)

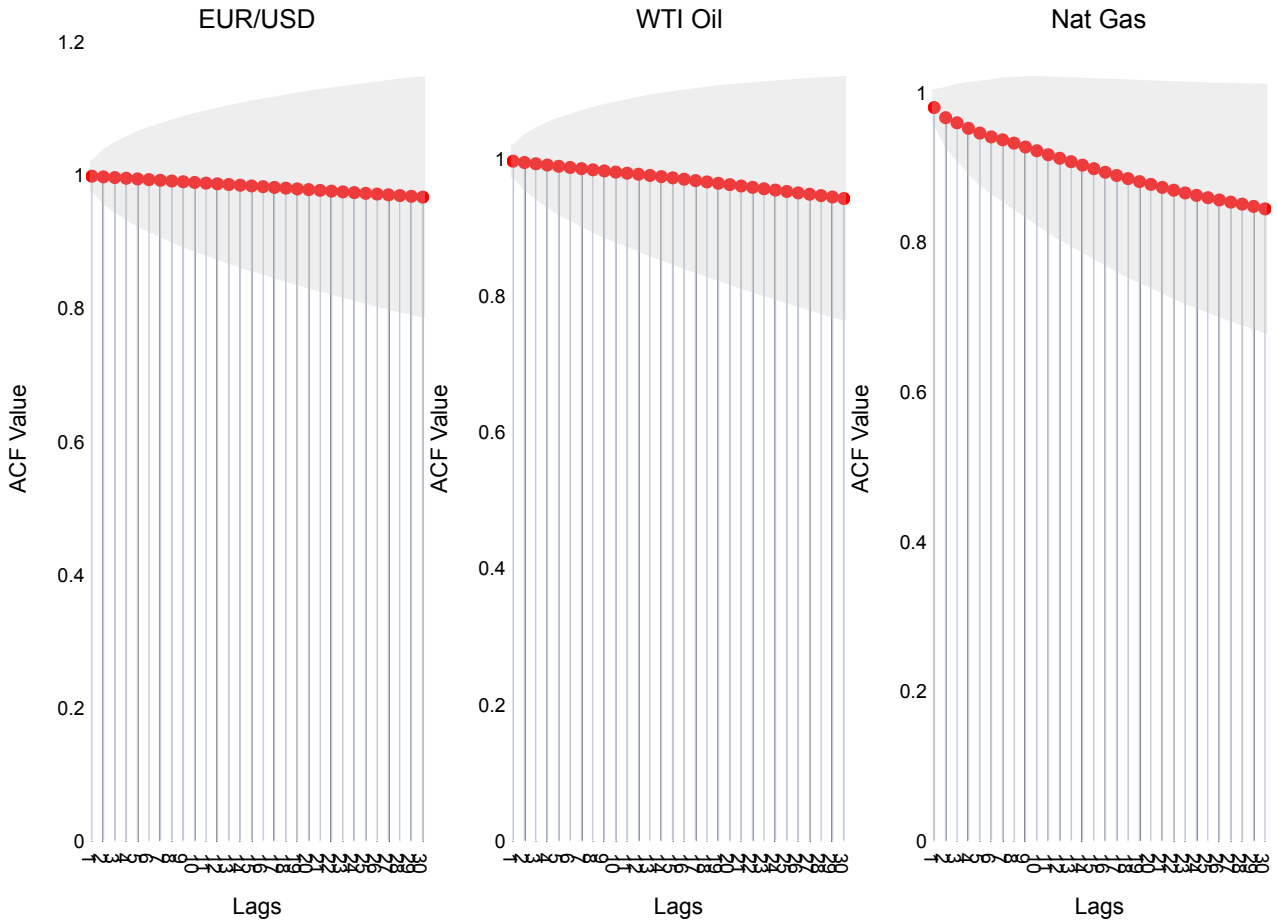


Figure: ACF values for daily observations of the EUR/USD spot exchange rate, oil and gas over the time: 1999 - 2025.[29]

[29] Own Illustration based on XX, page X, tests used from: [**seabold2010statsmodels**] and data taken from [**fred˙2025˙federal˙reserve˙data**], last accessed 24.10.25.

Tests for Autocorrelation (log first differences)

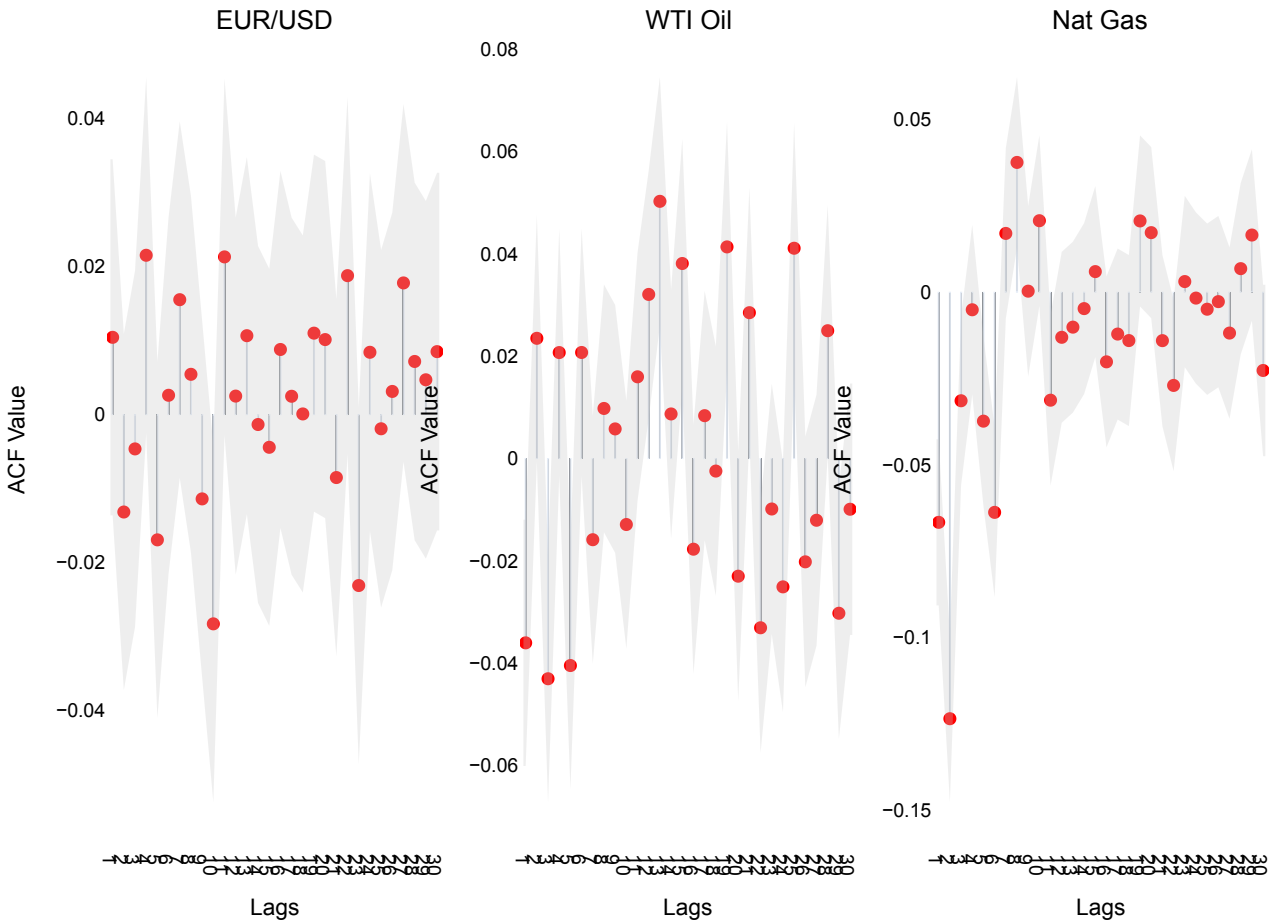


Figure: ACF values for daily observations (log first differences) of the EUR/USD spot exchange rate, WTI Oil and Natural Gas over the time: 1999 - 2025.[30]

[30] Own Illustration based on XX, page X, tests used from: [**seabold2010statsmodels**] and data taken from [**fred:2025:federal:reserve:data**], last accessed 24.10.25.

Tests for Partial Autocorrelation (raw data)

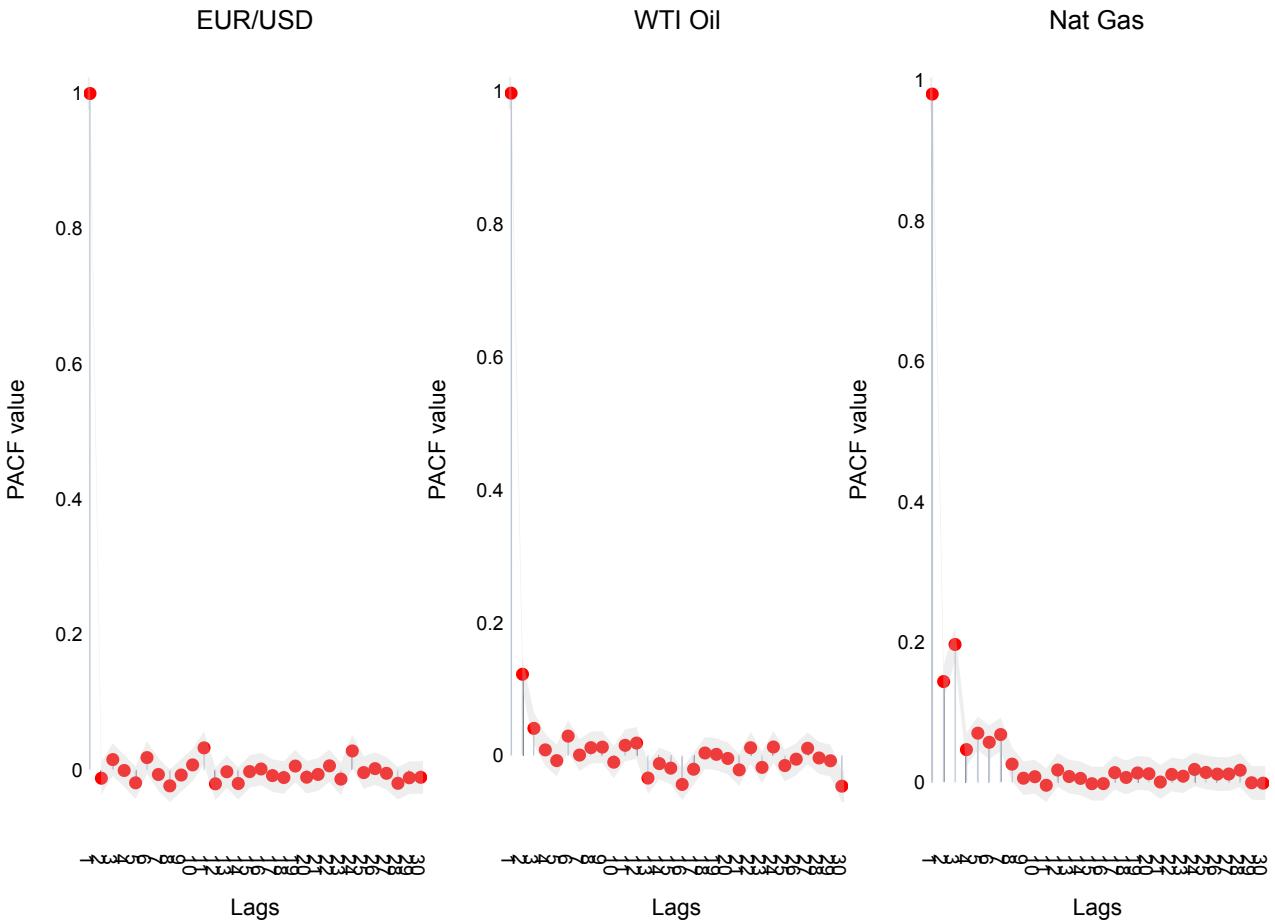


Figure: PACF values for daily observations of the EUR/USD spot exchange rate, WTI Oil and Natural Gas over the time: 1999 - 2025.[31]

[31] Own Illustration based on XX, page X, tests used from: [**seabold2010statsmodels**] and data taken from [**fred:2025:federal:reserve:data**], last accessed 24.10.25.

Tests for Partial Autocorrelation (log first differences)

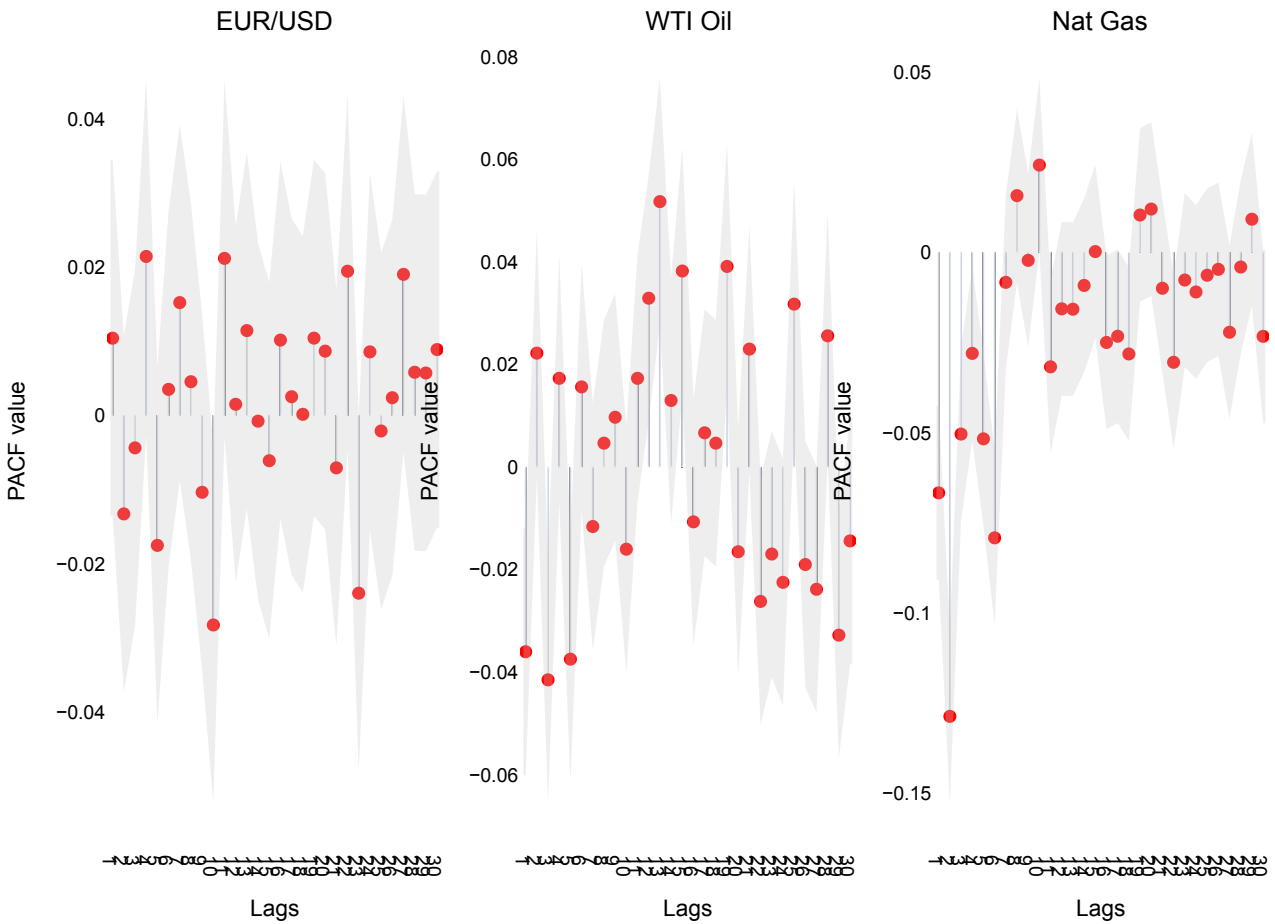


Figure: PACF values for daily observations (log first differences) of the EUR/USD spot exchange rate, WTI Oil and Natural Gas over the time: 1999 - 2025.[32]

[32] Own Illustration based on XYZ (2000), page 7 and data accessed 10.09.25.

Granger Causality Tests - EUR/USD and oil (raw data)

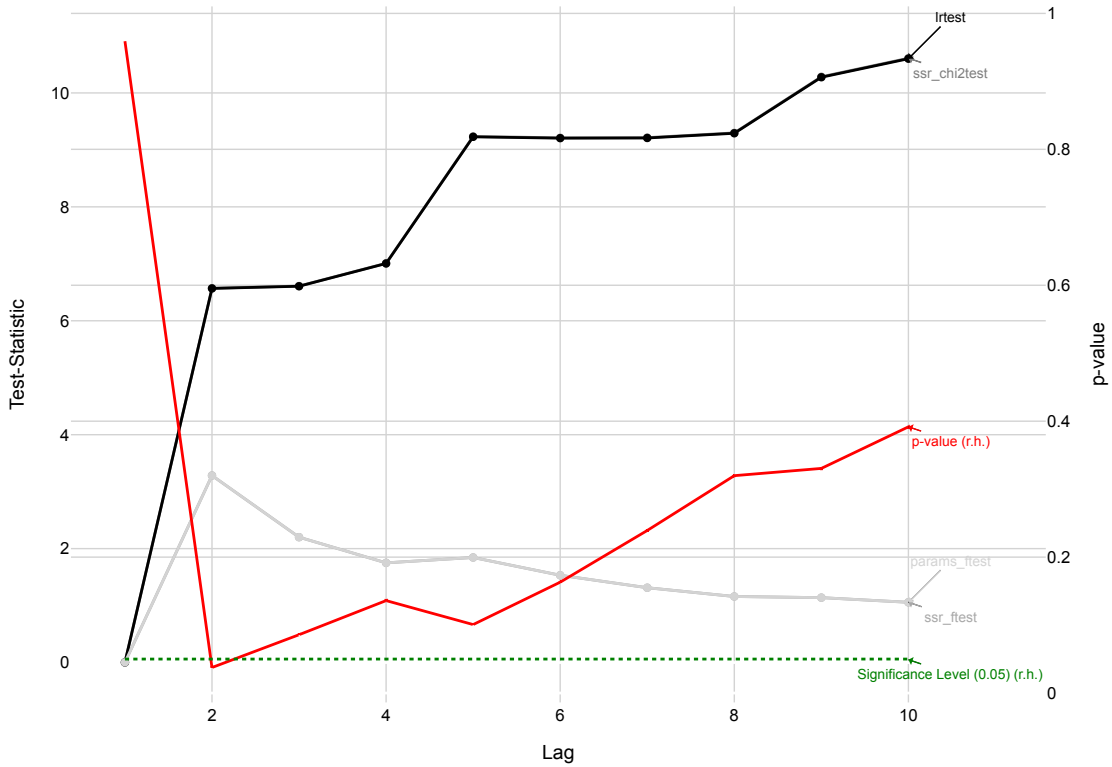


Figure: Granger causality test results testing granger causality of daily observations of oil for EUR/USD spot exchange rate over the time: 1999 - 2025.[33]

[33] Own Illustration based on XX, page X, tests used from: [**seabold2010statsmodels**] and data taken from [**fred˙2025˙federal˙reserve˙data**], last accessed 24.10.25.

Granger Causality Tests - EUR/USD and gas (raw data)

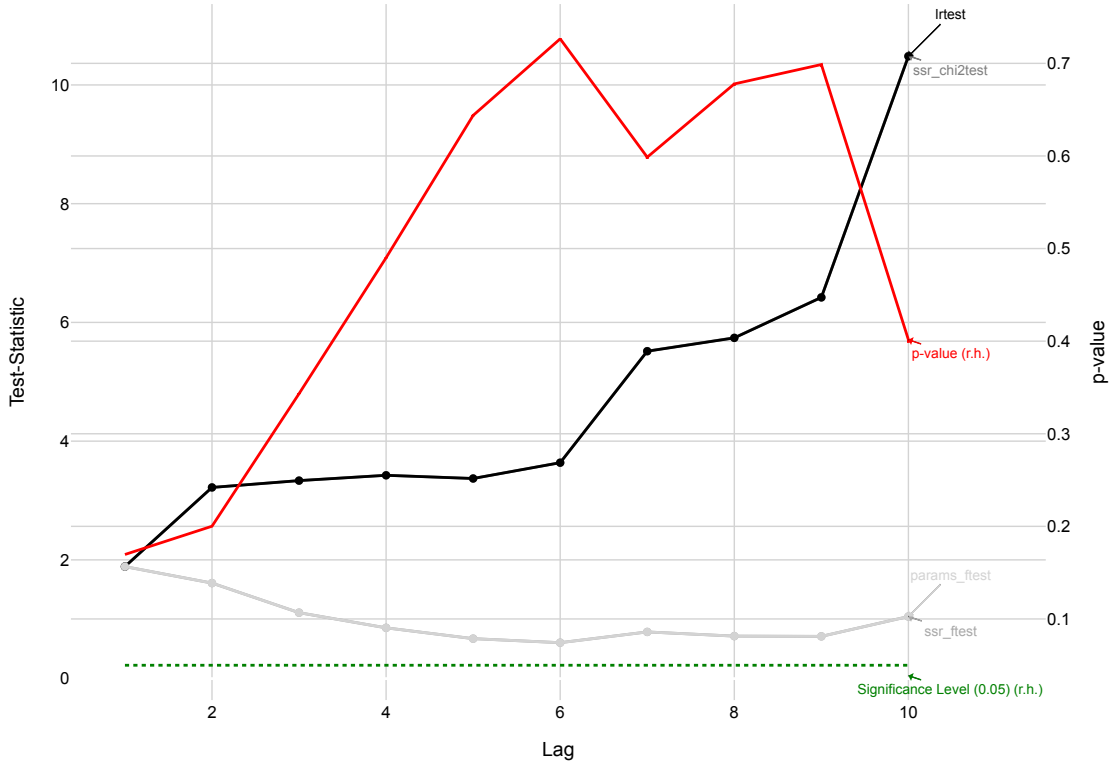


Figure: Granger causality test results testing granger causality of daily observations of gas for EUR/USD over the time: 1999 - 2025.[34]

[34] Own Illustration based on XX, page X, tests used from: [**seabold2010statsmodels**] and data taken from [**fred˙2025˙federal˙reserve˙data**], last accessed 24.10.25.

Granger Causality Tests - EUR/USD and oil (log first differences)

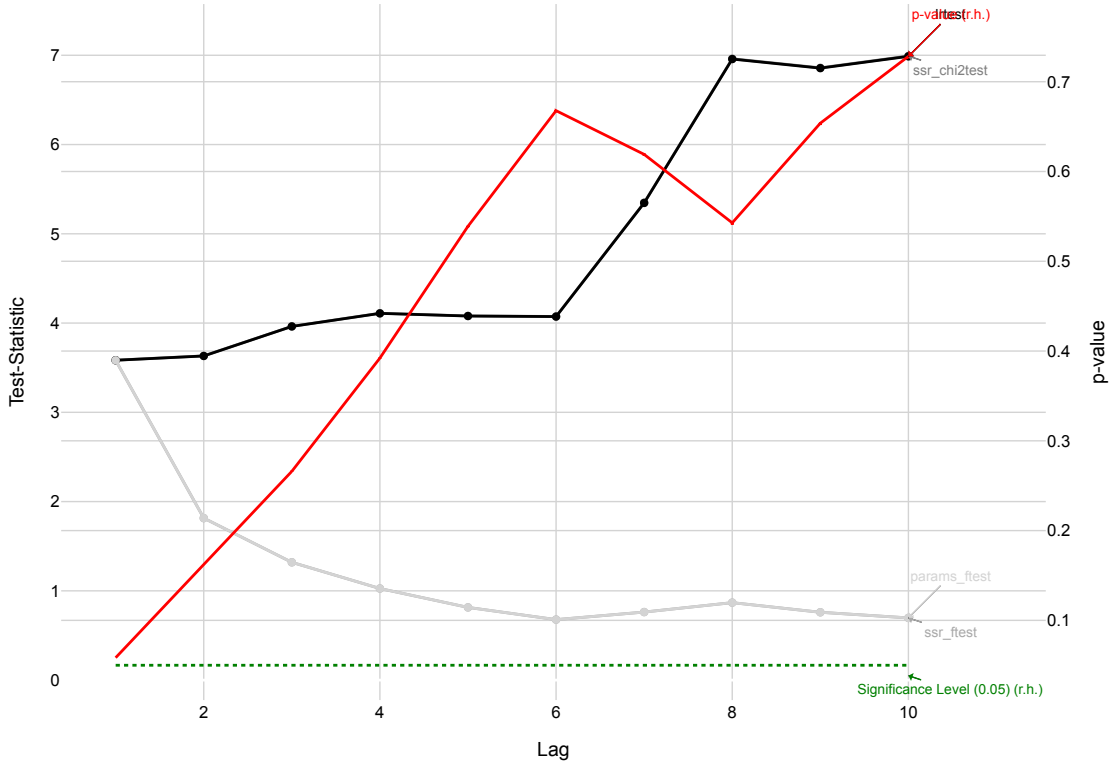


Figure: Granger causality test results testing granger causality of daily observations (log first differences) of WTI Oil for EUR/USD over the time: 1999 - 2025.[35]

[35] Own Illustration based on XX, page X, tests used from: [**seabold2010statsmodels**] and data taken from [**fred˙2025˙federal˙reserve˙data**], last accessed 24.10.25.

Granger Causality Tests - EUR/USD and gas (log first differences)

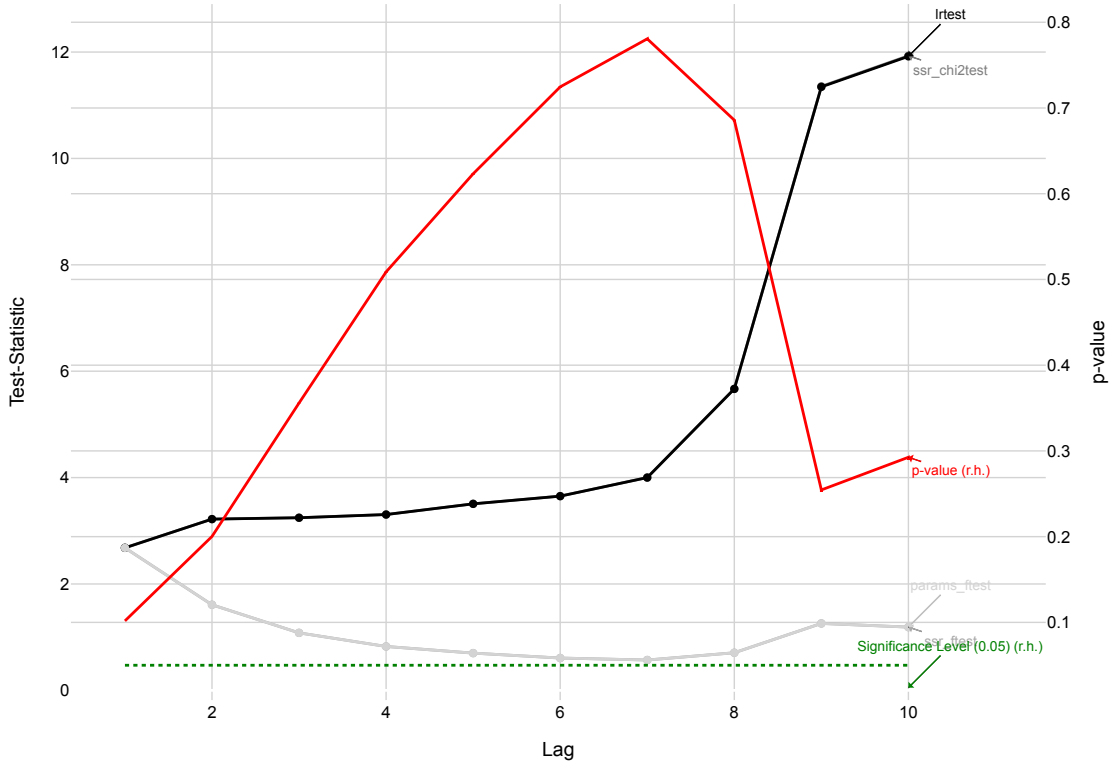


Figure: Granger causality test results testing granger causality of daily observations (log first differences) of Natural Gas for EUR/USD over the time: 1999 - 2025.[36]

[36] Own Illustration based on XX, page X, tests used from: [**seabold2010statsmodels**] and data taken from [**fred˙2025˙federal˙reserve˙data**], last accessed 24.10.25.

Appendix - Figures and Tables

No.	Exchange Rate	Basket	Data Frequency	Data Availability	Exchange Rate Type	Bilateral/Multilateral	Source	Data ID	Link
1	Real effective exchange rate	broad	monthly	1994-01	REER	Multilateral	[**bis˙2025˙effective˙exchange˙rates**]	M.R.B.US	Link
2	Real effective exchange rate	narrow	monthly	1964-01	REER	Multilateral	[**bis˙2025˙effective˙exchange˙rates**]	M.R.N.US	Link
3	Nominal effective exchange rate	broad	monthly	1994-01	NEER	Multilateral	[**bis˙2025˙effective˙exchange˙rates**]	M.N.B.US	Link
4	Nominal effective exchange rate	narrow	monthly	1964-01	NEER	Multilateral	[**bis˙2025˙effective˙exchange˙rates**]	M.N.N.US	Link
5	Nominal effective exchange rate	narrow	daily	1983-10-03	NEER	Multilateral	[**bis˙2025˙effective˙exchange˙rates**]	D.N.N.US	Link
6	Nominal effective exchange rate	broad	daily	1996-04-11	NEER	Multilateral	[**bis˙2025˙effective˙exchange˙rates**]	D.N.B.US	Link
7	USD-EUR Spot Rate	-	daily	1999-01-04	NER	Bilateral	[**fred˙2025˙federal˙reserve˙data**]	DEXUSEU	Link
8	Nominal Broad U.S. Dollar Index	broad	daily	2006-01-02	NEER	Multilateral	[**fred˙2025˙federal˙reserve˙data**]	DTWEXBGS	Link
9	Real Broad Dollar Index	broad	monthly	2006-01-01	REER	Multilateral	[**fred˙2025˙federal˙reserve˙data**]	RTWEXBGS	Link

Table: Various exemplary exchange rate data sources for the USD.[37]

[37] Own illustration based on

Appendix - Data and Definitions

Appendix - Definitions

Appendix - Definitions

PPP Deviation Calculation

$$\text{PPP Deviation} = \log(REER_{country} - REER_{USD}) \tag{1}$$

Source: [**oecd˙2025˙purchasing˙power˙parities**], data taken from [**fred˙2025˙federal˙reserve˙data**], last accessed 24.10.25.

CPI Component Distribution Calculation

$$\text{CPI Component Distribution} = \frac{\text{CPI}_{component}}{\text{CPI}_{total}} \times 100 \tag{2}$$

Source: Data taken from [**fred˙2025˙federal˙reserve˙data**] and [**ecb˙2025˙data˙portal**], last accessed 24.10.25.

Appendix - Definitions

Rolling Volatility Calculation

$$\text{Rolling Volatility} = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(x_t - \bar{x})^2} \tag{3}$$

Source: Data taken from [**fred˙2025˙federal˙reserve˙data**], last accessed 24.10.25.

Appendix - Definitions

	Event	Start-date	End-date	Event-Type	Source
1	US Recession 1970-1970	1970-01-01	1970-12-01	US-Recession	[**fred˙usrec**]
2	Bretton Woods Breakdown	1971-08-15	1973-03-19	Major Global Crisis	[**corzo2025exchange**]
3	Nixon Shock	1971-08-15	1973-03-19	Major Global Crisis	[**ehrmann2004exchange**]
4	Oil Crisis I	1973-10-17	1974-03-01	Major Global Crisis	[**trancoso2023beyond**]
5	US Recession 1973-1975	1973-12-01	1975-04-01	US-Recession	[**fred˙usrec**]
6	Oil Crisis II	1979-01-01	1981-03-01	Major Global Crisis	[**trancoso2023beyond**]
7	US Recession 1980-1980	1980-02-01	1980-08-01	US-Recession	[**fred˙usrec**]
8	US Recession 1981-1982	1981-08-01	1982-12-01	US-Recession	[**fred˙usrec**]
9	Black Monday Crash	1987-10-19	1987-10-19	Major Global Crisis	[**ehrmann2004exchange**]
10	US Recession 1990-1991	1990-08-01	1991-04-01	US-Recession	[**fred˙usrec**]
11	Asian Financial Crisis	1997-07-02	1998-12-31	Major Global Crisis	[**martins2023events**]
12	Russian Crisis	1998-08-17	1998-09-01	Major Global Crisis	[**martins2023events**]
13	Dot-com Bubble	2000-03-01	2002-10-01	Major Global Crisis	[**martinez2025equilibrium**]
14	US Recession 2001-2001	2001-04-01	2001-12-01	US-Recession	[**fred˙usrec**]
15	Global Financial Crisis	2007-08-09	2009-03-09	Major Global Crisis	[**martinez2025equilibrium**]
16	US Recession 2008-2009	2008-01-01	2009-07-01	US-Recession	[**fred˙usrec**]
17	US QE	2008-11-25	2014-12-31	Major Global Crisis	[**martinez2025equilibrium**]
18	European Debt Crisis	2009-10-01	2012-12-31	Major Global Crisis	[**martinez2025equilibrium**]
19	US Debt Ceiling Crisis	2011-08-20	2011-08-05	Major Global Crisis	[**martinez2025equilibrium**]
20	US-China Trade War	2018-03-22	2020-01-15	Major Global Crisis	Various
21	COVID-19 Pandemic	2020-02-20	2021-11-16	Major Global Crisis	[**martins2023events**]
22	US Recession 2020-2020	2020-03-01	2020-05-01	US-Recession	[**fred˙usrec**]
23	Russia-Ukraine War	2022-02-24	2022-06-01	Major Global Crisis	[**chandan2025currency**]

Table: Major global crisis periods (theoretical regimes).[38]

[38] Own illustration based on and data taken from [**fred˙2024˙us˙recession˙indicator**], last accessed 24.10.25.

Appendix - Definitions

Nominal effective exchange rate (NEER) (Slide 13):

$$\text{NEER} = \frac{1}{N} \sum_{i=1}^{N} \text{NER}_i$$

Source: Definition taken from [**bis˙2025˙effective˙exchange˙rates**].

Real effective exchange rate (REER) (Slide 13):

$$\text{REER} = \text{NEER} \times \frac{\text{CPI}_{domestic}}{\text{CPI}_{foreign}}$$

Source: Definition taken from [**bis˙2025˙effective˙exchange˙rates**].

Appendix - Definitions

Nominal exchange rate (NER) (Slide 13):

$$\text{NER} = \frac{\text{Units of Domestic Currency}}{\text{Units of Foreign Currency}} \tag{4}$$

Source: Definition taken from [**ca˙zorzi˙2018˙exchange˙rate˙forecasting**], page 5.

Real exchange rate (RER) (Slide 13):

$$\text{RER} = \text{NER} \times \frac{\text{CPI}_{domestic}}{\text{CPI}_{foreign}} \tag{5}$$

Source: Definition taken from [**ca˙zorzi˙2018˙exchange˙rate˙forecasting**], page 5.

Clustering Metrics: Silhouette Score (Slide 17):

$$\text{Silhouette Score} = \frac{b - a}{\max(a, b)} \tag{6}$$

Source: Definition taken from [**sklearn˙silhouette˙score**].

Appendix - Definitions

Standard UIP relationship - log terms (Slide XX):

$$\triangle s_{t+1} = i_t - i_t^* + u_{t+1} \tag{7}$$

Source: Definition taken from [**reitz˙2025˙applied˙econometrics˙fx**], section "Testing UIP conditions", page 7.

Calculation of the RCM (Slide 17):

$$\text{RCM} = 100\, S^2 \left(1 - \frac{1}{N} \sum_{t=1}^{N} \left[g_t p_t + (1 - g_t)(1 - p_t) \right] \right) \tag{8}$$

with $g_t = \mathbf{1}\{p_t \geq \tau\}$, p_t the smoothed probability at time t, N the sample size, and τ the classification threshold (e.g. 0.5).
Source: Definition taken from [**ang1998regime**].

References

References - Literature

References - Literature I

References - Data

References - Data I

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Thank you for your attention!

We await your Questions and/or Comments.

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Public GitHub Project Repository: `https://github.com/RobertHennings`

Discussion

1. Have you expected this outcome?
2. What do you think about the dynamics?
3. What other variables could be potentially included?

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Further Material for Illustrations - Questions