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Wirtschafts- und Sozialwissenschaftliche Fakultät

Identifying (volatility-)regimes in the EUR/USD spot exchange rate using clustering algorithms: An Oil and Gas Perspective on Parity Conditions.

Seminar in Applied Financial Economics: Applied Econometrics of FX Markets - Prof. Dr. Reitz

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GitHub: `https://github.com/RobertHennings/Seminar`

Kiel - 14.11.2025

Outline

Intro: Energy Commodities and Exchange Rates

*"This has led some to suggest that an **unidentified real** factor may be causing **persistent shifts** in real equilibrium exchange rates."*
— R.A. Amano, S. van Norden[1]

*"This may in fact be the case or it is also possible that the relationship between exchange rates and **oil shocks is non-linear** and not being detected by a **linear regression framework**."*
— S. A. Basher, A. A. Haug, P. Sadorsky[2]

*"The long-run real exchange rate of these 'commodity currencies' is **not constant** but is **time varying**, being dependent on **movements in the real price of commodity exports**."*
— P. Cashin, L. F. Cespedes, R. Sahay[3]

[1] [AN98], p. 301

[2] [BHS16a], p. 17

[3] [CCS04], p. 239

Intro: The PPP puzzle[5] and Commodity Currencies

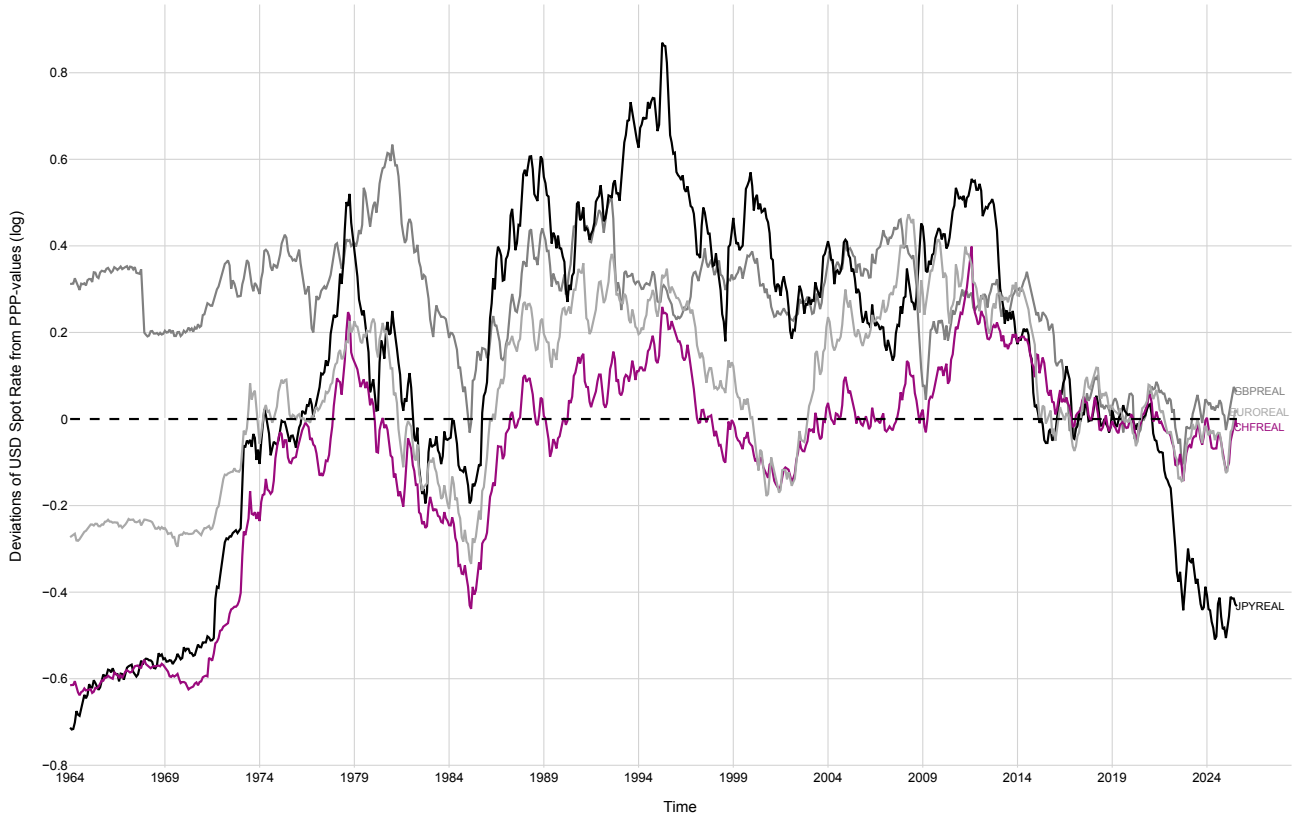

Figure 1: Monthly deviations of USD Spot Rate from PPP-values (in log terms) for the period: 1964 - 2025.[4]

[4] Own Illustration based on [Rei25], section "Modeling Trends: Unit Roots in Time Series", page 18/18 and data taken from [IS25], last accessed 24.10.25, own calculations.

[5] This puzzle concerns the finding of many researchers that the speed of mean reversion of real exchange rates is too slow to be consistent with PPP, which is the proposition that exchange rates are determined by movements in relative prices.

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Research Hypothesis

Energy Price Contributions to Inflation - USA

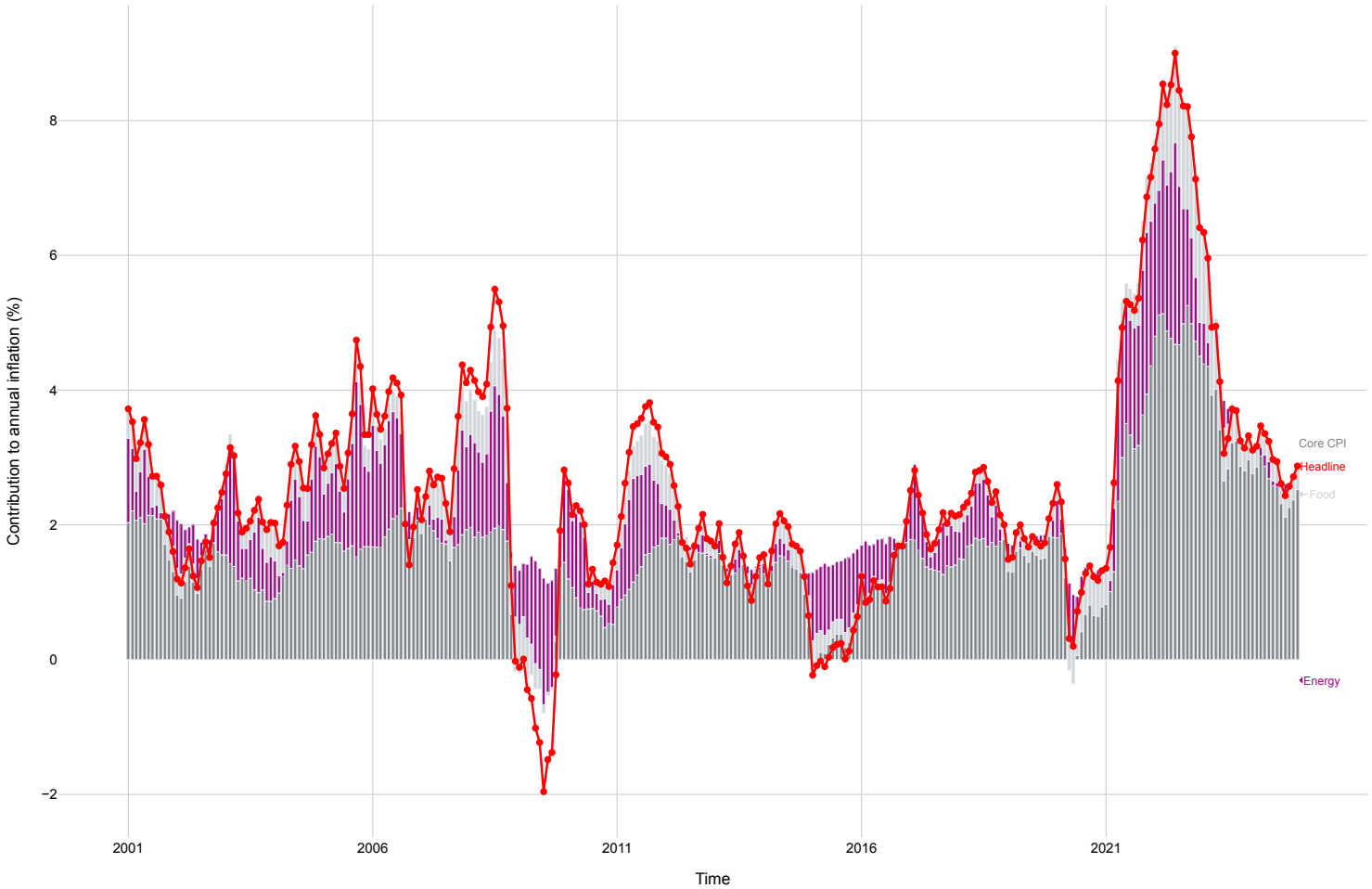

Figure 2: Monthly US CPI: Headline and component contributions for the period: 2001 - 2024.[6]

[6] Own Illustration based on Chart A, [Ban22], and data taken from [SL25a], last accessed 24.10.25, own calculations.

Energy Price Contributions to Inflation - EU Area

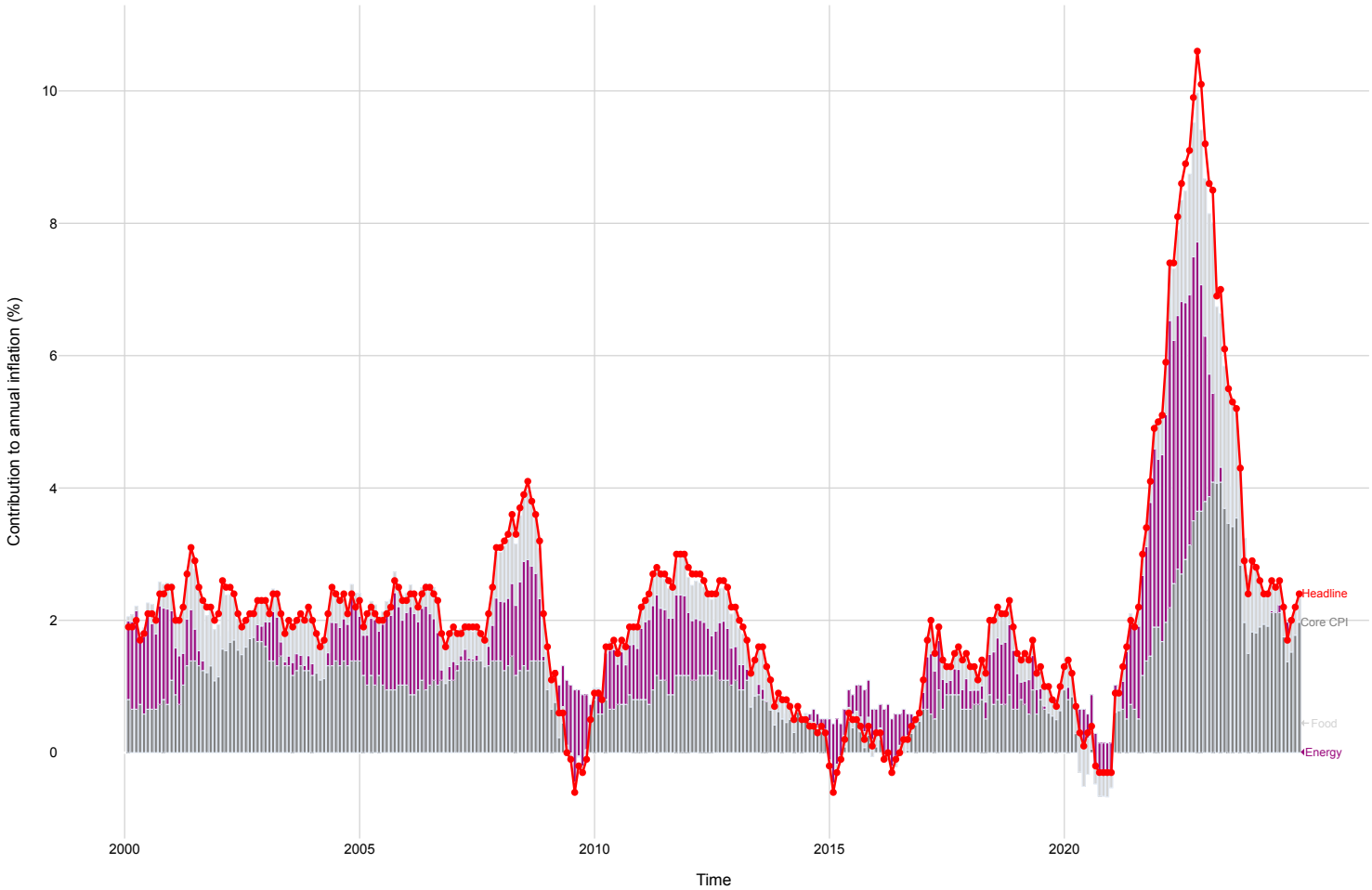

Figure 3: Monthly EU Area CPI: Headline and component contributions for the period: 2000 - 2024.[7]

[7] Own Illustration based on Chart A, [Ban22], and data taken from [Ban25], last accessed 24.10.25, own calculations.

Rolling Volatility of Exchange Rates and Energy Commodity Prices

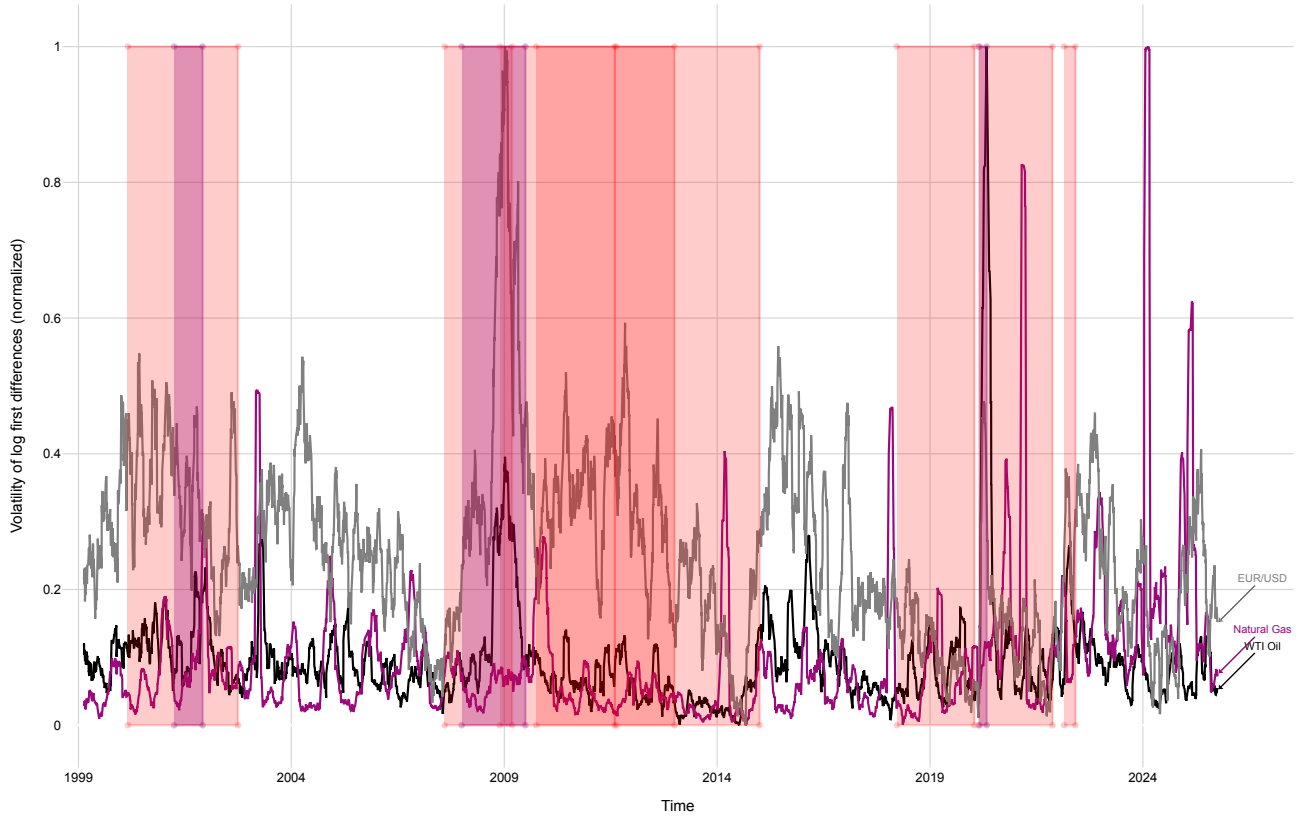

Figure 4: Daily normalized EUR/USD spot exchange rate, oil and gas log first differences volatility with highlighted crisis periods for the period: 2000 - 2024.[8]

[8] Own Illustration based on [Zub+21], page 7 and data taken from [SL25a], last accessed 24.10.25, own calculations.

Formulated Research Hypothesis

Main Research Hypothesis

*"The standard UIP-equilibrium condition is **time-dependent** and primarily controlled by **two main regimes**, characterised by either **high or low (market-) volatility**."*

Additional Research Hypothesis I

*"Monetary policy, i.e. **interest rates**, is partly driven by **energy commodity prices** that induce volatility through the inflationary pass-through channel, especially during **phases of market distress** in economies heavily relying on import/export of energy commodities."*

Additional Research Hypothesis II

*"Factoring in variables related to **energy commodity prices** in combination with using **alternative clustering techniques** improves the **identification of the regimes** to better pinpoint the time-dependent testing of the standard UIP relation, compared to Markov-Switching benchmark models."*

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Theoretical Framework

A simple model of exchange rates and commodity prices

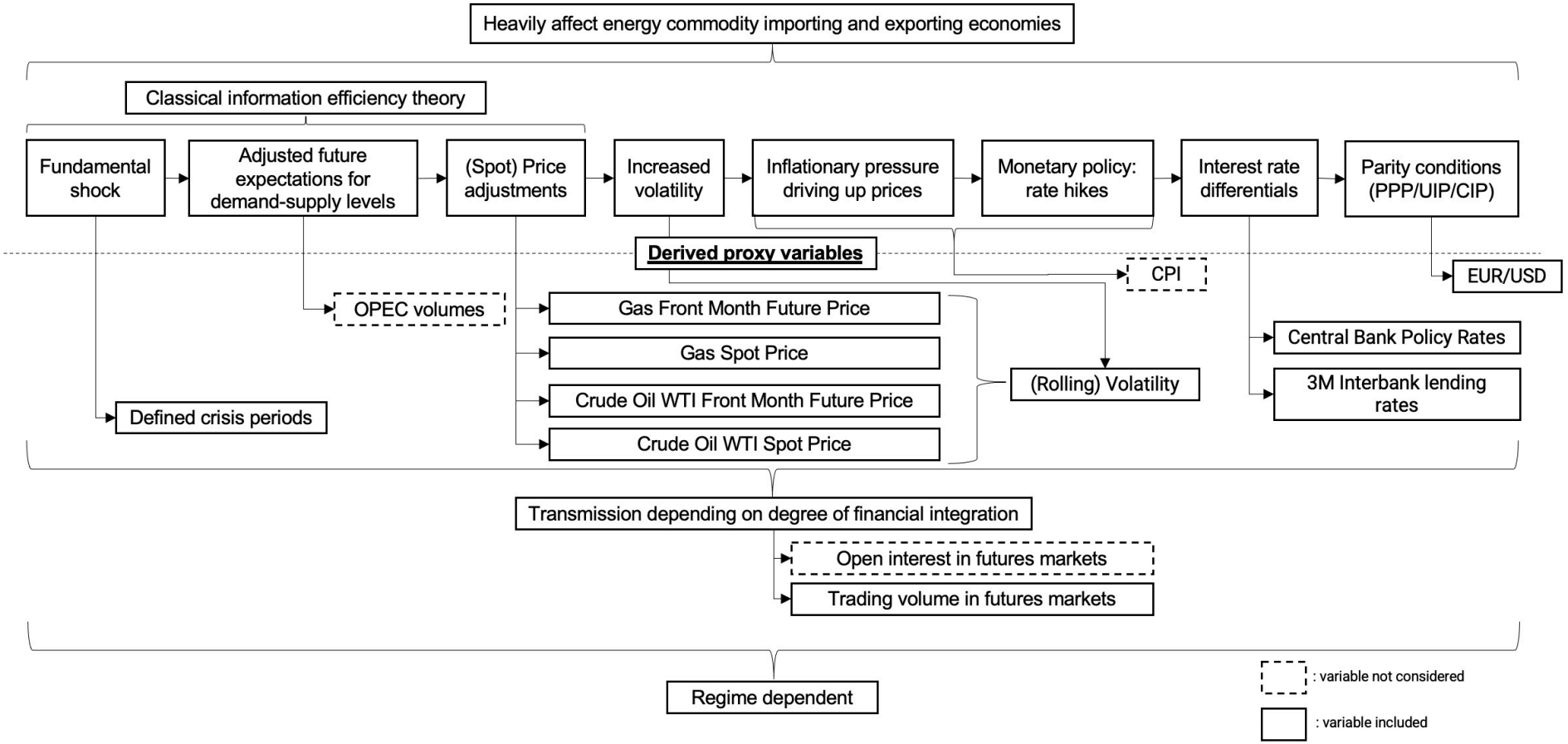
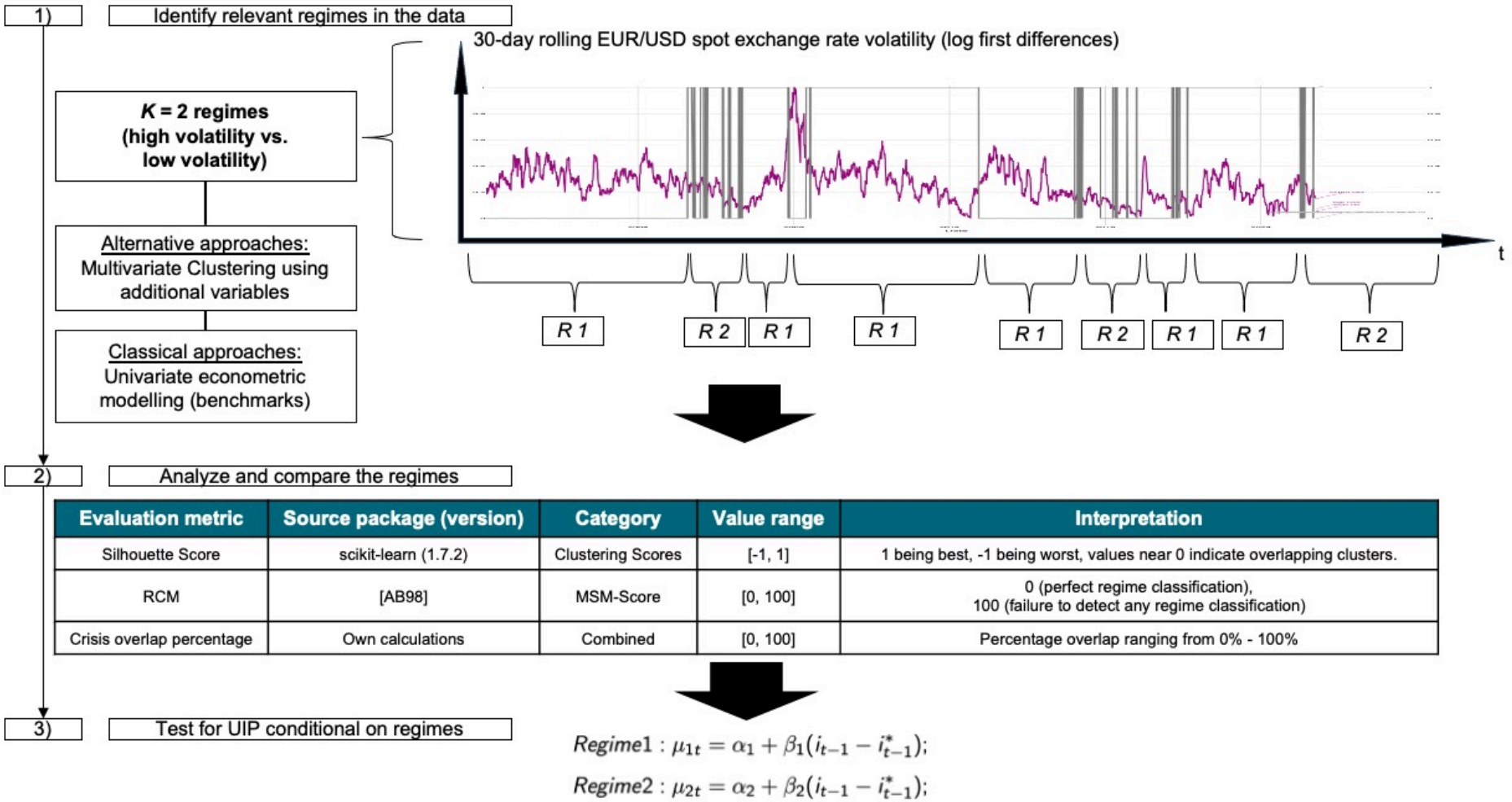


Figure 5: Main theoretical framework for the seminar project analysis, inflation pass-through effect of energy commodity prices.[9]

[9] Own Illustration based on Figure 3, page 3, [OUS20].

Theoretical Framework



Evaluation metric	Source package (version)	Category	Value range	Interpretation
Silhouette Score	scikit-learn (1.7.2)	Clustering Scores	[-1, 1]	1 being best, -1 being worst, values near 0 indicate overlapping clusters.
RCM	[AB98]	MSM-Score	[0, 100]	0 (perfect regime classification), 100 (failure to detect any regime classification)
Crisis overlap percentage	Own calculations	Combined	[0, 100]	Percentage overlap ranging from 0% - 100%

Figure 6: Main theoretical framework for the seminar project analysis, key project steps.[10]

[10] Own Illustration with formulas taken from [Rei25].

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Model Results

Model comparison and selection

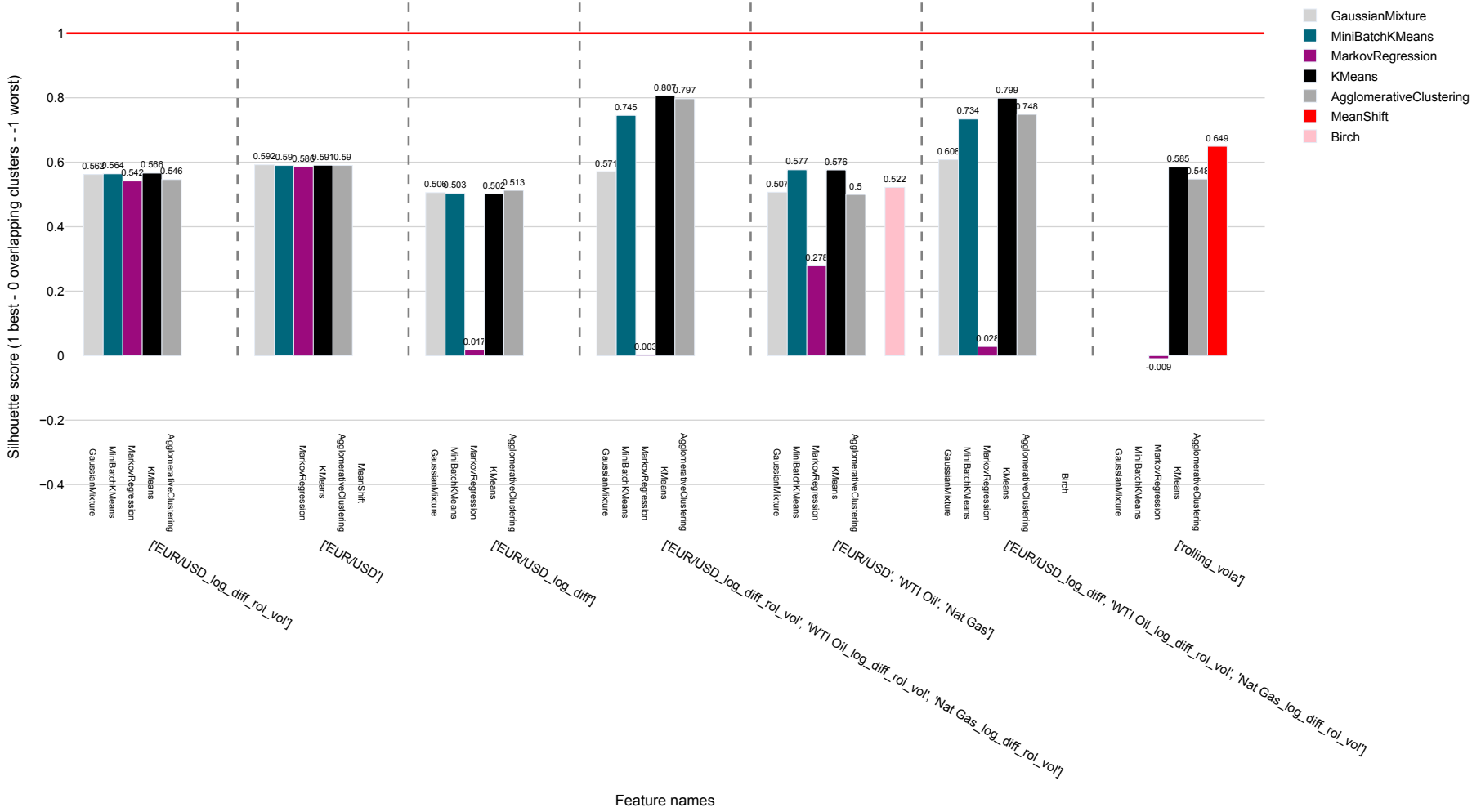

Figure 7: Model comparison using the silhouette score for various regime identification model configurations.[11]

[11] Own Illustration based on [Jah+24], page 9 and data taken from [SL25a], last accessed 24.10.25, own calculations.

Predicted Regimes - Evolution over time with highlighted Crisis Periods

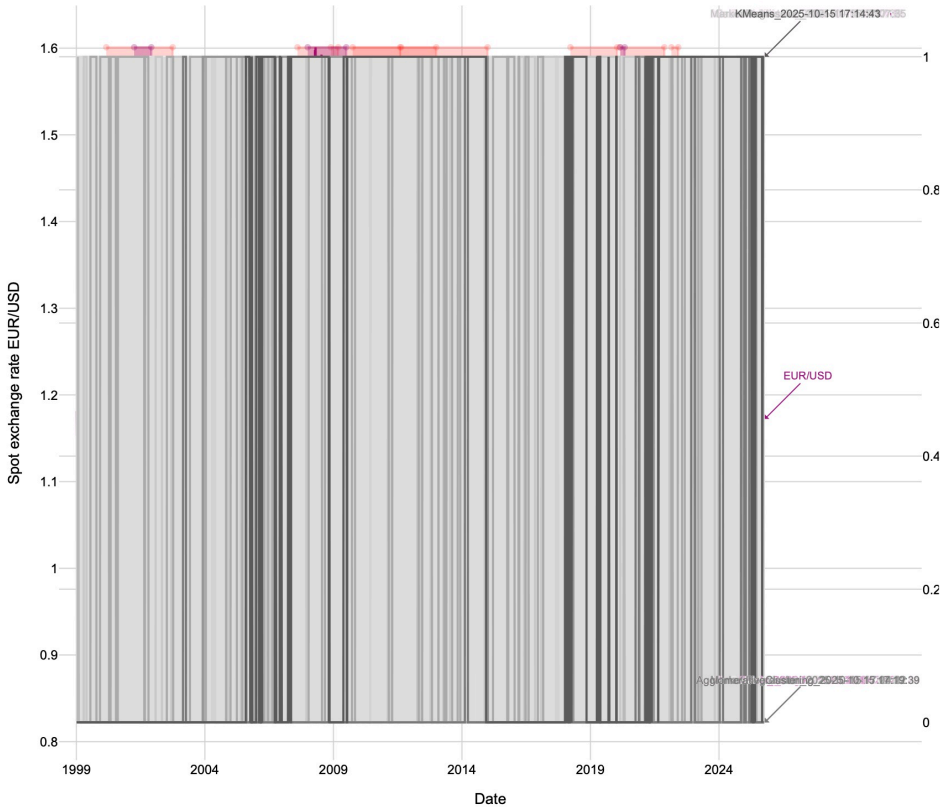

Figure 8: Time-series of predicted regimes with highlighted crisis periods.[12]

[12] Own Illustration based on [Ayo16], page 5 and data taken from [SL25a], last accessed 24.10.25, own calculations.

Predicted Regimes - Some excerpts (I)

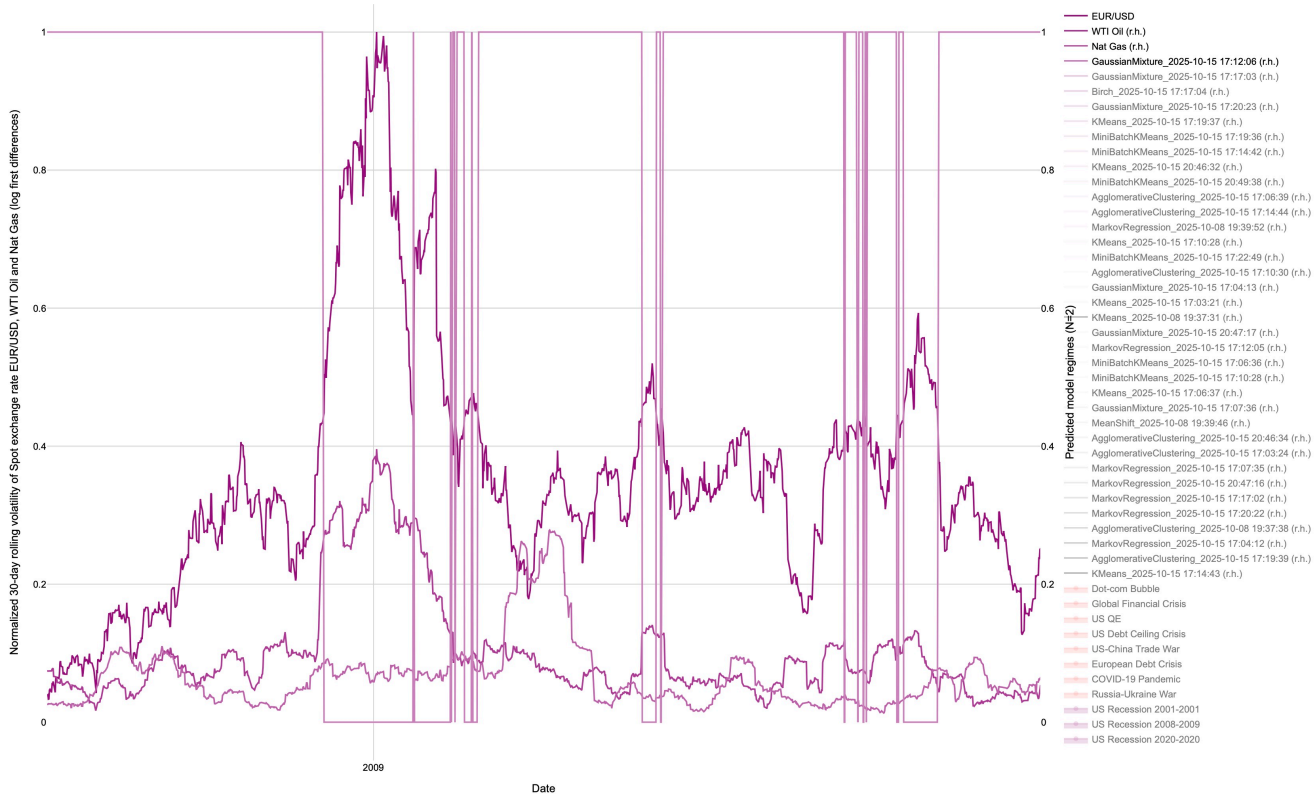

Figure 9: Time-series of predicted regimes with highlighted crisis periods, exemplary algorithm: GaussianMixture.[13]

[13] Own Illustration based on [Ayo16], page 5 and data taken from [SL25a], last accessed 24.10.25, own calculations.

Predicted Regimes - Some excerpts (II)

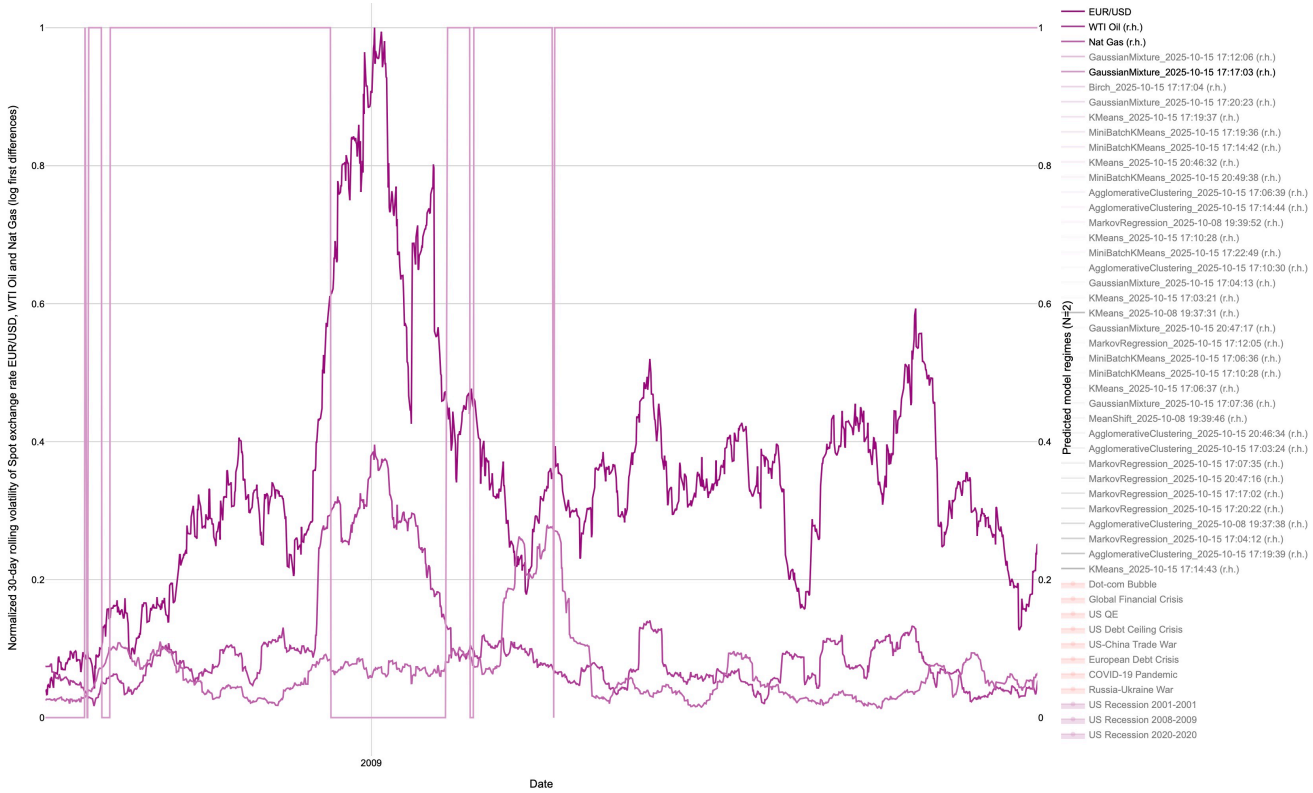

Figure 10: Time-series of predicted regimes with highlighted crisis periods, exemplary algorithm: GaussianMixture.[14]

[14] Own Illustration based on [Ayo16], page 5 and data taken from [SL25a], last accessed 24.10.25, own calculations.

Predicted Regimes - Some excerpts (III)

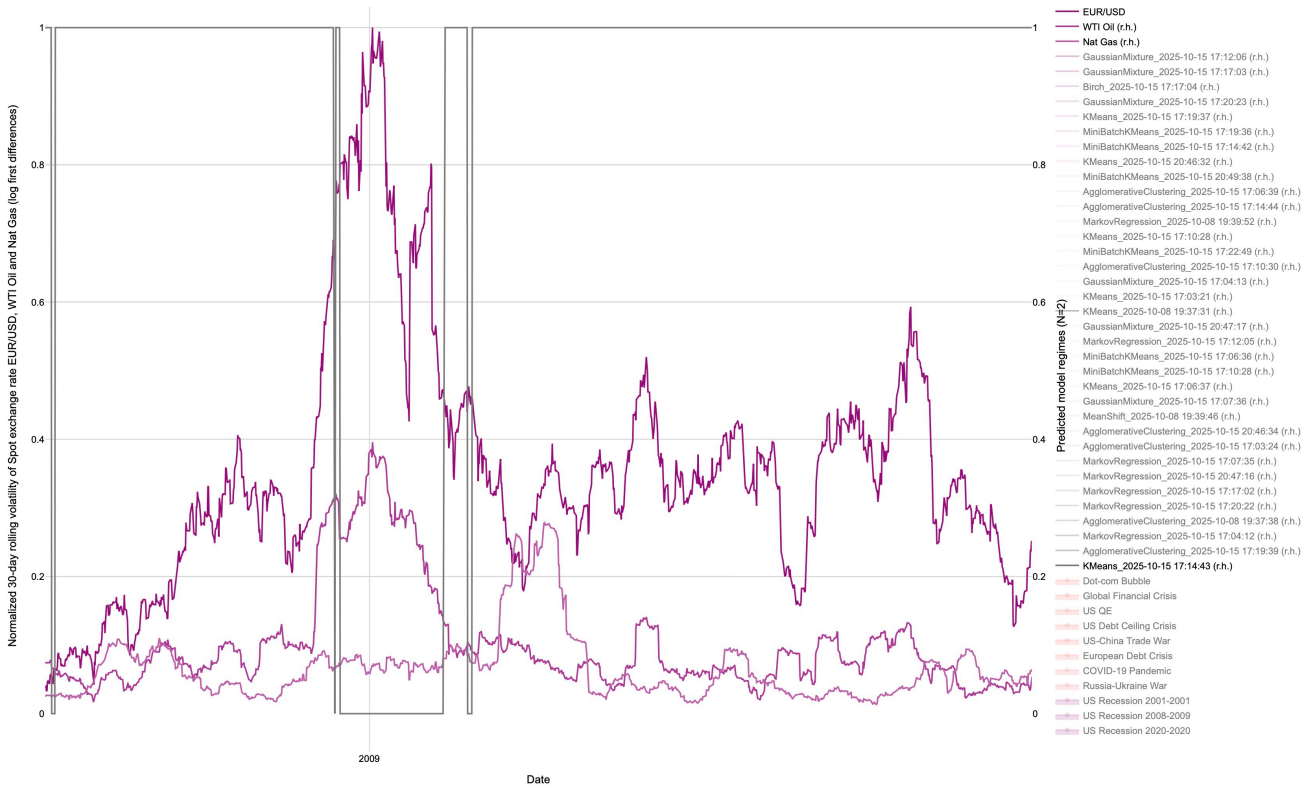

Figure 11: Time-series of predicted regimes with highlighted crisis periods, exemplary algorithm: KMeans.[15]

[15] Own Illustration based on [Ayo16], page 5 and data taken from [SL25a], last accessed 24.10.25, own calculations.

Predicted Regimes - Some excerpts (IV)

Figure 12: Time-series of predicted regimes with highlighted crisis periods, exemplary algorithm: AgglomerativeClustering.[16]

[16] Own Illustration based on [Ayo16], page 5 and data taken from [SL25a], last accessed 24.10.25, own calculations.

Predicted Regimes - Some excerpts (V)

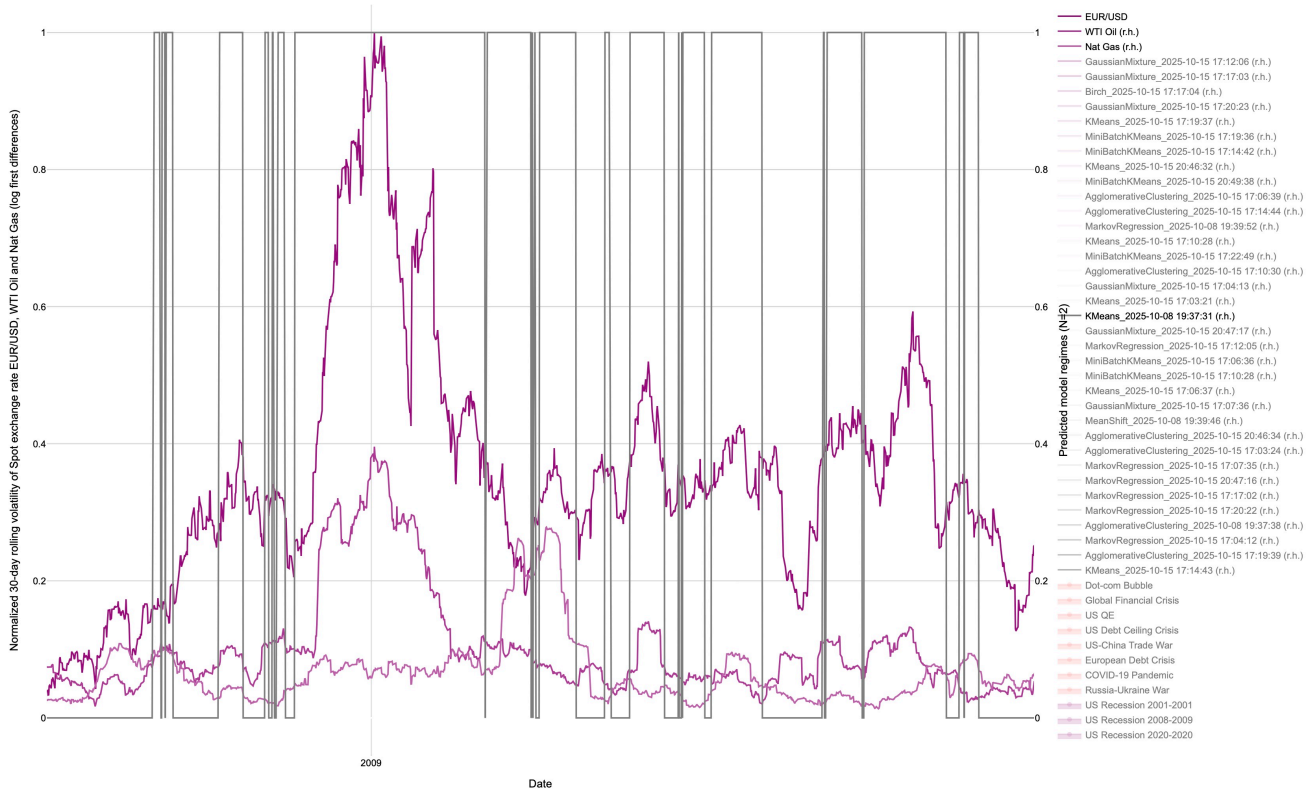

Figure 13: Time-series of predicted regimes with highlighted crisis periods, exemplary algorithm: KMeans.[17]

[17] Own Illustration based on [Ayo16], page 5 and data taken from [SL25a], last accessed 24.10.25, own calculations.

Predicted Regimes - Overlapping Comparison

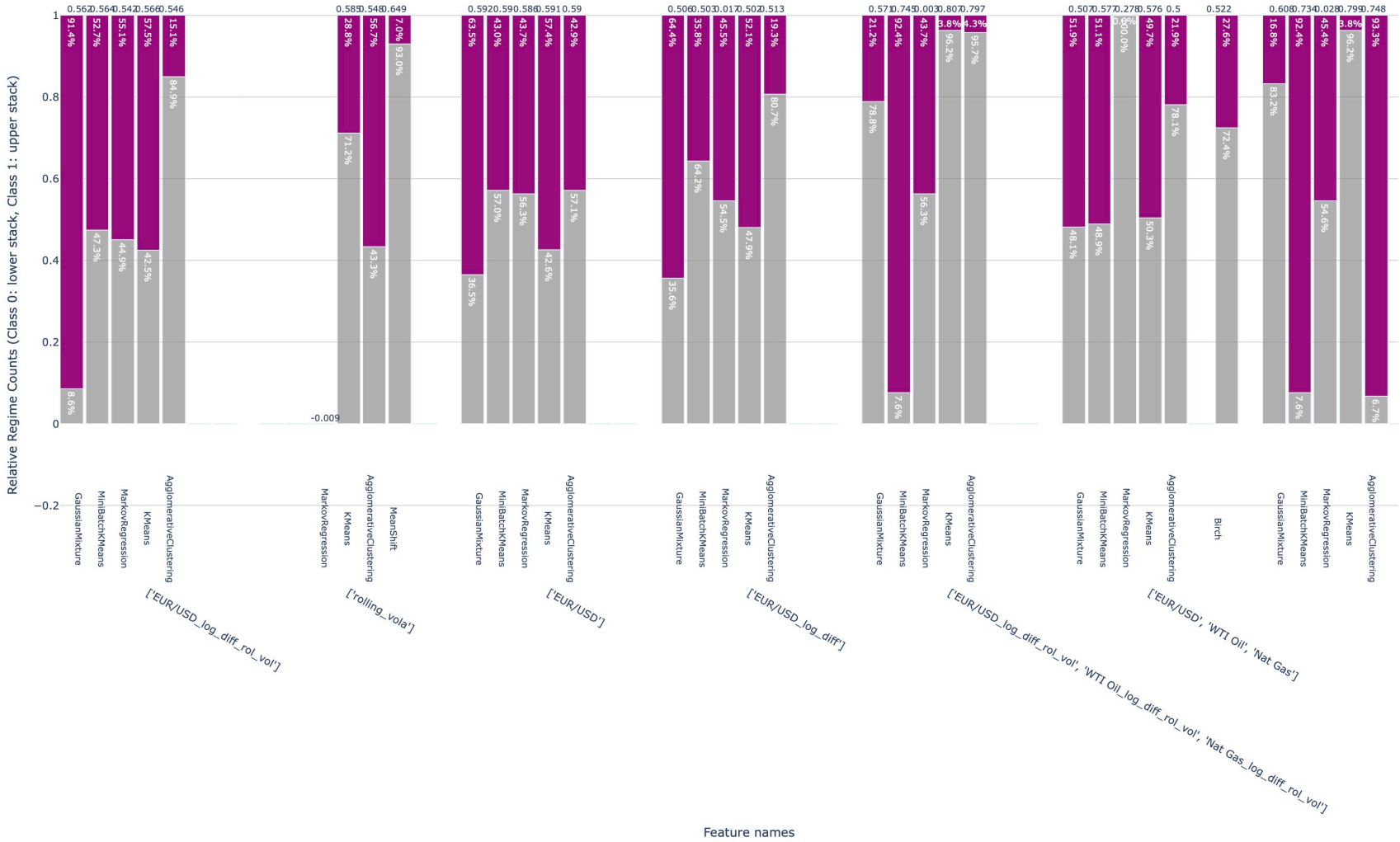

Figure 14: Relative share of regime classification per features per clustering algorithm.[18]

[18] Own Illustration based on [MB21], page 764 and data taken from [SL25a], last accessed 24.10.25, own calculations.

Predicted Regimes - Overlapping Comparison with theoretical crisis periods

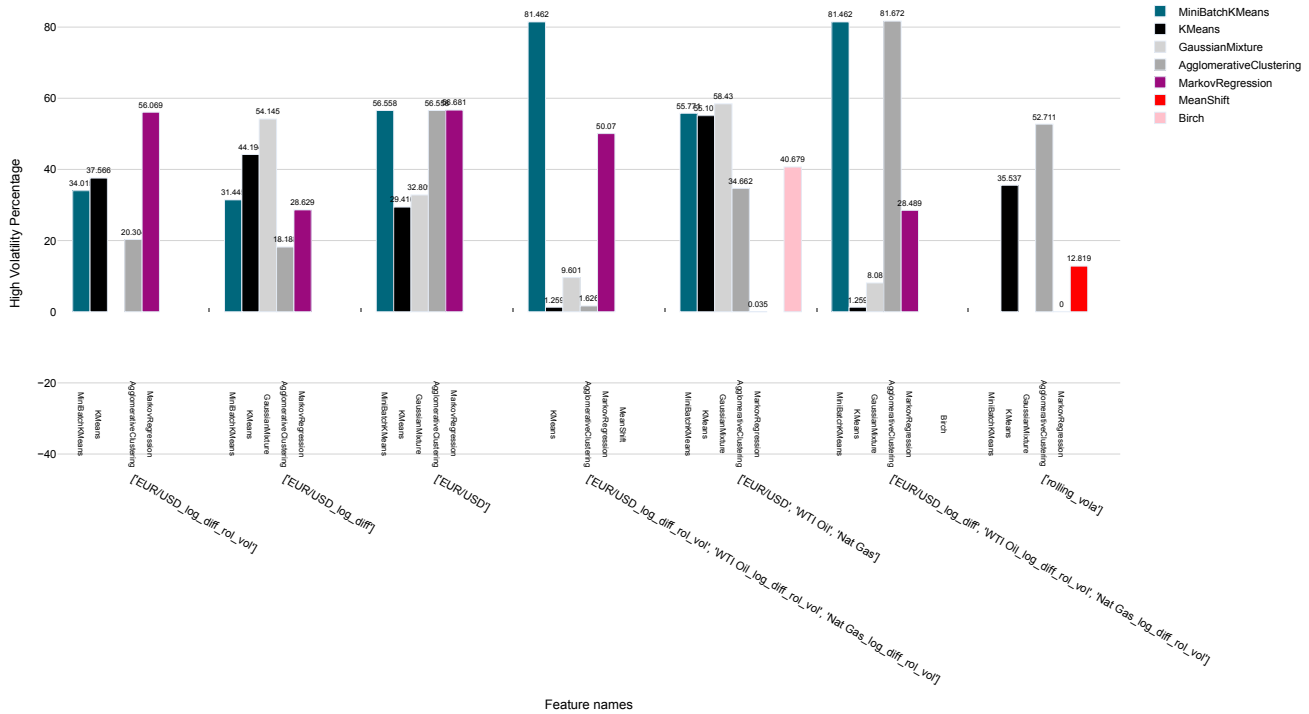

Figure 15: Relative share of observations classified into class 1 and falling into a defined theoretical crisis period per features per clustering algorithm.[19]

[19] Own Illustration based on [MB21], page 764 and data taken from [SL25a], last accessed 24.10.25, own calculations.

UIP estimation - benchmark models

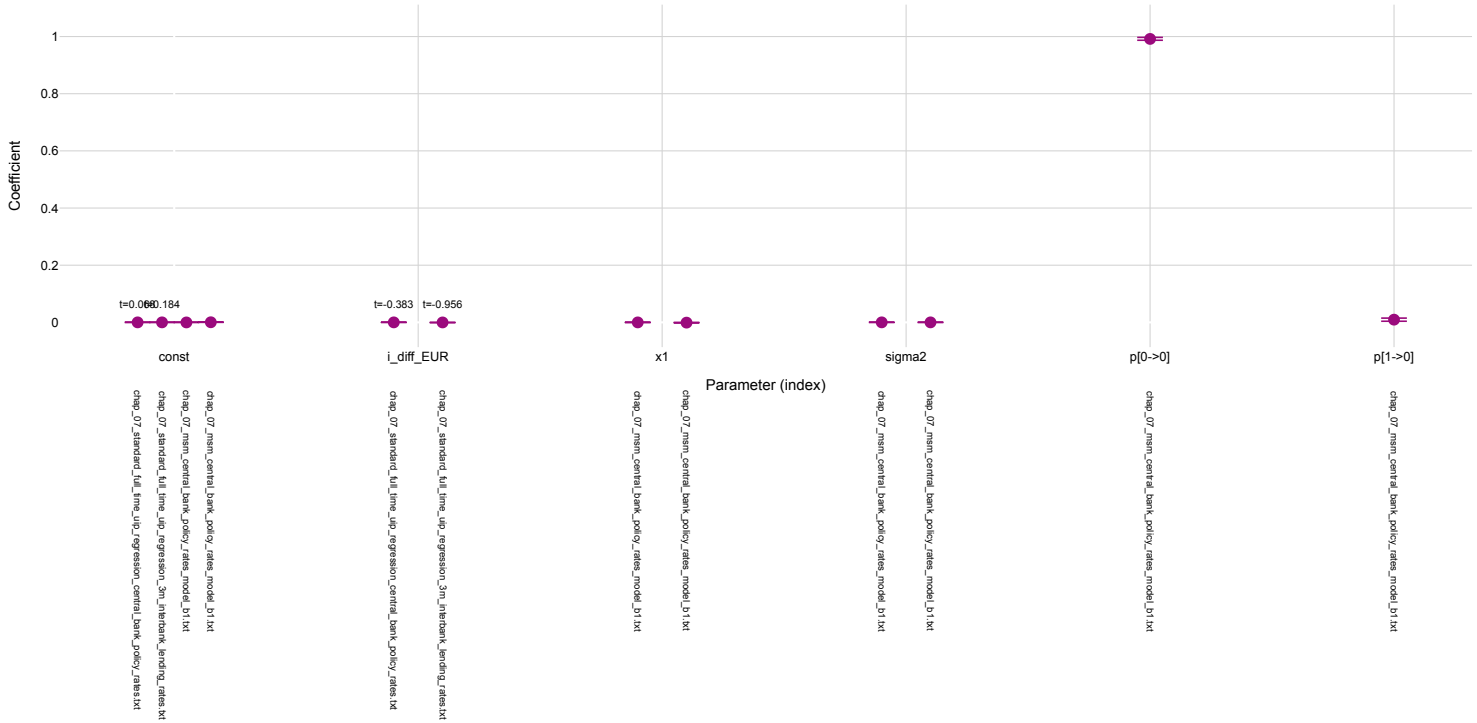

Figure 16: UIP estimation results for benchmark models - standard linear regression and Markov-Switching models.[20]

[20] Own Illustration based on data taken from [SL25a], last accessed 24.10.25, own calculations (used MacKinnon and White's (1985) HC1 heteroscedasticity robust standard errors [MW85]).

UIP estimation - identified regimes

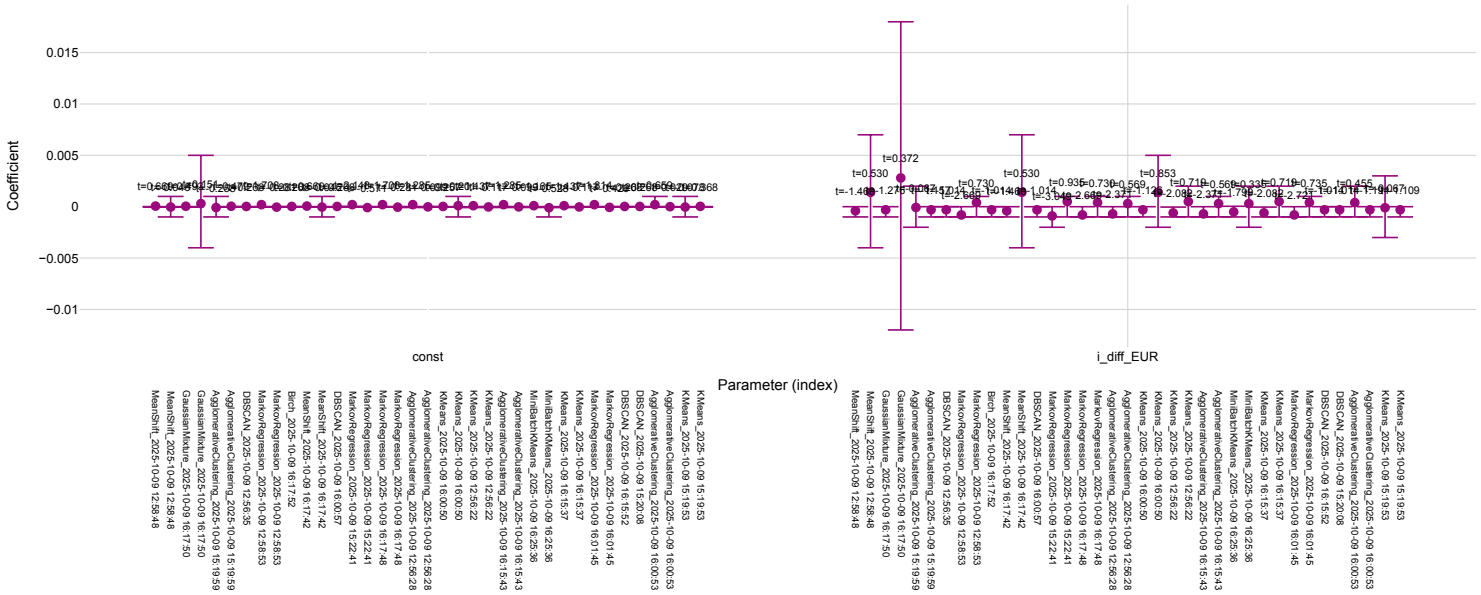

Figure 17: UIP estimation results for identified regimes using clustering algorithms.[21]

[21] Own Illustration based on data taken from [SL25a], last accessed 24.10.25, own calculations (used MacKinnon and White's (1985) HC1 heteroscedasticity robust standard errors [MW85]).

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Conclusion and Discussion

Seminar Project Summary

Main Research Hypothesis

Comparing different (configurations of) optimised clustering algorithms to identify (volatility driven) exchange rate regimes in the EUR/USD spot exchange rate yields promising results.

Additional Research Hypothesis I

Incorporating commodity prices (WTI Crude Oil Prices and Natural Gas Prices/ their rolling volatilities) as external variables improves the regime identification results.

Additional Research Hypothesis II

In comparison with the benchmark models, the clustering techniques seemed to be able to identify regimes more granular, but despite that were not able to yield more reliable/robust results for the standard UIP-relationship.

Seminar Project Limitations

Evaluation Metrics

Only used one metric for clustering (Silhouette Score).

Crisis Periods

May include more periods or find a suitable solution for overlapping crisis periods.

Hyperparameter Tuning

Hyperparameter tuning did not finish in time for all model configurations.

Future Research - Possible Extensions

Other Spot rate pairs

Extend the analysis to other spot exchange rate pairs, e.g. focus on the Commodity Currencies with a strong commodity linkage.

Other commodities

Extend the analysis to also include other commodities, e.g. Natural Gas, Brent Oil, Gold, etc..

Other macroeconomic/external variables

Extend the analysis by incorporating other macroeconomic variables, e.g. Interest Rates, Inflation Rates, etc..

Other data frequencies

Extend the analysis by using intraday data to capture more granular dynamics.

Thank you for your attention!

We await your Questions and/or Comments.

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Robert Hennings
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Public GitHub Project Repository: `https://github.com/RobertHennings/Seminar`

Discussion

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1. Have you expected this outcome?
2. What do you think about the dynamics?
3. What other variables could be potentially included?

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Appendix

Appendix - Abbreviations

Abbreviations

PPP	Purchasing Power Parity
CPI	Consumer Price Index
OPEC	Organization of the Petroleum Exporting Countries
BIS	Bank for International Settlements
RCM	Regime Classification Measure
UIP	Uncovered Interest Parity
NEER	Nominal Effective Exchange Rate
REER	Real Effective Exchange Rate
NER	Nominal Exchange Rate
RER	Real Exchange Rate
PPI	Producer Price Index
GDP	Gross Domestic Product
OI	Open Interest
ACF	Autocorrelation Function
PACF	Partial Autocorrelation Function

Appendix - Literature Review

Systematic Literature Overview: Main Approaches

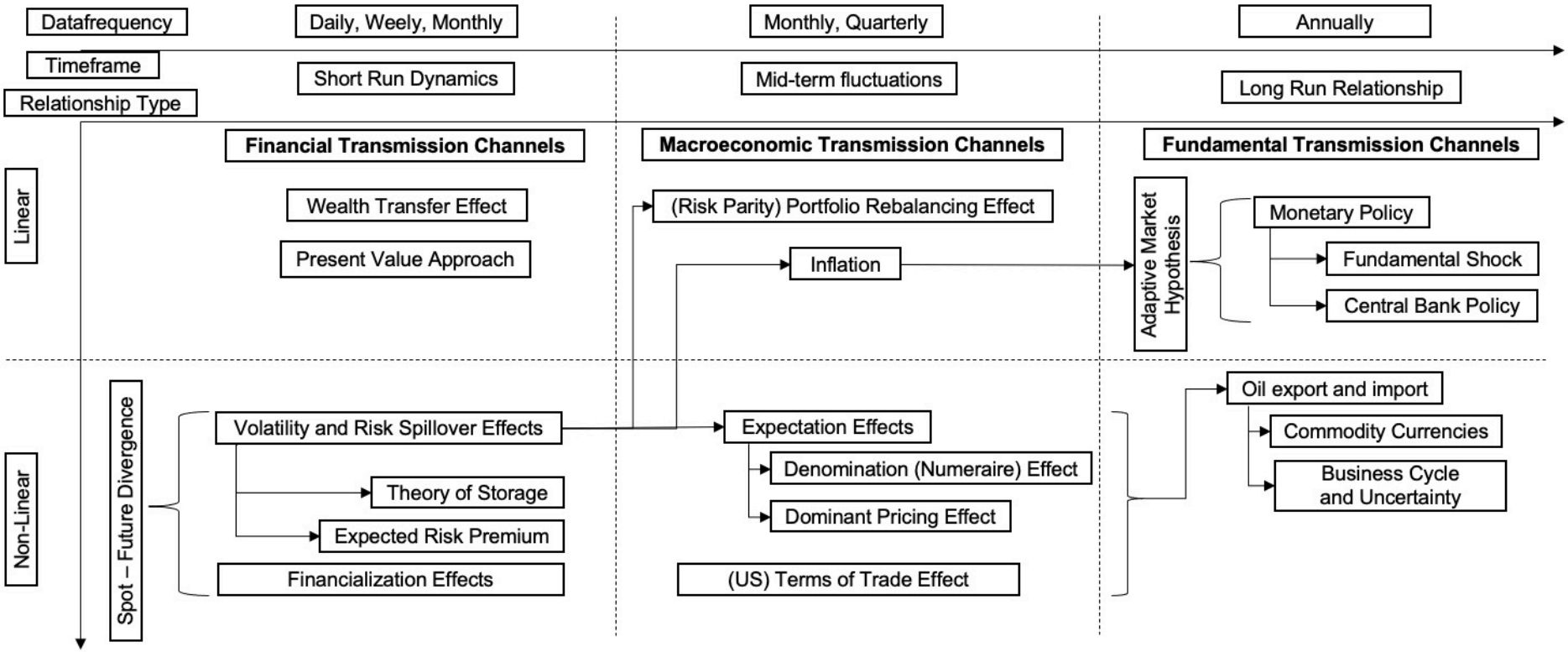

Figure 18: Systematic Overview about main theoretical approaches.[22]

[22] Own Illustration based on Figure 5, page 5, [OUS20], full list provided in the References section.

Appendix - Figures and Tables

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Full theoretical framework

Definitions - prices and measurements

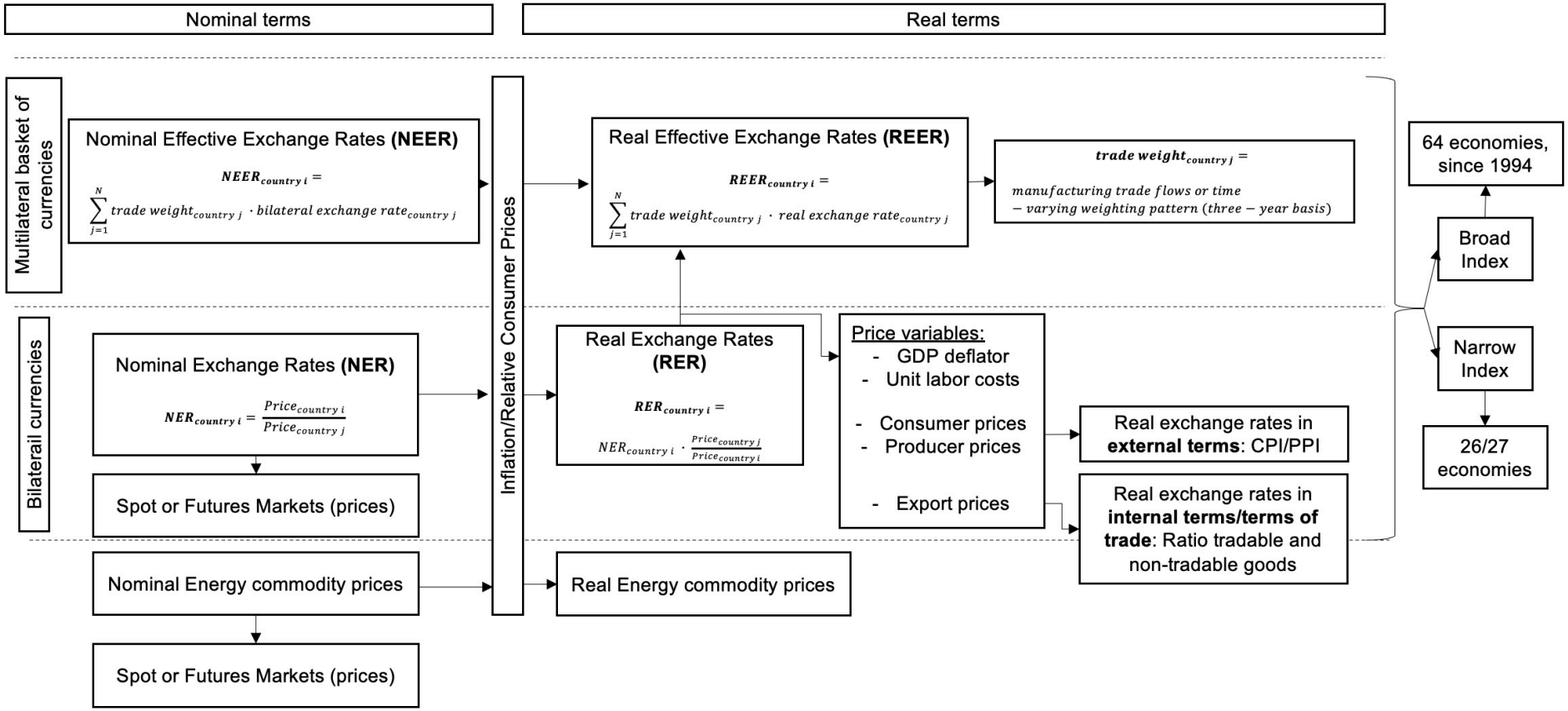
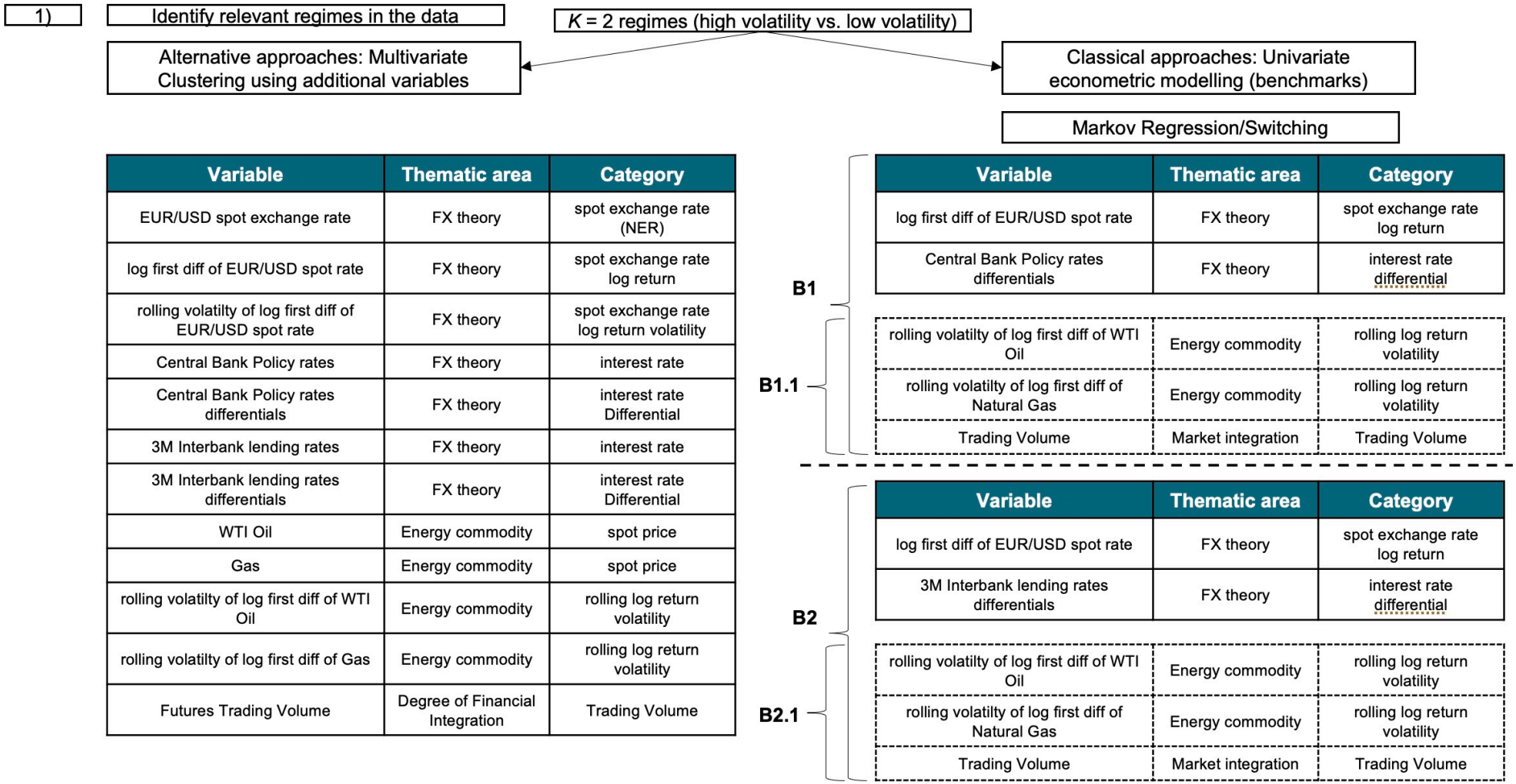


Figure 19: Schematic overview of prices and measurements of various exchange rate types.[23]

[23] Own Illustration based on [Ros03], page 8, exchange rates in natural logarithm (geometric averages) and data definitions from [IS25].

Theoretical Framework (I)



Variable	Thematic area	Category
EUR/USD spot exchange rate	FX theory	spot exchange rate (NER)
log first diff of EUR/USD spot rate	FX theory	spot exchange rate log return
rolling volatilty of log first diff of EUR/USD spot rate	FX theory	spot exchange rate log return volatility
Central Bank Policy rates	FX theory	interest rate
Central Bank Policy rates differentials	FX theory	interest rate Differential
3M Interbank lending rates	FX theory	interest rate
3M Interbank lending rates differentials	FX theory	interest rate Differential
WTI Oil	Energy commodity	spot price
Gas	Energy commodity	spot price
rolling volatilty of log first diff of WTI Oil	Energy commodity	rolling log return volatility
rolling volatilty of log first diff of Gas	Energy commodity	rolling log return volatility
Futures Trading Volume	Degree of Financial Integration	Trading Volume

B1

Variable	Thematic area	Category
log first diff of EUR/USD spot rate	FX theory	spot exchange rate log return
Central Bank Policy rates differentials	FX theory	interest rate differential

B1.1

Variable	Thematic area	Category
rolling volatilty of log first diff of WTI Oil	Energy commodity	rolling log return volatility
rolling volatilty of log first diff of Natural Gas	Energy commodity	rolling log return volatility
Trading Volume	Market integration	Trading Volume

B2

Variable	Thematic area	Category
log first diff of EUR/USD spot rate	FX theory	spot exchange rate log return
3M Interbank lending rates differentials	FX theory	interest rate differential

B2.1

Variable	Thematic area	Category
rolling volatilty of log first diff of WTI Oil	Energy commodity	rolling log return volatility
rolling volatilty of log first diff of Natural Gas	Energy commodity	rolling log return volatility
Trading Volume	Market integration	Trading Volume

Figure 20: Main theoretical framework for the seminar project analysis, main variables and benchmark models used.[24]

[24] Own Illustration.

Theoretical Framework (II)

1) Identify relevant regimes in the data

$K = 2$ regimes (high volatility vs. low volatility)

Alternative approaches: Multivariate Clustering using additional variables

Algorithm	Source package (version)	Category
KMeans	scikit-learn (1.7.2)	Clustering
AgglomerativeClustering	scikit-learn (1.7.2)	Clustering
DBSCAN	scikit-learn (1.7.2)	Clustering
SpectralClustering	scikit-learn (1.7.2)	Clustering
MeanShift	scikit-learn (1.7.2)	Clustering
GaussianMixture	scikit-learn (1.7.2)	Clustering
Birch	scikit-learn (1.7.2)	Clustering
AffinityPropagation	scikit-learn (1.7.2)	Clustering
OPTICS	scikit-learn (1.7.2)	Clustering
MiniBatchKMeans	scikit-learn (1.7.2)	Clustering

Hyperparameter Optimisation

Technique	Source package (version)	Category
GridSearchCV	scikit-learn (1.7.2)	Parameter Optimization

TimeSeriesSplit

CombinatorialPurgedCV

Classical approaches: Univariate econometric modelling (benchmarks)

Markov Regression/Switching

Algorithm	Source package (version)	Category
MarkovRegression	statsmodels (0.14.5)	Regime Switching Regression

$$Pr(I_t = 1|I_{t-1} = 1) = P$$
$$Pr(I_t = 2|I_{t-1} = 1) = 1 - P$$
$$Pr(I_t = 2|I_{t-1} = 2) = Q$$
$$Pr(I_t = 1|I_{t-1} = 2) = 1 - Q$$

Hamilton suggests that Bayesian inference is an unbiased and efficient way to infer regime probabilities

$$Pr(I_{t-1} = 1|\Delta s_{t-1}) = \frac{f(\Delta s_{t-1}|I_{t-1} = 1)p_{1t-1}}{f(\Delta s_{t-1}|I_{t-1} = 1)p_{1t-1} + f(\Delta s_{t-1}|I_{t-1} = 2)(1 - p_{1t-1})}$$

and

$$Pr(I_{t-1} = 2|\Delta s_{t-1}) = \frac{f(\Delta s_{t-1}|I_{t-1} = 2)(1 - p_{1t-1})}{f(\Delta s_{t-1}|I_{t-1} = 1)p_{1t-1} + f(\Delta s_{t-1}|I_{t-1} = 2)(1 - p_{1t-1})}$$

where p_{1t-1} and $p_{2t-1} = 1 - p_{1t-1}$ are called prior probabilities

Using the posteriors we can calculate an expectation of the next periods regime probability as

$$p_{1t} = P\ Pr(I_{t-1} = 1|\Delta s_{t-1}) + (1 - Q)\ Pr(I_{t-1} = 2|\Delta s_{t-1})$$

or

$$p_{1t} = P\left[\frac{f_{1t-1}p_{1t-1}}{f_{1t-1}p_{1t-1} + f_{2t-1}(1 - p_{1t-1})}\right] + (1 - Q)\left[\frac{f_{2t-1}(1 - p_{1t-1})}{f_{1t-1}p_{1t-1} + f_{2t-1}(1 - p_{1t-1})}\right]$$

and

$$p_{2t} = 1 - p_{1t}$$

Estimation of the model by maximizing the log-likelihood:

$$L = \sum_{t=1}^{T} log\left[p_{1t}\frac{1}{\sqrt{2\pi h_{1t}}}exp(\Theta_1) + (1 - p_{1t})\frac{1}{\sqrt{2\pi h_{2t}}}exp(\Theta_2)\right]$$

and

$$\Theta_1 = \frac{-(\Delta s_t - \mu_{1t})^2}{2h_{1t}}, \Theta_2 = \frac{-(\Delta s_t - \mu_{2t})^2}{2h_{2t}}$$

$$Regime1 : \mu_{1t} = \alpha_1 + \beta_1(i_{t-1} - i^*_{t-1});\ h_{1t} = \sigma_1^2$$
$$Regime2 : \mu_{2t} = \alpha_2 + \beta_2(i_{t-1} - i^*_{t-1});\ h_{2t} = \sigma_2^2$$

Figure 21: Main theoretical framework for the seminar project analysis, main algorithms used.[25]

[25] Own Illustration based on formulas taken from [Rei25], section "Modeling Nonlinearities I: Markov-Switching", page 7-15 and algorithms from [dev24a] and implemented in [Fou24].

Theoretical Framework (III)

2) Analyze and compare the regimes

Model evaluation and selection

Evaluation metric	Source package (version)	Category	Value range	Interpretation
Silhouette Score	scikit-learn (1.7.2)	Clustering Scores	[-1, 1]	1 being best, -1 being worst, values near 0 indicate overlapping clusters.
RCM	[AB98]	MSM-Score	[0, 100]	0 (perfect regime classification), 100 (failure to detect any regime classification)
Crisis overlap percentage	Own calculations	Combined	[0, 100]	Percentage overlap ranging from 0% - 100%

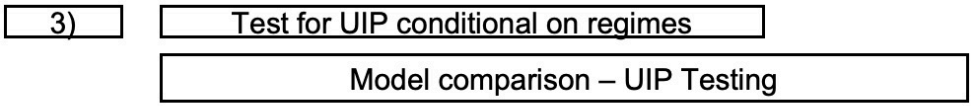
Theoretically affecting crisis periods - major global shocks and US recessions

No.	Start-date	End-date	Event-Type	Event	Source
1	1970-01-01	1970-12-01	US-Recession	US Recession 1970-1970	[St 25b]
2	1971-08-15	1973-03-19	Major Global Crisis	Bretton Woods Breakdown	[Cor+25]
3	1971-08-15	1973-03-19	Major Global Crisis	Nixon Shock	[EF04]
4	1973-10-17	1974-03-01	Major Global Crisis	Oil Crisis I	[T T23]
5	1973-12-01	1975-04-01	US-Recession	US Recession 1973-1975	[St 25b]
6	1979-01-01	1981-03-01	Major Global Crisis	Oil Crisis II	[T T23]
7	1980-02-01	1980-08-01	US-Recession	US Recession 1980-1980	[St 25b]
8	1981-08-01	1982-12-01	US-Recession	US Recession 1981-1982	[St 25b]
9	1987-10-19	1987-10-19	Major Global Crisis	Black Monday Crash	[EF04]
10	1990-08-01	1991-04-01	US-Recession	US Recession 1990-1991	[St 25b]
11	1997-07-02	1998-12-31	Major Global Crisis	Asian Financial Crisis	[ML23]
12	1998-08-17	1998-09-01	Major Global Crisis	Russian Crisis	[ML23]
13	2000-03-01	2002-10-01	Major Global Crisis	Dot-com Bubble	[MNR25]
14	2001-04-01	2001-12-01	US-Recession	US Recession 2001-2001	[St 25b]
15	2007-08-09	2009-03-09	Major Global Crisis	Global Financial Crisis	[MNR25]
16	2008-01-01	2009-07-01	US-Recession	US Recession 2008-2009	[St 25b]
17	2008-11-25	2014-12-31	Major Global Crisis	US QE	[MNR25]
18	2009-10-01	2012-12-31	Major Global Crisis	European Debt Crisis	[MNR25]
19	2011-08-20	2011-08-05	Major Global Crisis	US Debt Ceiling Crisis	[MNR25]
20	2018-03-22	2020-01-15	Major Global Crisis	US-China Trade War	Various
21	2020-02-20	2021-11-16	Major Global Crisis	COVID-19 Pandemic	[ML23]
22	2020-03-01	2020-05-01	US-Recession	US Recession 2020-2020	[St 25b]
23	2022-02-24	2022-06-01	Major Global Crisis	Russia-Ukraine War	[Cha25]

Figure 22: Main theoretical framework for the seminar project analysis, main evaluation metrics and crisis periods used.[26]

[26] Own Illustration based on data taken from [SL25a], last accessed 24.10.25.

Theoretical Framework (IV)



We may suggest that UIP holds in 'normal' times, but is violated in 'non-normal' times

The sequences of switching between the two states may help us learning what drives the UIP puzzle

Therefore we need two separate mean equations to be estimated

$$Regime1: \mu_{1t} = \alpha_1 + \beta_1(i_{t-1} - i^*_{t-1});\ h_{1t} = \sigma_1^2$$
$$Regime2: \mu_{2t} = \alpha_2 + \beta_2(i_{t-1} - i^*_{t-1});\ h_{2t} = \sigma_2^2$$

Figure 23: Main theoretical framework for the seminar project analysis, testing the regime dependent UIP-relationship.[27]

[27] Own Illustration based on formulas taken from [Rei25], section "Modeling Nonlinearities I: Markov-Switching", page 7-15.

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Modern Energy Commodity Markets
-
The current state

Oil: Global Production and Consumption over time

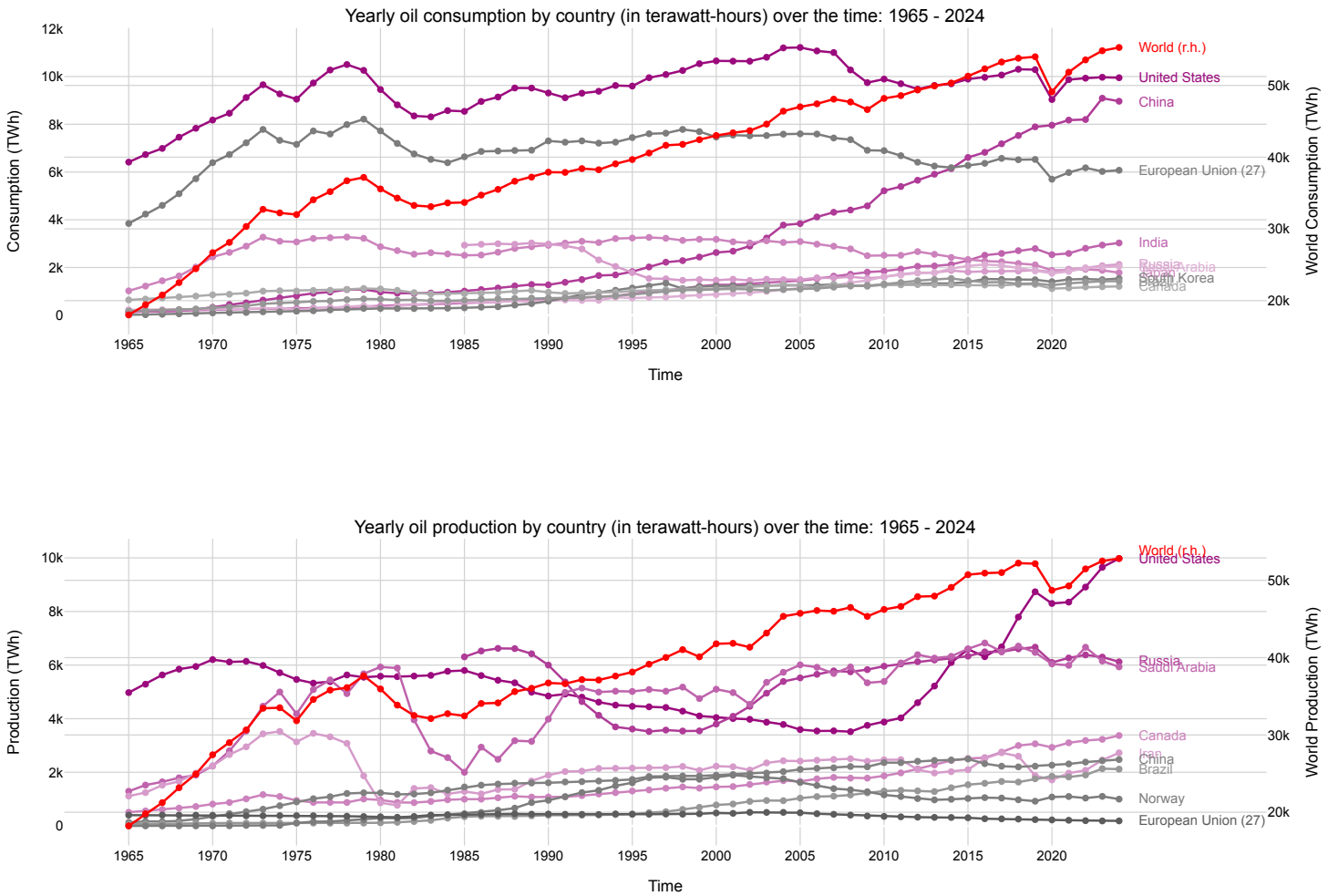

Figure 24: Yearly oil consumption and production by country for the period: 1965 - 2024 (in terawatt-hours).[28]

[28] Own Illustration based on [Haa+09], page 5 and data taken from [Ins25b], last accessed 24.10.25.

Gas: Global Production and Consumption over time

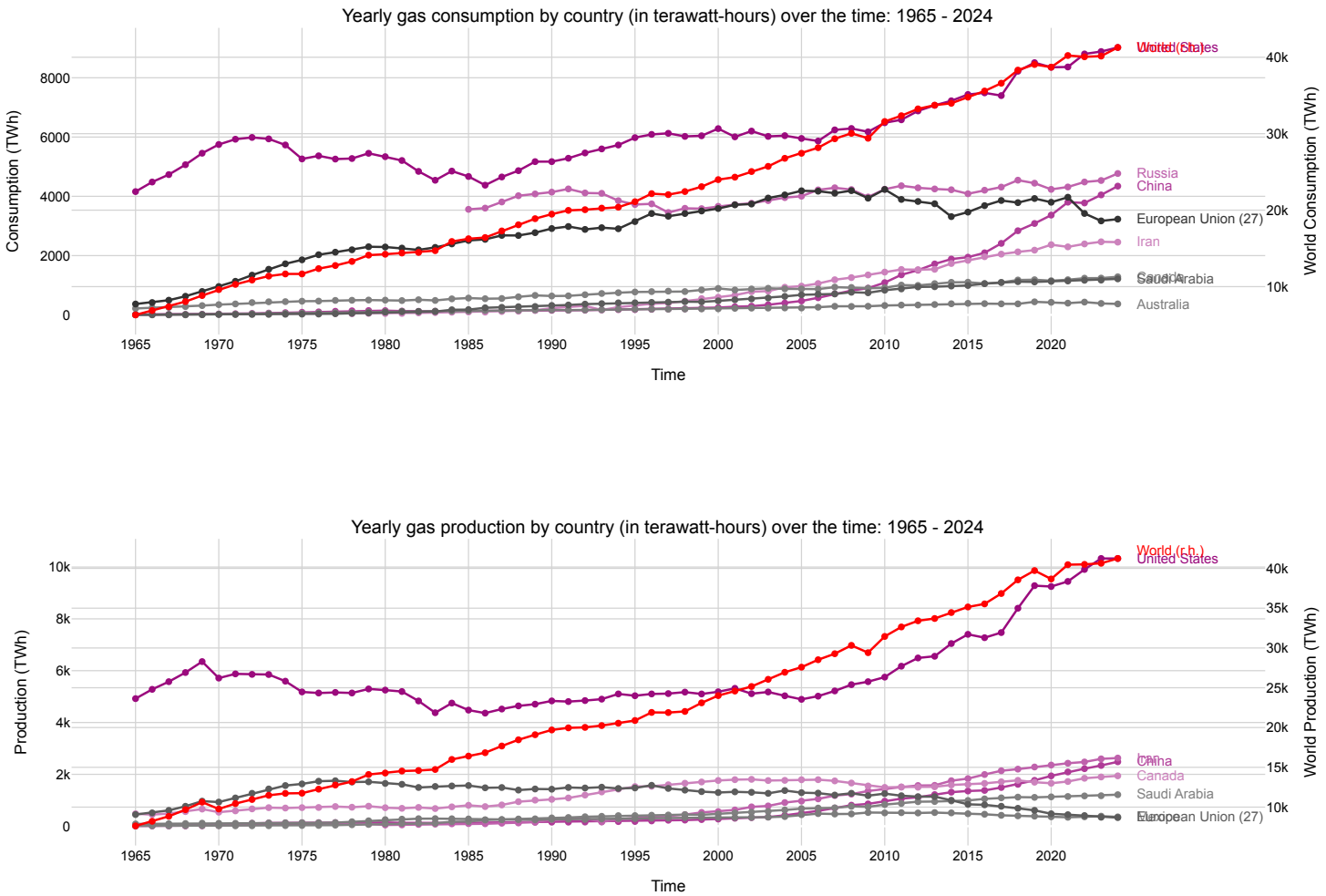

Figure 25: Yearly gas consumption and production by country for the period: 1965 - 2024 (in terawatt-hours).[29]

[29] Own Illustration based on [Haa+09], page 5 and data taken from [Ins25a], last accessed 24.10.25.

Financial Markets: Oil and Gas OI over time

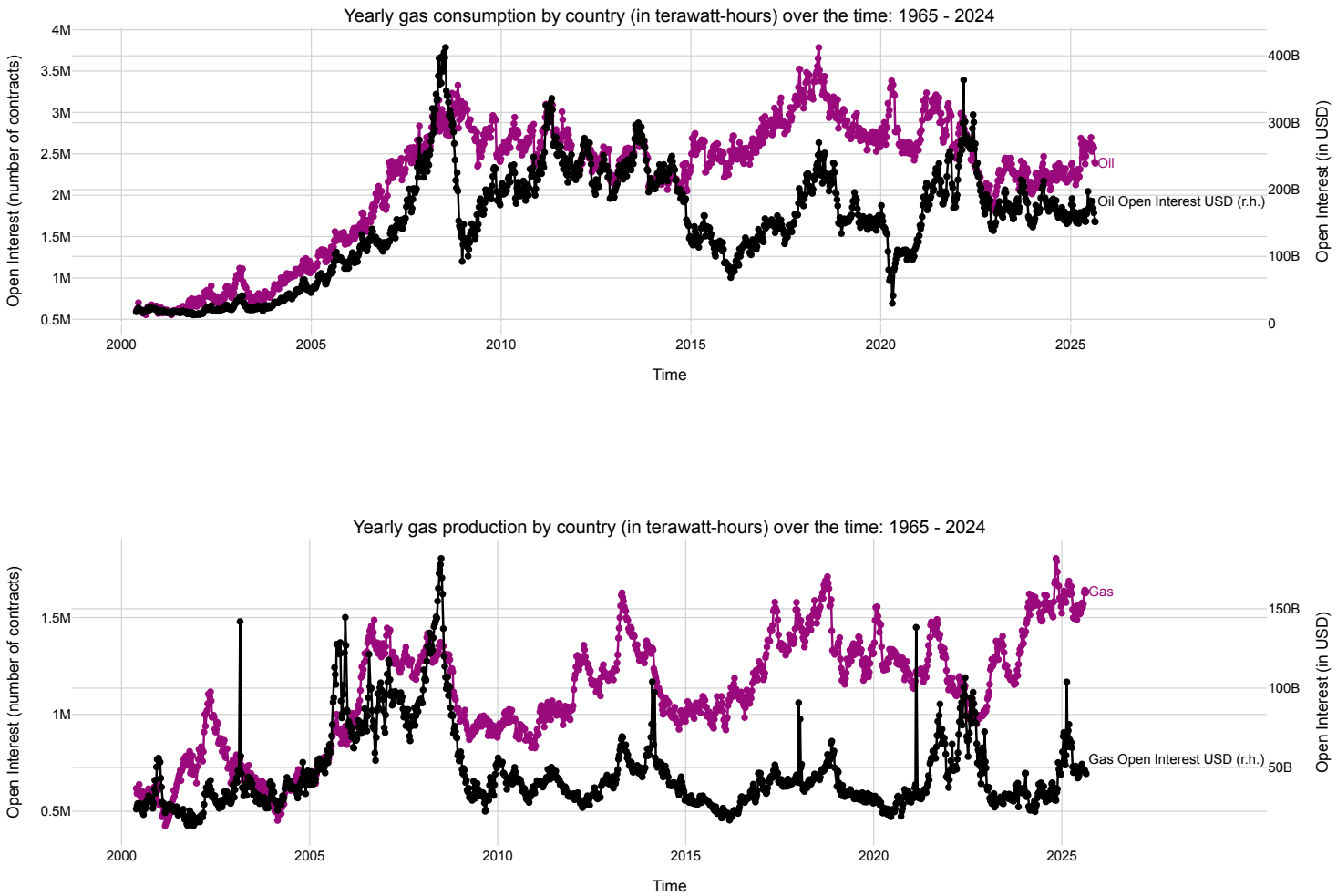

Figure 26: Weekly open interest of oil and gas products for the period: 2000 - 2024[30]

[30] Own Illustration based on [Aus11], page 53 and data taken from [Com25], last accessed 24.10.25.

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Data Characteristics and Stylized Facts

Interest Rate Benchmarks - Absolute Levels

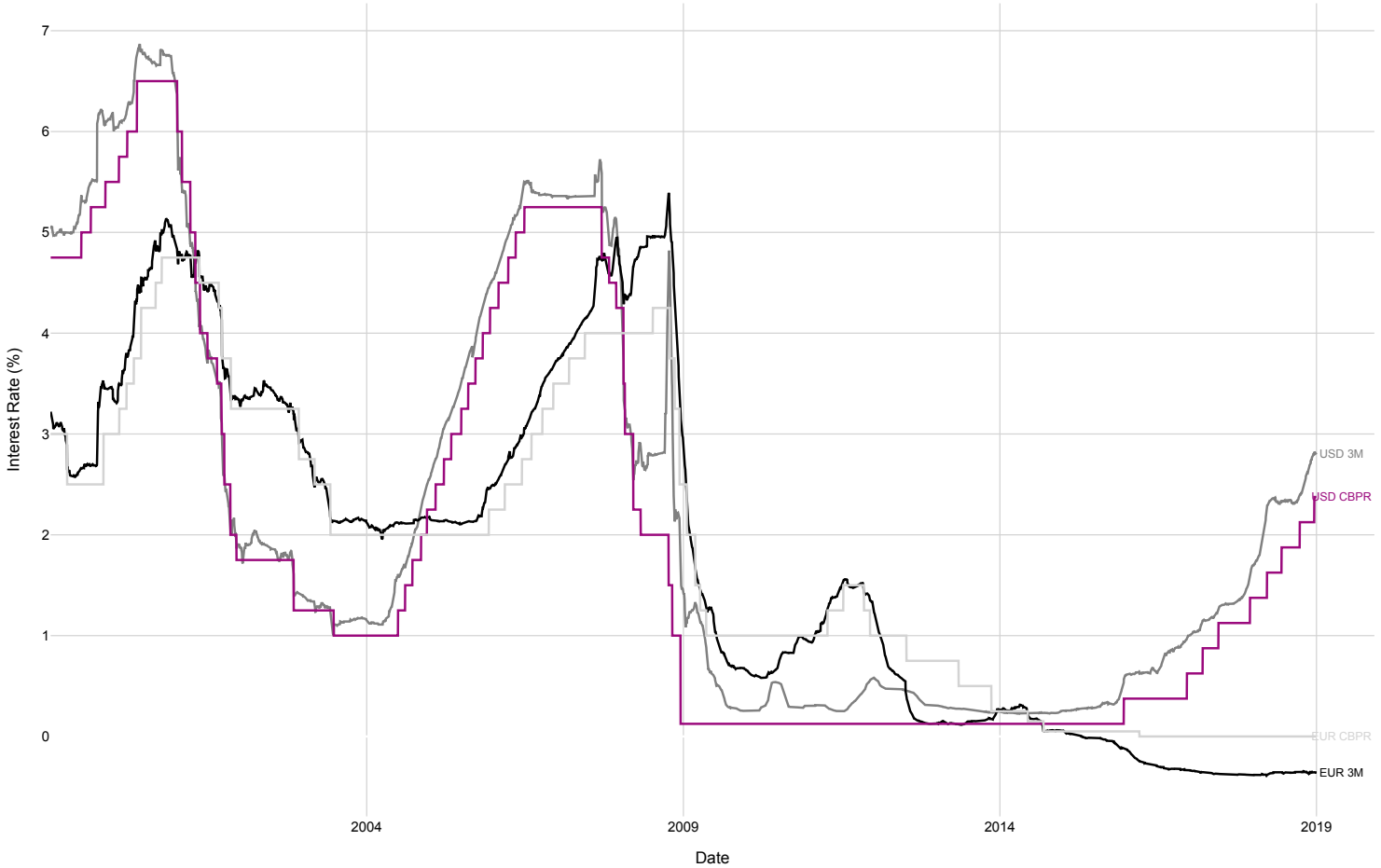

Figure 27: Daily BIS Central Bank Policy Rate and 3M Interbank Rates for the period: 1999 - 2019.[31]

[31] Own Illustration based on [ANT24], Figure 1, page 2 and data taken from [IS25] and [Rei25], last accessed 24.10.25.

Interest Rate Benchmarks - Relative Levels

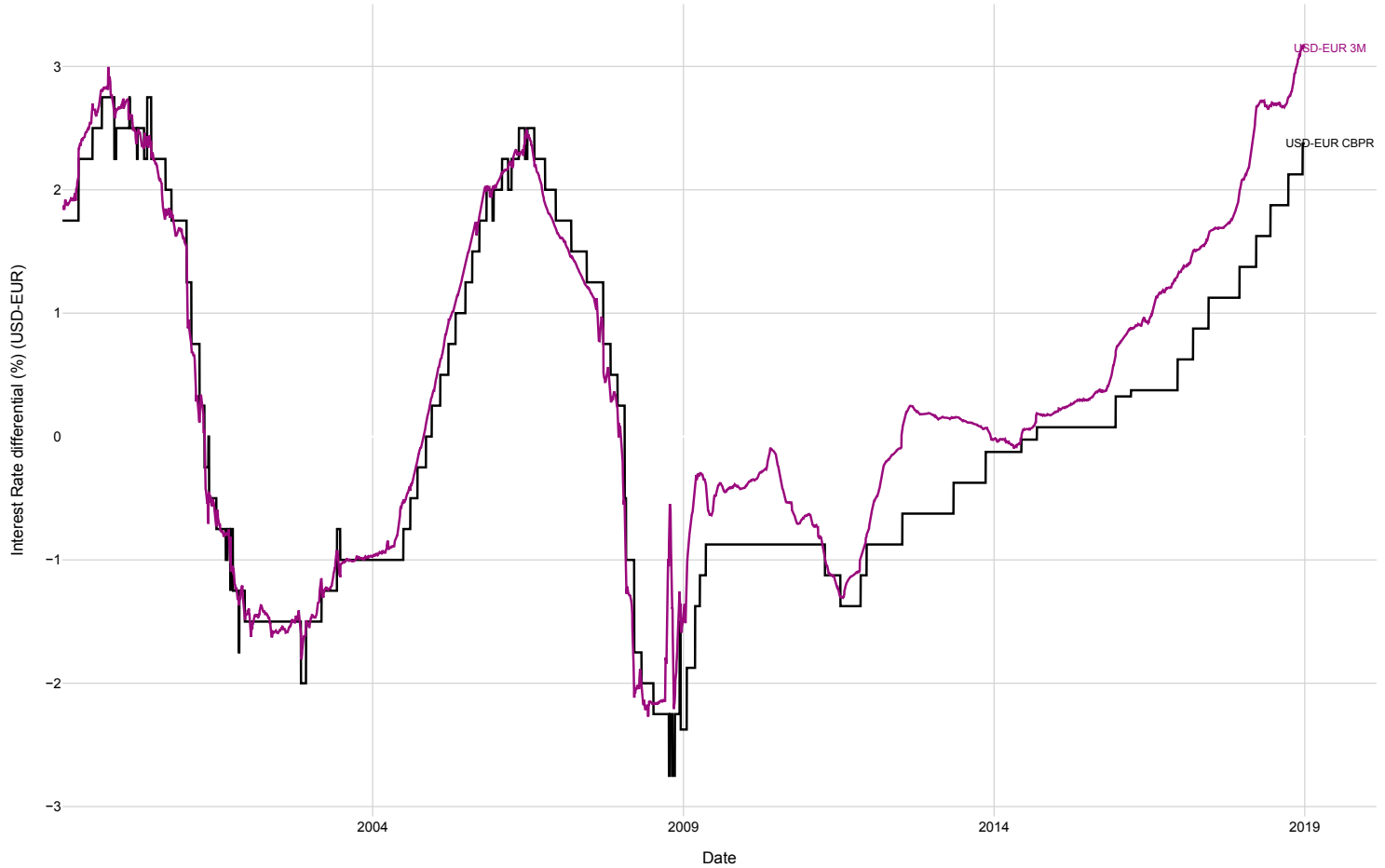

Figure 28: Daily BIS Central Bank Policy Rate and 3M Interbank Rates differentials (USD-EUR) for the period: 1999 - 2019.[32]

[32] Own Illustration based on [ANT24], Figure 1, page 2 and data taken from [IS25] and [Rei25], last accessed 24.10.25.

Main variables distributions (raw data - normalized)

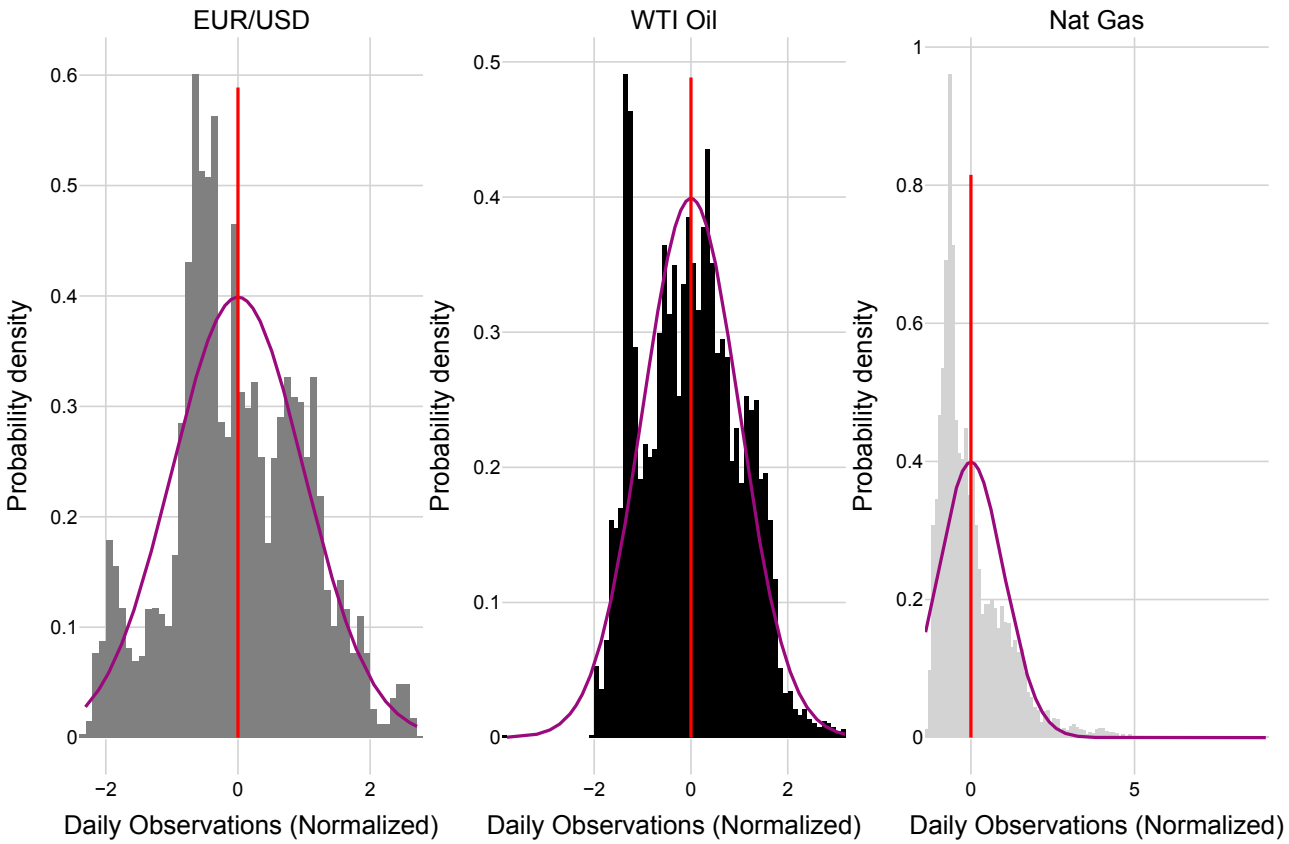

Figure 29: Normalized daily EUR/USD spot exchange rate, oil and gas for the period: 1999 - 2025.[33]

[33] Own Illustration based on [DLS11], page 5 and data taken from [SL25a], last accessed 24.10.25.

Main variables distributions (log first differences)

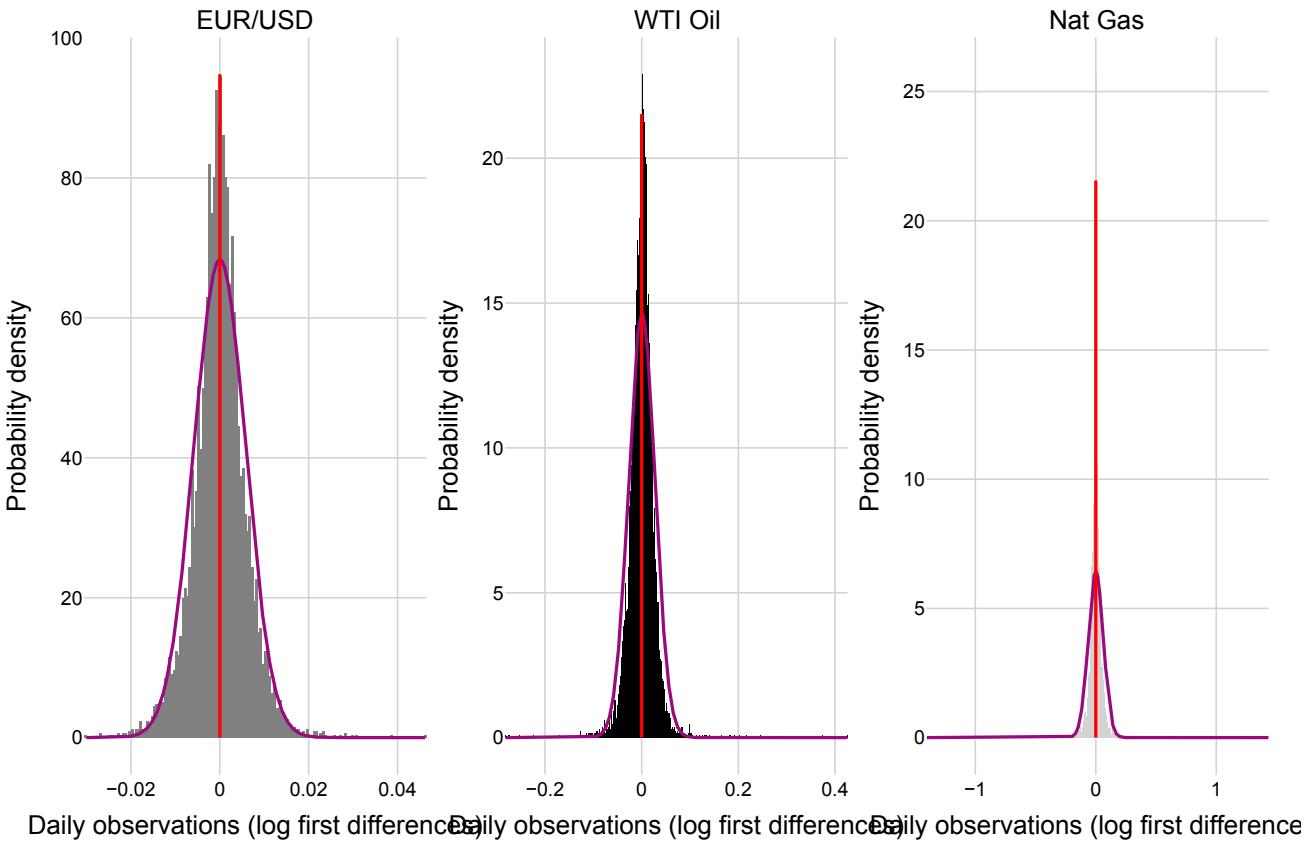

Figure 30: Log first differences of daily EUR/USD spot exchange rate, oil and gas for the period: 1999 - 2025.[34]

[34] Own Illustration based on [DLS11], page 5 and data taken from [SL25a], last accessed 24.10.25.

Tests for Normality (raw data)

Variable	Test	Statistic	p-value	Significance-level	p-value $<$ 0.05	Result
EUR/USD	Shapiro-Wilk	0.989	0.000	0.050	True	Not-Normal
EUR/USD	Kolmogorov-Smirnov	0.799	0.000	0.050	True	Not-Normal
EUR/USD	D'Agostino's K^2	55.280	0.000	0.050	True	Not-Normal
WTI Oil	Shapiro-Wilk	0.981	0.000	0.050	True	Not-Normal
WTI Oil	Kolmogorov-Smirnov	1.000	0.000	0.050	True	Not-Normal
WTI Oil	D'Agostino's K^2	340.085	0.000	0.050	True	Not-Normal
Nat Gas	Shapiro-Wilk	0.8640	0.000	0.050	True	Not-Normal
Nat Gas	Kolmogorov-Smirnov	0.938	0.000	0.050	True	Not-Normal
Nat Gas	D'Agostino's K^2	2130.008	0.000	0.050	True	Not-Normal

Table 1: Shapiro-Wilk, Kolmogorov-Smirnov, D'Agostino's K^2 test for normality for the EUR/USD spot exchange rate, WTI Oil and Natural Gas daily observations for the period: 1999 - 2025.[35]

[35] Own Illustration based on [Küh07], page 24, tests used from: [Vir+25] and data taken from [SL25a], last accessed 24.10.25.

Tests for Normality (log first differences)

Variable	Test	Statistic	p-value	Significance-level	p-value $<$ 0.05	Result
EUR/USD	Shapiro-Wilk	0.978	0.000	0.050	True	Not-Normal
EUR/USD	Kolmogorov-Smirnov	0.490	0.000	0.050	True	Not-Normal
EUR/USD	D'Agostino's K^2	406.730	0.000	0.050	True	Not-Normal
WTI Oil	Shapiro-Wilk	0.870	0.000	0.050	True	Not-Normal
WTI Oil	Kolmogorov-Smirnov	0.464	0.000	0.050	True	Not-Normal
WTI Oil	D'Agostino's K^2	2040.190	0.000	0.050	True	Not-Normal
Nat Gas	Shapiro-Wilk	0.677	0.000	0.050	True	Not-Normal
Nat Gas	Kolmogorov-Smirnov	0.439	0.000	0.050	True	Not-Normal
Nat Gas	D'Agostino's K^2	2698.020	0.000	0.050	True	Not-Normal

Table 2: Shapiro-Wilk, Kolmogorov-Smirnov, D'Agostino's K^2 test for normality for the EUR/USD spot exchange rate, WTI Oil and Natural Gas daily observations for the period: 1999 - 2025.[36]

[36] Own Illustration based on [Küh07], page 24, tests used from: [Vir+25] and data taken from [SL25a], last accessed 24.10.25.

Tests for Stationarity - ADF Tests (raw data)

ADF Statistic	p-value	Start Time	End Time	Regression Type	Observations	Variable	Result
-1.847	0.357	04-01-1999	04-01-1999	c	6640	EUR/USD	Non-Stationary
-1.846	0.682	04-01-1999	04-01-1999	ct	6640	EUR/USD	Non-Stationary
-2.655	0.480	04-01-1999	04-01-1999	ctt	6640	EUR/USD	Non-Stationary
-0.254	0.594	04-01-1999	04-01-1999	n	6640	EUR/USD	Non-Stationary
-2.789	0.059	04-01-1999	04-01-1999	c	6624	WTI Oil	Non-Stationary
-2.770	0.208	04-01-1999	04-01-1999	ct	6624	WTI Oil	Non-Stationary
-3.060	0.267	04-01-1999	04-01-1999	ctt	6624	WTI Oil	Non-Stationary
-0.697	0.413	04-01-1999	04-01-1999	n	6624	WTI Oil	Non-Stationary
-4.341	0.000	04-01-1999	04-01-1999	c	6633	Nat Gas	Stationary
-4.743	0.001	04-01-1999	04-01-1999	ct	6633	Nat Gas	Stationary
-4.742	0.003	04-01-1999	04-01-1999	ctt	6633	Nat Gas	Stationary
-1.897	0.055	04-01-1999	04-01-1999	n	6633	Nat Gas	Non-Stationary

Table 3: Augmented-Dickey-Fuller (ADF) test for stationarity in various variants for the EUR/USD spot exchange rate, WTI Oil and Natural Gas daily observations for the period: 1999 - 2025.[37]

[37] Own Illustration based on [Küh07], page 24, tests used from: [SP10] and data taken from [SL25a], last accessed 24.10.25.

Tests for Stationarity - ADF Tests (log first differences)

ADF Statistic	p-value	Start Time	End Time	Regression Type	Observations	Variable	Result
-80.613	0.000	05-01-1999	05-01-1999	c	6637	EUR/USD	Stationary
-80.607	0.000	05-01-1999	05-01-1999	ct	6637	EUR/USD	Stationary
-80.601	0.000	05-01-1999	05-01-1999	ctt	6637	EUR/USD	Stationary
-80.619	0.000	05-01-1999	05-01-1999	n	6637	EUR/USD	Stationary
-14.504	0.000	05-01-1999	05-01-1999	c	6603	WTI Oil	Stationary
-14.525	0.000	05-01-1999	05-01-1999	ct	6603	WTI Oil	Stationary
-14.547	0.000	05-01-1999	05-01-1999	ctt	6603	WTI Oil	Stationary
-14.463	0.000	05-01-1999	05-01-1999	n	6603	WTI Oil	Stationary
-20.094	0.000	05-01-1999	05-01-1999	c	6616	Nat Gas	Stationary
-20.102	0.000	05-01-1999	05-01-1999	ct	6616	Nat Gas	Stationary
-20.121	0.000	05-01-1999	05-01-1999	ctt	6616	Nat Gas	Stationary
-20.095	0.000	05-01-1999	05-01-1999	n	6616	Nat Gas	Stationary

Table 4: Augmented-Dickey-Fuller (ADF) test for stationarity in various variants for the EUR/USD spot exchange rate, WTI Oil and Natural Gas daily observations (log first differences) for the period: 1999 - 2025.[38]

[38] Own Illustration based on [Küh07], page 24, tests used from: [SP10] and data taken from [SL25a], last accessed 24.10.25.

Tests for Cointegration (raw data)

Cointegration Score	p-value	Start Time	End Time	Observations	Trend	Variable X	Variable Y	Result
-2.967	0.118	04-01-1999	01-10-2025	6641	c	EUR/USD	WTI Oil	Not Cointegrated
-3.364	0.134	04-01-1999	01-10-2025	6641	ct	EUR/USD	WTI Oil	Not Cointegrated
-3.635	0.167	04-01-1999	01-10-2025	6641	ctt	EUR/USD	WTI Oil	Not Cointegrated
-3.268	0.013	04-01-1999	01-10-2025	6641	n	EUR/USD	WTI Oil	Cointegrated
-2.416	0.317	04-01-1999	01-10-2025	6641	c	EUR/USD	Nat Gas	Not Cointegrated
-2.634	0.446	04-01-1999	01-10-2025	6641	ct	EUR/USD	Nat Gas	Not Cointegrated
-3.530	0.204	04-01-1999	01-10-2025	6641	ctt	EUR/USD	Nat Gas	Not Cointegrated
-4.182	0.001	04-01-1999	01-10-2025	6641	n	EUR/USD	Nat Gas	Cointegrated

Table 5: Engle and Granger Cointegration test for the EUR/USD spot exchange rate, WTI Oil and Natural Gas daily observations for the period: 1999 - 2025.[39]

[39] Own Illustration based on [Küh07], page 24, tests used from: [SP10] and data taken from [SL25a], last accessed 24.10.25.

Tests for Cointegration (log differences)

Cointegration Score	p-value	Start Time	End Time	Observations	Trend	Variable X	Variable Y	Result
-4.293	0.003	17-02-1999	01-10-2025	6609	c	EUR/USD	WTI Oil	Cointegrated
-4.805	0.002	17-02-1999	01-10-2025	6609	ct	EUR/USD	WTI Oil	Cointegrated
-4.830	0.006	17-02-1999	01-10-2025	6609	ctt	EUR/USD	WTI Oil	Cointegrated
-5.724	0.000	17-02-1999	01-10-2025	6609	n	EUR/USD	WTI Oil	Cointegrated
-4.001	0.007	17-02-1999	01-10-2025	6609	c	EUR/USD	Nat Gas	Cointegrated
-4.348	0.009	17-02-1999	01-10-2025	6609	ct	EUR/USD	Nat Gas	Cointegrated
-4.341	0.030	17-02-1999	01-10-2025	6609	ctt	EUR/USD	Nat Gas	Cointegrated
-4.246	0.000	17-02-1999	01-10-2025	6609	n	EUR/USD	Nat Gas	Cointegrated

Table 6: Engle and Granger Cointegration test for the EUR/USD spot exchange rate, WTI Oil and Natural Gas daily observations (log first differences) for the period: 1999 - 2025.[40]

[40] Own Illustration based on [Küh07], page 24, tests used from: [SP10] and data taken from [SL25a], last accessed 24.10.25.

Tests for Autocorrelation (raw data)

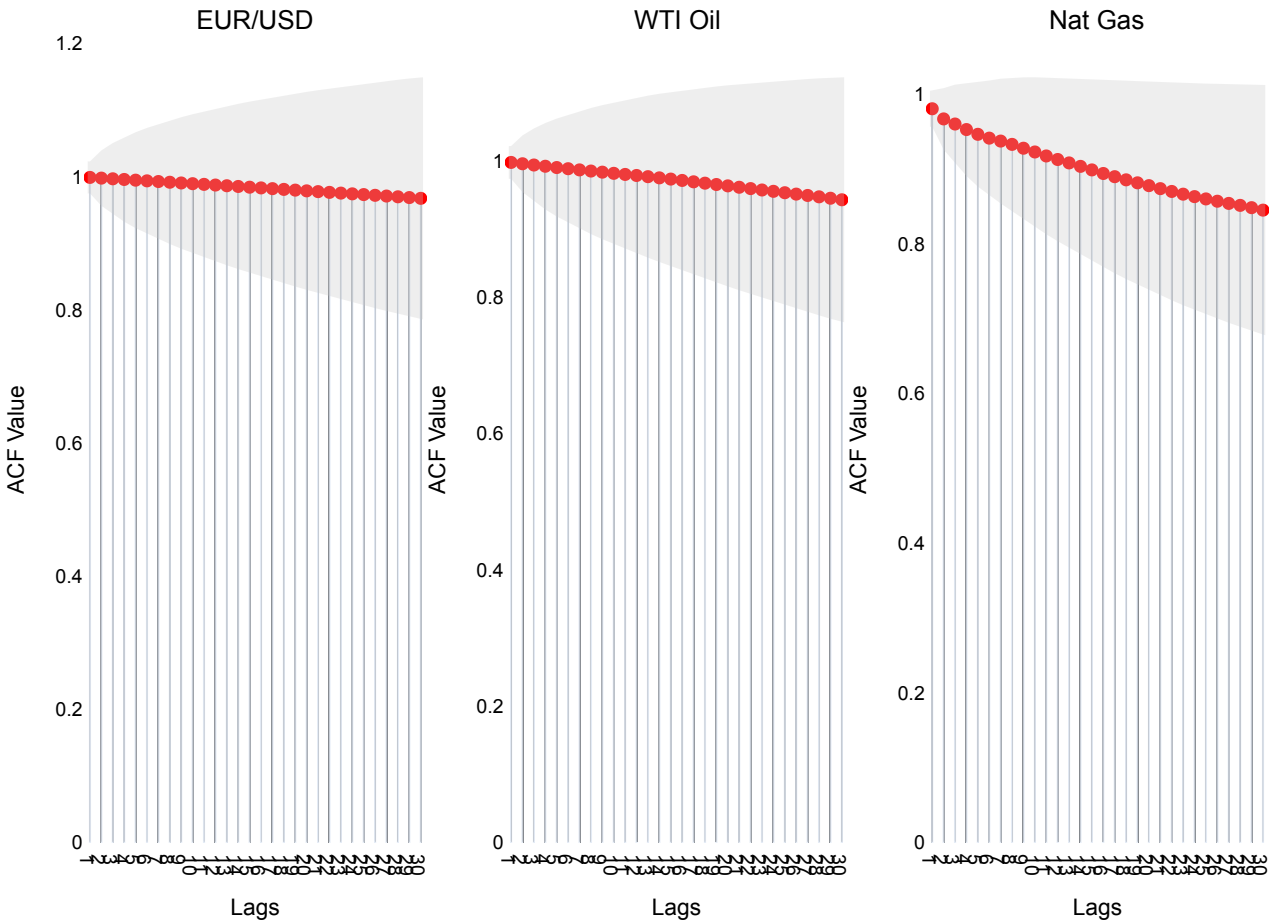

Figure 31: ACF values for daily observations of the EUR/USD spot exchange rate, oil and gas for the period: 1999 - 2025.[41]

[41] Own Illustration based on [SSN19], page 10, tests used from: [SP10] and data taken from [SL25a], last accessed 24.10.25.

Tests for Autocorrelation (log first differences)

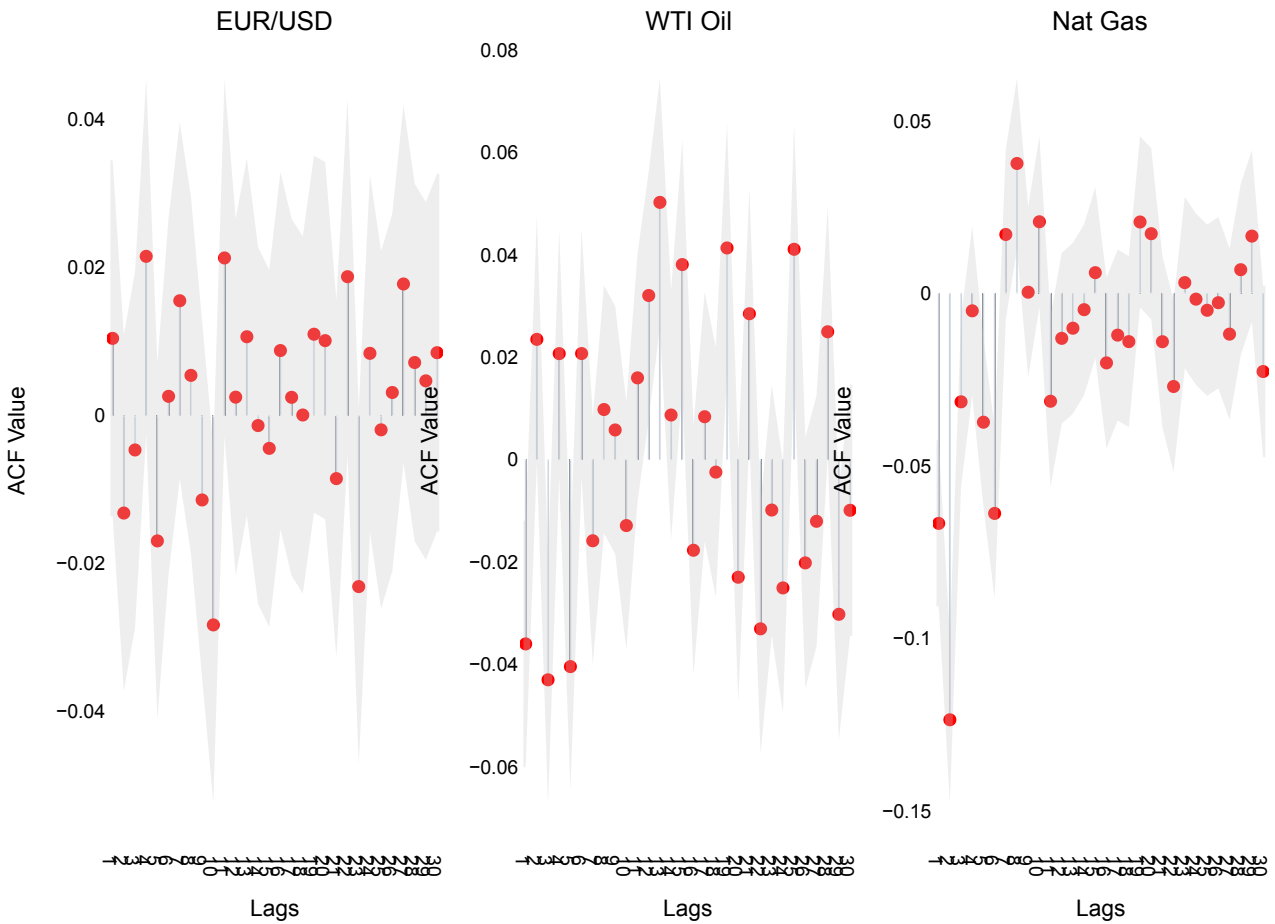

Figure 32: ACF values for daily observations (log first differences) of the EUR/USD spot exchange rate, WTI Oil and Natural Gas for the period: 1999 - 2025.[42]

[42] Own Illustration based on [SSN19], page 10, tests used from: [SP10] and data taken from [SL25a], last accessed 24.10.25.

Tests for Partial Autocorrelation (raw data)

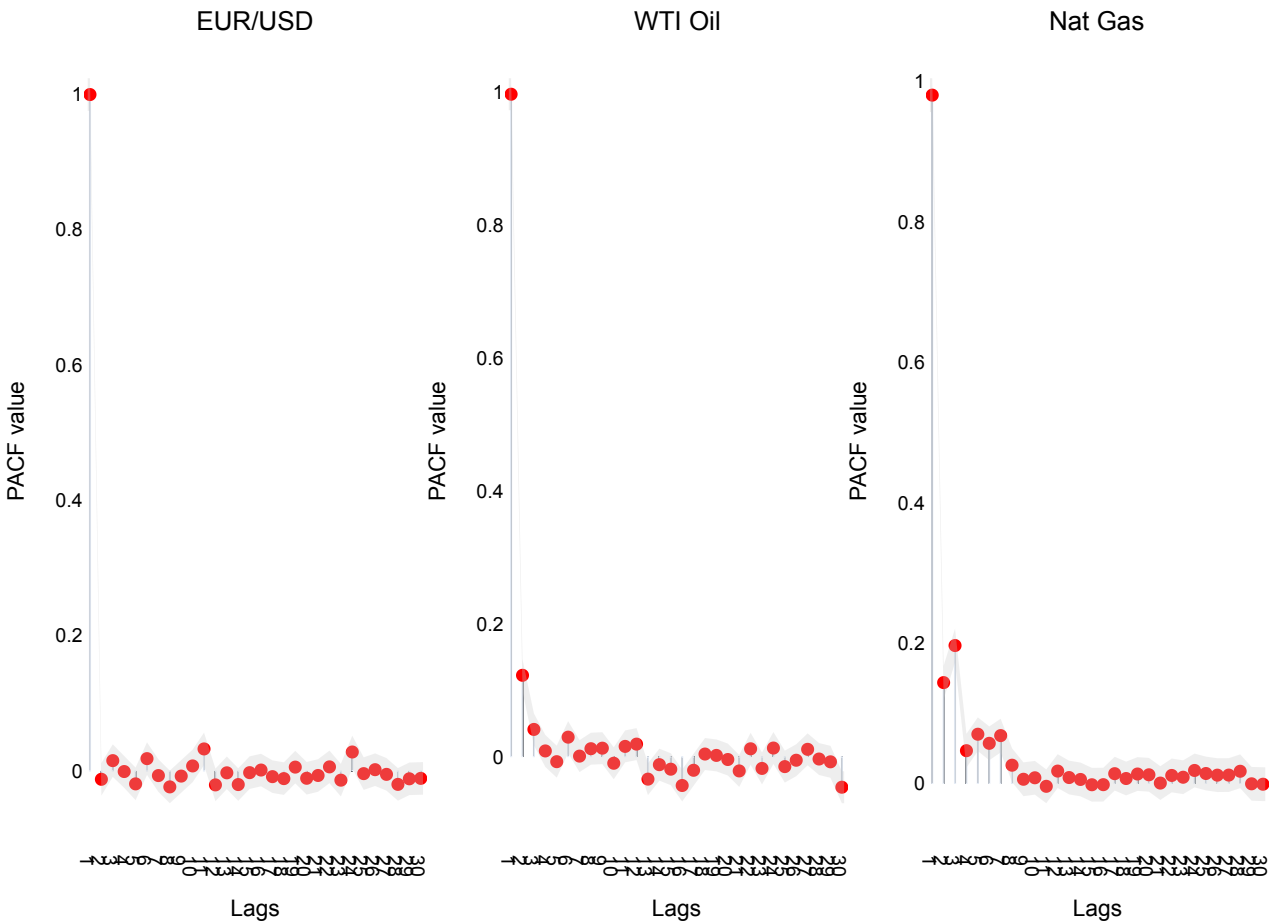

Figure 33: PACF values for daily observations of the EUR/USD spot exchange rate, WTI Oil and Natural Gas for the period: 1999 - 2025.[43]

[43] Own Illustration based on [SSN19], page 10, tests used from: [SP10] and data taken from [SL25a], last accessed 24.10.25.

Tests for Partial Autocorrelation (log first differences)

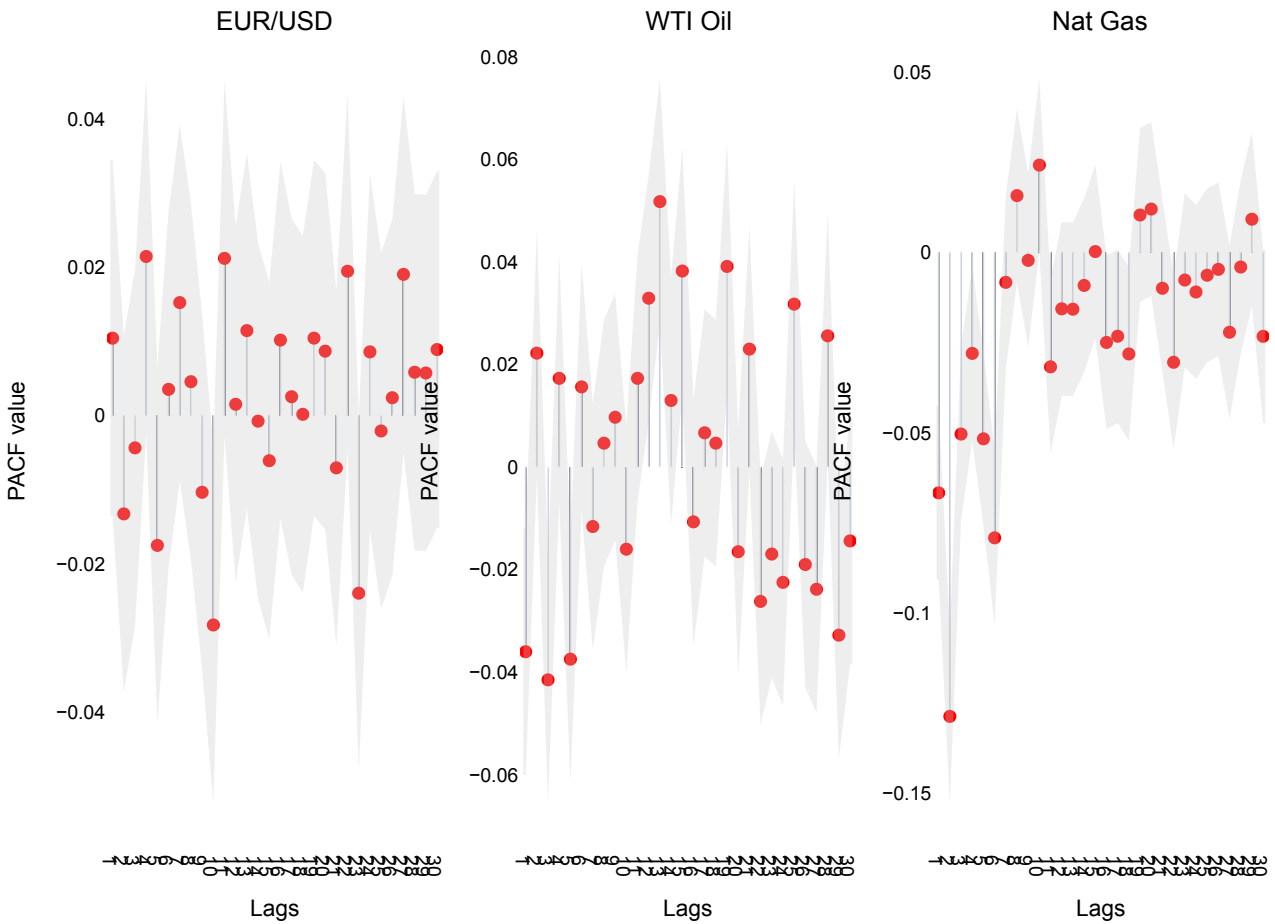

Figure 34: PACF values for daily observations (log first differences) of the EUR/USD spot exchange rate, WTI Oil and Natural Gas for the period: 1999 - 2025.[44]

[44] Own Illustration based on [SSN19], page 10, tests used from: [SP10] and data taken from [SL25a], last accessed 24.10.25.

Granger Causality Tests - EUR/USD and oil (raw data)

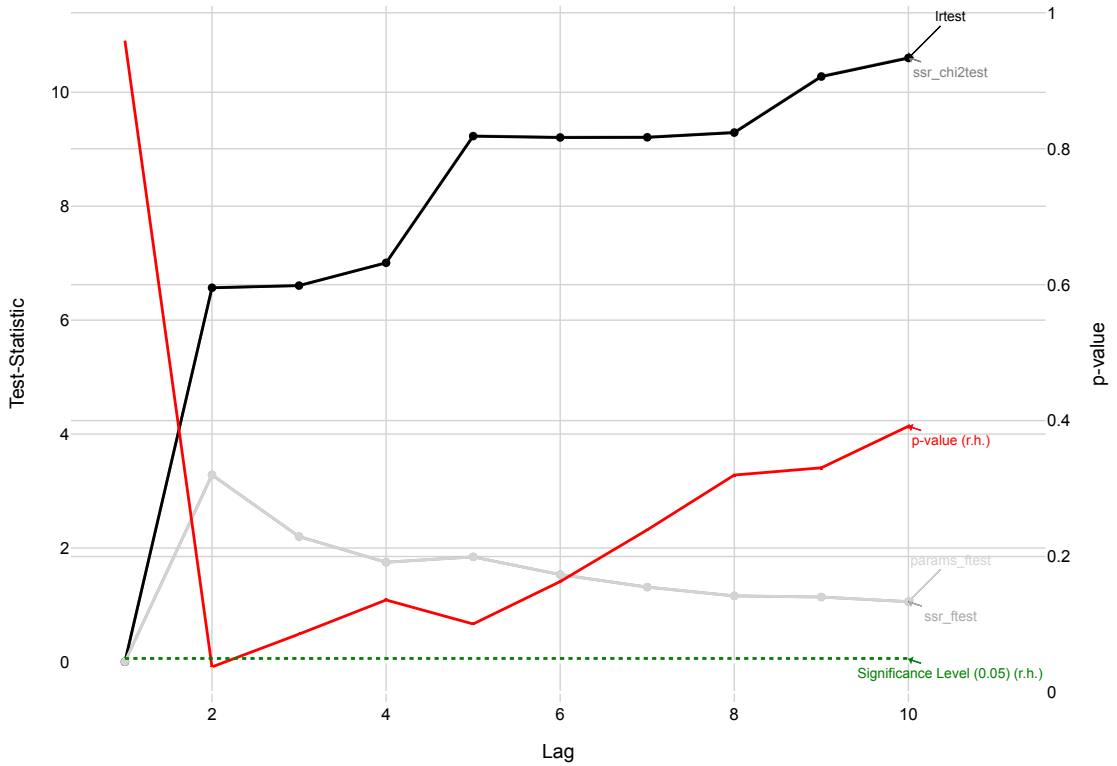

Figure 35: Granger causality test results testing Granger causality of daily observations of oil for EUR/USD spot exchange rate for the period: 1999 - 2025.[45]

[45] Own Illustration, tests used from: [SP10] and data taken from [SL25a], last accessed 24.10.25.

Granger Causality Tests - EUR/USD and gas (raw data)

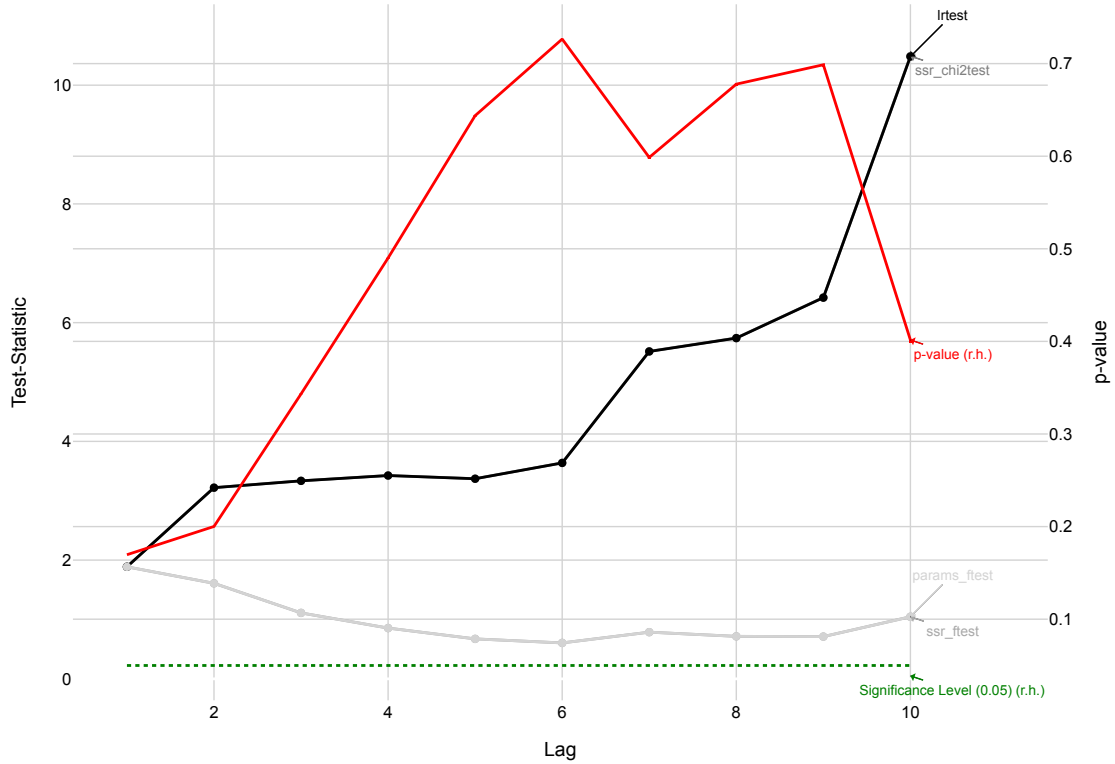

Figure 36: Granger causality test results testing Granger causality of daily observations of gas for EUR/USD for the period: 1999 - 2025.[46]

[46] Own Illustration, tests used from: [SP10] and data taken from [SL25a], last accessed 24.10.25.

Granger Causality Tests - EUR/USD and oil (log first differences)

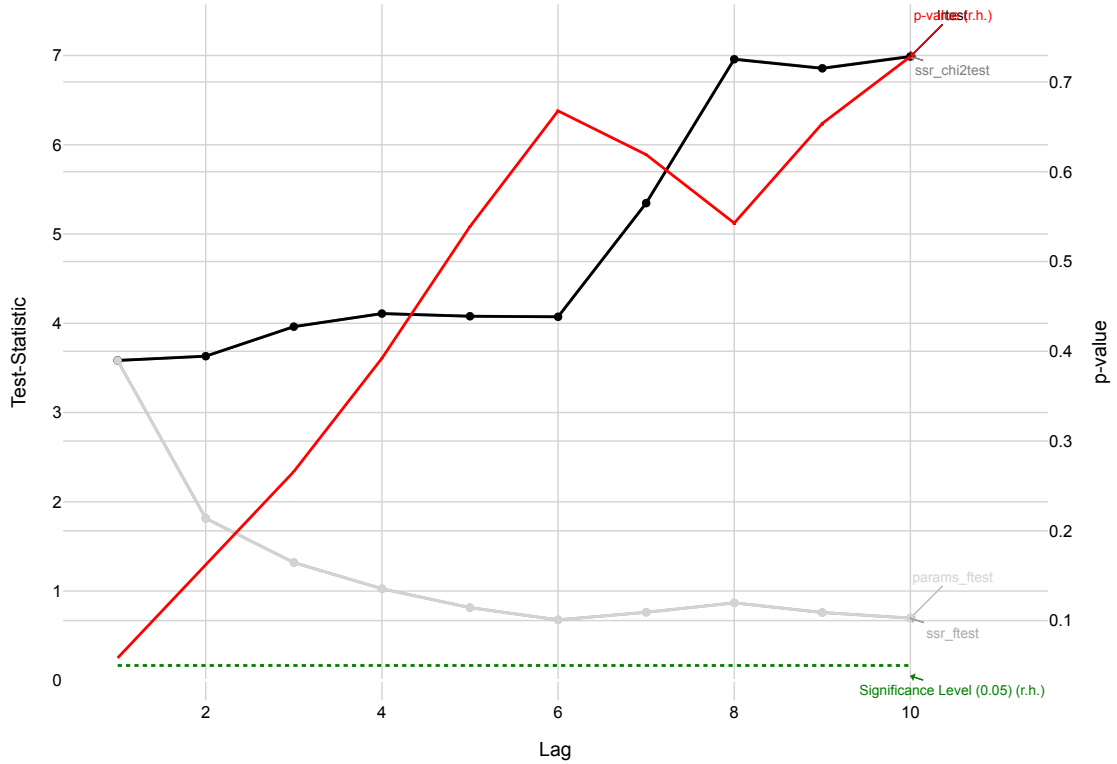

Figure 37: Granger causality test results testing Granger causality of daily observations (log first differences) of WTI Oil for EUR/USD for the period: 1999 - 2025.[47]

[47] Own Illustration, tests used from: [SP10] and data taken from [SL25a], last accessed 24.10.25.

Granger Causality Tests - EUR/USD and gas (log first differences)

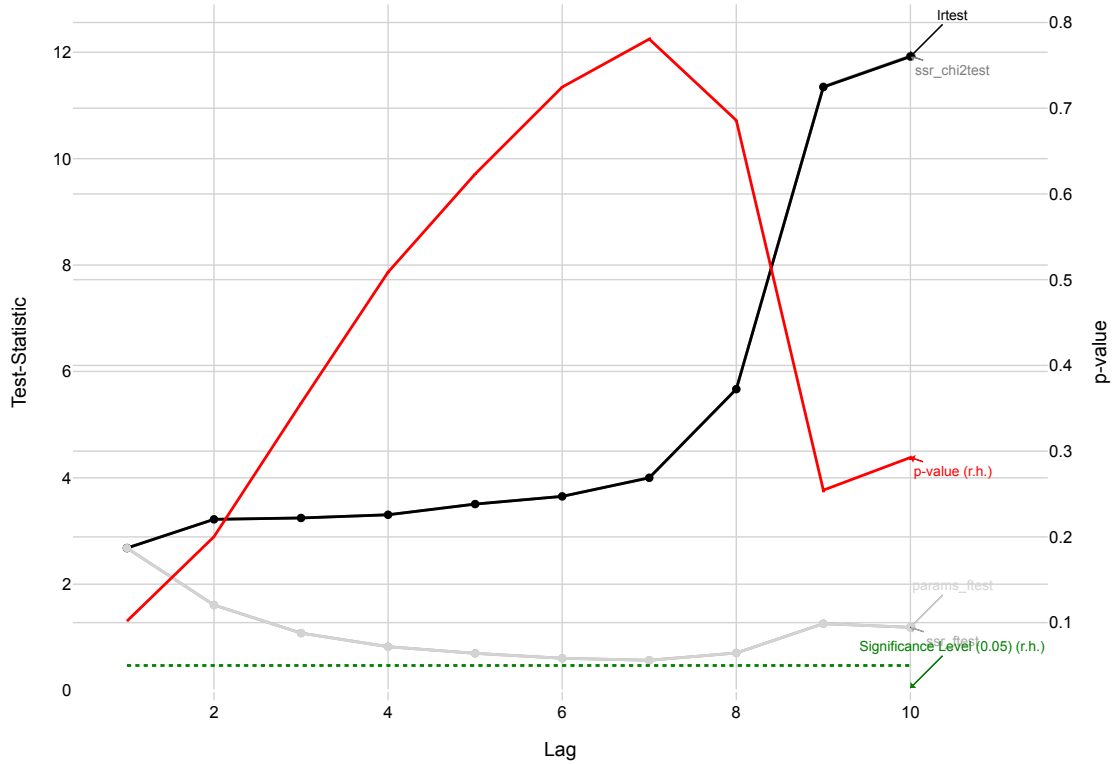

Figure 38: Granger causality test results testing Granger causality of daily observations (log first differences) of Natural Gas for EUR/USD for the period: 1999 - 2025.[48]

[48] Own Illustration, tests used from: [SP10] and data taken from [SL25a], last accessed 24.10.25.

Appendix - Figures and Tables

No.	Exchange Rate	Basket	Data Frequency	Data Availability	Exchange Rate Type	Bilateral/Multilateral	Source	Data ID	Link
1	Real effective exchange rate	broad	monthly	1994-01	REER	Multilateral	[IS25]	M.R.B.US	Link
2	Real effective exchange rate	narrow	monthly	1964-01	REER	Multilateral	[IS25]	M.R.N.US	Link
3	Nominal effective exchange rate	broad	monthly	1994-01	NEER	Multilateral	[IS25]	M.N.B.US	Link
4	Nominal effective exchange rate	narrow	monthly	1964-01	NEER	Multilateral	[IS25]	M.N.N.US	Link
5	Nominal effective exchange rate	narrow	daily	1983-10-03	NEER	Multilateral	[IS25]	D.N.N.US	Link
6	Nominal effective exchange rate	broad	daily	1996-04-11	NEER	Multilateral	[IS25]	D.N.B.US	Link
7	USD-EUR Spot Rate	-	daily	1999-01-04	NER	Bilateral	[SL25a]	DEXUSEU	Link
8	Nominal Broad U.S. Dollar Index	broad	daily	2006-01-02	NEER	Multilateral	[SL25a]	DTWEXBGS	Link
9	Real Broad Dollar Index	broad	monthly	2006-01-01	REER	Multilateral	[SL25a]	RTWEXBGS	Link

Table 7: Various exemplary exchange rate data sources for the USD.[49]

[49] Own illustration based on Table 1, page 3, [Ban21], last accessed 24.10.2025.

Appendix - Definitions and Data

Appendix - Definitions

Appendix - Definitions

PPP Deviation Calculation

$$\text{PPP Deviation} = \log(REER_{country} - REER_{USD}) \tag{1}$$

Source: [ECoD25], data taken from [SL25a], last accessed 24.10.25.

CPI Component Distribution Calculation

$$\text{CPI Component Distribution} = \frac{\text{CPI}_{component}}{\text{CPI}_{total}} \times 100 \tag{2}$$

Source: Data taken from [SL25a] and [Ban25], last accessed 24.10.25. Historical weighs can be taken from [Dat25].

Appendix - Definitions

Rolling Volatility Calculation

$$\text{Rolling Volatility} = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(x_t - \bar{x})^2} \tag{3}$$

Source: Data taken from [SL25a], last accessed 24.10.25.

Appendix - Definitions

	Event	Start-date	End-date	Event-Type	Source
1	US Recession 1970-1970	1970-01-01	1970-12-01	US-Recession	[SL25b]
2	Bretton Woods Breakdown	1971-08-15	1973-03-19	Major Global Crisis	[Cor+25]
3	Nixon Shock	1971-08-15	1973-03-19	Major Global Crisis	[EF04]
4	Oil Crisis I	1973-10-17	1974-03-01	Major Global Crisis	[Tra23]
5	US Recession 1973-1975	1973-12-01	1975-04-01	US-Recession	[SL25b]
6	Oil Crisis II	1979-01-01	1981-03-01	Major Global Crisis	[Tra23]
7	US Recession 1980-1980	1980-02-01	1980-08-01	US-Recession	[SL25b]
8	US Recession 1981-1982	1981-08-01	1982-12-01	US-Recession	[SL25b]
9	Black Monday Crash	1987-10-19	1987-10-19	Major Global Crisis	[EF04]
10	US Recession 1990-1991	1990-08-01	1991-04-01	US-Recession	[SL25b]
11	Asian Financial Crisis	1997-07-02	1998-12-31	Major Global Crisis	[ML23]
12	Russian Crisis	1998-08-17	1998-09-01	Major Global Crisis	[ML23]
13	Dot-com Bubble	2000-03-01	2002-10-01	Major Global Crisis	[MNR25]
14	US Recession 2001-2001	2001-04-01	2001-12-01	US-Recession	[SL25b]
15	Global Financial Crisis	2007-08-09	2009-03-09	Major Global Crisis	[MNR25]
16	US Recession 2008-2009	2008-01-01	2009-07-01	US-Recession	[SL25b]
17	US QE	2008-11-25	2014-12-31	Major Global Crisis	[MNR25]
18	European Debt Crisis	2009-10-01	2012-12-31	Major Global Crisis	[MNR25]
19	US Debt Ceiling Crisis	2011-08-20	2011-08-05	Major Global Crisis	[MNR25]
20	US-China Trade War	2018-03-22	2020-01-15	Major Global Crisis	[HR25]
21	COVID-19 Pandemic	2020-02-20	2021-11-16	Major Global Crisis	[ML23]
22	US Recession 2020-2020	2020-03-01	2020-05-01	US-Recession	[SL25b]
23	Russia-Ukraine War	2022-02-24	2022-06-01	Major Global Crisis	[Cha25]

Table 8: Major global crisis periods (theoretical regimes).[50]

[50] Own illustration based on Table 1, [Löy23], and data taken from [SL25b], last accessed 24.10.25.

Appendix - Definitions

Nominal effective exchange rate (NEER) (Slide 13):

$$\text{NEER} = \frac{1}{N} \sum_{i=1}^{N} \text{NER}_i \tag{4}$$

Source: Definition taken from [IS25].

Real effective exchange rate (REER) (Slide 13):

$$\text{REER} = \text{NEER} \times \frac{\text{CPI}_{domestic}}{\text{CPI}_{foreign}} \tag{5}$$

Source: Definition taken from [IS25].

Appendix - Definitions

Nominal exchange rate (NER) (Slide 13):

$$\text{NER} = \frac{\text{Units of Domestic Currency}}{\text{Units of Foreign Currency}} \tag{6}$$

Source: Definition taken from [CZR18], page 5.

Real exchange rate (RER) (Slide 13):

$$\text{RER} = \text{NER} \times \frac{\text{CPI}_{domestic}}{\text{CPI}_{foreign}} \tag{7}$$

Source: Definition taken from [CZR18], page 5.

Clustering Metrics: Silhouette Score (Slide 17):

$$\text{Silhouette Score} = \frac{b - a}{\max(a, b)} \tag{8}$$

Source: Definition taken from [dev24b].

Appendix - Definitions

Standard UIP relationship - log terms:

$$\triangle s_{t+1} = i_t - i_t^* + u_{t+1} \tag{9}$$

Source: Definition taken from [Rei25], section "Testing UIP conditions", page 7.

Calculation of the RCM (Slide 17):

$$\text{RCM} = 100\, S^2 \left(1 - \frac{1}{N} \sum_{t=1}^{N} \left[g_t p_t + (1 - g_t)(1 - p_t) \right] \right) \tag{10}$$

with $g_t = \mathbf{1}\{p_t \geq \tau\}$, p_t the smoothed probability at time t, N the sample size, and τ the classification threshold (e.g. 0.5).
Source: Definition taken from [AB98].

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Further Material for Illustrations - Questions

Why the silhouette score?

The silhouette score is a measure of how similar an object is to its own cluster compared to other clusters. It provides a way to assess the quality of clustering results.

$$\text{Silhouette Score} = \frac{b - a}{\max(a, b)} \tag{11}$$

where:

- a is the average distance between a data point and all other points in the same cluster.
- b is the average distance between a data point and all points in the nearest cluster.
- **Range:** The silhouette score ranges from -1 to 1. A score close to 1 indicates that the data point is well clustered, while a score close to -1 suggests that it may be in the wrong cluster.
- **Interpretation:** A high silhouette score indicates that the clusters are well separated and distinct, while a low score suggests overlapping or poorly defined clusters.
- **Advantages:** It provides a clear and interpretable metric for evaluating clustering performance, allowing for easy comparison between different clustering algorithms or parameter settings.
- **Disadvantages:** The silhouette score may not perform well with clusters of varying densities or non-convex shapes, and it can be sensitive to noise and outliers in the data.
- **Alternatives:** Other clustering evaluation metrics include the Davies-Bouldin index, the Calinski-Harabasz index, and the Dunn index, each with its own strengths and weaknesses.

What are other clustering evaluation metrics?

- **Davies–Bouldin Index:**

$$\mathrm{DB} = \frac{1}{K} \sum_{i=1}^{K} \max_{j \neq i} \frac{s_i + s_j}{d(c_i, c_j)}$$

where s_i is the average intra-cluster distance for cluster i and $d(c_i, c_j)$ is the distance between centroids.

- **Calinski–Harabasz Index:**

$$\mathrm{CH} = \frac{\mathrm{tr}(B_K)/(K-1)}{\mathrm{tr}(W_K)/(n-K)}$$

with $\mathrm{tr}(B_K)$ the between-cluster dispersion trace and $\mathrm{tr}(W_K)$ the within-cluster dispersion trace.

- **Dunn Index:**

$$\mathrm{Dunn} = \frac{\min_{i \neq j} \delta(C_i, C_j)}{\max_k \mathrm{diam}(C_k)}$$

where $\delta(C_i, C_j)$ is inter-cluster distance and $\mathrm{diam}(C_k)$ the diameter of cluster k.

- **Adjusted Rand Index (ARI):**

$$\mathrm{ARI} = \frac{\sum_{ij} \binom{n_{ij}}{2} - \frac{\sum_i \binom{a_i}{2} \sum_j \binom{b_j}{2}}{\binom{n}{2}}}{\frac{1}{2}\left(\sum_i \binom{a_i}{2} + \sum_j \binom{b_j}{2}\right) - \frac{\sum_i \binom{a_i}{2} \sum_j \binom{b_j}{2}}{\binom{n}{2}}}$$

with n_{ij} cell counts of the contingency table, $a_i = \sum_j n_{ij}$, $b_j = \sum_i n_{ij}$.

- **Normalized Mutual Information (NMI):**

$$\mathrm{NMI} = \frac{I(U; V)}{\sqrt{H(U)\, H(V)}}$$

where $I(U; V)$ is mutual information and $H(\cdot)$ are entropies (alternative normalisations like the arithmetic mean of entropies are also used).

Why exactly Volatility?

- Exchange rates often exhibit **volatility clustering**: high volatility periods followed by similar periods.
- Capturing these clusters helps identify different **regimes** in the market with distinct risk-return profiles.
- Regime detection aids in modeling and forecasting exchange rate dynamics better.
- Clustering methods group time periods with similar volatility patterns to discover these regimes.

Why Clustering Algorithms?

- Used clustering for unsupervised regime detection without prespecified labels.
- Gaussian Mixture Model (GMM) chosen for its ability to model overlapping regimes and handle mixed distributions.
- Hyperparameter tuning optimizes number of clusters and covariance structure for best fit.
- Silhouette Score used for evaluating cluster cohesion and separation.
- Limitations include sensitivity to initial parameters and possible overfitting.

Why including Commodity Variables in Regime Detection?

- Inclusion of WTI crude oil, natural gas prices, and their volatilities leverages the commodity-export-import link to EUR/USD.
- Commodity variables capture economic fundamentals related to inflation, trade balances, and macro stability.
- Results show improvement in regime identification quality, suggesting commodity influence on exchange rate volatility regimes.
- Reflects structural changes in markets driven by global commodity price shifts.

Why the CBPR and not the real 3M Interbank rates?

Using Reuters it was hard to identify the correct, needed 3-month interbank lending rates series for the countries involved. Especially the LIBOR or now the SOFR semed to not be accessible with the current university license. Also the EURIBOR seemed to not be available.

- The CBPR rates seemed to at least roughly proxy the behavior of the 3M interbank rates.
- The CBPR rates are not ‘tradable‘ in that sense or directly market-inferred, what makes them less robust/reliable in mirroring the real market states.
- The big advantage of using the CBPR rates is that they are freely available, easy to query, transparent in their calculation and cover all the needed currencies on a long daily historic frequency.