

This can be construed as a linear programming problem. Let a , b , c , and d be the hours allotted to factories A, B, C, and D.

Minimize:

$$a + b + c + d$$

subject to:

$$10a + 10b + 10c + 10d \geq 3000$$

$$10a + 20b + 20d \geq 5000$$

$$20a + 40c + 40d \geq 6000$$

$$a, b, c, d \geq 0$$

We convert this to standard form by adding surplus variables s_i :

Minimize:

$$a + b + c + d$$

subject to:

$$10a + 10b + 10c + 10d - s_1 = 3000$$

$$10a + 20b + 20d - s_2 = 5000$$

$$20a + 40c + 40d - s_3 = 6000$$

$$a, b, c, d \geq 0$$

$$s_1, s_2, s_3 \geq 0$$

To use the simplex method, create the initial tableau. Note we will maximize the negation of the original objection function:

$$Z = -(a + b + c + d)$$

$$Z + a + b + c + d = 0$$

$$\begin{bmatrix} a & b & c & d & s_1 & s_2 & s_3 & Z & \\ 10 & 10 & 10 & 10 & -1 & 0 & 0 & 0 & 3000 \\ 10 & 20 & 0 & 20 & 0 & -1 & 0 & 0 & 5000 \\ 20 & 0 & 40 & 40 & 0 & 0 & -1 & 0 & 6000 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

I'm too lazy to actually run the algorithm, so I put it into a solver. The results are here: <https://tinyurl.com/wb7x3wb2>

Looks like my solution to run factory D for 300 hours is as good as their solution to run B for 150, C for 50, and D for 100. However, the products will be done in 150 hours since the factories may run in parallel.