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Assignment 2 – September 29th, 2023

CS 5350 – Advanced Algorithms

1. The Fibonacci sequence can be defined by and . Binet Proposed a closed formula for the Fibonacci sequence:

Show that Binet’s formula is correct, i.e., that .

1. It’s easy to see that using and generates the same sequence.
   1. Thus,
2. The Fibonacci sequence is a linear recurrence with constant coefficients equations of degree 2.
   1. Thus, it can be written as , where and .
3. A solution to the recurrence relation is when is a root of the polynomial as shown below:
   1. .
   2. Using the quadratic formula, we have .
      1. Quadratic formula: .
      2. Substitution: .
4. Because the characteristic roots are distinct real solutions, we can write a general solution as where are real constants.
5. Solve for and using initial conditions:
   1. When :
   2. When :
   3. Therefore, and
6. Substituting the values of and in equation.
   1. Multiply numerator and denominator by -1: .
7. Therefore, .
8. The Toom-Cook Multiplication Algorithm splits the two input integers a and b of size n into three parts each.

combines the six parts through with operations, obtaining intermediate values through and though , each of which has size executes 5 recursive calls to compute five products through and finally combines these five products through in such a way to obtain the complete product , using operations.

1. Using the Master Theorem, show that the complexity of the Toom-Cook Multiplication Algorithm is .
   1. The relevant points for time complexity analysis from the Toom-Cook Multiplication Algorithm are:
      1. Size decreases by a factor of 3.
      2. Five recursive calls are performed with the new size.
      3. Division and merge time take time.
   2. Thus, the recurrence equation is .
   3. Show that case 1 of the master theorem applies to this recurrence equation.
      1. Let and .
      2. Substitute :
   4. Therefore, .
2. Determine whether the Toom-Cook or the Karatsuba Algorithm is faster.
   1. Toom-Cook Algorithm: .
   2. Karatsuba Algorithm: .
   3. Therefore, the Toom-Cook multiplication algorithm is faster than the Karatsuba multiplication algorithm.
3. Determine whether the Toom-Cook or the Schoolbook Multiplication Algorithm is faster.
   1. Toom-Cook Algorithm: .
   2. Schoolbook Multiplication Algorithm: .
   3. Therefore, the Toom-Cook multiplication algorithm is faster than the schoolbook multiplication algorithm.