Q: Prove case 3 of Master Theorem.

Lemma 4.2 states:

Let and be constants and let be a function defined over real numbers . Then the recurrence has solution .

Lemma 4.3 states:

Let and be constants and let be function defined over real numbers . Then the asymptotic behavior of the function , defined for , can be characterized as follows:

Case 3: If there exists a constant in the range s.t. , then .

Proof of case 3:

1. Let , , and .
2. Let and .
3. We can see that because is a term of .
4. Choose s.t. .
5. Show that . Proof by induction.
   1. Base case: Let .
   2. Inductive case: Assume is true.
      1. .
6. Now show that .
   1. Use : .
   2. Extract from the sum:
   3. Increasing the sigma upper bound would make the right side bigger, holding the inequality.
   4. Thus: .
   5. The infinite geometric series states that when .
   6. Therefore, we can use it to get: .
   7. Because is a constant, we have .
7. Therefore, because then .

Master theorem states:

Let and be constants and let be deriving function that is defined and nonnegative on all sufficiently large reals. Define the algorithmic recurrence on the positive real numbers by . Then the asymptotic behavior of can be characterized as follows:

Case3: If there exist a constant s.t. , and if additionally satisfies the regularity condition for some constant and all sufficiently large , then .

Proof of case 3:

1. Let , be constants, where and is satisfied for some .
2. By lemma 4.2 we know that if , the equation has solution .
3. Then, by lemmas 4.3 we know that .
4. Thus, .
5. Using , we have that .
6. Thus, .
7. Therefore, .