

1/Read/Maria

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Maria and the Math Fairies: Origins and Algebra

by

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Chapter 1. On the way to the Math Olympiad

Maria skipped beside her big brother Guillermo, making high narrow bounces to keep from getting ahead of him as he walked. This did not consume much of Maria's energy, so she kept talking continuously as their family entered the train station in Madrid.

"and did you know that 128 doubled is 256? I bet I can calculate doubles just as fast as anyone else in the Math Olympiad, what do you think? And I bet there will be children from all over Spain. They say a boy from Barcelona won it last year, and before that a girl from Andalusia, what do you think?"

In fact her brother thought very little about these things, because he was trying to keep her from bumping into people in

the crowd.

At the platform, train number 21435 had just closed its doors.

"We will wait for the next train," said Maria's father.

Maria skipped along the platform away from her family, and did math problems in time to her skipping. "Eight times twelve is ninety-six, left-left, fourteen times fourteen is 1-9-6, right-right."

Her parents proudly watched her skip to the end of the platform. Her big brother also watched her, just because he was used to looking after her.

The train slid into the dark tunnel at the end of the platform. As the last car was passing Maria, a light within it flashed. Maria had time to turn and look at the train. That is impossible, but that is what happened. Then the blast blew the windows and Maria against the brick walls of the station. Her family saw her slide down the wall.

All of us change a little bit each moment. We are happier, or sadder, or wiser, or have forgotten something small, but by the time Maria hit the platform she had forgotten quite a lot.

#

Six months later, Maria sits in her wheelchair at her old desk and goes through her old papers. It is hard for her to

understand what they mean.

"Put Maria's paper where she will find it!" whispers the grandmother fairy to the fairy boy.

The deep, melting chocolate sound of her brother playing the cello fills the room as Maria leans forward in her wheelchair to look through the papers.

"Mom, what is this strange paper?" asks Maria slowly, holding up one she has just selected.

"It is just another paper that you wrote before you were hurt."

"Mom, how could I write so many numbers and strange symbols, when now I can barely read?"

"Maria, you used to love numbers and arithmetic. You would think for hours about math. You would practice and practice. Always my clever little girl!"

"Can you read it to me?" asks Maria thoughtfully.

Maria's mother walks behind the wheelchair and leans down to study the page.

"It is a table of numbers. I don't know why you wrote it, but at the bottom you wrote:

My law is flawless. It always works. When I understand why, I will be a great mathematician, and smarter even than my brother!

Maria stares at the paper in her hands and as Guillermo plays the most dramatic part of his piece, she asks herself, "What did I mean by my law?"

The boy fairy whispers to his grandmother, "I understand it! May we help her?"

"Not only may we, we must."

The Mysterious Paper:

1	0 & 2	0	1
2	1 & 3	3	1
3	2 & 4	8	1
4	3 & 5	15	1
5	4 & 6	24	1
6	5 & 7	35	1
7	6 & 8	48	1
8	7 & 9	63	1
9	8 & 10	80	1
10	9 & 11	99	1

Chapter 2. The Boy Sledgehammer

A thousand years ago in the city of Zhenjiang in the Eastern part of China, the great polymath Shen Kuo invited his aged cousin, Shen Gun, to visit him at the Dream Brook Estate, his great mansion, to study with him.

"Come in disguise," he wrote, "for my old enemy, Neng Fa, and the Black Dragon Society have been active recently in

Zhenjian. I do not believe he would dare to oppose me openly, because of the favor of the Emperor which I now enjoy, but he might try to harm you."

Shen Gun, who was famed for her curiosity and love of knowledge, hoped this was a chance to study with her famous cousin, even if it was dangerous. So she wrote out a note and gave it to a servant girl, telling her, "You must not lose this note, for on it I have written where I have gone, and that I demanded that you trade clothes with me. You must give me your clothes, and you must put my fine clothes on and go to market. If you are caught wearing my clothes and accused of stealing, this note will get you out of trouble."

Shen Gun set out on foot to the great mansion of Shen Kuo. She carefully bowed to everyone who looked important, and kept her eyes down on the road in front of her. After a long while she thought, "This is an awful way to travel about, not seeing anything or looking anyone in the eye." But she played her part well. Her disguise might have worked perfectly---if she had been the only person to read the note from Shen Kuo.

Just then she heard the murmur of a crowd, and she saw onlookers centered around something on the street corner.

"This is a chance," thought Shen Gun, "to see a street performer, without worrying about them asking me for money! So I

suppose that being a servant has at least one advantage."

Shen Gun worked her way to the front of the crowd that had gathered to see Bao Yung, the Boy Sledgehammer.

"Come generous people, and see the great Bao Yung, boy master of the ancient Kung Fu art of Brick-smashing!" cried Master Li, the man who ran the show. A boy, about eight, stood on a little rug surrounded by brick shards. On either side of him were two stout posts, about one meter apart. A brick balanced on one of these poles. With a great shout, Bao Yung leaped into the air and his leg shot out sideways, smashing the brick into a cloud of dust and brickbats that flew out a few meters to the right.

Shen Gun applauded along with delighted crowd. Two men dressed in black silk worked their way closer to the disguised woman.

Shen Gun was an old woman, and a fever had killed her two daughters and her only grandson. "My grandson would be about this boy's age," thought the old woman.

Bao Yung went to a great chest behind him and retrieved two more bricks, placing one on top of each of the poles, as the man running the show collected money from the crowd.

"Come, generous people, the Boy Sledgehammer must eat to keep up such great strength!" cried Master Li collecting money.

In a dramatic pause in Master Li's patter, Bao Yung leaped into the air and both his feet shot out sideways, simultaneously smashing the two bricks. Bao Yung hooked his arms above his head, mimicking antennae. The clouds of dust from the two destroyed bricks temporarily made Bao Yung look like a great moth with brown wings. As the crowd roared with delight, the two men in black silk moved closer to Shen Gun.

"Now for our finale, we must have a volunteer from the audience, yes, You! Thank you, sir," said Master Li, selecting the richest-looking man from the crowd, and starting to drag him forward."

"No!" shouted Bao Yung.

Master Li and the rich man both looked at him astounded.

"This lady," said the small boy, walking up to Shen Gun and taking her hand, "shall be our volunteer today."

Shen Gun was worried now that she had been recognized, and that the boy would somehow beg her for a great sum of money. She resisted the gentle pull of the boys hand.

"I am sure," said Bao Yung, "that the two gentlemen escorting you will not mind."

Shen Gun was now very confused, and turning to her left to get away from the boy she collided with the hard chest of a man in black silk who was looking directly down at her with steady,

unblinking eyes. She turned to the right, and there was another man in black silk staring at her in that direction, with his hands hidden inside his sleeves.

Seeing this, Shen Gun allowed the boy to drag her in front of the crowd and to the great chest.

"Old woman," said Bao Yung in a loud voice, "you must select two bricks from our box." Then the boy bent over into the chest as if to help her.

But as Shen Gun bent to select the bricks, Bao Yung whispered to her, "I have seen those men in black before. They are thugs from the Black Dragon Society. When two men like that come from nowhere and lurk right behind you in a crowd, your very life is in danger."

"What can we do? Can you defend me with your Kung Fu?" whispered Shen Gun, after she made a great show of handing the first brick to Bao Yung.

"Alas, madam, I do not know any Kung Fu. Master Li has taught me only the art of making bricks which are easy to shatter. Before Master Li bought me, I was a tumbler, and before that a juggler, but I have never been given a chance to truly learn the martial arts."

Chapter 3. Maria Tries to Multiply

The next day, as her brother practices, Maria lies on her bed and racks her brains over a math problem. The boy fairy and his grandmother are watching over her.

"Now, how much is five times five?" she asks herself.

Maria answers herself, "It is five plus five plus five plus five, and five plus five equals ten, and ten plus five equals fifteen, and fifteen plus five equals twenty," and as she works out the complicated math her face feels hot.

The boy fairy, who is named Bao Yung, whispers, "But you need one five more!"

Maria says to herself, "But I need one five more, so five plus twenty is twenty-five, and that is the answer. Fiddlesticks! That takes a lot of time!"

The boy fairy says, "You just have to memorize the multiplication table."

Maria asks Guillermo, "Guillermo, did I once know the whole multiplication table by heart?"

Guillermo stops playing and hangs his cello bow on his pinky, as he does whenever Maria interrupts his practicing. "Oh, yes, and much more! You could multiply three-digit numbers in your head like lightning. In fact you were rather obnoxious about it—"

"Guillermo," Maria interrupts him, "how can I relearn it? Memorizing things is hard for me now, and I can't even read very well!"

Guillermo tilts his head to the left and looks a hundred miles a way, as he always does when he is thinking hard.

The grandmother fairy, who is named Shen Gun, whispers to him, "If you truly love her, find a way to make multiplication a game for her."

Guillermo brightens and says, "I have an idea. We will throw dice, and you will try to multiply the numbers that show face-up. We will do it everyday, when I help you with your physical therapy. I am sure that after a month you will have the whole table by heart. The doctors say you need to exercise your mind as much as possible."

Maria's jaw drops open. "You would do that for me? Thanks! I thought you hated the time that you spend helping me exercise."

Guillermo answers, "Yeah, I hate everything that keeps me from practice, because I am afraid that I will never play as beautifully as Pablo Cassals or Yo-Yo Ma if I don't practice. Still, I love you even more than I love music. I'm sorry that I grumble when I help you exercise your legs; I know that you still can't do it by yourself."

"Oh, you are the best brother anyone could ever have!"

The boy fairy says to his grandmother, "Oh, grandmother, you succeeded very well!"

The grandmother smiles and says, "We will use whomever we can to help her."

Chapter 4. Escape

Master Li grasped Bao Yung quite roughly by the arm.

"Right now," emphasized Master Li, "in a great demonstration of martial art handed down from ancient sages, Bao Yung, the Boy Sledgehammer, will smash these heavy bricks from the shoulders of this frail old woman without hurting her in the least."

Master Li led Shen Gun to between the posts.

Once again, Bao Yung left his script, and surprised Master Li.

"Now I must summon great quantities of Chi!" said the boy, and he walked to the front of the crowd. Turning his back on the crowd and the two tight-lipped and flinty-eyed men who watched angrily, he began chanting nonsense and making strange gestures with his hands. However, Shen Gun saw that whenever his hands passed in front of his body, he beckoned to her with a finger, while his other hand made mystical chi gestures to distract the crowd.

Each time he beckoned, Shen Gun took one step away from the posts and toward the boy. The boy looked very small standing in front of the two thugs, whose faces revealed their anger at this interruption of their sinister plans.

Finally, when Shen Gun was just six feet away, Bao Yung did a handspring to his feet right in front of Shen Gun. With this momentum, he leaped high into the air, forming a tumbling ball with his body right over her head. Doing a half-twist and landing behind her facing the men in black, he bounced up again to her height. His small right fist shot out, smashing the brick into a cloud of dust that flew directly into the face of the startled assassin on their left. The other man charged at the old woman, but the boy jumped again and his left fist sent the brick on her right shoulder into the face of the charging killer.

Chapter 5. The Very Official and Unpleasant Meeting

Maria has to go to a very official meeting at the school.

"To determine what grade you will be in, since you have missed so much school," says her mother.

"To determine how stupid I am," thinks Maria.

Maria wheels herself in behind her mother. There is a lively man behind a desk and a brown man in a chair who looks

like his skin is made of fallen oak leaves.

They say a lot of things that are meant to make Maria and her mother feel better, but don't. Maria thinks about her old friends, some of whom came to see her in the hospital. They are in the ninth grade now, but none of them will be at this school, even if she can trick these old men into believing she belongs in the ninth grade.

"Now Maria," says the lively man, "we just need to ask you a few questions."

"Do you remember who Christopher Columbus was?"

"Yes, of course," lies Maria.

"And what did he do?"

Bao Yung whispers, "He sailed the ocean blue."

Maria says, "He discovered the New World, in 1942."

The lively man smiles and looks straight at Maria.

Maria knows she has done something wrong.

"450 years too high! It was 1492!" whispers the boy fairy.

"I mean 1492. For some reason I always add 450 years to that date," says Maria, surprising herself.

The lively man's eyebrows move up and down and closer together. Maria steals a glance at the wrinkled brown man in the chair. He is watching her the way a cat watches a sparrow.

"Very good, Maria, very good," says the man behind the

desk.

"And what is four times five?"

Maria takes the time to add five to itself four times; it ends in a zero, which is good—she knows it must end in a zero or a five.

"Twenty," says Maria.

"Very good!" says the man at the desk. Maria feels a rush of hope.

"Too slow," says the man in the chair in a dry voice, barely moving his lips, "she is adding."

"Now Maria, what is four times seven?"

Bao Yung starts to say, "It is—" but his grandmother, Shen Gun, stops him.

"We cannot help her here," says the ancient fairy.

"But she needs us!" says the boy.

"She needs us to learn, not to cheat, my dear boy," says the grandmother.

"26!" guesses Maria.

Silence fills up the room, like water filling a glass. When the water gets above her head, Maria starts to cry.

"Mrs. Gomez, we feel that Maria would best be served by being placed in the fifth grade. But don't worry—Mr. Garza is an expert at adjusting his curriculum to match the strengths and

weaknesses of his students."

Maria thinks, "5th Grade? Nine minus five is four. The kids will be four years younger than me." Then she starts to cry harder.

Chapter 6. A Undignified Way to Travel

Bao Yung fled out of the crowd and down the street. Shen Gun followed him as quickly as she could. Seeing that she could not run fast, Bao Yung ducked into a market, where they could thread their way threw a maze of stalls. He ran to a row of hanging duck cages and struck them with his hand to set them quacking, and then reversed direction and ran the other way. Shen Gun followed him to one particular stall.

Behind them, they heard the sound of the men in black crashing through the market and asking where they had gone.

The two short figures crouched as low as they could and scooted into the stall of a fortune teller. The boy spoke in the language *Luoli*¹ to the strange woman there. Shen Gun thought that neither the language he spoke nor the woman and the two men in the stall were Chinese. Without a moment's hesitation, the first man picked a rug off the ground and wrapped Bao Yung in it

1 *Luoli* is the Chinese name for Roma people, usually called Gypsies in English. The Luoli are known to have been in China during the Yuan dynasty, about two centuries after the Song Dynasty. Shen Kuo lived during the Song Dynasty and served as a military general for the Song Emperor Shenzong before he was placed under house arrest.

and put him over his shoulder. The second man picked up a heavy, tall basket containing colorful scarves and clothes and dumped its contents onto the floor, then popped the basket over the old woman. He turned her upside down, frightening the old woman terribly, and slung the basket on to his back as if it were a backpack. No one had ever dared to treat Shen Gun in such a rough manner since she was a little girl, but she knew better than to complain and give herself away.

The two men carried them away as quickly as they could without arousing suspicion. Shen Gun could see a little through the woven bamboo of the basket. When they entered the street and the sounds of the throng died down, the boy, who could see nothing, whispered, "Madam, are you hurt?"

"No," whispered Shen Gun, whose head was beginning to hurt from being upside down for so long, "where are these men taking us?"

"Where would you like to go, madam? This ride is not so comfortable as a rickshaw, but safer, it would seem," said the man who was carrying Shen Gun.

"To the mansion of Shen Kuo," said Shen Gun, "and get the boy their safely also, and you shall have a purse of silver."

"This is not a good day for us to take silver from Shen Kuo or his friends. But I will tell your fortune now--you will

arrive safely behind the walls of Shen Kuo's estate. If this fortune comes true for you, come back in a few weeks and give the silver to our mother."

The men fell silent then, and two men in black silk raced past them.

Chapter 7. Guillermo and the Game

Guillermo helps Maria move her legs and feet up and down and back and forth. Maria throws dice into a tray.

"A six and a four. I should know it. I can add it up, but..."

Guillermo says, "Don't worry, it is twenty-four."

Maria says, "Fishtickles! I can add, and the fives are pretty easy, since anything times a five ends in a five or a zero, but fours are hard and sixes impossible."

Guillermo say, "Don't worry. Your wise old brother foresaw this problem. I can make it even simpler. We can use just the numbers up to four."

"You mean just not count the fives and sixes?"

Guillermo replies, "We could do that, but you would have to re-roll often. No, I mean use a die with only four faces! Technically, this shape is called a tetrahedron."

He hands the plastic shape to Maria.

Maria chuckles, "Hey, they look like little pyramids! But how do you read them? On cubes, the number is the one that is on the face that is up, but these land with the point up!"

"Yes, you're right, the choice is the face that touches the ground, the one that is out of sight. But since you can't see the one that is down, its number is printed at the top on every face, so that the number that is straight up and down and not tilted is the choice the die has made."

Maria turns and twists the die in her hand. "I understand now. And each face is the same, so that whichever falls down is completely random, like on a cube."

Guillermo responds, "Clever little one, we will use two tetrahedrons¹ this week for you to memorize the fours, and when you know the fours by heart, we will use two cubes to work on sixes. After that, we can use two octahedrons that have eight faces to work on sevens and eights. Then dodecahedrons that go up to twelve, and finally icosahedrons that go up to twenty."



"But I don't understand how we will ever be able to do

¹ Most mathematicians prefer to use the word *tetrahedra* to mean more than one tetrahedron.

that!"

Guillermo answers, "Oh, it will be easy. A friend of mine plays a game that uses all of the Platonic solids. They were discovered by the Greeks twenty-three centuries ago."

The boy fairy says, "Never fear, Maria, we will help you, and learning depends more on the heart than on the head!"

Chapter 8. Entering Shen Kuo's estate

The sun was setting as they approached Shen Kuo's walled estate from the rear, after wandering, it seemed to Shen Gun, through every possible part of Zhenjiang. Just as they turned to the West to go around to the main entrance, the man carrying Bao Yung casually said, "They are behind us, brother. Run after we turn this corner!"

The two ran as fast as they could to a certain spot on the wall. Shen Gun, who was still upside down in the basket, was bounced on her head without mercy.

"I think the shrubbery is up there," panted the man carrying Bao Yung.

"Yes, see the marks on the wall?" said the second man.

"We'll throw the boy over the wall first; he is tougher than the old woman, and if he survives, we will have found the privet shrubs, rather than the hard statues of Shen Kuo's

ancestors."

The second man set down the basket and on a count of three they tossed the carpet containing Bao Yung over the wall.

Bao Yung wriggled free of the carpet and found himself about five feet off the ground, in a dense, well-manicured shrub.

"I'm alive!" called out Bao Yung.

The two men in black were running toward them down the long wall of the estate now, with silver daggers in their hands. The men took no time to explain the situation to Shen Gun, but lifted the basket by both ends and began swinging.

As they shouted "Three!", Bao Yung heard the old woman scream and saw the basket fly over the wall to land right on top of him, pushing him further down into the soft shrub.

Shen Gun never found out what happened to the two men who saved her life, for when she went back later to pay them, their stall was gone. No one would say where they had gone, and Shen Gun never knew if the street people conspired to hide the family, or if in truth they had left without telling anyone where they were going.

A dozen of Shen Kuo's gardeners, cooks, servants, and bodyguards had now come and surrounded the shrub, brandishing broomsticks, mops, and swords. Shen Kuo stood among them.

Chapter 9. Square Numbers

A few weeks later, Maria is throwing the little mountains, which show a three and a three, and thinks, "Three times three equals---oh, fudgebuckets! Since my accident, I am the only girl in the world who can't memorize anything at all!"

The boy fairy, Bao Yung, says "Be brave! Billions of souls live and die who never even get a chance to learn how to multiply."

Maria throws the dice again and says, "Look --- four times four, the biggest that I have to learn now. I have already thrown it many times. How to memorize it, how to get it by heart? I can add it up, one after the other, but even if I get the correct answer, I won't have got it by heart."

Bao Yung says, "Four times four and three times three are special, aren't they?"

Maria, throwing the dice again, says, "Well, it will be easier to remember the simpler product of a number times itself, because there are only four: one times one, two times two, three times three, and finally four times four."

Bao Yung says, "And one times one is trivial¹, is it not?"

Maria thinks, "Guillermo told me that there is a law

¹ Mathematicians often say "That's *trivial*," when they mean "That's easy" or "That's really simple". They sometimes say it when it isn't easy at all.

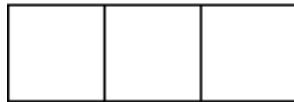
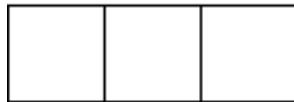
stronger than the laws of the police: one times any number at all is just that number. So of course, one times one is just one."

Bao Yung says, "Two times two seems easy, but is it?"

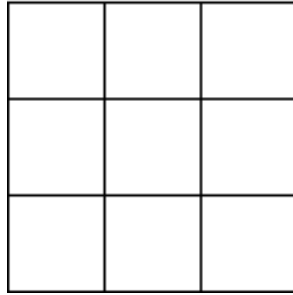
Maria thinks, "And two times two is easy, since two times any number is just that number added to itself. So two times two is four, since two plus two is four."

The boy fairy says, "But the threes are too hard, without some kind of help."

Maria says, "But three times three is...is...I don't have a rule for them! I will have to draw three groups of three:



Maria thinks to herself, "But that is not very pretty, I can draw three times three as a beautiful square:



Maria thinks, "Now I must count the little squares to know what three times three is, like this:

1	2	3
4	5	6
7	8	9

Maria thinks, "So now I can easily see that three times three is nine. That's all good. I guess any number times itself will form a square when drawn like this. Now for four times four, I just have to add some squares."

Bao Yung touches her hand and says, "But like this, my friend:"

Maria thinks to herself, "And now I just have to write in the numbers...but wait...I have to write in and count. I see that it is nine plus two groups of three, one up and down, and

1	2	3	
4	5	6	
7	8	9	

one side to side, plus...one more. I know how to do that, it is nine plus six plus one, or sixteen! Four time four is sixteen. But I will write it in to be sure..."

1	2	3	10
4	5	6	11
7	8	9	12
13	14	15	16

Maria draws in 10, 11, and 12 going down, and then 13, 14, and 15 going to the right, and finally the last number, 16, which is the number of little squares.

Maria says, "Yes, I was right! So that is the rule of square numbers: the next square number is the previous square number, plus two times the previous number you are squaring, plus one!"

The boy fairy says, "But slow down, how do you know that it

always works?"

Maria thinks, "But I need to rethink this. I will check that this rule works for two squared and three squared."

Bao Yung says, "Shouldn't you check all of them?"

Maria begins drawing, and ends up with a four by four square. This time, she draws each layer of the square by adding to the right and bottom of the last square, to check her rule.

1	2	5	10
3	4	6	11
7	8	9	12
13	14	15	16

"Sure enough," thinks Maria, "on the diagonal you can find each number times itself, the last number in the square. If I had enough patience and paper, I could go on forever."

Maria makes a table up to four squared in order to check

her idea.

Number	How to add it up	The number times itself, a square number
0		0
1	$0 + 2 \times 0 + 1$	1
2	$1 + 2 \times 1 + 1$	4
3	$4 + 2 \times 2 + 1$	9
4	$9 + 2 \times 3 + 1$	16

Maria thinks, "Well, the rule seems to work. I wonder how high I can go?"

She adds some more rows to her table:

Number	How to add it up	The number times itself, a square number
0		0
1	$0 + 2 \times 0 + 1$	1
2	$1 + 2 \times 1 + 1$	4
3	$4 + 2 \times 2 + 1$	9
4	$9 + 2 \times 3 + 1$	16
5	$16 + 2 \times 4 + 1$	25
6	$25 + 2 \times 5 + 1$	36
7	$36 + 2 \times 6 + 1$	49
8	$49 + 2 \times 7 + 1$	64
9	$64 + 2 \times 8 + 1$	81
10	$81 + 2 \times 9 + 1$	100

By the time she gets to 9×9 , Maria is so excited that she is holding her breath. When she get to 100 she stops. "What

big numbers I have created, and doubling 9, which is just adding 9 to 9, was the hardest single thing I had to do! I would hate to have to draw a square with 100 little squares inside it."

Bao Yung says, "How wonderful that you know that $10 \times 10 = 100$, and yet you don't know what 4×3 is!"

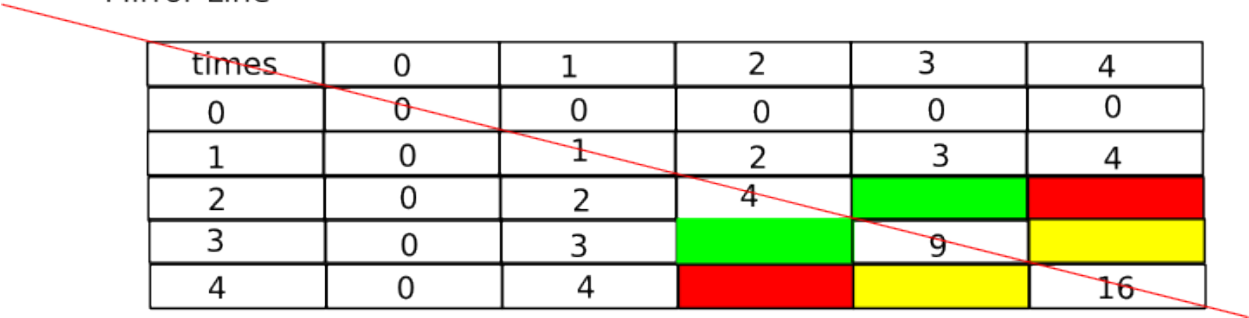
Chapter 10. Mirrors and Multiplication

Maria, taking some more paper, thinks, "So I can create a table that shows the numbers up to four that I can multiply quickly."

times	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4		
3	0	3		9	
4	0	4			16

Maria thinks, "Look, there are only six locations that are empty. And if I can only remember that the diagonal is like a mirror and that everything below the diagonal is a reflection of what is above the diagonal, then there are really only three empty spaces! Tomorrow I will start again...if I learn only one location everyday, I will have it all by heart in less than a week, and I can start throwing the cubes."

Mirror Line



times	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4		
3	0	3		9	
4	0	4			16

Chapter 11. Shen Kuo's Offer

"Gun, my esteemed cousin," began Shen Kuo, "I asked you to come in disguise, not to frighten my servants with an airborne invasion of my garden! Are you experimenting with rocketry, or did you have yourself thrown over my wall just so you would be above me when we met? I hope you are not putting on airs by assuming so high a position in my household! Please come down at once, and introduce this young boy."

The servants helped the bruised and smiling old woman out of the basket and to the ground. Bao Yung hopped lightly down.

"Kuo, this boy saved my life. My disguise was useless. Two thugs crept up behind me. This boy saw the danger while I was uncomprehending. By his brave action and the help of a Luoli family, we have reached the Dream Brook Estate, shaken but unhurt. You are famously rich in wisdom and friends, neither has old age robbed you of enemies."

"Indeed, honored cousin Gun, I must apologize for not offering you a more hospitable entrance to my estate. In a life as a long as mine, one accumulates enemies as the face accumulates wrinkles. It is more likely that a traitor in one of our households informed the Black Dragon Society of our plan, than that assassins happened to see through your disguise in the street, for, believe me, in my long life I never saw a more slovenly and wretched-looking serving woman than yourself in your present condition."

With that, Shen Gun ordered every courtesy and luxury for the two escapees.

Bao Yung had heard that people bathed in water scented with rose petals, and that a very rich boy might be allowed to eat as much as he wanted from a platter of pastries so numerous that he would have some difficulty counting them all, but he had not really believed it until now.

When they were fully refreshed, Shen Kuo came to them and started a little speech that had none of his previous teasing manner.

"Honored cousin," began Kuo in his beautiful garden, "I have fought in wars, been arrested, impeached, and finally restored to honor. The Dao has granted to no man greater opportunity for happiness than I have had here. As is my duty, I

have tried, by my books, to teach the rude glimpses of the eternal Dao that have been granted to me so that others might marvel at the splendor of the world. In this way, I have hoped my reputation might live after me when I have died. I feel that time is coming soon."

"Let us hope that unhappy day is far off," replied Shen Gun.

"Let us hope it is far enough away that I can transfer my teachings to you, for that is why I have summoned you. If you will allow me, I would pour my thoughts into you, who are indeed worthy, and this boy, who is younger and may prove a sturdier vessel."

These words surprised Bao Yung, who looked to Shen Gun. His first thought was that if he were a student of the great Shen Kuo, he would never be hungry or cold again. His second thought was that he would surely learn something more useful than smashing bricks.

"I would of course have offered to teach your little grandson as well, had he survived. Since you tell me you already feel as a grandmother to this resourceful boy, it will be a pleasure to teach him as if he were our own family. It may be that there are a few things he can teach us," said Shen Kuo, winking, "for he certainly took Neng Fa to school today!"

Bao Yung bowed low, saying "I will study very hard, Master."

Shen Gun asked, "We are grateful, but why spend your remaining time on an old woman and a poor boy? Are not your books of greater importance?"

"Call it an old man's folly. A book may gain passing fame and praise. In a few years, rats may eat it. A flood may dissolve it, or a fire may burn it up. It may simply be forgotten. But teaching those who value knowledge directly, face-to-face, mind-to-mind, is like planting a fruit tree in a barren waste. If the tree survives and bears seed, the one tree may become two, and the two four, and the four sixteen and eventually the waste is a pleasant and nourishing grove. A thousand years hence some one may enjoy the fruit of the unbroken chain of the descendants of that first tree, although they never think to ask what hand planted the first tree. It may even be that I have similarly enjoyed such fruit thoughtlessly. If you will promise to teach others what little you can learn from me, I will have repaid a debt to those spirits who have helped me so much in my own life."

Chapter 12. The Bathroom

Because of her injury, Maria has to spend more time in the

bathroom than the other kids, who rush out to play leaving her alone in silence. Sometimes Maria cries when she is alone, and sometimes she practices her multiplication tables.

One day, a girl comes into the bathroom. Maria stops crying when she hears the door open. The other girl is crying too, but she stops when she sees Maria's chair. The girl goes into a stall. From her shoes, Maria knows it is Amaris, a very pretty girl.

Maria doesn't know if you are supposed to talk in the bathroom. When she was younger, she did, so she guesses it is all right.

"Hi. Is anything wrong?" says Maria.

"The boys are calling me ugly."

"That's not so bad. They called me stink-bomb once. And I think you're very pretty."

"I know...but they won't stop. I didn't mind too much at first, but they just won't stop," says the girl.

"Even after you ask them nicely?"

"Even then, only more so."

"I always used to just ignore them."

"If I do that, they follow me around and say it," says the girl.

"Fumfummary! That is so cruel! I hate boys!" says Maria.

"They call me toad-face."

"The little monsters!" says Maria.

"...and that's not even the worst, you should hear what they call you, but at least they do it behind your back!"

Maria felt very cold inside.

"What do they call me?" she asks.

"Oh, Maria, I should never have mentioned it."

"Well you have to tell me now!" says Maria.

"They call you, 'push-toy'."

Chapter 13. A Lesson for Bao Yung

One day, Shen Kuo felt that Shen Gun had learned a lesson well enough to teach it to Bao Yung. The old man wanted to watch the grandmother teaching the grandson, not because he doubted her understanding, but because he knew that teaching something is the best way to understand it deeply. This lesson was so important that Shen Kuo had taught it to anyone who would listen, and now wanted to see it taught by another.

"This lesson," began Shen Gun, "is called Clarity of Thought. The thought of a scholar must be as clear as a fast-running icy cold stream in winter. That is a fancy way of saying that it must be easy for another person to understand it."

Shen Kuo smiled, for he preferred poetic and flowery speech

and metaphor like "Clarity of Thought" and "clear as a winter stream", but he knew that Shen Gun's modest way of explaining it was more practical for the young boy. The old man realized that his metaphor was in fact a bit muddy, and frowned at that, for but rarely did another improve his work.

"To make our thought easy to understand, we must break it into small pieces. Each piece should be so simple that there can be no confusion as to what it means. Do you understand that, Bao Yung?"

Bao Yung answered, "I think so."

"But one must do more. One must take each piece, and arrange them in an orderly fashion. Do you understand that, Bao Yung?"

"Grandmother, I do not think so."

"Good! Because I do not expect one so young to understand. The only way to become good at it is to practice. But let me ask you some questions, Bao Yung. When we use paper and ink to solve a math problem, for whom do we write?"

"For ourselves, so that we will let the paper remember for us those things which are too hard for us to remember!" replied Bao Yung proudly.

Shen Gun replied, "Indeed, that is what most people do. But that is not what a great scholar should do. We should strive to

write so that others may understand what we write."

"Grandmother, that is all very good for Master Kuo, whose thoughts are valued far and wide. But who cares how I reach a sum or a product? Why should I take pains to write so neatly when, since I well understand what I mean when I am writing, I need only jot down a few notes?" asked Bao Yung.

"To that, Bao Yung, there are two answers, the first will be a mystery to one so young, but the second you can well understand. The first reason is that the boy who does the writing is not the same boy who will do the reading a year later, or a week later, or a minute later. A boy's head is as leaky as a sieve. Do you deny it?"

"No, Grandmother, I do not deny it," said the boy. Shen Kuo suspected the boy did not really believe it.

"The second reason is that how well you practice will determine how well you perform. This is true for a musician, a mathematician, or even just the act of thinking. If you do your mathematical exercises in a minimal and sloppy way, you will make mistakes. But far worse, you will develop the bad habit of being sloppy, and the mistakes caused by that will make you go slower in the end."

Bao Yung looked at his small fists and remembered the painful hours of practice shattering bricks of greater and

greater size. The bricks had taught him that practice makes perfect. But he was not convinced that mathematics should be as hard as that. Nothing could be as hard as those bricks, except for real ones.

"Shen Kuo," asked Bao Yung, "when you were a boy were you neat and logical, arranging your thoughts into a strong chain of clear expressions, or were you sloppy as I am now?"

Shen Kuo chuckled at the audacious boy.

"I did not learn to order my calculations or my thoughts properly until I was 30 years old. But do not use that as an excuse! For though I seem rich and famous to you, I assure I would have accomplished much more if I had learned this great lesson of Clarity of Thought when I was a boy. I greatly regret that I did not."

The idea that wealth and fame could be had just by clear thinking intrigued Bao Yung, who had often been hungry and cold, and had never before imagined that great wealth such as that which Shen Kuo had was based on ordered thought, rather than cunning.

Chapter 14. Multiplication

Maria sits at her desk thinking about the girl she used to be.

"It was two months ago that I started practicing multiplication with dice. Now I can multiply one-digit numbers easily. So, in a way, I am not the same person I was two months ago, or, before that. No one lives forever. So, who am I, really? Just a push-toy? I can barely remember the girl I was. Is it greedy to want to get her back? Or should I be a new person?"

The grandmother fairy, Shen Gun, knows that it is time for her to learn something new. She asks, "What do you love? You are lucky---your love shall be your destiny."

Maria thinks, "I love Mom and Dad and Guillermo---and math."

Shen Gun says, "Even before the accident, you loved math."

Maria thinks to herself, "When I understand why I wrote this mysterious paper, then I will understand my old self better."

Shen Gun says, "You will not be the old Maria until you surpass her."

Maria thinks, "Well then, I'd better get started. Why can't I multiply two-digit numbers? Guillermo says 2 times 13 is 26, but why? Everyone says that I could do that in my head easily before the bomb wounded me."

Shen Gun says, "Draw it on paper."

Maria thinks, "So what, really, is two times thirteen? It

is two groups of thirteen.

--	--	--	--	--	--	--	--	--	--	--	--	--

--	--	--	--	--	--	--	--	--	--	--	--	--

And if I put them together:

"But how many little squares there are! Too many! I can count them up. I know that I would find 26, but that is too much time and writing," thinks Maria.

The elderly fairy says, "But what is a two-digit number?"

Maria thinks, "Why are two-digit numbers so hard? They are just something times ten plus something else times one. If I draw ten and three like so:

--	--	--	--	--	--	--	--	--	--

--	--	--

--	--	--	--	--	--	--	--	--	--

--	--	--

Then if I were to put them back together, I would get right back where I started, and have made no progress, unless I do it differently like this:

Maria thinks, "Well, the group on the right is easy, since it is just 2 times 3, and I know without counting that it equals 6. But the group on the left is much more..."

"...But less than 26, certainly..." says Grandmother fairy.

Maria thinks, "Oh, it is just 2 times 10, and anything times 10 is easy, since you just add a zero: 20! And 20 plus 6 is 26, of course, and that is the answer! So, if I think that thirteen is one ten, plus three ones, it's easy enough to multiply by two!"

"Now you understand."

Maria thinks, "Now I understand that in order to multiply a two-digit number by a one-digit number, I only have to split the two-digit number, multiply, and then add the resulting product to the ones. Let me try it on some bigger numbers---do I dare?"

"Be bold!"

"Well, suppose I took 46×4 . Forty-six is just four tens plus six ones. I know that $4 \times 4 = 16$, that's a square number. I know $4 \times 6 = 24$. Ten times 16 is 160, so I just add 24 to that, which is easy: 184. It is very simple, once you have seen it..."

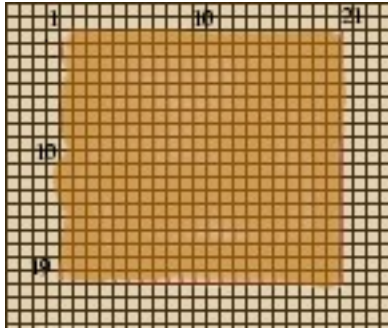
"...like every valuable magic," says Shen Gun.

For the first time in a long time, Maria feels deeply happy.

Chapter 15. Bricks and Squares

Maria's mother pushes her slowly down the platform at the train station.

The boy fairy says, "Look, Maria, at the patched bricks right here:



Mary says to her mother, "Look, Mom, there are different-colored bricks right here."

Maria's mother responds without looking up, "Yes, they had to repair the wall after the explosion, and new bricks never match exactly the same color as old bricks."

Maria says to her mother, "Maybe a piece of an old brick is still inside me, right, Mom?"

Maria's mother moves in front of her and squats so that her eyes are level with Maria's.

"Yes. They took much from you, and left you a little bit of brick."

Maria says happily, "Don't be sad for me, Mom. I am relearning so much. I feel like everyone is helping me."

Bao Yung says, "How many new bricks, I wonder?"

Maria says, "Mom, let me count the new ones!"

"No, dear, the train is coming, and it is too many to count." says Maria's mother.

Maria says, "But I only need to count the rows and columns, wait---it is 19 columns and 21 rows."

Maria's mother asks, "Can you really multiply that high already?"

Maria, fingering the folded old sheet of paper in the other Maria's handwriting that she always keeps in her pocket, says "Not yet."

Bao Yung whispers, "Soon."

Chapter 16. The Silk Road

Shen Kuo found the company of his cousin, Shen Gun, as pleasant as that of any of his friends. Often he would sit in the garden and let her mind soak up his thoughts as a sponge might soak up the tea they drank.

One day, when Shen Kuo felt particularly weak, he mentioned, in an offhand way, an idea to his cousin. Shen Kuo was very shy about this idea, for it was very dangerous. He was doubly shy because Shen Gun was a woman, although she seemed particularly well-suited to the task. She was too old to bear children, and

she had no family except for young Bao Yung, as they had all succumbed to fever. As Shen Kuo's mind got weaker, he was more willing to listen to his inner thoughts, which he thought of as the murmurings of the eternal Dao.

"Shen Gun, if I were a young man, I would take my books and travel Westward, on the Silk Road, perhaps to the very end, if it has one. I would see the things of this world and see what beauty they offer. Above all else, I would learn what other peoples might teach me, and I would write that down and bring it home, to the greater honor of the Emperor."

"Shen Kuo, that would be a most noble adventure. But I fear, dear cousin, that illness must be clouding your mind, for do you not perceive that a great advantage of this voyage would be in teaching your own wisdom to other peoples as well, to magnify the honor of the Emperor. For surely no one has a greater store of knowledge than you."

"Shen Gun, you are clever even in your flattery. But I suppose such a voyage is impossible."

"Shen Kuo, such a voyage is not impossible, but necessary. Do you not perceive how bored I am of your miserable garden and your weak tea? Do you imagine that your inane ramblings can enthrall a vibrant young woman such as myself forever? No, Shen Kuo, I'm afraid I have been waiting for an opportunity to be rid

of you once and for all, and this is it. This might be my last chance to get away! I suppose you will want me to wait until you die, since your life will be so dull without me to explain things to you, but you might linger for decades here in squalor while I grow old. No, no, I insist that Bao Yung and I leave as soon as we have the proper provisions, and a cart sufficient to carry your writings. It will have to be a stout cart, as well, for you have ruined a lot of paper in this long life of yours."

Shen Kuo sat very still, and Shen Gun was afraid that her teasing of the old man had gone to far. Finally, the old man laughed.

"Shen Gun, this is the greatest honor I have ever received in my life. You shall have not only a cart, but horses and servants, and a chest of silver and four bodyguards to guard it and the two of you."

Chapter 17. Mr. Garza

The man made out of oak leaves, Mr. Garza, teaches math. Maria tries to make sure he never ever notices her in any way. The only thing Maria likes about Mr. Garza is that the boys are also terrified of him.

"Today, class, we will prepare to get ready for pre-algebra. Who can tell me what algebra is?"

"The know-it-alls will shout out stupid answers," thinks Maria.

"Really big numbers?"

"A town in Morocco?"

"Something you need to get into a University?"

Mr. Garza says, "Algebra is the study of structure, relation and quantity. There is nothing more basic or pure in all the world." Mr Garza seems to look out the window. The folds in his face change into a curve. Maria realizes it is a smile, and she begins to be less afraid of Mr. Garza.

After a few seconds he coughs. "But for our purposes algebra means doing arithmetic with something called a *variable*. A *variable* is a quantity that can change. That means it can represent different numbers at different times. We will use letters instead of numbers, to represent variables."

Maria notices that very few people were paying attention.

"Up until now, when I have given you a problem in arithmetic, I have just written it like this:

$$5 \times 7$$

and expected you to understand that this means "five multiplied seven times" and to write down the correct answer."

"35!" shouts one of the boys.

Mr. Garza takes a chart out of his desk drawer and writes

down a number besides the boy's name. The scratching of his pencil seems very loud.

He continues. "Sometimes, we have used an empty line and an equals sign to represent the answer, like this:

$$_ = 3 \times 7$$

"But now, children, we need to be able to think more precisely and more powerfully about math, and so we have to give a name to the answer, and to many other quantities. We will use letters for that, like this:

$$x = 3 \times 7$$

"Can anyone see a problem with this?"

Maria raises her hand, and Mr. Garza's face stirs around and he nods.

Maria says, "Well, it looks like you have two X's, but one is the variable, and one means multiply."

"Precisely!" says Mr. Garza. "From now on, we will no longer use a crossed diagonals multiplication sign, which looks like an X, but will use a dot instead, like this:

$$x=3\cdot7$$

(Mr. Garza makes a tiny mark between the 3 and the 7.)

"Be sure to make the dot on the middle of the line of letters, so that it cannot be confused with the dot we use to separate thousands, and which the crazy English the Americans

use to separate the fractional part of a number, which looks like this: $x=3.701$,” says Mr. Garza. “Are there any questions?”

Maria raises her hand. “Mr. Garza, why do we use something so small as a little dot to mean something as big and important as multiplication?”

“That is a very good question, Maria. For your homework, think about this: do we use long words to mean things we use everyday, like air and food and paper, or things that we rarely use, like airplanes, locomotives, and penicillin?”

Chapter 18. Maria's Birthday

There are some short words that aren't used everyday. 'Coma' is one of them. Maria was in a coma for 92 days. She regained consciousness on her thirteenth birthday.

When Maria's fourteenth birthday is near, Maria's mother asks her what she would like to do to celebrate it.

“I would like to have a party with some of my old friends,” says Maria.

“I'll see what I can do,” says her mother.

On her birthday, Maria helps her mother decorate the house.

“How many people are coming, Mom?” ask Maria.

“Well, your little cousins are coming, and three of your old friends.”

When the first friend comes to the door, Maria is scared that she won't remember the girl's name. When her mother says, "Glad you could make it, Linda," Maria is astonished. This girl does look like her old friend Linda, but she is much taller. She wears a short black skirt, a black long-sleeved shirt with ruffles, red fingerless gloves, and black boots. A short and very uncomfortable looking iron chain encircles her neck. She has a small silver ring in her nose. Her skirt seems to have four layers and it rustles as she walks. Maria doesn't have any clothes at all like any of these, and she never ever wears short skirts, because she doesn't want anyone to see how her leg muscles have shriveled up.

Linda starts talking to Maria, which is good, because Maria doesn't know what to talk about. Eventually Maria says, "I like your clothes," and then blushes because that is such a corny thing to say, but Linda doesn't seem to mind. Linda talks a lot about strange places and events, like "The Dark Hole" and "The March on Monday". Eventually, Maria realizes Linda is talking about musical groups.

The second girl to come is Antionette, who has not changed at all in the last two years. Maria is very happy to see her. They had sometimes been best friends at school.

Antionette also says Linda has nice clothes. Linda doesn't

seem to care much.

"So how do you like school?" Antionette asks.

"Oh, it's fine, except they put me back in the fifth grade for a while."

"That must suck. But I wasn't asking you, I was asking Linda," says Antionette.

Linda hesitates. "It's okay. The teachers are mean; but I have a boyfriend."

"I have a boyfriend too. What about you, Maria?"

Maria looks down at her legs. "No, I don't have a boyfriend," she says.

"Oh, well, of course not. Anyway, my boyfriend is 16 and has a scooter. What about yours, Linda?"

Linda's boyfriend has a good collection of music, and a keyboard.

Maria wishes that her wheelchair would sink into the floor with her in it and she would never be seen again.

"Mom, when do you think Patricia will get here?"

"I don't know, Maria, perhaps she will not be able to make it."

Finally, Linda asks Maria what she likes to do. Maria feels a wave of gratitude wash over her.

"Well, I still like math."

Antionette looks like that is the stupidest thing she has every heard. Linda says, "That's cool. I'm trying to learn German."

"Ooh, that's cool," say Maria and Antionette at the same time.

"My English is getting pretty good..." says Antionette.

"Linda, why do you want to learn German..." says Maria.

"I met some English boys at the beach..." interrupts Antionette.

"I bet there are lots of great German punk bands..." continues Maria.

"Maria if you keep talking while I'm talking..." says Antionette. But Maria does not want to quit talking at all.

"I wish I knew lots about music like you..."

"then I'm leaving..."

"well, it's supposed to be my party..."

"but it's not a fun party..."

"so I think you should let me talk..."

"okay then I'm leaving..."

"okay then why don't you..."

"okay, I will..."

As soon as she could without letting Maria's mother know how rude she was being, Antionette leaves.

Pretty soon Linda says she has to go too.

Maria begins thinking about ways she could get her wheelchair to the roof of a very high building.

Chapter 19. The Sinister Thought of Neng Fa

That night, Maria has a confused and disturbing nightmare about an evil man wearing yellow silk with weird writing embroidered on it in crimson thread. In her dream, the man is muttering to himself.

"...by the nine-headed black she-dragon...yet he remains in honor, and my sorceries rebound against me, and the venom I put into ink is harmless and embitters the noble against me...but soon he will misstep and show himself a coward and a traitor...what do you do with two cart horses and a wagon?

If you think like Shen Kuo...you spill secrets like a sick man spews filth...but he has vomited forth the paltry things he knows already, even to the edge of the Middle Kingdom...the edge, the edge at his throat, I wish I had one...the edge but not beyond...but why not? Why not beyond China itself? What arrogance! Just like him. That must be his plan. Who would have thought one so old and weak...so maybe not him himself, but who then? Surely not that crone he keeps like a pet...he wouldn't

dare...she has a whelp of a slave boy he calls her grandson...a bitch and its pup on the long road to Rome...over my dead body! No, over their dead bodies...

Chapter 20. The First Law

Maria keeps thinking about what she meant by "My law" when she wrote it so long ago. Maria asks, "Dad, what is the law of numbers?"

Her father puts down his book.

"Oh, there isn't just one law, there are many. But I forget them, for the most part."

Surprised, Maria asks, "Many? Can you give me one now?"

Her father says, "Yes: zero times anything at all is still zero."

Maria answers, laughing, "Dad, that is trivial! Give me an interesting law!"

Her father responds, "Is that so? Then I will give you a very simple, but very deep law. Mankind needed thousands of years to understand it. Here it is: *You can substitute equals for equals.*"

Maria frowns uncertainly, "Those are just words..."

Her father smiles and answers, "Every child thinks that, but in fact, this law, which is called 'the rule of Leibniz', is

powerful magic, because it permits us to use the other laws, for the most part."

Maria demands excitedly, "Show me how to use it, please!"

Her father responds, "Look: $4 = 2 + 2$, yes?"¹

Maria says, "Of course I know that!"

Her father says, "Quickly, then: what is $(2+2) \cdot 10$?"

Maria says, "I'm not sure..."

"Okay, quickly, what is $4 \cdot 10$?"

"Forty," answered Maria immediately.

Her father answers, "Right. But if you believed the law, you would immediately have known that $(2+2) \cdot 10 = 4 \cdot 10 = 40$.

Maria says, "I'm not sure I understand."

Her father says, "Anytime, anytime at all, that you know one thing is equal to another thing, you can write or think the second thing in place of the first in the formula or thought.

Therefore, $4=2+2$ and the Rule of Leibniz work together."

Maria responds, "So $4 \cdot 10 = 40$?"

Her father nods, "Yes."

Maria asks, "So $(2+2) \cdot 10 = 40$?"

Her father says, "Indeed! Usually we try to make things

1 Maria and her father are speaking out loud, not writing equations on paper, but it is easier to read " $4 = 2 + 2$ " than "four equals two plus two." In order to say $(2+2) \cdot 10 = 4 \cdot 10 = 40$, which is easy to write, you would have to say something like "the quantity two plus two times ten equals four times ten, which is furthermore equal to forty", which is not nearly as clear as writing it out and using parentheses.

simpler and shorter, but you can use the Rule of Leibniz to make things longer and more complicated, as you have."

Maria thinks out loud, "However, $10=5+5$. So,

$(2+2) \cdot (5+5)=40$?"

Her father says, "Bravo! Well done! Precisely!"

Maria goes on to say, "and $5=2+3$, so if I substitute

$2+3$ in place of 5, then $(2+2) \cdot (2+3+2+3)=40$?

"Yes, indeed! But that is a complicated way to think about 40, is it not?"

Maria says, "Yes. But still, it is very powerful. The law both builds up and tears down, depending on how you use it.

Thanks, Dad! Give me more laws!"

Her father scratches his head.

"Darling, you should go ask your mother."

Chapter 21. The Baleful Brotherhood

One day the servants bustled in and out of the small shrine that stood behind the fish pond in Shen Kuo's garden. Servants carried in eight heavy bronze pots with lids carved like dragons, lions, tigers and fish. Shen Kuo had excavated these objects from a buried town, and they were old when the First Emperor of the Song dynasty was young.

"Shen Kuo, what ritual are you preparing? Often have I heard you upbraid the superstitious who hold empty rituals to the great Dao with ancient bedpans and soup pots, believing them to be the magic relics of celestial sages," said Shen Gun.

"Indeed, Shen Gun, I have fought the conservatives who have not enough sense or logic to explain the world, but find only wizardries and magic in everything they cannot explain. Yet there is much that I also cannot explain. Perhaps mankind will never be able to explain much. Thus I am preparing a potent ritual to bless you and Bao Yung, and to protect you from the Baleful Brotherhood."

This change in Shen Kuo unsettled the scholarly woman, and she asked, "Shen Kuo, who are the Baleful Brothers you speak of? Do you mean the Black Dragon Society? Why did they attempt to kill or kidnap me?"

"No, Shen Gun, I have no magic to turn a dagger. By the Baleful Brotherhood I mean only those who oppose the progress of mankind, for the most part unwittingly. I mean those who do not affirm the beauty of the world. I mean those who prefer ignorance to knowledge. My enemy, Neng Fa, is surely one of them. But a conniving enemy like Neng Fa and his stealthy black-clad assassins are not so dangerous as the indifference to wisdom one can find in the street, or even in our own hearts,

every day. I call them a brotherhood, because they are most dangerous when acting in concert, as a mob, although they believe they are not members of any brotherhood."

When all was ordered as Shen Kuo directed, and Shen Kuo's sighting tube and armillary sphere indicated the planets were in the correct position, the ritual began.

Shen Gun and Bao Yung were blindfolded by the servants. Bao Yung thought this was silly, since he could peek beside his nose and see the well-worn garden path as they were led to the temple. In the temple, their blindfolds were removed.

White paper covered the floor, and on this paper were drawn huge geometric shapes, one inside the other. Along the line of each shape stood words in ink of many colors, some of them in Chinese, and some of them in other scripts which Shen Gun and Bao Yung had never seen before. Around the letters that formed this writing were yet more small letters written with a tiny brush. Bao Yung thought perhaps around these letters there were yet more letters written with a single cat's whisker dipped in ink, but he could not be sure.

Four men, dressed in sumptuous silk robes and striped hats sat on ornately carved wooden thrones that had been carried in to the temple. The highest throne was at true North. Shen Kuo was famous for having devised the mechanism of distinguishing

true North from magnetic North found with a compass. An enormous gong was struck three times.

Each man spoke in turn in a different language, from East to South to West to North. In the North throne was Shen Kuo, and he spoke in Chinese, though many of the words he used were unknown to Shen Gun and Bao Yung, or seemed to have secret meanings.

Bao Yung feared the dragons and lions might come to life, but nothing at all happened. He was bored and beginning to wish they would hurry up when he realized that he could see nothing at all outside of the temple. Where there should have been the garden and the lanterns from the house, there was only blackness, as if the temple were submerged in an ink lake.

Finally, Shen Kuo spoke.

"Here in the presence of the eternal Dao, make you now a solemn vow, to form an unbreakable link in the chain."

Shen Kuo had not told Shen Gun what to do at this part of the ritual. She greatly feared embarrassing him, but could think of nothing more than her most fervent desire.

"I promise to teach."

"Bao Yung, is that your vow also?"

"Shen Kuo, I promise both to learn and to teach."

"The sages have written that a heart-felt oath taken

between the Nine Heavens and the Middle Kingdom in the presence of the Jade Emperor will not be broken, even by death. So let it be."

Then the gong sounded three times again, and Bao Yung was relieved to see the lanterns lighting the garden once again.

Chapter 22. Mom's Law

The next morning, Maria wheels herself into the kitchen, her mother is there reading a book on biology while stirring a soup pot.

"Mom, what are the laws of Mathematics?"

Still reading, her mother responds, "Guillermo says that you already know the law of distribution, and that is very important."

Maria says, "Really? I doubt that I know it...I can do very little, except multiply two-digit numbers."

Her mother smiles at her daughter, and, taking a sheet of paper, says, "But you understand this:

$$\begin{aligned}
 &2 \cdot 13 \\
 &= \{ 13 = 10 + 3 \} \\
 &2 \cdot (10 + 3) \\
 &= \{ \text{distribution} \} \\
 &2 \cdot 10 + 2 \cdot 3
 \end{aligned}$$

= { multiplication }

20 + 6

= { addition }

26?"

Maria says, "Yes, although that is a weird way to write a big bunch of equations!"

Her mother continues, "And, to know that $2 \cdot (10+3)=26$, you have to know to double 10 to have 20, and and to double 3 to have 6, right?"

Maria responds, "Yes, but those are just facts, not laws."

Smiling, her mother says, "You're right---but why is

$2 \cdot (10+3)=2 \cdot 10+2 \cdot 3$?"

"It just IS...one can show that by drawing squares a certain way," replies Maria.

"Is that true? I'm surprised. Sometime you will have to show me how. However, I know this law, called *distribution of multiplication over addition*, by the formula:

$A \cdot (B+C)=(A \cdot B)+(A \cdot C)$."

"Mom! I love magic formulas, but you can't have a math formula without a single number in it!" exclaims Maria as if she has been tricked.

Her mother acts surprised, and says, "You can't? How then

can you do it by drawing squares?"

"The squares are not numbers...the numbers represent the squares," says Maria.

"Good---are pictures of boxes more or less real than numbers?" asks Mom.

Maria responds, "More real---numbers are magic symbols, that name real amounts of things."

Her mother says, "Correct---and letters can be even more abstract and magic than numbers. In a true equation, the same letter always means the same number but that number can be anything at all."

Maria asks, "Anything? Then I can substitute any number at all for A, B, and C in your equation: $A \cdot (B + C) = (A \cdot B) + (A \cdot C)$, and the equation will still be true?"

"As long as you remember to put the same number in for each letter where ever it occurs, no matter how many times. A letter can't represent two separate numbers at the same time in the same equation."

"No way! I don't believe it."

Her mother responds, "Okay, here is some homework for you: Guess any three numbers, and write them down, for example: A = 3, B = 1, C = 4. Substitute the numbers into the equation, and do the arithmetic until the left and right sides of the equation

are simple numbers. If you can find three numbers for which that equation is not true, I will make *paella*¹ for you everyday for as long as I live."

Maria says, "But there are numbers without limit...when will I ever finish?"

"When you are bored, or when you believe that the law is true."

Maria says thoughtfully, "Thanks, Mom, I certainly will think about your law."

Chapter 23. A Warning

Shen Kuo and his cousin and her charge spent many days planning the Westward trip. The actual goal was to reach a city called *Constantinople*². Shen Kuo had read hints that the people there might have texts as old as those of China which might contain lost knowledge in Philosophy, Mathematics, and especially Geometry.

Long did they ponder a fragmentary map of the Silk Road, the dangerous and long trade route Westward. They knew that it might take several years to reach Constantinople. They would not return for many years, since even if they reached

1 *Paella* is a delicious national dish of Spain made with rice, meat, seafood and vegetables.

2 Constantinople was the capital of the Eastern Roman Empire, from 395 C.E. until 1204 C.E. In 1453 C.E. It became the capital of the Ottoman Empire. In 1930 it was renamed Istanbul.

Constantinople, they would have to begin the task of learning whatever outlandish languages those ancient books of wisdom were written in, and of course they should also attempt to translate Shen Kuo's books and other Chinese classics into whatever useful languages they found there¹.

Shen Kuo was unlikely to live many years longer, so the two old friends knew they would never see each other again. Sickness had not yet crept up on Shen Gun, but even she might not live long enough to return to China. They both hoped, however, that she would survive to guide Bao Yung to manhood, and that, having both sown and reaped the greatest wisdoms known to mankind, he would return to China.

One day, a letter came that put an end to their hopeful and patient planning. The letter warned that Neng Fa and the Black Dragon Society knew of their plans, and would stop at nothing to prevent them.

Shen Kuo wondered how his plans were known. But he reflected that his purchase of the wagon, maps, provisions and books might be enough for a thoughtful watcher to guess his plans.

Shen Kuo was sick and feeble, but he was still decisive. He summoned Bao Yung and Shen Gun.

1 In fact, Shen Kuo is thinking of the 13 books called *Elements* collected by the Greek mathematician Euclid who died in 283 B.C.E. Although Euclid's *Elements* were written in Greek, by 1093 C.E. they would have been translated into Latin and Arabic, the most important languages for international communication of that time.

"Bao Yung, you are wise in the coarse ways of thugs and murderers. What is the safest way for you and Shen Gun to begin your journey, since we believe our plan has been discovered, and that the Black Dragon Society intends to stop it?"

In Shen Kuo's estate, Bao Yung felt safe and comfortable. He had almost forgotten how to survive in the harsh world beyond Shen Kuo's walls. Almost, but not quite.

"Sir, send the cart you bought South, to Shanghai, to some friend you trust to return it. Place large chests and boxes of supplies on it, and send your most able and vigorous servants. Have them keep the wagon closed, as if there is someone in it. Then send Shen Gun and I alone and by boat Westward to Nanjing, with enough money to buy a new wagon and horses in a different town, and send bodyguards and servants carrying your books and provisions by a different route to meet us in Nanjing."

Shen Gun replied, "Bao Yung, if I had been as clever as that when I commanded an army, many lives might have been saved."

Chapter 24. A Shorter Way to Write Squares

Maria says to her brother very sweetly, "Guillermo, Mom gave me an arithmetic law. At first I didn't believe it, but I tried 20 examples, and it was never false."

Guillermo says, "That's the way mathematical laws are."

Stronger than steel. Stronger than the army."

Maria asks, "I use this law when I multiply two-digit numbers, don't I?"

"I guess," says Guillermo. "I guess the way anyone multiplies two-digit or bigger numbers does depend on it. But I never think that way."

Maria asks, "Is it better to use laws consciously, like on paper, or to do it automatically, without thinking, like I used to be able to do?"

Guillermo answers, "Both...it depends...I don't know. In music, one has to play according to the rules, before one can play freely, by memory, from the heart. Perhaps mathematics is the same; once you have practiced enough, you can do it simply, automatically, but if it is hard, you should use the rules consciously, or on paper."

Maria says, "Thank you, Guillermo, for helping me with my exercises and relearning."

Guillermo says, "Mother tells me you are now studying algebra."

"Certainly not!" says Maria. "I am just practicing the use of the laws."

Guillermo says, "It is the same thing. Do you know how to write the squaring operation algebraically?"

Maria, shaking her head, says, "No."

Guillermo shows her a sheet of paper. "Look: $x \cdot x = x^2$. Do you see how the '2' is small and high?"

Maria says, "Sure. So is that a new law? That is another one for me! But what good is it? It's simple enough to write:

$x \cdot x$ when I have to."

Guillermo answers, "Yes, but how about $(x+y) \cdot (x+y)$ instead of $(x+y)^2$, or even worse, $(x+(a+b)) \cdot (x+(a+b))$?"

Mary answers, "Well, I often grow tired of writing so many symbols, and each time you write one, there is a chance you make a mistake. But are you saying that I can substitute not just numbers, but letters and whole expressions in the equation

$x \cdot x = x^2$?

Guillermo answers, "Certainly, any mathematical expression at all. The little '2' really just means "Multiply the thing by itself."

Maria pauses, and then timidly asks, "Did I understand this before the bomb?"

Guillermo looks Maria in the eye and says, "Before, you calculated more rapidly, but now you understand more thoroughly."

Maria responds, "Well, I have to. And now I have Dad's law, which is "equals for equals", and Mom's law, which is

"distribution", and I will call the law of squares Guillermo's law."

Guillermo says, "Well, it is just notation, really, not a law, but thank you! But why are you so worried about laws?"

"I do two hours of physical therapy every day, and the Doctors say therapy for my brain is even more important."

Chapter 25. A Chase in the Desert

Shen Gun and Bao Yung journeyed from Zhenjiang to Nanjing as they had planned, and then on to Kashgar, more than half the way to Persia. From Persia, they intended to travel to Constantinople. Shen Gun required Bao Yung to write the events of each day in a journal, as part of his education in composition and calligraphy. Bao Yung thought it was already a pretty good story, for many adventures and strange occurrences had befallen them. The boy pleased Shen Gun very much, for he worked very hard for his age. He had begun to make his own observations, particularly in mathematics and algebra, which went well beyond the tasks that Shen Gun set him to do.

In the desert three days West of Kashgar, the sky is like black velvet and the stars twinkle like little diamonds against the glowing background of the Milky Way. Bao Yung lay awake studying geometry by lamp light in the wagon. He had spent the

daylight hours reading a recently published travel story by Su Shi. Bao Yung thought this very educational, at least about the way men think, but Shen Gun did not agree, so she had made him promise he would do an extra hour of study when the sun went down. Bao Yung thought this was a bit unfair, since Shen Gun seemed to waste hours staring at people, the horizon, and even rocks, as if they were startling objects of great beauty that she had never seen before.

Bao Yung and Shen Gun had no clock, but they had a beautiful brass astrolabe, which can be used to tell latitude, and also to tell time by measuring the angle of bright stars or the Sun with the horizon. Because the Earth rotates completely in 24 hours and there are 360 degrees in a complete rotation, every hour the stars overhead wheel 15 degrees Westward. If you could always see a star lined up with the equator, then you could just measure it changing its angle in the sky to tell time. However, as you go North or South in latitude the heights of all the stars change. An astrolabe converts the angle into time for you, though it must be adjusted if your travels take you different to latitudes.

Bao Yung would hang the astrolabe from an iron hook at the curtain of the wagon so that it hung vertically, but he could peek out into the chilly air and see the stars. He would aim the

rule at a bright star, and then where the rule crossed the markings on the rim of the astrolabe, he would read off the precise angle of the star against the horizon. Bao Yung's favorite star, Altair, moved slowly across the inky sky. Bao Yung was impatient, and also loved playing with the beautiful instrument, so he checked the time by sighting Altair on the astrolabe every five minutes. Sometimes, when Bao Yung was nervous, he secretly thought that the astrolabe was the most valuable object he had ever held, and that if some disaster occurred on the journey, he could buy enough food to survive and get back to Zhenjiang if only he could sell the astrolabe.

As he sighted down the rete for the last time that night, he saw a tiny orange star shoot up from the ground and then slowly fall back to earth in a graceful arc.

The next morning, he agitated Shen Gun by asking her about this shooting star.

"Bao Yung, that was undoubtedly a rocket, used to send a signal."

"Who would be sending a signal out in the middle of the desert, Grandmother?"

"That is a good question. I hope we do not learn the answer."

The next night, the bodyguards kept careful watch. When the

sky was fully dark, they saw an orange rocket to the Northeast. Bao Yung thought it was closer than the night before. Shen Gun and the bodyguards began discussing this quietly, until Bao Yung noticed another rocket to the Southwest. The group silently scanned the sky until a third rocket was seen from the Southeast.

"Madam, we are surrounded," said the bodyguard.

Bao Yung slipped the astrolabe under his jacket.

Shen Gun argued with the fearful bodyguards for a long time, and finally the men mounted the wagon and drove on, leaving Shen Gun and Bao Yung alone in the desert.

"Bao Yung, we will walk quietly to the North. Hopefully our pursuers will follow the wagon, and we will be safe until the bodyguards return to us."

Bao Yung was very frightened at first. After an hour walking in the ditches of the desert he felt very small and unimportant, and was sure now one was looking for him. By listening to the wind sigh and whistle through the rocks, he could keep his mind off how much his feet hurt. The wind sometimes sounded like tiny shouts or yelps very far away.

Bao Yung turned back and stood very still with his mouth open to hear better. The baying of a pack of hounds came to him quite distinctly on the wind, from the direction where they had left the wagon at the trail.

There was no hope of hiding from hounds, who could smell the footprint of a fox in a pine forest. As the chilling music of the hounds got closer and louder, they got out all of their food to give to the dogs, in hopes of befriending them. When the thin, shaggy dogs arrived they were not fierce, but try as they might they could not keep them from baying. Soon hoof beats were added the baying of the hounds. So it was that Shen Gun and Bao Yung were captured by horsemen of the Black Dragon Society without a bodyguard and without a fight. When Bao Yung offered to give the astrolabe to the men if they would release them, the leader flung it away into the desert.

Chapter 26. Problems

Mr. Garza says to the class, "So your homework is this list of multiplication problems." Mr. Garza draws some products on the board.

Solve for x :

$$x=8\cdot16$$

$$x=7\cdot19$$

$$x=3\cdot12$$

$$x=9\cdot14$$

$$x=5\cdot13$$

$$x=5\cdot8$$

Then disaster strikes. Mr. Garza says, "Maria, would you

be so kind as to wait after the class."

Fighting the urge to cry as the class leaves, Maria thinks, "Fustymusticans! I am the stupidest girl in Spain! And now mean Mr. Garza knows it, and will punish me."

The boy fairy, Bao Yung, says, "No, no, he is not mean or unkind, he will help you."

Very mildly, Mr. Garza says, "Maria, please don't cry. You are not in trouble."

With a trembly voice, Maria says, "I know that I am stupid and make lots of mistakes, but I swear I work very hard!"

Smiling, Mr. Garza answers, "Don't worry, In fact, your score wasn't the worst in the class; but you indeed missed four out of six, and, because you didn't show your work, I don't know why."

Bao Yung says, "Tell him the reason."

Maria, more calmly, says, "Sir, I know that before, I mean a year ago, I always did everything in my head, and never used paper, and...I want to be the way I was."

The boy fairy says, "Well said! How honest you are!"

After a pause, Mr. Garza answers, "Of course, of course, very understandable. But even a genius uses paper to show how they reach a solution."

Looking at the floor, Maria says, "Before, everyone thought

I was a clever girl---and now, I can't walk, and everyone thinks I'm stupid, and they say I smell bad, and I am stupid, since I can't even remember numbers for five seconds, and I get confused on every problem, even the simplest."

Bao Yung says, "You are stupid when you call yourself a stupid girl."

Mildly, Mr. Garza answers, "Maria, in just a few months you have regained an ability with math that takes many people years to learn. Allow me to teach you how to use paper to help. That way, you will be able to solve harder problems, and someday you won't need to use paper, and may even be a famous mathematician."

"Okay," says Maria.

"Come to my tutoring session at the high school an hour early, and I will show you some things."

Maria felt suddenly scared. She hadn't understood that agreeing to this meant going to her old school, and being seen being tutored. The old Maria had never, ever, gone to a tutoring session.

Chapter 27. The First Master

After many days of traveling as prisoners, they came to a cloud-shrouded mountain range. Their captors walked the horses up a steep narrow road until the air was thin and hard to

breathe, then they dismounted and climbed higher. Shen Gun had to be carried.

After being given water and food, they were brought into a huge, strange room more opulent than any palace they had seen or heard of.

Small panes of colored glass made up the entire South wall. The setting sun turned each into a glowing lamp of scarlet, blue, green, or gold. It would have been quite beautiful, if it had not depicted a horrible winged lizard gripping in one huge foreclaw a lady with yellow hair and the body of dying man in metal armor in the other.

The shifting and multi-colored light from the windows fell upon an enormous black dragon facing them with its jaws agape. They soon perceived by the stillness of the dragon and the calmness of their guards that this monstrosity was a statue wrought in black iron. Its eyes were iron-lidded fire opals. Bao Yung thought it must be a real dragon turned to iron by some curse, because it was so like a living dragon, and he shuddered. Shen Gun thought no artist could conceive of such a beautiful and detailed sculpture without working from memory of a living sight, and she shuddered.

A man in yellow silk with scarlet embroidery stood on a platform near the dragon's mouth. Beside him was a little silver

cart stacked with books. To the right of the platform, a great water wheel stood, unmoving but ready to receive water from a spout formed in the shape of a great brass carp. This wheel was fully 8 meters in diameter, and only the top three meters was visible. The axis and the lower part were below the floor.

The guards directed them across the floor tiled in intricate patterns of lapis lazuli and carmeline. As they walked they saw that the great room was lined from floor to ceiling with cabinets displaying books, scrolls, skulls, and instruments of unfathomable purpose.

The man in yellow turned as they approached and spoke to them, in the first Chinese they had heard since being captured.

"Welcome to the fortress of the Black Dragon Society. I am Neng Fa, the humble 66th Master of the Society. Would you care for some plum wine, Shen Gun? I assure you it is from the Emperor's best cellar." Without waiting for an answer, Neng Fa clapped his hands.

"You almost escaped me Shen Gun, but you gave yourself away in Kashgar. What woman but yourself could possibly explain Shen Kuo's ridiculous theory of land formation by erosion and deposition? I thank you for the chest of silver, though your men made it costly to retrieve it."

With that remark a smiling woman brought a golden goblet of

wine on a tray of ivory. With horror, Shen Gun realized that the smirking young woman was the very servant girl with whom she had traded clothes so long ago.

"I have something special planned for you tomorrow. Since you are so interested in teaching, I will allow you to teach a math lesson to the initiates of the Black Dragon Society. You see, we are always learning. In fact, our entire Society is dedicated to the principle that knowledge is power, and therefore knowledge is for us, and for us alone."

Neng Fa took the wine which Shen Gun refused, and quaffed it. "In this room is collected knowledge beyond your imagining. The combined secrets and sorcery of Greece, Atlantis, Indus, Sumeria, Mu, Hyboria, Caral, Leng, Lemuria, Egypt, and civilizations too dark and secret to name while the sun still shines, wait here for our exclusive use. These," said Neng Fa, motioning to the stack of books and scrolls, "are our most recent additions. They have been translated for our exclusive use into the cryptic language of power taught us by the First Master, whose effigy we honor here," Neng Fa indicated the iron dragon.

"These books, being written in languages haltingly understandable by the common rabble such as yourself, which might provide some pitiful little orts and shards of knowledge

to mankind at large, will be fed to the First Master. It is a beautiful ceremony, the destruction of mankind's feeble efforts to shake off its yoke of ignorance, but you will not survive to see it."

Chapter 28. At the High School

The idea of the tutoring sessions was bad enough, but Maria had never imagined that her mother would actually make her go! When Maria's mother asks Maria that night why she is crying and finds out that Mr. Garza had asked her to come to the tutoring session, she insists on taking Maria, no matter how many times Maria explains that she doesn't want to go and that she shouldn't have to.

Maria wears a baseball cap and a jacket, even though it isn't cold. But no one recognizes her, because no one is there.

"I must be the only person stupid enough to need tutoring," thinks Maria as she rolls down the empty halls to the classroom where Mr. Garza is reading a book with Greek letters on the cover.

Without even saying hello, Mr. Garza says, "Yesterday, I gave you a problem, $7 \cdot 17$, and your answer had a small mistake. Your answer was 109, but 119 is the correct solution."

Maria wonders, "Where did I go wrong?"

Mr. Garza says, "Since you didn't show your work, I don't know. But look how I prefer to reason on paper."

Mr. Garza, taking paper, "First, we write the problem, usually the expression that we have to simplify:

$$7 \cdot 17$$

after that we write an equals sign, and empty curly braces, likes so:

$$7 \cdot 17 = \{ \quad \quad \quad \}$$

Maria wonders out loud, "Why use the little curly braces that look like smiles, or my Dad's mustache?"

Mr. Garza answers, "The braces hold the reason or at least a hint for the next step. You can't use parentheses, because they are used in the equations themselves."

"Why do we need a hint?" asks Maria.

Mr. Garza says, "Because when someone is checking over your work, they need to be able to see how and why you are allowed to make each step. If your hint doesn't explain it, it is like a magician pulling a rabbit out of a hat---very impressive, but very mystifying! When I studied at the Sorbonne¹, there was a rhyme we used:

Be a mathematician,

¹ *La Sorbonne* is a famous University in Paris, France.

= { Hint }

[New Equivalent Expression]

where we know that the New Expression is equal to the Original Expression because of the reason we write into the Hint."

Maria says, "Oh, and we have two links in our chain, and two equal signs. Now we can multiply 7 by 10 and then 7 by 7, by Mom's Law?"

Mr. Garza answers, "Mom's Law? Usually it is called The Law of Distribution or The Distributive Property of Multiplication over Addition, but it doesn't matter what you call it. Now we are ready to write down the second hint, and the third link in our chain:

$$\begin{aligned}
 & 7 \cdot 17 \\
 = & \{ 17 = 10 + 7 \} \\
 & 7 \cdot (10 + 7) \\
 = & \{ \text{Distribution} \} \\
 & 7 \cdot 10 + 7 \cdot 7 ,
 \end{aligned}$$

and go ahead and write the fourth link in the chain, just to get ready:

$$\begin{aligned}
 = & \{ \quad \quad \quad \} \\
 & \underline{\hspace{2cm}}, \text{ and we will finish the}
 \end{aligned}$$

fourth link by writing the next expression on the line.

Maria, excitedly, "But $7 \cdot 10 = 70$, that's easy, and $7 \cdot 7 = 49$."

"Okay, so we write in the next line, so that it looks like this:

$$\begin{aligned}
 & 7 \cdot 17 \\
 = & \{ 17 = 10 + 7 \} \\
 & 7 \cdot (10 + 7) \\
 = & \{ \text{Distribution} \} \\
 & 7 \cdot 10 + 7 \cdot 7 \\
 = & \{ 7 \cdot 10 = 70 \text{ , } 7 \cdot 7 = 49 \} \\
 & 70 + 49 \dots \text{and we write out the fifth link in}
 \end{aligned}$$

the chain to get ready, like this:

$$= \{ \quad \quad \quad \}$$

Maria says, "and $70 + 49 = 109 \dots$ "

Mr. Garza says, "So we complete the chain:

$$\begin{aligned}
 & 7 \cdot 17 \\
 = & \{ 17 = 10 + 7 \} \\
 & 7 \cdot (10 + 7) \\
 = & \{ \text{Distribution} \}
 \end{aligned}$$

$$\begin{aligned}
 & 7 \cdot 10 + 7 \cdot 7 \\
 = & \{ \quad 7 \cdot 10 = 70 \quad , \quad 7 \cdot 7 = 49 \quad \} \\
 & 70 + 49 \\
 = & \{ \text{addition} \quad \} \\
 & 109
 \end{aligned}$$

... and that chain of equations means that the product
 $7 \cdot 17$ equals 109.

Maria scratches her head in wonder and says, "But, you said the answer is 119."

Mr. Garza says, "In fact it is, yes. There is an error somewhere in the chain, but now that we have a chain, we can easily check each step, and each step is much simpler than the whole problem."

Maria immediately starts checking, "Well, certainly
 $17 = 10 + 7$, and certainly Mom's Law is correct. I didn't miswrite any numbers, as I sometimes do. Certainly 7 times 10 is 70 (that's easy!) and 7 times 7 is a square that I have memorized, I know it is 49. And $70 + 49$ is, is, uh oh, I'm not sure..."

Mr. Garza says, "Yes! You have already found the problem. Therefore, use this method to add 70 and 49..."

Mary says, "But that is very simple, you shouldn't have to

use a chain on paper for that.."

Mr. Garza says, "But Maria, you got it wrong, didn't you? So I don't think it is too simple for you right now. This method is not a religion; it is a tool. One should use it when it is needed, and if you are making too many mistakes, then obviously you need it."

Maria says, "Yes, sir. So:

$$\begin{aligned}
 &70 + 49 \\
 &= \{ \text{arithmetic} \} \\
 &70 + 40 + 9 \\
 &= \{ \text{association of addition} \} \\
 &(70 + 40) + 9 \\
 &= \{ \text{addition} \} \\
 &110 + 9 \\
 &= \{ \text{addition} \} \\
 &119
 \end{aligned}$$

is the right answer!"

Mr. Garza says, "Indeed, yes."

But Maria looks troubled and says, "However, I now have two chains..."

Mr. Garza answers, "Yes, you can create a long chain out of the two smaller chains, but you don't have to. You just have to

correct the first one:

$$\begin{aligned}
 & 7 \cdot 17 \\
 = & \{ \quad 17=10+7 \quad \} \\
 & 7 \cdot (10+7) \\
 = & \{ \text{Distribution} \quad \} \\
 & 7 \cdot 10 + 7 \cdot 7 \\
 = & \{ \quad 7 \cdot 10 = 70 \quad , \quad 7 \cdot 7 = 49 \quad \} \\
 & 70 + 49 \\
 = & \{ \text{see the other chain I made} \} \\
 & 119
 \end{aligned}$$

Maria says, "Thanks, Mr. Garza. This method seems very useful. But, it requires a lot of writing, and I've never seen the other students write that much."

"Indeed they do not, but they often mistakes, as you do right now. But you are playing a greater game than they are, are you not? You are trying to do something more than just complete your homework---you are trying to really learn."

Maria had not thought about the possibility that she could do something better than other students since before she was wounded. For months she had been doing physical therapy and studying, her "mental therapy" as the doctors called it, just to

be normal again.

Mr. Garza says, "Please, use this method I have shown you this week, and we will see if your homework improves. After that, you can decide yourself when you should use it."

"Yes sir!"

As Maria turns away from Mr. Garza, a stream of students who had been waiting politely in the hall pour into the room. Maria yearns to zoom out of the room and not face these people she had once known, but they are coming in through the only exit, so she slowly goes up a narrow side aisle. She stares straight ahead and tries to look as bored as possible. Against her will she locks eyes with a flat-faced girl with frizzy brown hair and the girl's face ignites a fire in Maria's mind.

She knows her---Clarissa. She had forgotten her, but now she remembers. This is a girl she had made of fun before the accident. The names she called the girl flood back to her now. She can see herself: a very cruel, arrogant girl, saying those things to the slower girl, and wonders who she was. Clarissa holds her gaze until Maria drops her eyes.

Chapter 29. Of Squares and Curves

As she reaches the back of the room, Bao Yung makes sure she catches her right wheel on the leg of an overturned chair,

spinning her to face Mr. Garza at the head of the class.

"You're not supposed to do things like that, Bao Yung. Let her go."

"No, it will work, you will see Grandmother."

Maria fears making a racket and drawing attention to herself, so she pretends to be interested in Mr. Garza's lesson.

"Many of you have asked for a review of parabolas. A parabola is a shape that is the graph of a quadratic function. Who can give me an example of a quadratic function?"

" $y=x^2$," says a girl.

"Precisely, that is the simplest parabola and the easiest to draw. Would you graph it for me?" said Mr. Garza, holding out a marker to the girl.

On a board like a giant sheet of graph paper, Mr. Garza tells the uncertain girl the points to plot:

(0,0)

(1,1)

(-1,1)

(2,4)

(-2,4)

(3,9)

(-3,9)

(4,16)

(-4,16)

Maria no longer needs to pretend to be interested. Bao Yung allows her to slip the wheel free.

"Now who wants to draw the curve?"

"I don't understand the math, but I love the shape," says Linda.

Maria watches fascinated as a curve of surprising grace emerges from the first girls dots and Linda's steady hand.

"What I don't understand," says a boy, "is how some thing like that comes from squares. I mean, squares are so hard and pointy and sharp."

"Well," says Mr. Garza, "in this case the word 'square' refers to the fact that the exponent of the term x^2 is two, or as we say 'the square of x'. It doesn't have anything to do with actual squares."

"But a square number is the area of an actual square, right?"

"Yes, but that has nothing to do with this graph."

"Oh, but of course it does, Mr. Garza!" says Maria.

The room clatters and screeches as everyone turns around in their chairs to look at Maria, who is turning red.

"What do you mean, Maria?"

Maria can't tell if Mr. Garza is angry or not.

"Well, I guess I mean you could draw the squares in on the graph."

"What squares?"

"The squares whose area is equal to the height of each point you plotted."

Mr. Garza looks at Maria, then at the graph, then at Maria. "You'd better show us, Maria," he says, holding out the marker there at the head of the class.

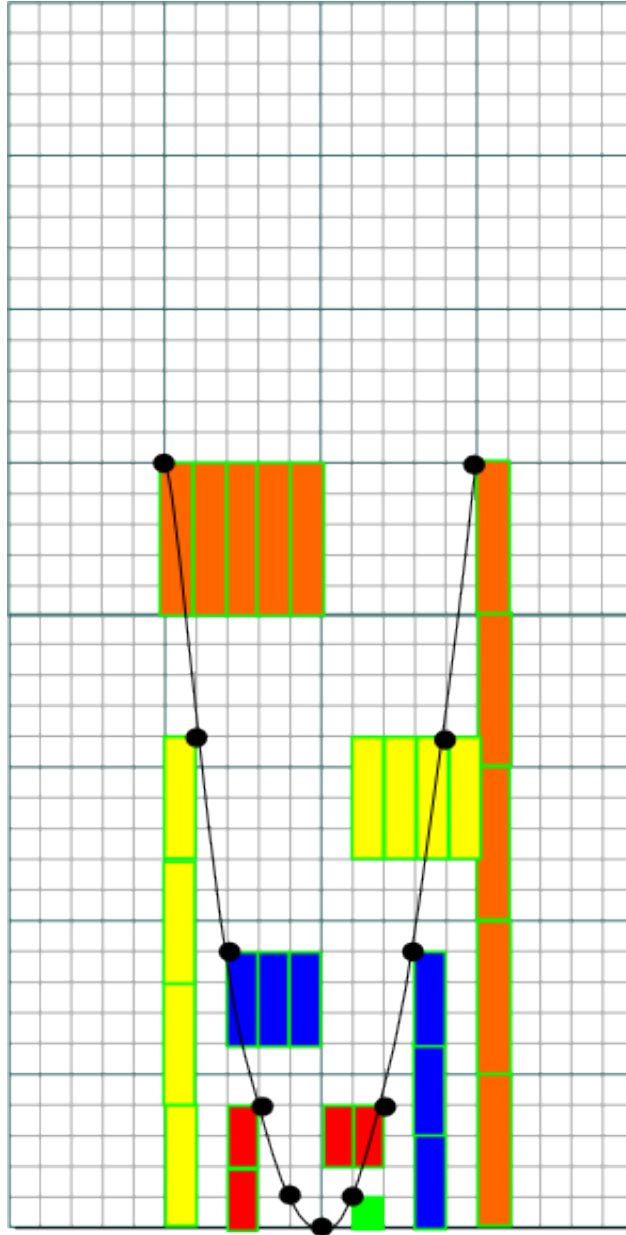
Maria has never been more afraid of anything than going to the head of the class now, except refusing to go.

As she rolls her chair down the aisle, people scoot their chairs back, either to give her room or, as it seems to Maria, to avoid being near her. Maria sees in her mind exactly what she wants to draw, "I understand squares perfectly", she thinks, "funny I have never thought of graphing a parabola before" she thinks, then the squares in her mental image disappear and she is about to scream, but Mr. Garza puts the marker in her hand the squares come back into view, just as she imagined them.

"What I mean," says Maria, "is that the height of each point is the area of a square, so if we draw a square and slice it into skinny rectangles each one square thick, then we can draw a little tower that is really a sliced up square that reaches the height of each point we plotted, and the same square unsliced

right next to it."

This is what Maria drew:



"So you can see," says Maria, "the curve is soft and pretty, but it is that way because that's how the areas of squares increase, even though the squares themselves are hard and

pointy."

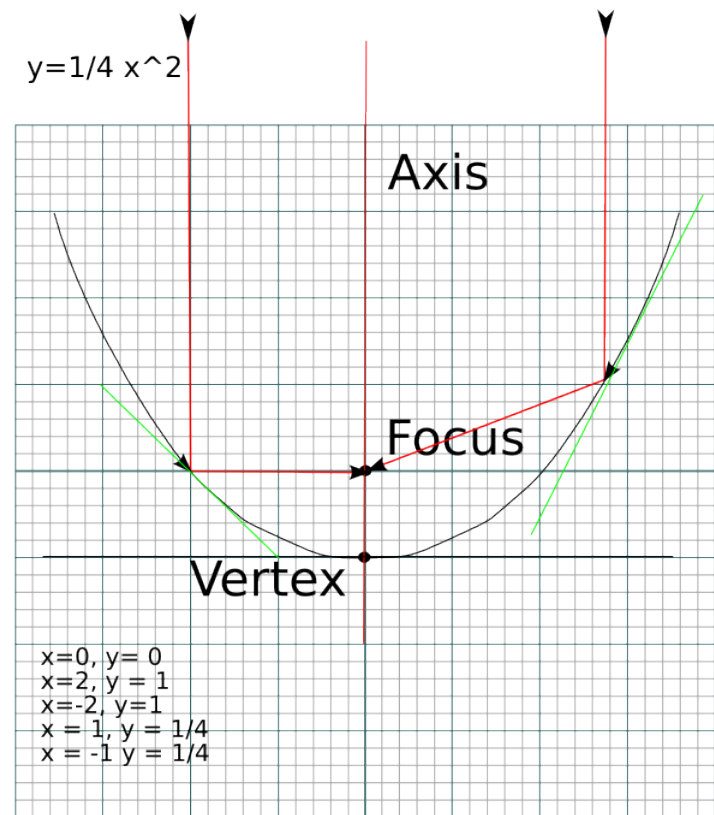
"Very good, Maria, thank you. Any questions?"

There are no questions about that, and as Maria rolls back to the back of the class, Linda flashes her a thumbs-up sign. Clarissa doesn't look at her at all.

Chapter 30. The Sun Cave

At sunrise, they were taken to a great pit cut into the hillside below the fortress. The pit, however, was not black, but silver. Its smooth walls had been lined with silver leaf. Bao Yung was afraid they would be thrown into the pit. Although the sides curved gently, it was so deep that one would have to be lucky to survive such a fall.

Neng Fa stood below the mouth of this pit. About 100 members of the Black Dragon Society looked up at him from the hill side. Neng Fa yanked a silk covering from a rectangular frame. The frame appeared to be strung with copper wires in a regular grid, and onyx beads emphasized the intersection of the vertical and horizontal wires. A graceful silver curve spanned most of the frame. Silk ribbons were threaded into the grid, apparently representing letters or numbers in some secret language. Neng Fa commenced his lesson.



"Today's lesson is on the focative power of the parabola.

Who can tell me the most basic equation for a parabola?"

" $y=x^2$," Said a man.

"Correct!" A red-bearded man with pink skin in blue silk gave the man a silver coin.

"What is the focus of such a parabola?"

"X = 0, Y = 1," said a woman.

"Incorrect!" said Neng Fa, and a black-bearded man with white skin in red silk struck her with a sword made of bamboo.

"It is $(0, \frac{1}{4})$ said Neng Fa, "because the focus is $y=\frac{1}{4}$. Why is it called the focus?"

"Because rays of light that enter the parabola are reflected to the focus!"

"Inexact! Two strokes!"

Neng Fa waited until the man had been struck twice to continue.

"Because every ray of light entering the parabola parallel to the axis is reflected to the focus, because the slope of a parabola at each point is precisely the slope needed to reflect a ray parallel to the axis to the focus. Now today is the summer solstice, and we are at 37 degrees North latitude. So the sun is at what angle in the sky at its highest point?"

"60 degrees to the North horizon!" said a man, who was given a coin.

"Correct, because the Earth's axis is tilted 23 degrees on its axis, at the height of summer the sun is 23 degrees higher in the sky than the latitude, and in winter it is 23 degrees lower. Since this is midsummer and we are at 37 degrees and $23+37=60$, the maximum height of the sun today is 60 degrees.

"Observe the silver cave. It is like a parabola, but revolved on its axis to form a shape in space, known as a paraboloid. This shape has been tilted 60 degrees so that, on this day, it matches the angle of the sun in the sky. Notice that the cave has been carefully, and with great expense that

was recently recovered from the captives, lined with silver leaf. On this day at noon, all the sun's rays will be, for a brief time, focused at a single point."

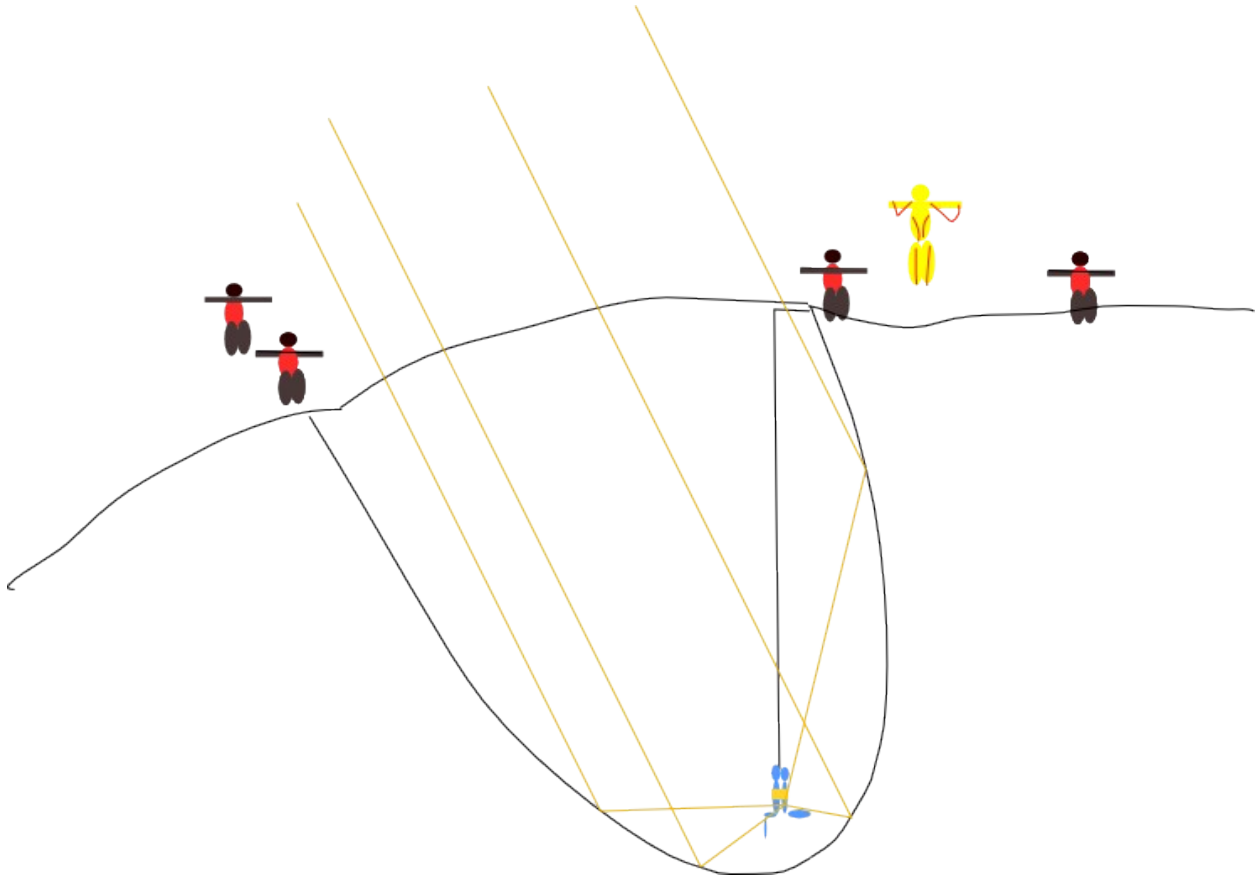
"The opening of this cave is approximately 24 meters across, and it is approximately 16 meters deep. The formula defining this shape is that the width from the wall to the axis, squared, is proportional to eight times the height. Where then, is the focus?"

"Two meters from the bottom of the pit!" cried out a tall black woman.

"Correct. Stand the miscreants up back to back and bind them around the arms and bodies with the rope between their heads so they will hang upright as you lower them down to the focus."

Chapter 31. Solar Power Abused

A plank of about 3 meters was projected over the North edge of the cave and weighted with a large green stone statue of a flying dog with a head like an octopus. The rope was attached to a ring at the end of the plank, and then Bao Yung and Shen Gun were slowly lowered.



"Grandmother, I don't understand---what's happening!"

"You heard Neng Fa. All the rays of light from the sun that enter the mouth of the cave will be reflected to focus of this pit. When a ray of light strikes a shiny surface, like a flat piece of silver, it bounces away at the same angle. A parabola changes its slope such that the reflection of all rays parallel to the axis, no matter where they hit the parabola, are aimed at the focus. How big did Neng Fa say the mouth is?

"24 meters across," says Bao Yung.

"And what is the area of a circle?"

"About $22/7$ times the square of the radius."

"Correct! And so then how much sunlight will the focus receive?"

"The radius is half the diameter, or 12 meters, and 12 squared is 144, divided by 7 is a little more than 20, and 20 times 22 is 440, so, whatever sunlight would normally land on about 450 square meters."

They were about half way down the cave now. The northern wall had curved away from them to about 4 meters, but was now getting closer.

"Right. And if we are hanging at the focus right at noon?"

"It will be as if 450 suns are shining on us? We will be burned to ash!"

"You are a smart boy, Bao Yung. Now help me think of some way out of this!"

Then they had reached end of the rope, and members of the Black Dragon Society tied off the rope to the end of the plank. They now dangled at the focus of the silver cave.

Bao Yung thought and thought, but nothing happened in his mind. Finally, he began to cry, and then to thrash about, as much as he could while tied up. Bao Yung had never had the kind of life in which he was allowed to have temper tantrums, but this tantrum saved his life, temporarily.

Shen Gun hissed, "Bao Yung, be still, I have a plan. It must snow on this cave, correct?"

Bao Yung tried to control his sobs. "Yes, it must."

"And the snow must melt, see the line down there where the silver down at the bottom is a bit tarnished?"

"Yes, Grandmother!"

"Where does the water go?"

"There must be a drain?"

"Indeed, I think I can see it."

Just as the yellow sun came into the blue disk of the mouth of the cave, the glow on the Western wall became a shine and showed the silver-covered grill on the drain at the bottom.

"That is our only hope---but it looks very small."

"You, Bao Yung, are slender enough to fit through it."

"But we cannot reach it."

Second by second, the walls of the cave were becoming brighter and brighter.

"True, but in your clever thrashing about, I believe I could feel that the plank the rope is tied to is a bit springy. Perhaps we can start bouncing, and bounce down enough to hook our feet in the grill."

"It is our only chance."

"We will wait till the sun is almost in the center of the

blue circle."

As the Earth turned, aiming the cave more directly at the Sun, the shine from the silver walls got brighter and brighter, and even the Eastern wall of the cave began to glow. Mocking faces lined the bottom of the blue disk formed by the cave's mouth.

Finally, they began to feel very warm, and had to shut their eyes tight. With their eyes shut they saw pink rather than black.

"Now!" whispered Shen Gun.

And they started pumping their legs up and down. Bao Yung, an acrobat, raised his legs into an perfect L, but Shen Gun could only raise her knees. At first, they made no progress, and they could feel the sunburn on their faces and hands. But then Shen Gun began chanting a steady rhythm, "Up-down-up-down" and the cruel laughter at the circle of the cave died.

"Master, they have found the resonant frequency of the plank!"

"One coin!" said Neng Fa. "It matters not! There soon will be enough power to slay them even as they pass up and down through the focus, however briefly.

Shen Gun and Bao Yung could well believe this, for now their faces burned as if seared by a cooking fire each time they

rose or fell through the focus.

Finally, Shen Gun's feet touched the grill. All of the Black Dragon Society were laughing hysterically, until they saw the up-down motion of the two suddenly stop at the lowest point.

The heavy grill, held by their four feet, slowly rose from its setting and they bounced gently up and down, with the focus of the gigantic paraboloid just above their heads.

Neng Fa was screaming in multiple languages, ordering people to jump into the cave, bring bows and arrows, and to haul them up all at the same time. A moment later they smelled the hemp rope holding them begin to smoke, then the rope burst into flame and broke.

Bao Yung became a wriggling tornado. No deer escaping a python every wriggled so hard. Soon the ropes worked loose and he was free. He tossed the grill aside. He measured the old woman's hips with his hands, and then, finding the hole where the grill was to be too small, smashed the bricks on either side with his fists to make it wide enough for her to pass through. He lowered the old woman, still wrapped in ropes into the hole. After about three meters the rope went slack. Bao Yung slid his feet into the hole. The men and women of the Black Dragon Society were forming a living chain, linked hand-to-hand, down the South wall of the cave to reach them. He slid in further

catching himself on his elbows. Arrows clattered past him. He feared falling on Shen Gun, so he tried to ease himself down the hole. When his elbows could no longer support him, he fell and landed on top of the old woman. He had no choice, for he had shattered his hands to save her.

The thugs of the Black Dragon Society came to the hole, but were too big to enter it.

"Should we pry up the bricks to follow them, master?"

"No---there is no need. Replace the grill and place one of the stone gargoyles that only four strong men can lift top of it. There are things in the sewers of our fortress far nastier than those two are."

After a while a terrible thud echoed through the dark tunnel, entombing them in complete darkness.

Chapter 32. Trapped in the Dark

"Bao Yung, are you hurt?"

"Yes, Grandmother, my hands are broken, but I can move them a little. Are you hurt?"

"Only a little. When we are rested we must go on."

"I will explore while you rest, Grandmother."

Bao Yung felt his way North, the only way he could go, back towards the fortress.

Soon the tunnel bent downward, and the air became moist and the floor damp and slippery. But in the absolute darkness, he began to see a glow. Coming upon it he saw it was a pool of water. The tunnel dipped into it completely. A feeble light from it told him it could not remain submerged forever, and that a light, not too far away, must shine upon it.

His heart soared. The water seemed cool and fresh and alive with gentle swirlings that played with his sight. He thought there were two particularly bright spots about a foot apart that were coming closer very slowly. Just as slowly, it dawned on Bao that he was looking at a pair of eyes, and began slowly backing up the tunnel. The eyes moved faster and now the boy could see a tail moving back and forth about three meters behind the eyes. He started backing faster, even though he should be safe from a fish, however big. The fish's great head lifted out of the water, supported by two heavy fins, and then the fish slithered forward, surprising Bao Yung with its speed. Bao Yung fell back on the slippery stone, and as the fish tossed its head from side to side to snap at him, he kicked it in the teeth. The fish slid back a few feet, and Bao crawled up a few feet. Four times Bao Yung had to drive it off, for being in air didn't seem to trouble it.

At the base of the slope the fish finally stopped, watching

him with goggle eyes and a toothy grin that seemed more than merely hungry to Bao.

"Grandmother, the way forward will be difficult," said Bao Yung.

Chapter 33. A List of Laws

A few weeks later, Guillermo is practicing a cello piece by Bach when Maria is writing out the laws that she knows:

Dad's Law (Rule of Leibniz)	You can substitute equals for equals
Mom's Law (Distribution)	$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$
Guillermo's Law (Definition of square)	$x^2 = x \cdot x$
Mr. Garza's Law	Don't pull rabbits out of hats!

"Well, those are the most fun," thinks Maria, but I guess there are some others...they aren't as much fun, but together with Dad's law, you need them in order to solve problems."

Associativity of Addition	$A + (B + C) = (A + B) + C$
Associativity of Multiplication	$A \cdot (B \cdot C) = (A \cdot B) \cdot C$
Commutativity of Addition	$A + B = B + A$
Commutativity of Multiplication	$A \cdot B = B \cdot A$

Maria stops writing to watch her brother playing. Numberless notes are rushing out his cello so fast they seemed to form a tiled mosaic in the air. When he finally takes a break, she asks, "Guillermo, how did you get so fast at the cello?"

"Thousands of hours of practice."

Maria thinks, "Well, maybe some things are that simple. I had better practice too. Now that I know algebra, there is always plenty to practice using the laws."

Maria thinks back to her trick for computing a square number from the previous square number. "I wonder if I can express that with algebra now?"

She remembers that her rule was that any square number was the previous square number, plus the previous number times two, plus one.

"I guess I can half-way write it mathematically like: (a number)² = (the previous number)² + (the previous number) X 2 + 1.

"Well," thinks Maria, "previous number isn't very mathematical. What I really mean by "previous number" is "x - 1", if x stands for the number I'm trying to square. So, if previous number = x - 1, I guess I can use Dad's Law to write a better equation: $x^2 = (x-1)^2 + 2 \cdot (x-1) + 1$."

Bao Yung says to her, "Mr. Garza would want you to do it on paper."

Maria takes a sheet of paper and writes at the top

$(x-1)^2 + 2 \cdot (x-1) + 1$, then she makes a whole bunch of equal signs

and blank lines and curly braces and then at the bottom she writes x^2 .

"So the top is my starting point, and the bottom is my target, and I bet it will take a bazillion laws to get from one to the other."

$$\begin{aligned}
 & (x-1)^2 + 2 \cdot (x-1) + 1 \\
 = & \{ \quad \quad \quad \} \\
 & \text{-----} \\
 = & \{ \quad \quad \quad \} \\
 & \text{-----} \\
 = & \{ \quad \quad \quad \} \\
 & \text{-----} \\
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 & \text{-----} \\
 = & \{ \quad \quad \quad \} \\
 & \text{-----} \\
 = & \{ \quad \quad \quad \} \\
 & \text{-----} \\
 & x^2
 \end{aligned}$$

Maria thinks, "Where to begin! The only thing I know is that I want to go from complicated at the top to simple at the bottom---but I know sometimes you have to complicate things to make them simpler."

So Maria uses Guillermo's Law.

$$\begin{aligned} & (x-1)^2 + 2 \cdot (x-1) + 1 \\ &= \{ \text{Guillermo's Law} \} \\ & (x-1) \cdot (x-1) + 2 \cdot (x-1) + 1 \end{aligned}$$

Maria thinks, "Well, there's no point in going back now, so I guess I should go on. That means multiplying things out more using Mom's law:

$$\begin{aligned} & (x-1) \cdot (x-1) + 2 \cdot (x-1) + 1 \\ &= \{ \text{Mom's Law (Distribution of 2 over } x - 1) \} \\ & (x-1) \cdot (x-1) + 2 \cdot x - 2 \cdot 1 + 1 \\ & \dots \text{but I can use Mom's law one more time.} \\ &= \{ \text{Mom's Law (Distribution } x - 1 \text{ over } x - 1) \} \\ & (x-1) \cdot x + (x-1) \cdot -1 + 2 \cdot x - 2 \cdot 1 + 1 \end{aligned}$$

Maria thinks, "Funtoosh! That doesn't look like progress.

At least I can add the $-2 \cdot 1$ and 1 together!"

$$\begin{aligned} & (x-1) \cdot x + (x-1) \cdot -1 + 2 \cdot x - 2 \cdot 1 + 1 \\ &= \{ \text{Arithmetic} \} \\ & (x-1) \cdot x + (x-1) \cdot -1 + 2 \cdot x - 1 \end{aligned}$$

Maria thinks, "That's a little better, and well worth the time it takes to write it out, but oh, how my hand hurts. I thought it might be hard, but this is as hard as a stone! What if I have made a mistake? One single mistake would make it

impossible to finish."

"Maria?" asks her mother. "I have been calling you. There is a phone call for you."

Chapter 34. The Weakness of a Fish

"What then is the weakness of a fish?" mused Shen Gun.

"Fire, but we have no fire. Poison, but we have no poison. A spear, but we cannot make one. Air. No, not air itself, but the lack of water. Dryness! Dryness is its weakness!"

In the complete darkness, Shen Gun began slowly grinding bricks together to make a fine dust. After hours of work she had a large pile.

"Bao Yung, if it escapes and we cannot draw it from the water we are doomed. If it tries to reach the water, you must grab it, or jump on it, or stop it somehow. If I get a chance, I will bash its head with a stone."

Shen Gun took off her jacket and filled it with dust and a heavy stone, and marched resolutely down the corridor with one hand on the wall.

Halfway down the slope they went, then Shen Gun began stamping and gently calling to the fish.

"It is coming, Grandmother!" whispered Bao.

"How far?" whispered Gun.

"3 meters. 2 meters? Can't you see it?"

"Just tell me where it is."

"1 meter. It's right there!"

"In front of me?"

"Right in front of you! Look out!"

As the fish slithered forward, Gun threw handfuls of dust in its direction. Bao blocked it by kicking it, but it snapped at his leg, and its sharp teeth became entangled in the cloth of his pants.

The fish was tremendously strong and began hurling the boy from side to side, striking Shen Gun in the legs. Shen Gun toppled, spilling her jacket directly onto the fish's head.

The fish began to shudder and cough. Shen Gun got to her knees and brought the stone down with both arms again and again until the fish lay completely still.

"Well done, Grandmother!" said Bao.

After she had caught her breath, Shen Gun said, "Time to eat!" She rubbed the scales off with the stone, and then bruised the flesh with the stone until she could tear off pieces with her fingers and feed them to the boy.

After eating a little of the raw chewy flesh, Shen Gun

said, "Oh, so it was only an uncommonly large snakehead fish after all, like the kind I used to buy in the market!"

Chapter 35. Completing a Proof

"Hello?" says Maria. Nobody except relatives has called her since her accident.

"Hi, Maria?" says Linda. "I just wanted to let you know I thought it was pretty cool what you showed Mr. Garza and they way you stood up to that bitchy snot Antionette. Listen, I know you don't have a lot of music, would you like to come over on Saturday and listen to some music? If your Mom can get you here?"

Maria says, "Yes", she would, "very much!", and feels that a golden beam of light rains down on her and lifts her chair a few inches off the ground.

After a while, she wheels herself back to her desk. She looks at where she left off.

Bao Yung says to Maria, "Don't give up hope just because it is a lot of writing. You have made great progress already!"

Maria thinks, "I have at least simplified one thing, I guess I should try to continue. Now I guess I have to use Mom's law again---no, again twice!"

$$(x-1) \cdot x + (x-1) \cdot -1 + 2 \cdot x - 1$$

$$= \{ \text{Mom's Law } x \text{ over } x - 1 \}$$

$$x^2 + -1 \cdot x + (x-1) \cdot -1 + 2 \cdot x - 1$$

$$= \{ \text{Mom's Law } -1 \text{ over } (x - 1) \}$$

$$x^2 + -1 \cdot x + x \cdot -1 + -1 \cdot -1 + 2 \cdot x - 1$$

"Goodness, it would be easy to make a mistake here, I'm glad I'm writing out every step. So now at least I can do some arithmetic to simplify it, and try to stick the x's together."

$$x^2 + -1 \cdot x + x \cdot -1 + -1 \cdot -1 + 2 \cdot x - 1$$

$$= \{ \text{Mom's Law (in reverse!)} \}$$

$$x^2 + (-1 + -1) \cdot x + -1 \cdot -1 + 2 \cdot x - 1$$

$$= \{ \text{Arithmetic} \}$$

$$x^2 + (-2) \cdot x + 1 + 2 \cdot x - 1$$

"Oh, look, I can add 1 and -1 together to get 0. I guess it is always better to simplify when you see an opportunity like that."

$$x^2 + (-2) \cdot x + 1 + 2 \cdot x - 1$$

$$= \{ \text{Arithmetic} \}$$

$$x^2 + (-2) \cdot x + 2 \cdot x + 0$$

"Well this is looking a lot better...and now the X's also cancel out to become zero!"

$$= \{ \text{Arithmetic, } -2X + 2X = 0 \}$$

$$x^2 + 0 + 0$$

Maria can barely believe it when she sees the two zeros like

eyes staring wide at her. It is just one more step until she reaches the target.

$$x^2+0+0$$

$$= \{ \text{Arithmetic} \}$$

$$x^2$$

"Bullseye!" thinks Maria. She goes back and checks every step. "Not a single rabbit," says Maria. Maria leans back in her chair. She has never felt so lucky and so happy. "It must be magic that is helping me," she thinks.

"Hey! I didn't know you could do stuff like that, Maria!" says Guillermo, looking over her chair onto her paper.

"I didn't either," says Maria.

"I did," says Bao Yung.

Chapter 36. Rectangles

When they had enough strength and courage to swim the little pool and Bao Yung helped Shen Gun follow the glow in the water under the submerged stone archway they found that the corridor continued, but rose upward out of the pool and became dry. A bright yellow light was emitted by some chamber ahead of them.

"Grandmother!" whispered Bao Yung, moving ahead of her and blocking her with his arm, "That light may be an escape, but it

may be a danger!"

"What light, Bao Yung? We are still in total darkness."

The light seemed quite strong to Bao Yung. He waved his aching hands in front of the old woman's face, and then the ache in his heart made him forget his hands. She had not moved even a muscle in her face, not even her eyes.

"Grandmother, the concentrated sunlight must have dazzled your eyes. The light here is strong."

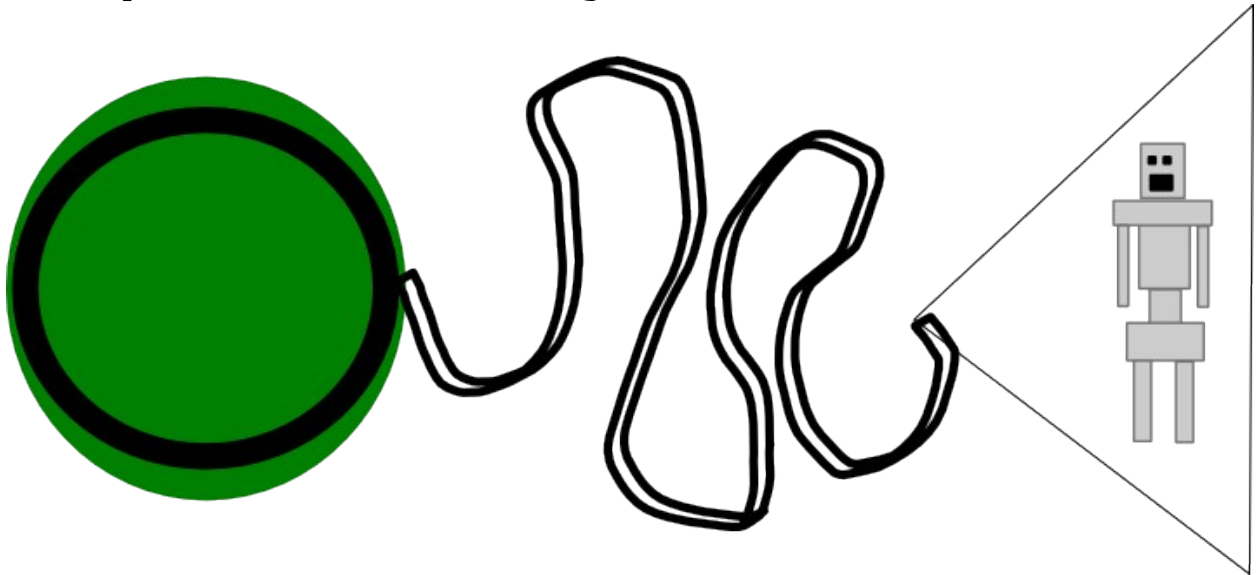
Shen Gun said nothing.

After a while, Bao Yung said, "There is a chamber ahead of us. It may be an escape, but it may be danger. We will sneak up there quietly."

Shen Gun put her hand on the boy's shoulder, and slowly they advanced into an empty chamber. The room measured eight meters wide, and eight meters high, and twenty-four meters long. Glass jars that contained a brightly glowing yellow liquid hung on chains from the ceiling to a height of four meters. The floor near their entrance contained a great circle spanning wall to wall made of close-set onyx tiles containing writing of a kind Yung had never seen inlaid in copper so old it had rusted to green. The inside of the circle was a huge slab of malachite.

Beyond the green circle lay a great chasm crossed by a

narrow path winding like the wriggling of a snake. If the pits on either side of the path had a bottom, they could not be seen in this light. The path led to a great equilateral triangle on the floor that pointed at the center of the circle. The triangle was a red mineral, perhaps cinnabar, and inlaid inside this triangle was a great blocky figure made of a gray metal that looked like tin. Beyond the triangle was a passage that would briefly flicker with a blue light.



As they moved further into the room, Yung could see the figure better and better and saw that it was the figure of a man, so much as one could make a human shape out of nothing but rectangles. The chest was a square set on a smaller square that was like a belly, and great long rectangular legs ending in blocky square feet came out of square hips. Great unjointed

rectangular arms paralleled the torso from square shoulders.

His head was a great square.

His eyes where two great square holes, set above a great square mouth that seemed to be permanently screaming.

As they advanced to the center of the circle, Bao Yung felt a deep grating vibration, like a heavy ship striking a dock. As he and Shen Gun stopped, the figure began to rise.

The tin man rose and rose right out of the floor, thicker than Bao Yung was tall. Bao Yung thought the mechanical man must be made of lead, rather than tin, for his rise seemed almost to push the floor down.

Then the great face, with eyes like holes into midnight, turned toward them.

Bao Yung screamed and pulled Shen Gun backward, but the old woman resisted his pull.

"What is it, Bao Yung? Why do you carry on so?"

"Grandmother we must flee! It is a monster!"

But now it was a monster who was standing, and each slow movement seemed to Bao Yung like small castle or a great bridge falling. The great man stood as tall as any two men, and must outweigh a herd of camels. It stomped toward them like some great iron-bound engine of war. It cared nothing for the

serpentine path, but seemed to float in space, except that each step shook the floor.

Bao Yung was so afraid that he let go of Shen Gun's hand and ran away, back across the circle. The old woman kept walking, groping with her arms in front of her, and calling his name.

Then, before she had quite gotten to the edge of the black circle, Bao Yung saw the huge cubical head of the lead man fall like a great hammer onto the figure of Shen Gun. The great mouth engulfed her, and the blow of the face striking the floor knocked Bao Yung off his feet and shook dust from the corridor walls.

When Bao Yung scrambled to his feet, Shen Gun was lying on the ground right at the edge of the circle, trying to get up.

"Bao Yung, return to me this instant or I shall have you whipped for an hour!" said the old woman.

Forgetting his fear, Bao Yung ran to her.

"Grandmother! What happened? The monster man fell right on top of you!"

"What? Nothing has happened to me, except that you scared me nearly out of my wits, and floor shifted a bit and I slipped down. What was that thumping noise?"

As Bao Yung turned to look at the triangle, he once again saw the mechanical man rising from his place in the floor. Bao Yung took hold of Shen Gun's hand, saying "Grandmother, forgive me for leaving you. There is some trick or illusion in this chamber. Take my hand and follow me."

Then Bao Yung closed his eyes tightly and, holding one hand up to Shen Gun, knelt to feel the edge of the path with his other hand. Together they walked and crawled right through the massive man of metal and into the next chamber.

Looking back into the room, Bao Yung was surprised to see that it did not seem rectangular at all from this point of view, but gracefully curved. Instead of stone walls, it appeared to have walls of silvered glass that reflected the strange floor in odd distortions.

Chapter 37. Insights

The grandmother fairy, Shen Gun, says, "Maria, are you ready to understand the paper you keep in your pocket?"

Taking out her paper, Maria says "It is time to think about this mysterious paper I wrote so long ago."

Shen Gun asks, "What is 5.7 ?"

Maria thinks " 5.7 equals 35 , and that is what I wrote. Apparently, the second column and the third are multiplication."

The grandmother fairy suggests, "Why did you spend the time and space for the first and fourth columns?"

Maria thinks, "However, what did I mean by the first column?"

Shen Gun suggests, "Use your new know-how to represent it with algebra."

Maria continues, "If I use X for the first column, the table means:"

X	$X - 1 \ \& \ X + 1$?	1
-----	----------------------	---	---

Shen Gun put in, "How clever!"

Maria says, "Look, that row captures the whole table---and concisely!"

The grandmother fairy says, "You wrote a question mark, but you already know the meaning of the third column."

Maria slowly says, "Well, I could also, and more precisely write:

X	$X - 1 \ \& \ X + 1$	$(x-1) \cdot (x+1)$	1
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That is clearer, ...perhaps...but now, clearly the second column is unnecessary...maybe I should draw a diagram."

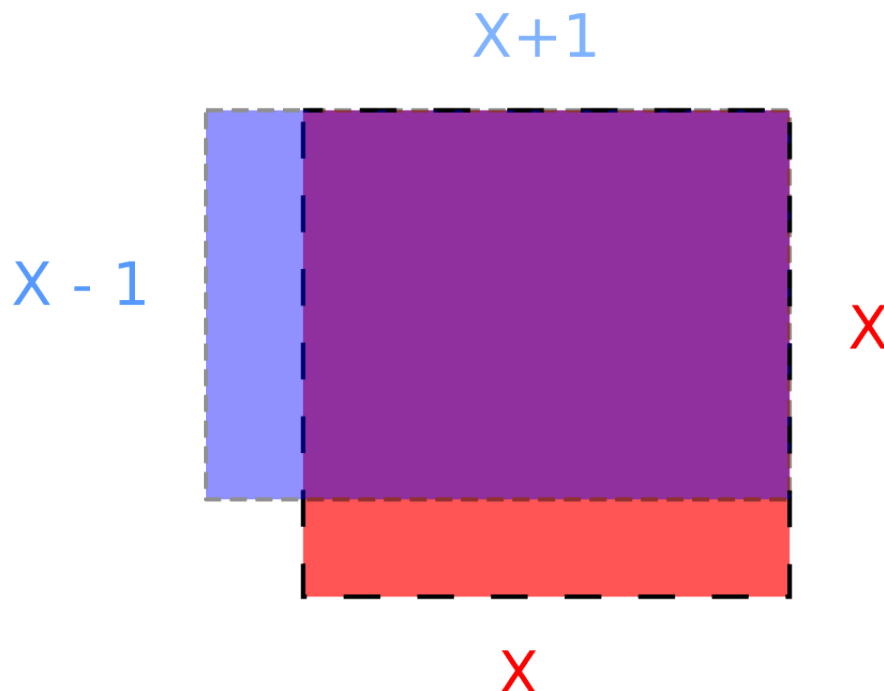
Shen Gun places her hand on Maria's hand and whispers, "Let me guide you. Draw a blue rectangle that is $X+1$ long and $X-1$ high."

Maria draws a rectangle and labels the long side $X+1$ and the shorter height $X-1$.

Now the grandmother fairy says, "But X is less than $X + 1$, and greater than $X - 1$, so if you drew a square area of size X in the same space, what would it look like?"

Maria draws in a red square that shares one corner with the blue rectangle. The diagram fascinates her. It seems to pulse with possibilities and hidden meanings. The red and blue color seem to vibrate. She likes the purple they make where they overlap.

The fairy says, "Now relate that to your table."



Maria thinks, "Well, the part colored blue (and purple!) is $(x-1) \cdot (x+1)$, and the part in red (and purple!) is x^2 ."

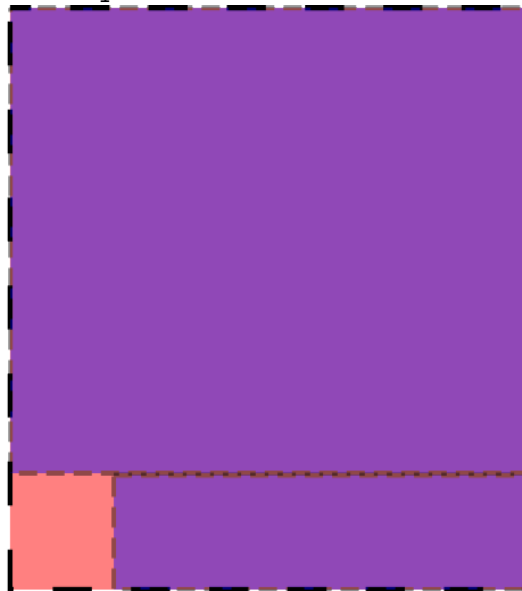
Shen Gun, the ancient scholar, uses all her magic.

"Maria, what would happen if you cut out the blue strip and laid it over the red?"

Maria thinks to herself, "Well, I think the blue would make everything purple, except maybe for a little square."

And then, without any more prompting from Shen Gun, Maria takes a pair of scissors and cuts out the blue strip and lays it over the red strip, and carefully traces it and then fills in the tracing with her blue crayon.

$X - 1$



Shen Gun says, "Very good! Now you have just the same amount of blue as before. How much red is left?"

Maria thinks, "Well, this is quite funny. What is now purple has an area of $(x-1)(x+1)$, and it is almost but not quite x^2 . I wonder how different it is?"

Now it is Shen Gun's turn to be excited. "Very, very good! Now how big is that little square of red?"

Maria thinks, "Well, that little red square could be anything."

Shen Gun says, "Could it really be 2 by 2?"

Maria lays the strip she cut back over it. "But actually, it must be exactly 1 by 1, because adding it in must make a strip $x-1$ long exactly equal to x !"

Shen Gun says, "Look at your paper."

The Mysterious Paper:

1	0 & 2	0	1
2	1 & 3	3	1
3	2 & 4	8	1
4	3 & 5	15	1
5	4 & 6	24	1
6	5 & 7	35	1
7	6 & 8	48	1
8	7 & 9	63	1
9	8 & 10	80	1
10	9 & 11	99	1

Maria sees that she had repeated that little 1 in the last column every time. "Why would I have written something so

boring as the number 1 on each and every line if it never changed?" thinks Maria.

"Because you were trying to prove that something always came out to 1."

Maria thinks, "Well, my picture says that x^2 is always 1 greater than $(x-1) \cdot (x+1)$, or, mathematically, $x^2 - (x-1) \cdot (x+1) = 1$. Can that be right, I wonder?"

Shen Gun asks, "What use is that equation?"

Maria thinks, "Well, if we think of this equation as $(x-1) \cdot (x+1) = x^2 - 1$, this is a good way to multiply two numbers that differ by exactly two, if you already know the square in the middle. And it is always good to know the square numbers."

Shen Gun, "You mean like 9 and 11?"

Maria stares at her drawing and the mysterious paper. She sees 9 and 11 are the last numbers she wrote. The highest square she knows is $10^2 = 100$. She knows 9 and 11 differ by 2. She is so excited that after a while her hands start to hurt from gripping the arms of her wheelchair.

"Could it be that that is the law that meant so much to me back then?"

Shen Gun suggests, "Try some values."

Maria, writing carefully because she is trembling with

excitement, writes $1=4\cdot4-(3\cdot5)=16-15=1$ ---YES! $1=7\cdot7-(6\cdot8) =$

$49-48=1$, indeed! Now I'm doing the exactly same thing that I wrote on the paper before the explosion!"

Shen Gun says "Yes, you are!"

Maria, sitting back, thinks, "So I have found my law---
Maria's law."

Chapter 38. The Dragon's Belly

In the occasional flicker of light in the long, low room, they saw an enormous wooden shaft ran the breadth of the room. It was rapidly turning, evidently driving a system of brass gears on the right that translated the slow, heavy rotation into a rapid one.

They could hear Neng Fa above the gear noise.

"Behold, the last copy of the seventh book by Aeschylus, taken from the library of Alexandria itself! Now it will be lost to the world, and known only to us!"

With that they heard a great clattering and sliding, until the scroll came to rest above a little iron door above them. Moving a wooden step ladder beneath the door they opened it to see the scroll of Aeschylus burning on a raised grate of thick, parallel iron bars. They perceived now that this must be the belly of the First Master. Soon another tremendous crackling of

blue electric fire burst from the grate to the top piece of the dragon sculpture, and even the wooden rod that held papyrus scroll by Aeschylus was consumed by fire.

"Grandmother, we must stop this!"

Shen Gun leaned her forehead against the top step of the ladder.

"Grandson, I am exhausted. I am an old blind woman, in the fortress of a thousand murderers with no way out below us. You have been very brave, but your hands are broken. Even if we were to escape, we would be ten thousand feet of ice and stone above anyone who might help us."

"Grandmother, I am also tired. But let us not sit here helpless as the best books in the world are burned. I will tell you where they are and you can try to pluck them through the grate.

"Why would we do that, Bao Yung? They are bound to find us eventually."

"We will need something valuable with which to pay people for food and help when we escape, Grandmother!"

At that Shen Gun began to cry, and then to laugh, and then climbed up, until she could put her hands through the grill.

Neng Fa triumphantly shouted, "The History of the Second

Blue Ascension of Atlantis!" Then the heavy codex, made of leaves of tarnished silver with letters of red lacquer bound in orichalcum, clanged down the dragon's throat. Shen Gun, with Bao Yung steadying her legs on the ladder, found it with her groping hands and turned it to fit through the grate, but it became stuck because the cover was decorated with a raised boss of some great lady or goddess rising from a lake. Before she could work it through the blue lightning flashed again.

With no pain at all, Shen Gun and Bao Yung were set free of the material world, and remained bound only by their vows.

Chapter 39. The Clever Girl Victorious

Bao Yung asks, "Maria, do you still feel stupid now?"

Maria thinks, "So now we have Mom's Law, and Dad's Law, which is fundamental, and now Maria's Law: $x^2-1=(x-1)\cdot(x+1)$, at least for all the numbers that I have tried. But why should I believe that Maria's Law is really true?"

Bao Yung asks, "Can you prove it?"

Maria thinks, "Based on the diagrams I have already drawn, it should not be so hard to make a logical chain that proves my law without pulling a rabbit from a hat:

$$\begin{aligned} & (x-1)\cdot(x+1) \\ &= \{ \text{distribution of } (x-1) \text{ or } (x+1) \} \end{aligned}$$

$$\begin{aligned}
& x \cdot (x+1) + -1 \cdot (x+1) \\
= & \{ \text{distribution of } x \text{ over } (x+1) \text{ and } -1 \text{ over } (x+1) \} \\
& (x \cdot x + x \cdot 1) + (-1 \cdot x + -1 \cdot 1) \\
= & \{ \text{arithmetic } -1 \cdot 1 = -1 \} \\
& x \cdot x + x \cdot 1 + -1 \cdot x + -1 \\
= & \{ \text{association of addition allow parentheses to be moved} \} \\
& x \cdot x + (x \cdot 1 + -1 \cdot x) + -1 \\
= & \{ \text{canceling equal positives and negatives} \} \\
& x \cdot x + 0 + -1 \\
= & \{ \text{arithmetic} \} \\
& x \cdot x - 1 \\
= & \{ \text{definition of square} \} \\
& x^2 - 1
\end{aligned}$$

Maria thinks, "So now I am certain it is really a law. It is a law stronger than a horse, as strong as any other law."

Bao Yung asks, "When is your law useful?"

Maria thinks, "My law, Maria's law, is not very useful, although it is very beautiful. You can only use it when you need to multiply two numbers that differ by 2."

"Like the bricks in the train station."

Maria says, "For example, those patched bricks were 21 rows by 19 columns or $19 \cdot 21$, so if I know $20 \cdot 20$, then I would know

the answer, but hey, that's easy, 20 squared is 400!"

"Yes, so how many bricks were replaced?" asks Bao Yung.

Maria says, " $400-1=399$. Imagine $19\cdot21=399$ --- and you hardly have to multiply to know it!"

"Those bricks have taught me something, though it took me a long time to understand it fully. I may never walk well. I will never be the same girl I once was. But with enough work, I can learn whatever I want to learn, just as well or better than before. I know who I am now. I am and have always been Maria, and I love math."

Bao Yung says, "And we love you, Maria!"

"Come, Grandson. Maria can heal herself now. Others need us."

END