

Annual Return

If you are considering investing in a mutual fund, you will look up the fund and find the average annual return of the fund for the last 1 year, the last 5 years, and the last 10 years. However, these are surprisingly deceptive measures of fund performance.

A much better measure is a matrix that gives the average annual return $A(y_b, y_s)$ for an investment in the fund bought in year y_b and sold in year y_s , for all $y_{min} \leq y_b \leq y_s \leq y_{max}$, where y_{min} and y_{max} are at least 20 years apart. This matrix answers questions of the form: if I bought in year y_b and sold in year y_s , what would my average annual return be.

From this you will find that there are some good years to buy and some bad years to buy, and some good years to sell and some bad years to sell. But if the average annual return is good enough for you for most purchase years and most sell years, perhaps the fund is a good investment.

You are being asked to compute such average annual return matrices.

To compute $A(y_b, y_s)$ we need to know the value v_b of some investment in year y_b and the value v_s of the same investment in year y_s . Then v_s/v_b is the total return as a ratio, and if $v_s/v_b = B^{(y_s - y_b)}$, then B is the average annual return as a ratio. But we do not want a ratio; we want a percentage. So we compute $A(y_s, y_b)$ so that $B = 1 + A(y_s, y_b)/100$.

Input

The first line is a title line, that names the fund. It is at most 80 characters.

After the title line there are $n \leq 30$ lines, with the i 'th line containing two numbers, y_i and v_i for $1 \leq i \leq n$. The y_i are year numbers, and are integers, $0 < y_i \leq 9999$, such that $y_{i+1} = y_i + 1$ for $1 \leq i < n$. The v_i are value numbers, and are floating point such that $0 < v_i \leq 1,000,000$.

The value numbers are the value of some investment made in the fund before year y_1 , assuming that once the investment was made no more money was ever put into the fund and no money was ever taken out. More specifically, any money made by the investment was automatically reinvested in the fund, which is a typical thing to do with mutual funds.

Its not important how big the initial investment was or how long ago it was made, as the only thing we are interested in is ratios v_j/v_i for $j > i$.

Input ends with an end of file.

Output

First output the title line, exactly as it is in the input.

Then output a matrix of n rows each with n columns, with each row on one line, and each column taking exactly 6 spaces.

For the first row, its first column is blank, and the $k + 1$ 'st column contains y_k for $1 \leq k < n$.

For the $j + 1$ 'st row, $1 \leq j < n$, the first column contains y_{j+1} , and the $k + 1$ 'st column contains $A(y_k, y_{j+1})$ for $1 \leq k \leq j$.

All numbers must be right adjusted in their 6 space wide column. The A numbers must have exactly 2 decimal places.

Sample Input

```
CONSTANT
1990 1.0000
1991 1.0001
1992 1.0002
1993 1.0003
1994 1.0004
1995 1.0005
```

Sample Output

```
CONSTANT
      1990  1991  1992  1993  1994
1991  0.01
1992  0.01  0.01
1993  0.01  0.01  0.01
1994  0.01  0.01  0.01  0.01
1995  0.01  0.01  0.01  0.01  0.01
```

Author: Bob Walton <walton@acm.org>

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