Equivariant Systems Theory and Observer Design for Autonomous Systems

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Outline

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Overall goals

This week-long course has four primary goals:

Goal 1: To provide students with an introduction to matrix calculus and the mathematical tools to work with matrix dynamical systems.

Goal 2: To provide a rigourous introduction to equivariant systems theory and contextualise that with examples from autonomous systems.

Goal 3:. To teach students the skills needed to go about actually designing observers for equivariant systems.

Goal 4: To provide an introduction to the state-of-the-art equivariant systems filters and observers in the literature at the moment.

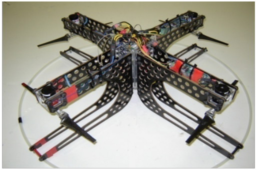
Presenters

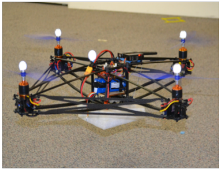
Robert Mahony: Professor at the Australian National University. My primary research interest is in the systems theory of robotic systems. In particular, I am interested in understanding and exploiting the fundamental structure of systems.

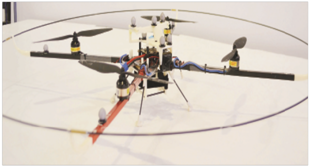
Tarek Hamel: Professor at the University of Nice. His primary research interest is in nonlinear systems theory of robotic vehicles. He has done extensive work on control and observer design for aerial robotic systems.

Jochen Trumpf: Professor at the Australian National University. He is a specialist in observer theory. His interest is to develop the fundamental system theory of observer design that goes beyond linear systems.

Quadrotor Aerial Vehicles







Outline

Why are observers important 1

- Autonomous Robotic vehicles are highly dynamic systems with low actuation to inertia ratios.

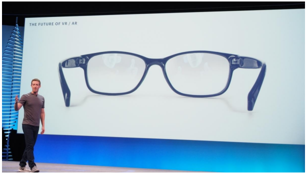
  High performance control must plan, anticipate and act early: $\rightarrow$ Feedforward.

- Feed-forward control requires system modelling and good state estimation.

Why are observers important 2

- State estimates are also critical for payload modules
  - Payload systems require both *accuracy* as well as *precision* of a state estimate.
  - Attitude estimation specifications for payload requirements are often far more restrictive than the requirements for control.
  - State estimation for payload operation can take advantage of payload sensors - cameras, radar, hyperspectral cameras, etc.
  - State variable requirements may be different than for the vehicles, e.g. tracking image homographies for image rectification, mosaicing, etc.

Why is observer design important 1



- Typical robotics applications involve unstructured environments.
- Limited payload leads to limited sensing capability.
- Limited payload also limits computing power!
    - Monte Carlo particle filters are not feasible to implement, nor is solving global solutions of Hamilton-Jacobi-Bellman equations in real time.
    - Small robotic systems and consumer electronics like Virtual reality very limited compute power available.

Why is observer design important 2

The natural state spaces for autonomous vehicles estimation are *not* vector spaces. **Requires nonlinear analysis**

- Estimators should have global (or almost global) basins of attraction. (most control loops do not need the same global properties).
- Estimators must be robust to system modelling error.
- Estimators should run 10-100 times faster than control loops. Need to be simple.
- Estimators must be reliable. Either simple algorithms and code that is easily verified or lots and lots and lots of testing.

There are a number of different approaches or philosophies that can be taken for observer design for autonomous robotic vehicles. The reason that we want to present this course is because we believe in the particular philosophy that we will be presenting.

Avionics

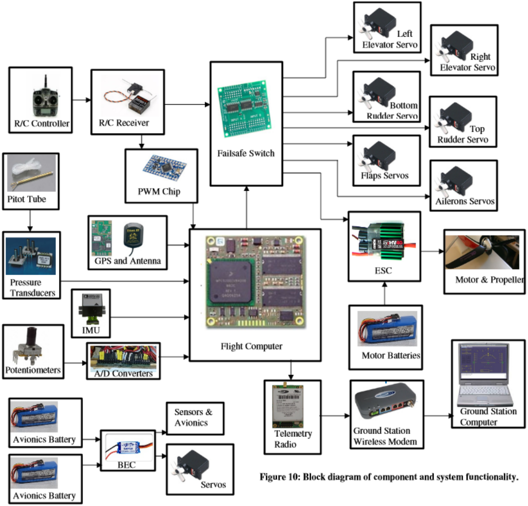


Figure 10: Block diagram of component and system functionality.

One of the key and motivating applications is avionics for Remote Piloted Aerial Systems (RPAS).

DfR solutions
`http://www.dfrsolutions.com/white-papers/improving-unmanned-aerial-vehicle-uav-reliability/`

Sensor characteristics 1

Sensors on aerial robotic vehicles are subject to harsh operating conditions:

- Vibration of the airframe.
- Temperature variation.
- Onboard communications protocols.
- Supply power noise and variation, particularly if the main battery is used for avionics and motors.
- Pressure changes.
- High temperatures in the avionics unit.
- Significant time varying magnetic fields due to power cables, etc.

Sensor characteristics 2

The resulting measurements have particular characteristics that pose specific challenges for observer design

- High noise levels: accel, gyro
- Large time varying bias: accel, gyro, baro, magneto
- Low sample rates and delays: GPS and vision
- Missing measurements: optic flow, vision
- Asynchronous measurements: vision, GPS, pitot tube
- Signal quantisation and resolution: Motor power sensors, pressure, transducer, optic flow.

These characteristics make the observer problem difficult.

Why are aerial robotic systems special?

Autonomous robotic systems have a number of special properties that are often under exploited and under utilised in developing state estimation.

The state of typical robotic vehicle is associated with an attitude and position of the rigid-body.

- **Attitude:** The relative orientation of the body-fixed-frame to a reference.
- **Position:** The location of the body-fixed-frame origin in a reference frame.
- **Pose:** The combined attitude and position.

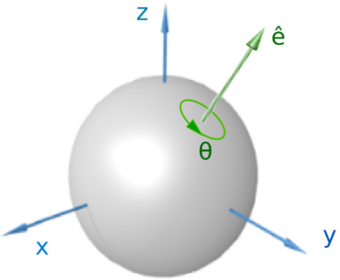
These state spaces have significant structure associated with the symmetry of rigid transformations of space. We can exploit this symmetry to obtain highly robust observers that address the challenges of the sensing paradigms.

Equivariant Systems Theory

Equivariant System Theory: is the theory of non-linear **control** systems that submit to a symmetry.

A symmetry of the sphere is a rotation $Q(\theta) : S^2 \to S^2$

$$\eta \mapsto Q^\top \eta$$



Equivariant Systems Theory

Symmetry of a dynamical system is a property of the defining equations of motion of a system.
Direction kinematics for $\eta \in \mathrm{S}^2$

$$\dot{\eta} = -\Omega \times \eta$$

This system is equivariant

$$\frac{\mathrm{d}}{\mathrm{d}t}(Q^\top \eta) = Q^\top \dot{\eta} = Q^\top(\Omega \times \eta) = (Q^\top \Omega) \times (Q^\top \eta).$$

New system. New state $\eta' := Q^\top \eta$, new input $\Omega' := (Q^\top \Omega)$.

$$\dot{\eta}' = -\Omega' \times \eta'$$

Input transformation: $\Omega' = Q^\top \Omega$

Invariance versus Equivariance

Consider a system on a manifold $\mathcal{M}$ as a physicist would think

$$\dot{\xi} = f(\xi)$$

Imagine a symmetry $\phi : \mathcal{M} \to \mathcal{M}$. Then the system is **invariant** (to ϕ) if

$$\mathrm{D}\phi|_{\xi}\, \dot{\xi} = f(\phi(\xi))$$

Consider a **control** system, with inputs $u \in \mathbb{L}$ on a manifold $\mathcal{M}$

$$\dot{\xi} = f(\xi, u)$$

Then the system is **invariant** (to ϕ) if

$$\mathrm{D}\phi|_{\xi}\, \dot{\xi} = f(\phi(\xi), u)$$

The system is **equivariant** if

$$\mathrm{D}\phi|_{\xi}\, \dot{\xi} = f(\phi(\xi), \psi(u))$$

for some **input transformation** $\psi : \mathbb{L} \to \mathbb{L}$

Outline

Sessions

Trondheim	Equivariant Systems Theory and Observer Design for Autonomous Systems				
	Robert Mahony, Jochen Trumpf and Tarek Hamel				
M16	17/06/2022	18/06/2023	19/06/2022	20/06/2022	21/06/2023
Time	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
9:00-9:30		**THEORY - RM**	**CASE STUDY - JT**	**THEORY - JT**	**THEORY - RM**
9:30-10:00	Registration	Foundations	Homography on SL(3)	Lie Theory and Symmetry	Kinematic Systems
10:00-10:30	Coffee	Symmetry and state			
10:30-11:00	**INTRO - RM**	break	break	break	break
11:00-11:30	**THEORY - JT**	**CASE STUDY - TH**	**PRACTICAL - TH**	**CASE STUDY - TH**	**THEORY - JT**
11:30-12:00	Matrix Calculus	Attitude on SO(3)	observer design	Velocity aided attitude	Observer design
12:00-12:30	Matrix ODE		for homography		Equivariant Filter
12:30-13:00					
13:00-13:30					
13:30-14:00	Registration				
14:00-14:30	**PRACTICAL - TH**	**THEORY - JT**		**PRACTICAL - TH**	**CASE STUDY - RM**
14:30-15:00	Matrix Calculus	Numerical implementation,		Velocity aided attitude	INS, VIO.
15:00-15:30		bias, time delays			
15:30-16:00	break	break		break	
16:00-16:30	**PRACTICAL - TH**	**PRACTICAL - TH**		**THEORY - RM**	
16:30-17:00	Motivated examples	Observer design SO(3)		Tangent symmetries and bias,	
17:00-17:30				Outer symmetries and group	
17:30-18:00				affine system	

Workload

- 10.5 hours of theory
- 6 hours of Case studies: Attitude, Homography, Velocity aided attitude, INS, VIO.
- 8.5 hours of practical work: Matrix calculus, Attitude observer, homography observer, Velocity aided attitude observer.
- 6 hours of self directed work.
  - Matrix Calculus exercises due Tuesday morning.
  - Attitude observer code: due Thursday morning
  - Homography code: due Thursday morning
  - Example questions for examination available on Thursday.
- Dinner Wednesday night.
- Oral Examination: Friday early morning.

Questions are welcome at any time.

Technical discussion during the breaks is expected and welcomed.