Engineering Mathematics

F17/2054/2022 - ROBERT ODHIAMBO

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Basic mathematical Concepts

Sets

Definition 1 *A set is a well defined collection or group of objects.*

These objects are also referred to as members of a set

1. Requirements of a set

   (a) A set must be well defined, i.e, must not leave room for any ambiguity.

   (b) The elements of a given set must be distinct, i.e, each element should appear only once.

   (c) The order of representing elements of a set is immaterial, different arrangement of the same elements does not showany difference.

2. Specifying or naming of sets
   By convention, sets are specified (named) using a capital letter. Further, the elements of a set are designated by either listing all the elements or by using a descriptive characteristic or pattern. The elements of a set are enclosed using curly brackets. We can represent them in three ways:

   – Listing of all elements
     $A = \{0, 1, 2, 3, 4, 5, 6\}$

   – Using a descriptive characteristic
     $A = \{A \textit{ such that } X \textit{ is a positive integer from 0 to 6 inclusive}\}$

   – Using a pattern
     $A = \{1, 2, \ldots, 6\}$

3. Set membership
   This is expressed by using the symbol $\in$. Considering set
   A in which 3 is a member Expressed as $3 \in A$

4. Finite set
   A set that consists of a limited or countable number of elements.

5. Subset
   Any set S is a subset of set A if all elements in
   S are members of A and is denoted
   by $\subset$ and is read as "S is a subset of A"
   A is said to be the superset of S denoted by $\supset$, $A \supset S$

6. Equality of Sets
   If all elements in set D1 are in D2 and all the elements in D2 are
   textin D1 then they are equal D1 = D2
   Can be denoted as $D1 \supset D2 \text{or} D2 \subset D1$, i.e,
   textthey are subsets or supersets of each other.

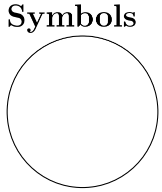
7. Universal Set
   Set that contains all the elemnts under consideration,denoted U.

8. Null or empty set
   Is a set with no elements and is denoted by $\{\}$ or $\emptyset$.

9. Complement of a set
   Given U and $A \subset U$ then the complement of A, denoted by A'
   or A^c represents all elements
   in U that are not in A.

10. Seys are pictorially represented using Venn Diagrams

    Symbols

    Circles: used to represent ordinary sets

Rectangle: Used to represent Universal set

11. Singleton Set
    Set with only one element

12. Disjoint sets
    Are two sets with no elements in common

Set Operations and Algebra

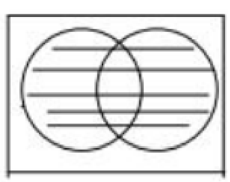
Definition 2 *These are operations where sets are combined to obtain other sets of interest*

Given two sets P and Q

They include:

1. **Union of Sets,** $\cup$
   Consists of elements in P or Q or both

P ∪ Q

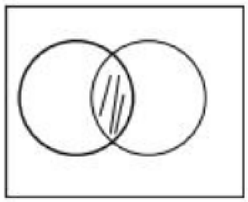


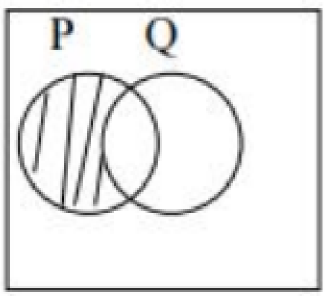
2. **Intersection of sets,** $\cap$
   Consists of elements in both P and Q(common elements)

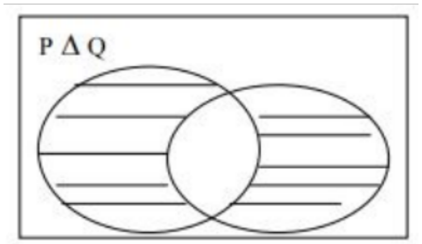
3. **Set difference/Injunction,** $\setminus$
   Consists of elements in P but not in Q

4. **Symetric difference,** Δ
   Consists of elements in P but not in Q and thos in Q but not in P

P ∩ Q







Laws of Set aAlgebra

1. Commutative Laws
   The order in which sets are combined in union or intersection is irrelevant, i.e $P \cup Q = Q \cup P$ and $P \cap Q = Q \cap P$

2. Associative Laws
   The selection of 3 or more sets for grouping in a union or intersection is immaterial, i.e $(P \cup Q) \cup R = P \cup (Q \cup R)$
3. Distributive Laws
   For any 3 sets P, Q and R:
   $P \cup (Q \cap R) = (P \cap Q) \cap (P \cup R)$
4. Impotent Laws
   For a set Q
   $Q \cup Q = Q$ and $Q \cap Q = Q$
   Other Laws:
5. $P \cup \emptyset = P$
6. $P \cap \emptyset = \emptyset$
7. $P \cup U = U$
8. $P \cap U = P$
9. $P \cup P' = U$
10. $P \cap P = \emptyset$
11. De Morgan's Laws
    For any two sets Q and R

    i $(Q \cup R)' = Q' \cap R'$

    ii $(Q \cap R)' = Q' \cap R'$

Boolean Algebra

Can be used to describe the manipulation and processing of binary information It's two-valued and has applications in the design of modern computer systems. It is common to interprete the digital values 0 as false and 1 as true.

Definitions

1. Boolean Expression: Combining the variables and operation yields Boolean expressions.
2. Boolean Function: A Boolean function typically has one or more input values and yields a result, based on these input value, in the range 0, 1.
3. A Boolean operator can be completely described using a table that list inputs, all possible values for these inputs, and the resulting values of the operation.
4. A truth table shows the relationship, in tabular form, between the input values and the result of a specific Boolean operator or function on the input variables.

Inputs		Outputs
x	*y*	*xy*
0	0	0
0	1	0
1	0	0
1	1	1

Figure 1: Truth table for AND

5. The AND operator is also known as a Boolean product. The Boolean expression xy is equivalent to the expression x * y and is read "x and y." The behavior of this operator is characterized by the truth table shown below

Inputs		Outputs
x	y	$x+y$
0	0	0
0	1	1
1	0	1
1	1	1

Figure 2: Truth Table for OR

Inputs	Outputs
x	$\overline{x}$
0	1
1	0

Figure 3: Truth table for NOT

6. The OR operator is often referred to as a Boolean sum. The expression x+y is read "x or y". The truth table for OR is shown below

7. Both $\bar{x}$ and x' are read as NOT x

8. The rule of precedence for Boolean operators give NOT top priority, followed by AND, and then OR

DeMorgan's law provides an easy way of finding the complement of a Boolean function. Boolean algebra is used in implementing digita computer circuits called **gates**.

Identity Name	AND Form	OR Form
Identity Law	$1x = x$	$0+x = x$
Null (or Dominance) Law	$0x = 0$	$1+x = 1$
Idempotent Law	$xx = x$	$x+x = x$
Inverse Law	$x\bar{x} = 0$	$x+\bar{x} = 1$
Commutative Law	$xy = yx$	$x+y = y+x$
Associative Law	$(xy)z = x(yz)$	$(x+y)+z = x+(y+z)$
Distributive Law	$x+yz = (x+y)(x+z)$	$x(y+z) = xy+xz$
Absorption Law	$x(x+y) = x$	$x+xy = x$
DeMorgan's Law	$(\overline{xy}) = \bar{x}+\bar{y}$	$(\overline{x+y}) = \bar{x}\bar{y}$
Double Complement Law	$\bar{\bar{x}} = x$	

Figure 4: Basic Identities of Boolean Algebra

Cartesian Products and Relations

For two sets A and B, the Cartesian Product of A and B is

$$A \times B = \{(a, b); a \in A, b \in B\}$$

We say that the elements of $A \times B$ are ordered pairs

Definition 3 *: For sets A, B, any subset of $A \times B$ is called a binary relation from A to B. Any subset of $A \times A$ is called a* ***binary relation*** *on A.*

A binary relation from A to B is a set $\mathcal{R}$ of ordered pairs where the first element of each ordered pair comes from A and the second element comes from B. We use the notation $a\mathcal{R}b$ to denote that $(a, b) \in \mathcal{R}$ and a $\not\mathcal{R}b$ When $(a, b) \in \mathcal{R}, a$ is said to be related to b by $\mathcal{R}$.
Let $A = \{0, 1, 2\}$ and $B = \{a, b\}$ then $\mathcal{R} = \{(0, a), (0, b), (1, a), (2, b)\}$ is a relation from A to B.
This means for instance that $0\mathcal{R}a, 1\mathcal{R}a$,etc
The above relation can be represented graphically using arrows to represent ordered pairs:

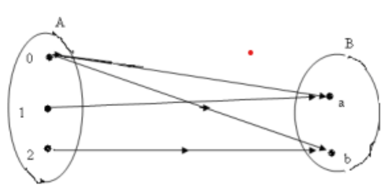


Figure 5: Graph for the relation explained above

Relations on a set

Definition 4 *A relation on a set A is a relation from A to A*

Example 1 *Let A=$\{1,2,3,4\}$. Which ordered pairs in the relation $\mathcal{R} = \{(a,b) : a \text{ divide } b\}$*

Solution Since (a,b) is in $\mathcal{R}$ if and only if a and b are positive integers not exceeding 4 such that a divides b,wesee that:
$\mathcal{R} = \{$(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)$\}$

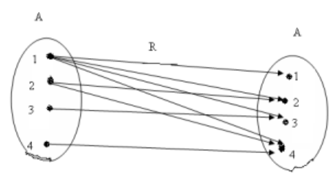


Figure 6: Graph of the relation in example 1

On a set A with n elements, a relation on A is a subset of $A \times A$. Since $A \times A$ has n^2 elements, when A has n elements and a set with m elements has 2^m subsets, there are 2^{n2} subsets of $A \times A$. Thus there are $2n^2$ relations on a set with n elements.

Properties of Relations

Definition 5 *A relation $\mathcal{R}$ on a set A is said to be **reflexive** if for all $x \in A, (x,x) \in \mathcal{R}$.*

Example 2 *For $A = \{1,2,3,4\}$*

A relation $\mathcal{R} \subseteq A \times A$ will be reflexive if and only if $\mathcal{R} \supseteq \{(1,1),(2,2),(3,3),(4,4)\}$
Consequently $\mathcal{R} = \{(1,1),(2,2),(3,3)\}$ is not aa reflexive relation on $A = \{(1,1),(2,2),(3,3)\}$ since $4 \in A$ but $(4,4) \notin \mathcal{R}$

$\mathcal{R} = \{(x,y) : x,y \in A, x \leq y\}$ is reflexive in $A = \{1,2,3,4\}$