Engineering Mathematics

F17/2054/2022

November 5, 2023

Basic mathematical Concepts

Sets

Definition 1 *A set is a well defined collection or group of objects.*

These objects are also referred to as members of a set

1. Requirements of a set

   (a) A set must be well defined, i.e, must not leave room for any ambiguity.

   (b) The elements of a given set must be distinct, i.e, each element should appear only once.

   (c) The order of representing elements of a set is immaterial, different arrangement of the
   same elements does not showany difference.

2. Specifying or naming of sets
   By convention, sets are specified (named) using a capital letter. Further, the elements of a s
   are designated by either listing all the elements or by using a descriptive characteristic or pa
   The elements of a set are enclosed using curly brackets. We can represent them in three way

   – Listing of all elements
     $A = \{0, 1, 2, 3, 4, 5, 6\}$

   – Using a descriptive characteristic
     $A = \{A \textit{ such that } X \textit{ is a positive integer from 0 to 6 inclusive}\}$

   – Using a pattern
     $A = \{1, 2, \ldots, 6\}$

3. Set membership
   This is expressed by using the symbol $\in$. Considering set A in which 3 is a member
   Expressed as $3 \in A$

4. Finite set
   A set that consists of a limited or countable number of elements.

5. Subset
   Any set S is a subset of set A if all elements in S are members of A and is denoted by $\subset$ and is read as "S is a subset of A"
   A is said to be the superset of S denoted by $\supset$, $A \supset S$

6. Equality of Sets
   If all elements in set D1 are in D2 and all the elements in D2 are in D1 then they are equal
   Can be denoted as $D1 \supset D2\text{or}D2 \subset D1$, i.e, they are subsets or supersets of each other.

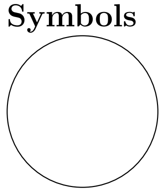
7. Universal Set
   Set that contains all the elemnts under consideration,denoted U.

8. Null or empty set
   Is a set with no elements and is denoted by $\{\}$ or $\emptyset$.

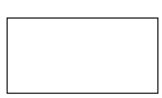
9. Complement of a set
   Given U and $A \subset U$ then the complement of A, denoted by A' or A^c represents all elements in U that are not in A.

10. Seys are pictorially represented using Venn Diagrams

    Symbols

    Circles: used to represent ordinary sets

Rectangle: Used to represent Universal set

11. Singleton Set
    Set with only one element
12. Disjoint sets
    Are two sets with no elements in common

Set Operations and Algebra

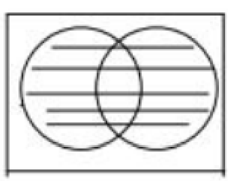
Definition 2 *These are operations where sets are combined to obtain other sets of interest*

Given two sets P and Q

They include:

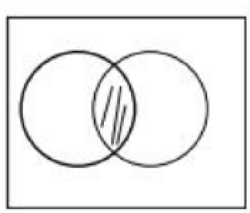
1. **Union of Sets,** $\cup$
   Consists of elements in P or Q or both

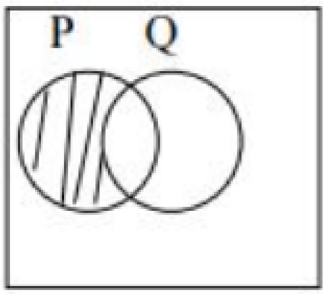
$P \cup Q$



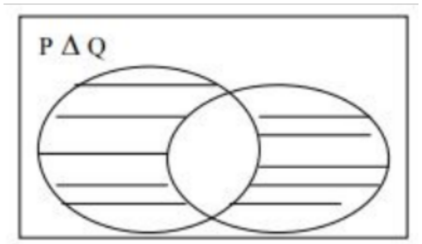
2. **Intersection of sets,** $\cap$
   Consists of elements in both P and Q(common elements)

$P \cap Q$





3. **Set difference/Injunction, **
   Consists of elements in P but not in Q
4. **Symetric difference, Δ**
   Consists of elements in P but not in Q and thos in Q but not in P



Laws of Set aAlgebra

1. Commutative Laws
   The order in which sets are combined in union or intersection is irrelevant, i.e $P \cup Q = Q \cup P$ and $P \cap Q = Q \cap P$

2. Associative Laws
The selection of 3 or more sets for grouping in a union or intersection is immaterial, i.e
$(P \cup Q) \cup R = P \cup (Q \cup R)$

3. Distributive Laws
For any 3 sets P, Q and R:
$P \cup (Q \cap R) = (P \cap Q) \cap (P \cup R)$

4. Impotent Laws
For a set Q
$Q \cup Q = Q$ and $Q \cap Q = Q$
Other Laws:

5. $P \cup \emptyset = P$

6. $P \cap \emptyset = \emptyset$

7. $P \cup U = U$

8. $P \cap U = P$

9. $P \cup P' = U$

10. $P \cap P = \emptyset$

11. De Morgan's Laws
For any two sets Q and R

    i $(Q \cup R)' = Q' \cap R'$

    ii $(Q \cap R)' = Q' \cap R'$

Boolean Algebra

Can be used to describe the manipulation and processing of binary information
It's two-valued and has applications in the design of modern computer systems.
It is common to intepréte the digital values 0 as false and 1 as true.
Definitions

1.

Cartesian Products and Relations

For two sets A and B, the Cartesian Product of A and B is

$$A \times B = \{(a, b); a \in A, b \in B\}$$

We say that the elements of $A \times B$ are ordered pairs

Definition 3 *: For sets A, B, any subset of $A \times B$ is called a binary relation from A to B. Any subset of $A \times A$ is called a* ***binary relation*** *on A.*