

Engineering Mathematics

F17/2054/2022 - ROBERT ODHIAMBO

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Basic mathematical Concepts

Sets

Definition 1 *A set is a well defined collection or group of objects.*

These objects are also referred to as members of a set

1. Requirements of a set

- (a) A set must be well defined, i.e, must not leave room for any ambiguity.
- (b) The elements of a given set must be distinct, i.e, each element should appear only once.
- (c) The order of representing elements of a set is immaterial, different arrangement of the same elements does not show any difference.

2. Specifying or naming of sets

By convention, sets are specified (named) using a capital letter.

Further, the elements of a set are designated by either listing all

the elements or by using a descriptive characteristic or pattern. The

elements of a set are enclosed using curly brackets. We can represent them in three ways:

- Listing of all elements

$$A = \{0, 1, 2, 3, 4, 5, 6\}$$

- Using a descriptive characteristic

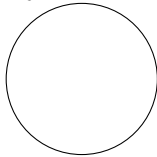
$$A = \{A \text{ such that } X \text{ is a positive integer from } 0 \text{ to } 6 \text{ inclusive}\}$$

- Using a pattern

$$A = \{1, 2, \dots, 6\}$$

3. Set membership
This is expressed by using the symbol \in . Considering set A in which 3 is a member Expressed as $3 \in A$
4. Finite set
A set that consists of a limited or countable number of elements.
5. Subset
Any set S is a subset of set A if all elements in S are members of A and is denoted by \subset and is read as " S is a subset of A "
 A is said to be the superset of S denoted by \supset , $A \supset S$
6. Equality of Sets
If all elements in set $D1$ are in $D2$ and all the elements in $D2$ are in $D1$ then they are equal $D1 = D2$
Can be denoted as $D1 \supset D2$ or $D2 \subset D1$, i.e., they are subsets or supersets of each other.
7. Universal Set
Set that contains all the elements under consideration, denoted U .
8. Null or empty set
Is a set with no elements and is denoted by $\{\}$ or \emptyset .
9. Complement of a set
Given U and $A \subset U$ then the complement of A , denoted by A' or A^c represents all elements in U that are not in A .
10. Sets are pictorially represented using Venn Diagrams

Symbols



Circles: used to represent ordinary sets



Rectangle: Used to represent Universal set

11. Singleton Set
Set with only one element
12. Disjoint sets
Are two sets with no elements in common

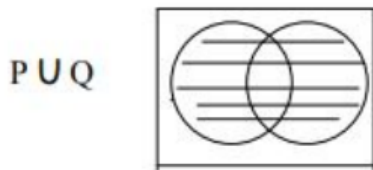
Set Operations and Algebra

Definition 2 *These are operations where sets are combined to obtain other sets of interest*

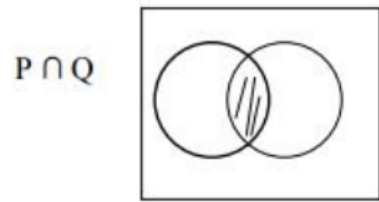
Given two sets P and Q

They include:

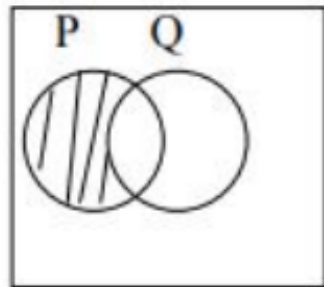
1. **Union of Sets, \cup**
Consists of elements in P or Q or both



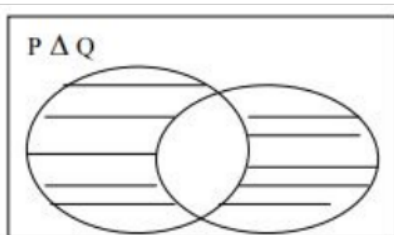
2. **Intersection of sets, \cap**
Consists of elements in both P and Q (common elements)
3. **Set difference/Injunction, \setminus**
Consists of elements in P but not in Q
4. **Symetric difference, Δ**
Consists of elements in P but not in Q and thos in Q but not in P



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Laws of Set Algebra

1. Commutative Laws

The order in which sets are combined in union or intersection is irrelevant, i.e
 $P \cup Q = Q \cup P$ and $P \cap Q = Q \cap P$

2. Associative Laws

The selection of 3 or more sets for grouping in a union or intersection is immaterial, i.e $(P \cup Q) \cup R = P \cup (Q \cup R)$

3. Distributive Laws

For any 3 sets P, Q and R:

$$P \cup (Q \cap R) = (P \cup Q) \cap (P \cup R)$$

4. Impotent Laws

For a set Q

$$Q \cup Q = Q \text{ and } Q \cap Q = Q$$

Other Laws:

5. $P \cup \emptyset = P$

6. $P \cap \emptyset = \emptyset$

7. $P \cup U = U$

8. $P \cap U = P$

9. $P \cup P' = U$

10. $P \cap P' = \emptyset$

11. De Morgan's Laws

For any two sets Q and R

i $(Q \cup R)' = Q' \cap R'$

ii $(Q \cap R)' = Q' \cup R'$

Boolean Algebra

Can be used to describe the manipulation and processing of binary information
It's two-valued and has applications in the design of modern computer systems.
It is common to interpret the digital values 0 as false and 1 as true.

Definitions

1. Boolean Expression: Combining the variables and operation yields Boolean expressions.
2. Boolean Function: A Boolean function typically has one or more input values and yields a result, based on these input value, in the range 0, 1.
3. A Boolean operator can be completely described using a table that list inputs, all possible values for these inputs, and the resulting values of the operation.
4. A truth table shows the relationship, in tabular form, between the input values and the result of a specific Boolean operator or function on the input variables.

Inputs		Outputs
x	y	xy
0	0	0
0	1	0
1	0	0
1	1	1

Figure 1: Truth table for AND

5. The AND operator is also known as a Boolean product. The Boolean expression xy is equivalent to the expression $x * y$ and is read “x and y.” The behavior of this operator is characterized by the truth table shown below

Inputs		Outputs
x	y	$x+y$
0	0	0
0	1	1
1	0	1
1	1	1

Figure 2: Truth Table for OR

Inputs	Outputs
x	\bar{x}
0	1
1	0

Figure 3: Truth table for NOT

6. The OR operator is often referred to as a Boolean sum. The expression $x+y$ is read “x or y”. The truth table for OR is shown below
7. Both \bar{x} and x' are read as NOT x
8. The rule of precedence for Boolean operators give NOT top priority, followed by AND, and then OR

DeMorgan’s law provides an easy way of finding the complement of a Boolean function. Boolean algebra is used in implementing digital computer circuits called **gates**.

Identity Name	AND Form	OR Form
Identity Law	$1x = x$	$0+x = x$
Null (or Dominance) Law	$0x = 0$	$1+x = 1$
Idempotent Law	$xx = x$	$x+x = x$
Inverse Law	$x\bar{x} = 0$	$x+\bar{x} = 1$
Commutative Law	$xy = yx$	$x+y = y+x$
Associative Law	$(xy)z = x(yz)$	$(x+y)+z = x+(y+z)$
Distributive Law	$x+yz = (x+y)(x+z)$	$x(y+z) = xy + xz$
Absorption Law	$x(x+y) = x$	$x+xy = x$
DeMorgan's Law	$(\overline{xy}) = \bar{x} + \bar{y}$	$(\overline{x+y}) = \bar{x}\bar{y}$
Double Complement Law	$\overline{\bar{x}} = x$	

Figure 4: Basic Identities of Boolean Algebra

Cartesian Products and Relations

For two sets A and B , the Cartesian Product of A and B is

$$A \times B = \{(a, b); a \in A, b \in B\}$$

We say that the elements of $A \times B$ are ordered pairs

Definition 3 : For sets A, B , any subset of $A \times B$ is called a *binary relation* from A to B . Any subset of $A \times A$ is called a **binary relation** on A .

A binary relation from A to B is a set R of ordered pairs where the first element of each ordered pair comes from A and the second element comes from B